Dynamic thermal admittance properties

There are two parameters that govern the dynamic thermal (storage) properties of a construction member: (i) thermal admittance, *Y*, and related quantities as specified in EN ISO 13786:2007; and (ii) surface heat capacity, *k*, as specified in EN ISO 13790:2004. These were determined for a 100 mm-thick solid concrete exposed external wall, based on ISO 13790: 2004, and the 'Dynamic Thermal Properties Calculator' software tool. The calculations assumed a vertical wall with horizontal heat flow and conventional surface boundary layer heat transfer coefficients of (i) inside surface coefficient R_{si} = 0.13 m^2 K/W, and (ii) outside surface coefficient R_{so} = 0.04 m^2 K/W, both are taken from ISO 6946: 2007 . The sol-air mean temperature variation was set at +/- 1K for direct comparison between different materials. The thermal admittance parameters and surface heat capacities are described as follows:

Thermal admittance parameters

Thermal admittance, *Y* (W/m^2K), describes the amount of energy leaving the internal surface of a construction member such as wall, floor or slab, i.e. the rate of heat flow into the room per unit degree of temperature swing. In other words, it describes the ability of a material or a structural element to exchange heat under a variety of temperature conditions over a specific period of time, usually 24 h. This is under theoretical conditions, where it is assumed that internal environmental temperature is varying and the external environmental temperature is constant. Thermal admittance is sometimes taken as a measure of thermal storage, but in reality, this is not absolutely true as it describes the heat flow into an internal surface.

A higher *Y*-value of a material/construction element means that indoor temperature fluctuations are reduced. It is mainly affected by thermal conductivity and specific heat capacity; therefore, heavyweight (dense) materials such as concrete have higher thermal admittance than lightweight materials like thermal insulating materials. Note, the *Y*-value and *U*-value have the same units, however, it is possible to have different construction elements with the same *U*-value but different thermal damping properties as measured by the *Y*-value, i.e. *U* is the rate of heat transfer across a construction component, whereas *Y* is the rate of transfer to the fabric. The *Y*-value is calculated using a set of matrices including density, heat capacity, thermal conductivity and thickness (see CIBCE Guide A for more explanation). Admittance for one dimension heat flow can be calculated using the temperature distribution equation

(Eq. 1) in a homogeneous construction element subject to one dimensional heat flow is given by the thermal diffusion equation.

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho C_p}{\lambda}\frac{\partial T}{\partial t} \qquad \text{Eq. 1}$$

Where, T = temperature (°C), x = distance in direction perpendicular to surface of slab (m^2), ρ = density (kg/m^3), C_p = specific heat capacity (J/kg K), λ = thermal conductivity (W/m K), t = time (s). For finite construction element and for sinusoidal temperature variations the temperature an energy cycles can be linked by using Eq. 2, Where, q=heat flux (W/m^2).

$$\begin{bmatrix} T_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix} \begin{bmatrix} T_2 \\ q_2 \end{bmatrix} \qquad \text{Eq. 2}$$

For a construction element of homogenous material of thickness d (m), the coefficients of Eq. 2 are given in Eq. 3 , Eq. 4 and Eq. 5.

$$\boldsymbol{m_1 = \cosh(p + ip)} \qquad \text{Eq. 3}$$

$$\boldsymbol{m_2} = \frac{\boldsymbol{d}\,\mathbf{sinh}(\boldsymbol{p}+\boldsymbol{ip})}{\boldsymbol{\lambda}(\boldsymbol{p}+\boldsymbol{ip})} \qquad \text{Eq. 4}$$

$$\boldsymbol{m_3} = \frac{\boldsymbol{\lambda}(\boldsymbol{p}+\boldsymbol{ip})\,\mathbf{sinh}(\boldsymbol{p}+\boldsymbol{ip})}{\boldsymbol{d}} \qquad \text{Eq. 5}$$

p is the periodic time and it is for 24 hours cycle, *i* is the complex number

$$p = \left(\frac{\pi d^2 \rho C_p}{86400\lambda}\right)^{1/2} \qquad \text{Eq. 6}$$

For a composite construction element e.g. wall, the matrices of each of the layers can multiplied together to give the relation between inside and outside, as shown in Eq. 7.

$$\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} 1- & R_{si} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} m_1 & m_2 \\ m_3 & m_1 \end{bmatrix} \begin{bmatrix} n_1 & n_2 \\ n_3 & n_1 \end{bmatrix} \cdots \begin{bmatrix} 1- & R_{si} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_e \\ q_{ie} \end{bmatrix} \qquad \text{Eq. 7}$$

Eq. 7 can be written as: $\begin{bmatrix} T_i \\ q_i \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \begin{bmatrix} T_e \\ q_{ie} \end{bmatrix}$ Eq. 8

Finally, the thermal admittance can be calculated using

$$Y = |Yc| = \frac{M_4}{M_2} \qquad \text{Eq. 9}$$

Time lag for thermal admittance, ø (h), represents the time difference between the timing of the peak heat flow at the internal surface of a construction member and timing of the peak internal temperature, i.e. it is the time between the response and the inducement. It can be calculated using Eq. 10.

$$Ø = \frac{12}{\pi} arctan\left(\frac{Im(Yc)}{Re(Yc)}\right) \qquad \text{Eq. 10}$$

Thermal decrement factor, *f* (dimensionless), represents the ratio of the peak heat flow out of the external surface of the element per unit degree of external temperature swing to the steady state heat flow through the element per unit degree of temperature difference between the internal and external environmental temperatures. In other words, it describes the rate of heat flow from the exterior surface of a constructional element. As well as the high thermal admittance, *f* is important in describing 'thermal mass' of a fabric construction member and it is calculated using Eq. 11.

$$f = |f_c| = \frac{1}{UM_2} \qquad \text{Eq. 11}$$

It is normally used to represent the reduction in temperature gradient formation (as a function of time) across a construction member due to heat storage within that member. Fabrics with high thermal decrements can be theoretically utilised in order to considerably decrease heating/cooling loads.

Decrement factor time lag, ω (h), is the time delay between the timing of the internal temperature peak and the peak heat flow out of the external surface and it is calculated using Eq. 12.

$$\omega = \frac{12}{\pi} arctan\left(\frac{Im(f_c)}{Re(f_c)}\right) \qquad \text{Eq. 12}$$

Surface factor, *F* (dimensionless), represents the ability of a surface element to gain heat by absorbing the incident radiative heat, e.g. from a sun patch on the surface. It is the ratio of the swing in heat flow from the internal surface of the element to the swing in heat flow received at the internal surface of the element.

Time lag for the surface factor, ψ (h), describes the time lag between the timing of the peak heat flow entering the surface of an element and peak heat flow leaving the surface and moving into the internal environment, e.g. room.

Surface heat capacities

Surface heat capacity, *K* (kJ/m²K), is the volumetric heat capacity of the 'thermally active' part of the construction up to a maximum thickness of 100 mm. It is a relatively simple way of characterising the thermal mass of a construction element as it does not take into account the impact of thermal conductivity in determining the timescale for heat to enter and leave the element. That is why the areal heat capacity provides a more accurate estimate of effective heat capacity of a construction member.

***K_{30}* (kJ/m²K),** is the equivalent of *K*, but for a maximum thickness of 30 mm for a construction member.