Week 11

Boolean Algebra and Logic Simplification

Logic Circuits Course
AIU-IE

Ch. 4

Boolean Algebra and Logic Simplification

Boolean Algebra and Logic Simplification

1. Boolean Operations and Expressions
2. Laws and Rules of Boolean Algebra
3. DeMorgan's Theorem
4. Boolean Expression for a Logic Circuit
5. Simplification Using Boolean Algebra
6. Standard Forms of Boolean Expressions
7. Boolean Expressions and Truth Tables

8. The Karnaugh Map
9. Karnaugh Map SOP Minimization
10. Karnaugh Map POS Minimization

1- Boolean Operations & Expressions

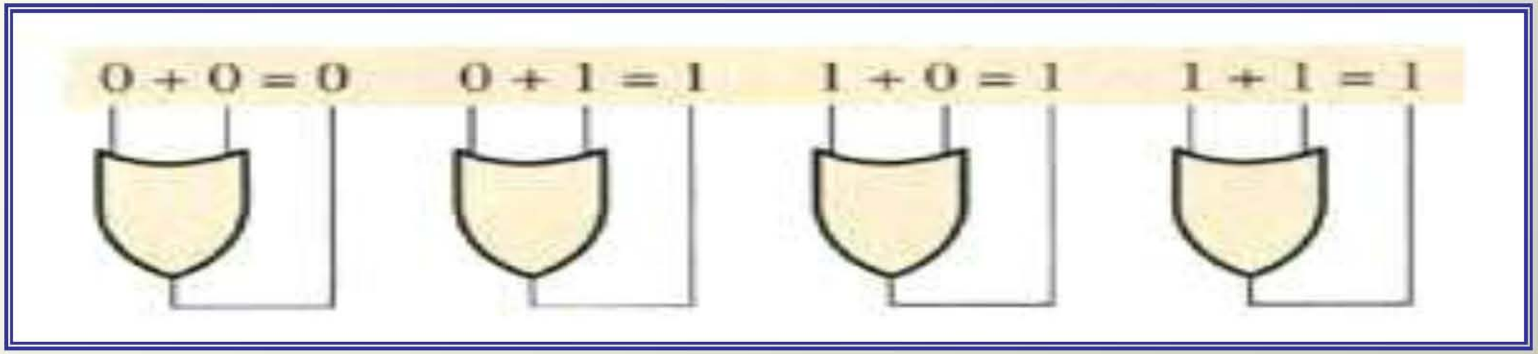
Boolean algebra:

- **Boolean Algebra is the mathematics of digital systems.**
- ***Variable, complement,*** **and** ***literal*** **are terms used in Boolean algebra.**
- **A variable is a symbol (usually an italic uppercase letter) used to represent a logical quantity. Any single variable can have a "1" or a "0" value.**
- **The complement is the inverse of a variable and is indicated by a bar over the variable .**
- **A literal is a variable or the complement of variable.**
- the complement of A is $\overline{A}$. If $A = 1$, then $\overline{A} = 0$.

  "not A" or "A bar" If $A = 0$, then $\overline{A} = 1$.

Boolean addition :

Boolean addition is equivalent to the OR operation and the basic rules are illustrated with their relation to the OR gate as follows:



- **In Boolean algebra, a sum term is a sum of literals.**
- **In logic circuits, a sum term is produced by an OR operation with no AND operations involved . Examples :**

$$A + B, A + \overline{B}, A + B + \overline{C}, \text{ and } \overline{A} + B + C + \overline{D}.$$

EXAMPLE

Determine the values of A, B, C, and D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0.

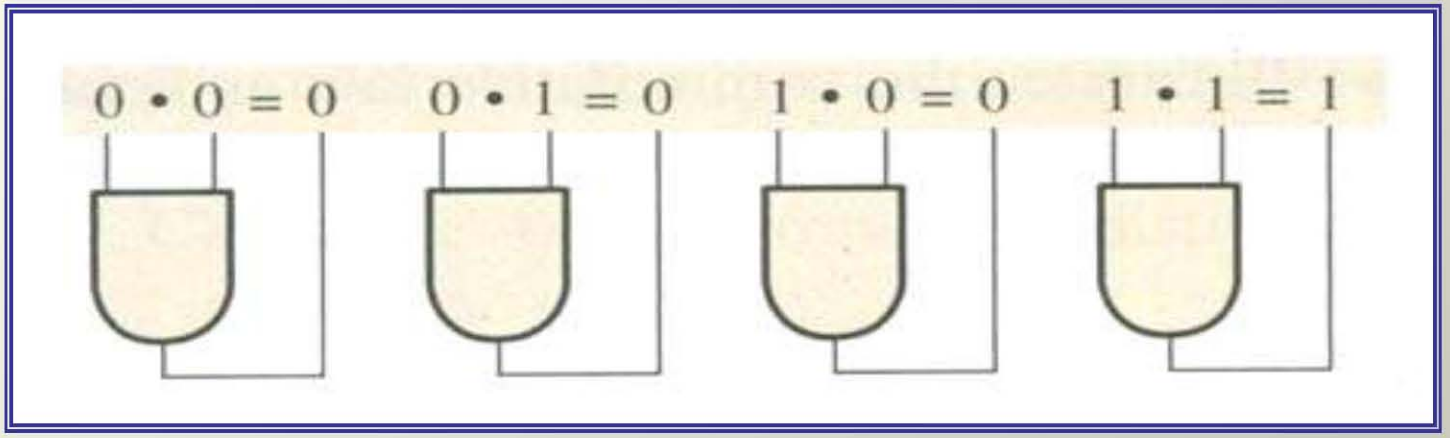
SOLUTION :

For the sum term to be 0, each of the literals in the term must be 0. Therefore, $A = \mathbf{0}$, $B = \mathbf{1}$ so that $\overline{B} = 0$, $C = \mathbf{0}$, and $D = \mathbf{1}$ so that $\overline{D} = 0$.

$$A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

Boolean multiplication:

Boolean Multiplication is equivalent to the AND operation and the basic rules are illustrated with their relation to the AND gate as follows:



- **In Boolean algebra, a product term is the product of literals.**
- **In logic circuits, a product term is produced by an AND operation with no OR operations involved.**

Example :

Determine the values of A, B, C, and D that make the product term $A\overline{B}C\overline{D}$ equal to 1.

Solution :

For the product term to be 1, each of the literals in the term must be 1. Therefore, A = **1**, B = **0** so that $\overline{B}$ = 1, C = **1**, and D = **0** so that $\overline{D}$ = 1.

$$A\overline{B}C\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

Determine the values of A and B that make the product term $\overline{A}\,\overline{B}$ equal to 1.

2- Laws and Rules of Boolean Algebra

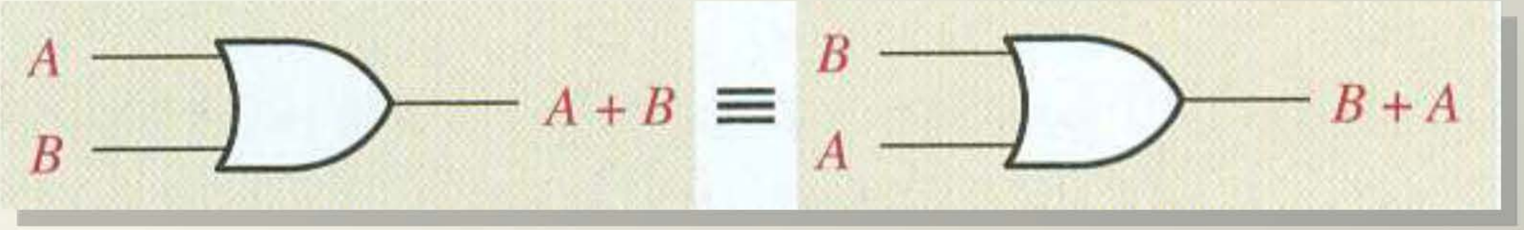
2-1 Laws Boolean Algebra

- Commutative Laws
- Associative Laws
- Distributive Law

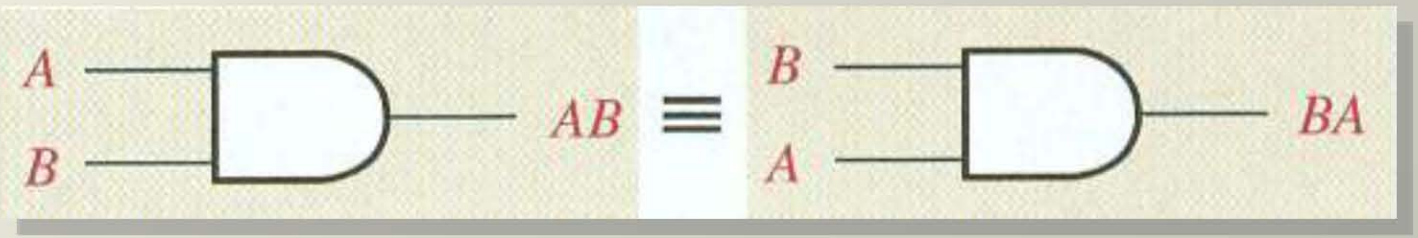
1- Laws Boolean Algebra

1- Commutative Laws :

For addition $\mathbf{A + B = B + A}$



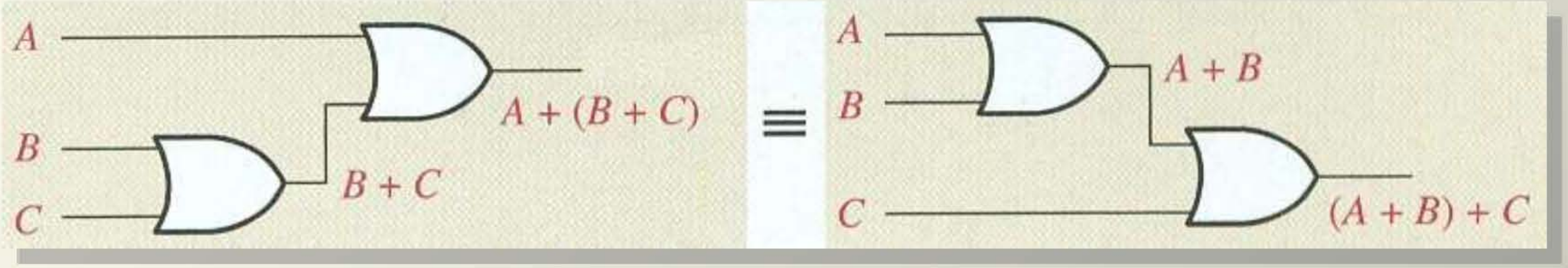
For Multiplication $\mathbf{A\,B = B\,A}$



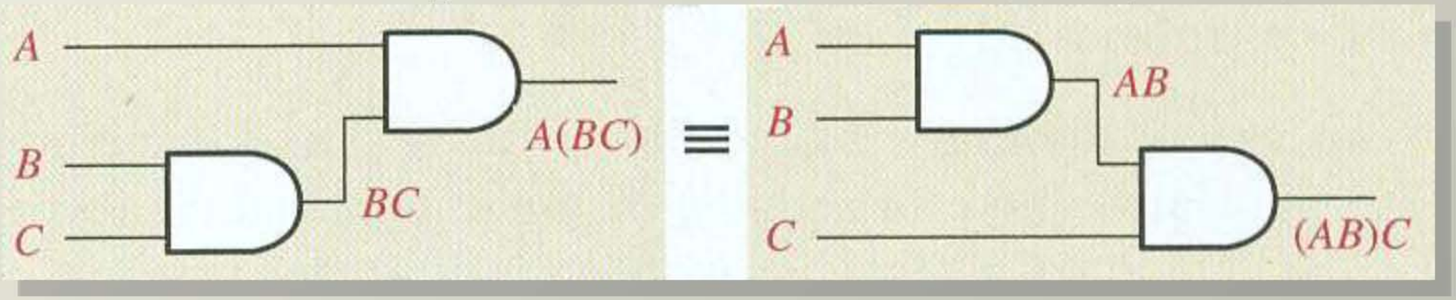
Laws Boolean Algebra (cont.)

2- *Associative Laws* :

For addition $A + (B + C) = (A + B) + C$



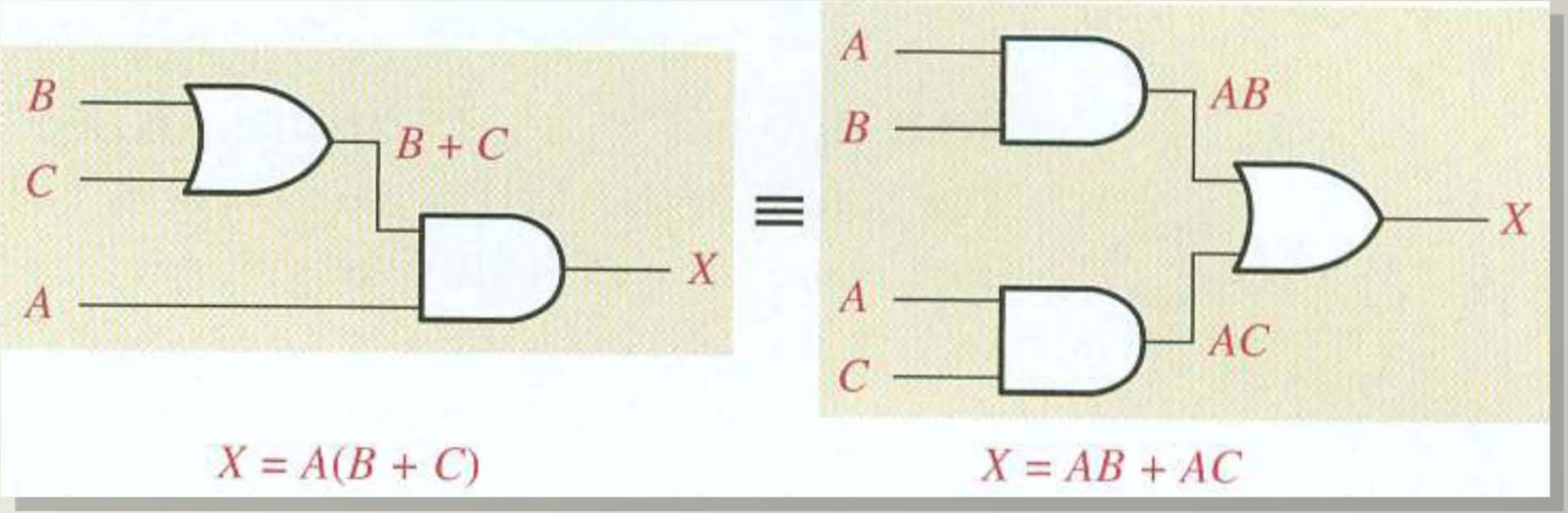
For Multiplication $A(BC) = (AB)C$



Laws Boolean Algebra (cont.)

3- Distributive Law :

$$A(B + C) = AB + AC$$



2- Rules of Boolean Algebra

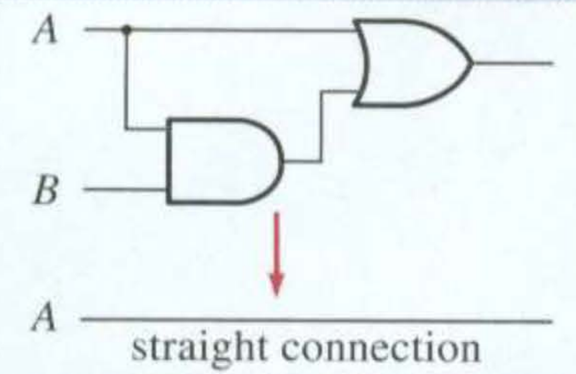
1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

Rules of Boolean Algebra (cont.)

- Rule 10: $A + AB = A$ >>> $A(1+B) = A$

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

Rules of Boolean Algebra (cont.)

- Rule 11: $A + \overline{A}B = A + B$

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

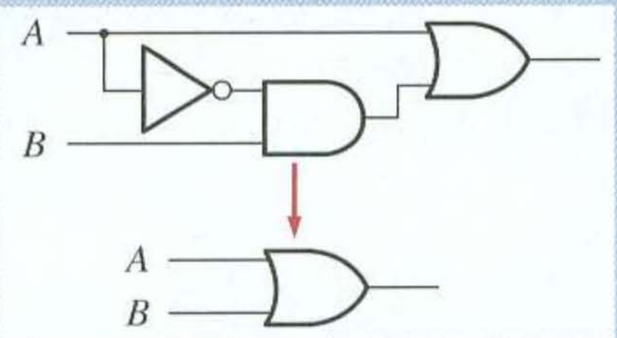
AND Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OR Truth Table

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



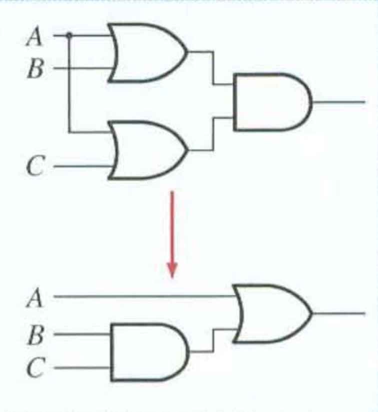
RULE 11: $A + \overline{A}B = A + B$ This rule can be proved as follows:

$A + \overline{A}B = (A + AB) + \overline{A}B$	Rule 10: $A = A + AB$
$= (AA + AB) + \overline{A}B$	Rule 7: $A = AA$
$= AA + AB + A\overline{A} + \overline{A}B$	Rule 8: adding $A\overline{A} = 0$
$= (A + \overline{A})(A + B)$	Factoring
$= 1 \cdot (A + B)$	Rule 6: $A + \overline{A} = 1$
$= A + B$	Rule 4: drop the 1

- Rule 12: (A + B)(A + C) = A + BC

A	B	C	A + B	A + C	(A + B)(A + C)	BC	A + BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



Rule 12. $(A + B)(A + C) = A + BC$ This rule can be proved as follows:

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$