Week 13

Standard Forms of Boolean Expressions

6- Standard Forms of Boolean Expressions

- The sum-of-product (SOP) form

  Example: X = AB + CD + EF

SOP expression can be implemented by AND-OR logic or by NAND-NAND logic

SOP expression is equal to 1 only if at least one of the product is equal to 1.

- **The product of sum (POS) form**

  Example: X = (A + B)(C + D)(E + F)

POS expression can be implemented by OR-AND logic or by NOR-NOR logic

POS expression is equal to 0 only if at least one of the sum terms is equal to 0.

1- The sum of product form (SOP) :

When two or more product terms are summed by Boolean addition. The resulting expression is a sum-of-products (SOP) .

Example :- $AB + ABC$

$ABC + CDE + \overline{B}C\overline{D}$

$\overline{A}B + \overline{A}B\overline{C} + AC$

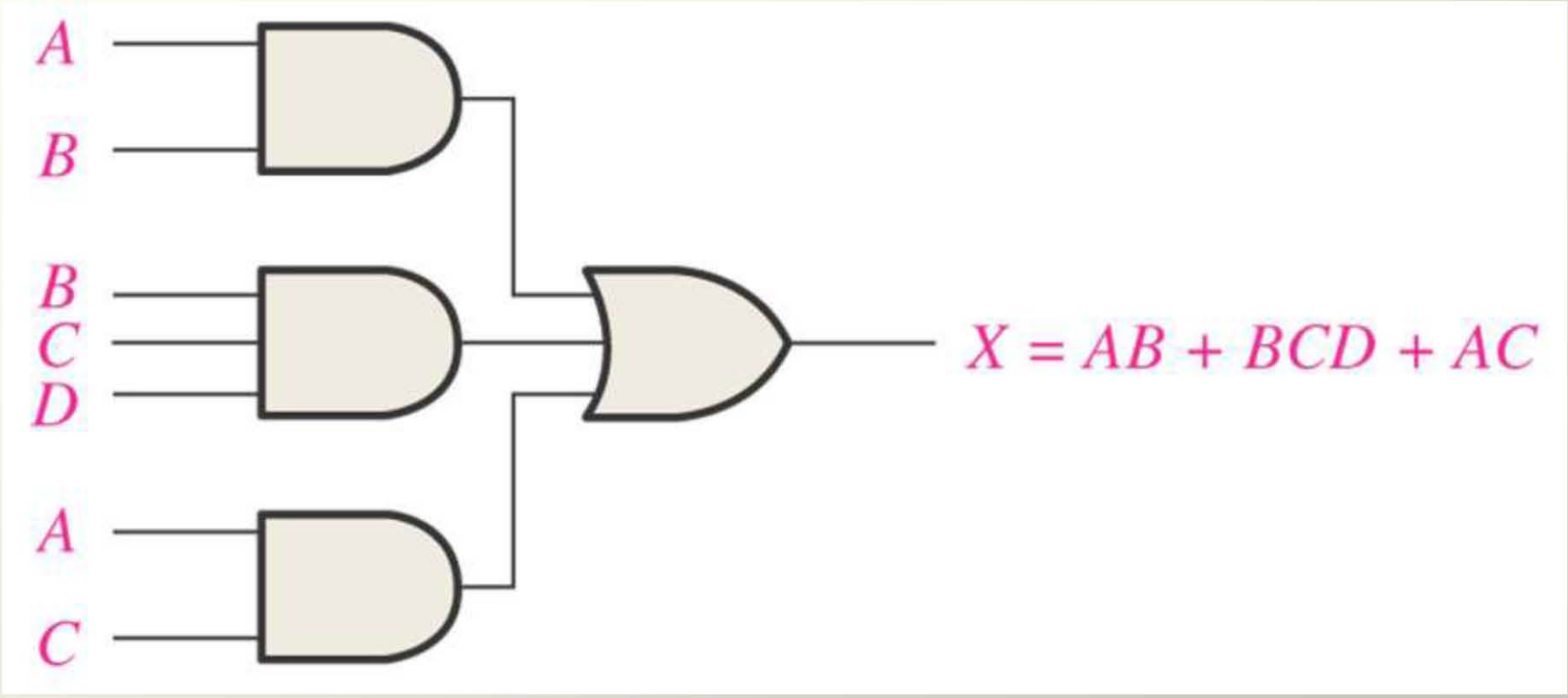
Conversion of a general expression to SOP form :

Any logic expression can be changed to SOP form by applying Boolean algebra techniques. For example the expression *A(B +CD)* can be converted to SOP form by applying the distributive law :

$$A(B+CD) = AB + ACD$$

(the expression has a domain made up of the variables A, B, C, and D)

Implementing the SOP expression AB + BCD + AC



Conversion Product Terms to standard SOP form:

A standard SOP expression is one in which all the variables in the domain appears in each product term in the expression.

Standard SOP expressions are important in constructing truth tables and in the Karnaugh map simplification methods.

Step 1: **Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms .**

Step 2: **Repeat step -1- until all resulting product term contain all variables in the domain in either complemented or un-complemented form .**

EXAMPLE 4–13

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Solution The domain of this SOP expression is A, B, C, D. Take one term at a time. The first term, $A\overline{B}C$, is missing variable D or $\overline{D}$, so multiply the first term by $D + \overline{D}$ as follows:

$$A\overline{B}C = A\overline{B}C(D + \overline{D}) = A\overline{B}CD + A\overline{B}C\overline{D}$$

In this case, two standard product terms are the result.

The second term, $\overline{A}\,\overline{B}$, is missing variables C or $\overline{C}$ and D or $\overline{D}$, so first multiply the second term by $C + \overline{C}$ as follows:

$$\overline{A}\,\overline{B} = \overline{A}\,\overline{B}(C + \overline{C}) = \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}$$

The two resulting terms are missing variable D or $\overline{D}$, so multiply both terms by $D + \overline{D}$ as follows:

$$\begin{aligned}\overline{A}\,\overline{B} &= \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} = \overline{A}\,\overline{B}C(D + \overline{D}) + \overline{A}\,\overline{B}\,\overline{C}(D + \overline{D}) \\ &= A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}\end{aligned}$$

In this case, four standard product terms are the result.

The third term, $AB\overline{C}D$, is already in standard form. The complete standard SOP form of the original expression is as follows:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

2- The product of sum form (POS) :

When two or more sum terms are multiplied. The resulting expression is a Product Of Sums (POS).

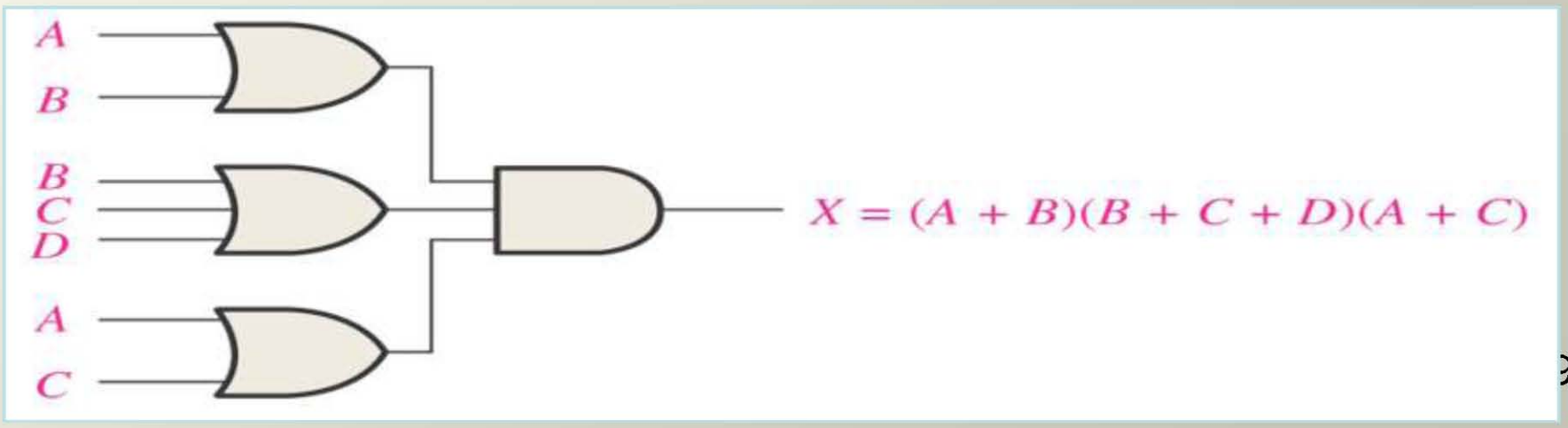
Examples :

$$(\overline{A} + B)(A + \overline{B} + C)$$

$$(\overline{A} + \overline{B} + \overline{C})(C + \overline{D} + E)(\overline{B} + C + D)$$

$$(A + B)(A + \overline{B} + C)(\overline{A} + C)$$

Implementing the POS expression (A+ B) (B+C+D)(A+C)



Converting a sum term to standard POS :

Step 1 : Add to each nonstandard product term a term made up of the product of the missing variable and its complement .

Step 2 : Apply rule 12 : A+BC = (A+B)(A+C).

Step 3 : Repeat step -1- until all resulting sum terms contains all variables in the domain in either complemented or un-complemented form .

EXAMPLE 4–15

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Solution The domain of this POS expression is A, B, C, D. Take one term at a time. The first term, $A + \overline{B} + C$, is missing variable D or $\overline{D}$, so add $D\overline{D}$ and apply rule 12 as follows:

$$A + \overline{B} + C = A + \overline{B} + C + D\overline{D} = (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})$$

The second term, $\overline{B} + C + \overline{D}$, is missing variable A or $\overline{A}$, so add $A\overline{A}$ and apply rule 12 as follows:

$$\overline{B} + C + \overline{D} = \overline{B} + C + \overline{D} + A\overline{A} = (A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})$$

The third term, $A + \overline{B} + \overline{C} + D$, is already in standard form. The standard POS form of the original expression is as follows:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D) =$$
$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

3- Converting Standard SOP to Standard POS:

Step 1 :Evaluate each product sum in the SOP expression. That is determine the binary numbers that represent the product form.

Step 2 : Determine all of the binary numbers not included in the evaluation in step 1 .

Step 3 : Write the equivalent sum term for each binary numbers from step and express in pos form.

EXAMPLE 4–17

Convert the following SOP expression to an equivalent POS expression:

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Solution The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

Since there are three variables in the domain of this expression, there are a total of eight (2^3) possible combinations. The SOP expression contains five of these combinations, so the POS must contain the other three which are 001, 100, and 110.

Remember, these are the binary values that make the sum term 0. The equivalent POS expression is

$$(A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + C)$$