Week 12

DeMorgan's Theorems

3- DeMorgan's Theorems

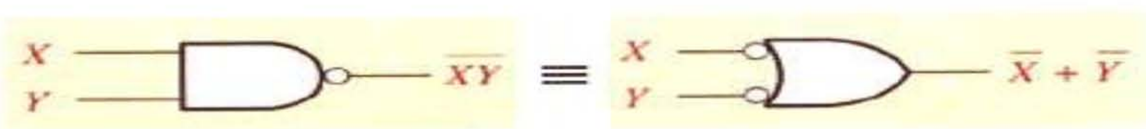
1. **The complement of a product of variables is equal to the sum of the complements of the variables.**

$$\overline{XY} = \overline{X} + \overline{Y}$$

2. **The complement of a sum of variables is equal to the product of the complement of the variables.**

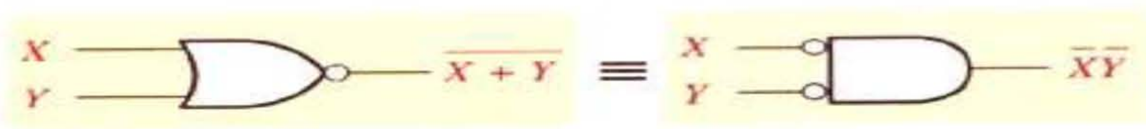
$$\overline{X + Y} = \overline{X}\,\overline{Y}$$

Remember: "Break the bar, change the sign"



NAND | Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



NOR | Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Example: application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B\overline{C}} + \overline{D(\overline{E + \overline{F}})}}$$

Step 1. Let $\overline{A + B\overline{C}} = X$ and $\overline{D(\overline{E + \overline{F}})} = Y$.

Step 2. Since $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$$\overline{(\overline{A + B\overline{C}}) + (\overline{D(\overline{E + \overline{F}})})} = (\overline{\overline{A + B\overline{C}}})(\overline{\overline{D(\overline{E + \overline{F}})}})$$

Step 3. Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A + B\overline{C}}})(\overline{\overline{D(\overline{E + \overline{F}})}}) = (A + B\overline{C})(\overline{\overline{D(\overline{E + \overline{F}})}})$$

Step 4. Applying DeMorgan's theorem to the second term,

$$(A + B\overline{C})(\overline{\overline{D(\overline{E + \overline{F}})}}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

Step 5. Use the rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E + \overline{F}$ part of the term.

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

EXAMPLE 4–5

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$ **(b)** $\overline{ABC + DEF}$ **(c)** $\overline{A\overline{B} + \overline{C}D + EF}$

Solution

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(A + B + C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(A + B + C)D} = \overline{A + B + C} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A + B + C}$.

$$\overline{A + B + C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

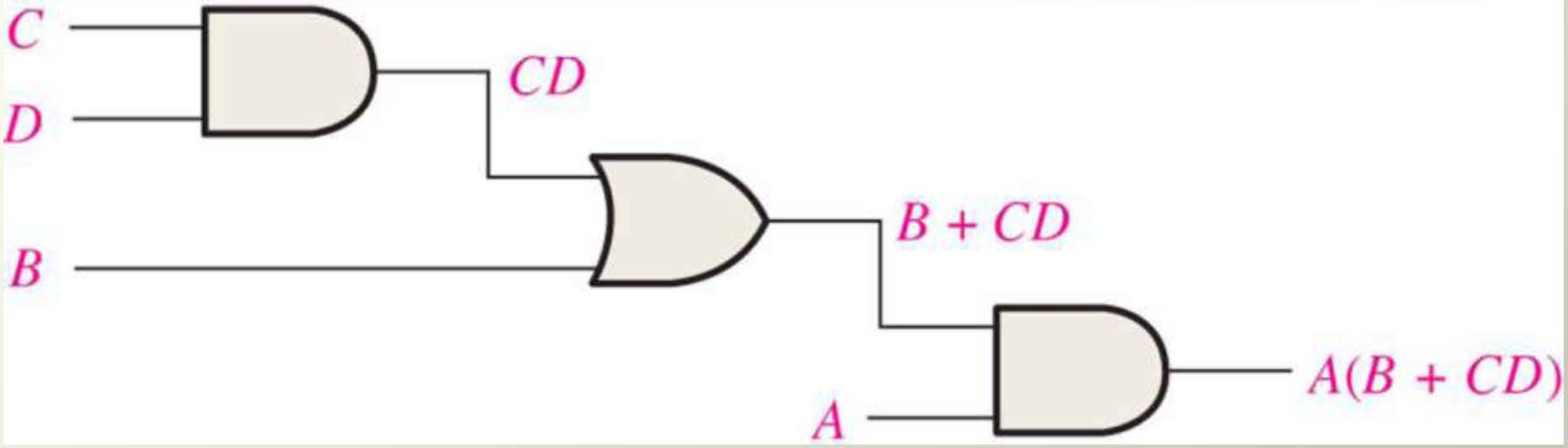
$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

4- Boolean Expression for a Logic Circuit

To derive the Boolean expression for a given combinational logic circuit, begin at the left most inputs and work toward the final output, writing the expression for each gate.

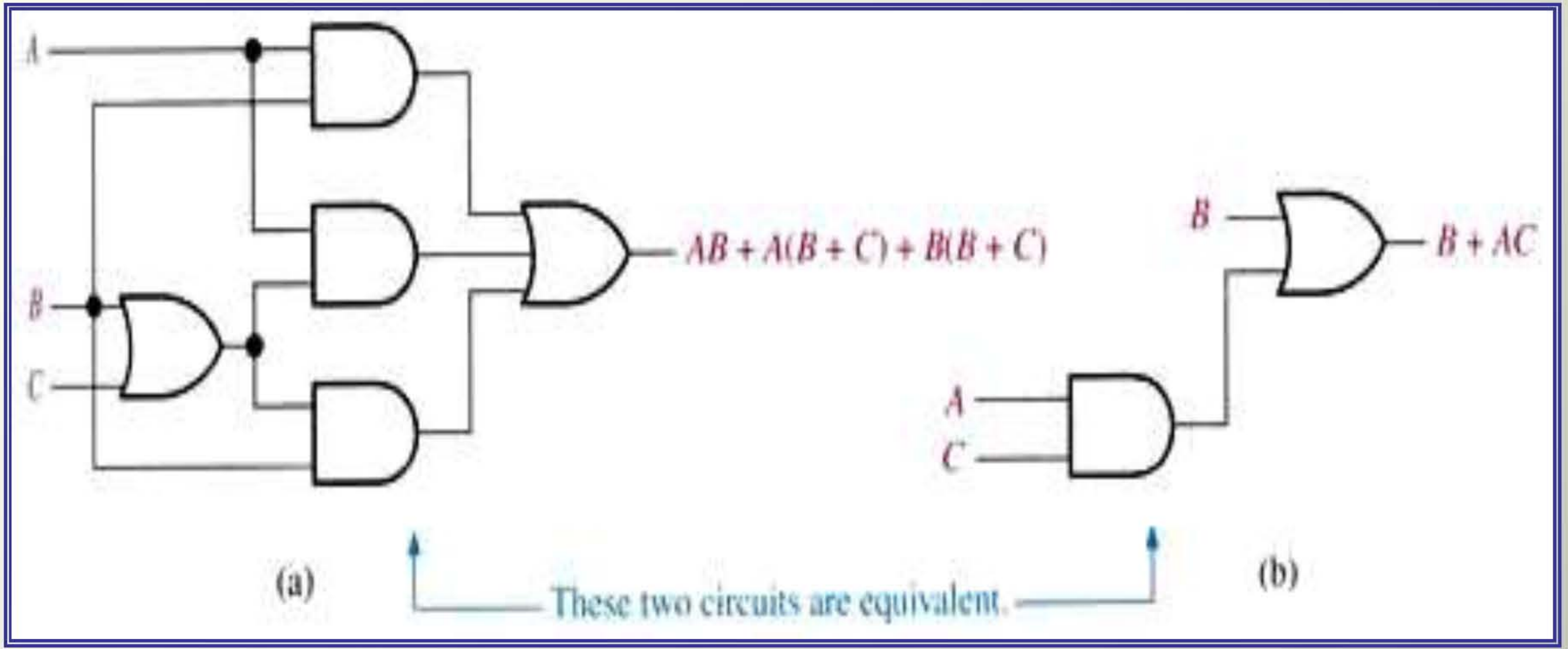


1- CD
2 - B + CD
3 - A . (B + CD)

Constructing a Truth Table for a Logic Circuit

5- Simplification using Boolean Algebra

Implementation of the Boolean expression : ***AB + A(B+C) + B(B+C) = B+AC***



AB + A(B+C) + B(B+C) =
AB+ AB + AC + BB + BC=
AB + AC + B + BC = AB + AC + B(1+c)=
AB+ AC + B = B(A+1) + AC
B + AC

EXAMPLE 4–9

Simplify the following Boolean expression: $[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$

Solution

Step 1. Apply the distributive law to the terms within the brackets.

$$(A\overline{B}C + A\overline{B}BD + \overline{A}\,\overline{B})C$$

Step 2. Apply rule 8 ($\overline{B}B = 0$) to the second term within the parentheses.

$$(A\overline{B}C + A \cdot 0 \cdot D + \overline{A}\,\overline{B})C$$

Step 3. Apply rule 3 ($A \cdot 0 \cdot D = 0$) to the second term within the parentheses.

$$(A\overline{B}C + 0 + \overline{A}\,\overline{B})C$$

Step 4. Apply rule 1(drop the 0) within the parentheses. $(A\overline{B}C + \overline{A}\,\overline{B})C$

Step 5. Apply the distributive law. $A\overline{B}CC + \overline{A}\,\overline{B}C$

Step 6. Apply rule 7 ($CC = C$) to the first term. $A\overline{B}C + \overline{A}\,\overline{B}C$

Step 7. Factor out $\overline{B}C$. $\overline{B}C(A + \overline{A})$

Step 8. Apply rule 6 ($A + \overline{A} = 1$). $\overline{B}C \cdot 1$

Step 9. Apply rule 4 (drop the 1). $\overline{B}C$

EXAMPLE 4-10

Simplify the following Boolean expression: $\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$

Solution **Step 1.** Factor BC out of the first and last terms. $BC(\overline{A} + A) + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$

Step 2. Apply rule 6 ($\overline{A} + A = 1$) to the term in parentheses, $BC \cdot 1 + A\overline{B}(\overline{C} + C) + \overline{A}\,\overline{B}\,\overline{C}$

Step 3. Apply rule 4 (drop the 1) to the first term and rule 6 ($\overline{C} + C = 1$) to the tern in parentheses.

$$BC + A\overline{B} \cdot 1 + \overline{A}\,\overline{B}\,\overline{C}$$

Step 4. Apply rule 4 (drop the 1) to the second term. $BC + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$

Step 5. Factor $\overline{B}$ from the second and third terms. $BC + \overline{B}(A + \overline{A}\,\overline{C})$

Step 6. Apply rule 11 ($A + \overline{A}\,\overline{C} = A + \overline{C}$) to the term in parentheses. $BC + \overline{B}(A + \overline{C})$

Step 7. Use the distributive and commutative laws to get the following expression: $BC + A\overline{B} + \overline{B}\,\overline{C}$

EXAMPLE 4–11

Simplify the following Boolean expression: $\overline{AB + AC} + \overline{A}\,\overline{B}C$

Solution

Step 1. Apply DeMorgan's theorem to the first term. $(\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$

Step 2. Apply DeMorgan's theorem to each term in parentheses.

$$(\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$

Step 3. Apply the distributive law to the two terms in parentheses.

$$\overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

Step 4. Apply rule 7 ($\overline{A}\,\overline{A} = \overline{A}$) to the first term, and apply rule 10 [$\overline{A}\,\overline{B} + \overline{A}\,\overline{B}C = \overline{A}\,\overline{B}(1 + C) = \overline{A}\,\overline{B}$] to the third and last terms.

$$\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Step 5. Apply rule 10 [$\overline{A} + \overline{A}\,\overline{C} = \overline{A}(1 + \overline{C}) = \overline{A}$] to the first and second terms.

$$\overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Step 6. Apply rule 10 [$\overline{A} + \overline{A}\,\overline{B} = \overline{A}(1 + \overline{B}) = \overline{A}$] to the first and second terms.

$$\overline{A} + \overline{B}\,\overline{C}$$