

# Week 14

## **Boolean Expressions and Truth Tables**

## 7- Boolean Expressions and Truth Tables

- for an expression of with a domain of  $n$  variables, there are  $2^n$  different combinations of those variables.

### Steps to construct truth table are:

- list all possible combination of binary value of the variables in the expression
- convert the (SOP or POS) expression to standard form if it is not already.
- For SOP expression:
  - place a 1 in the output column (x) for each binary value that makes the standard SOP expression a 1 and place 0 for all the remaining binary value.
- For POS expression:
  - place a 0 in the output column (x) for each binary value that makes the standard SOP expression a 0 and place 1 for all the remaining binary value.

## Determining SOP Standard Expression from a Truth Table

Truth table for a domain of 3 variables

INPUTS			OUTPUT
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

There are four 1 in the output column and the corresponding binary values are 011, 100, 110 and 111 . These binary values are converted to product term as :

$$011 \longrightarrow \bar{A}BC$$

$$100 \longrightarrow A\bar{B}\bar{C}$$

$$110 \longrightarrow A\bar{B}C$$

$$111 \longrightarrow ABC$$

The resulting standard SOP expression for the output X is

$$X = \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

## Determining POS Standard Expression from a Truth Table

Truth table for a domain of 3 variables

INPUTS			OUTPUT
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

For the POS expression, the output is 0 for binary values 000,001,010 and 101. These binary values are converted to sum terms as follows:

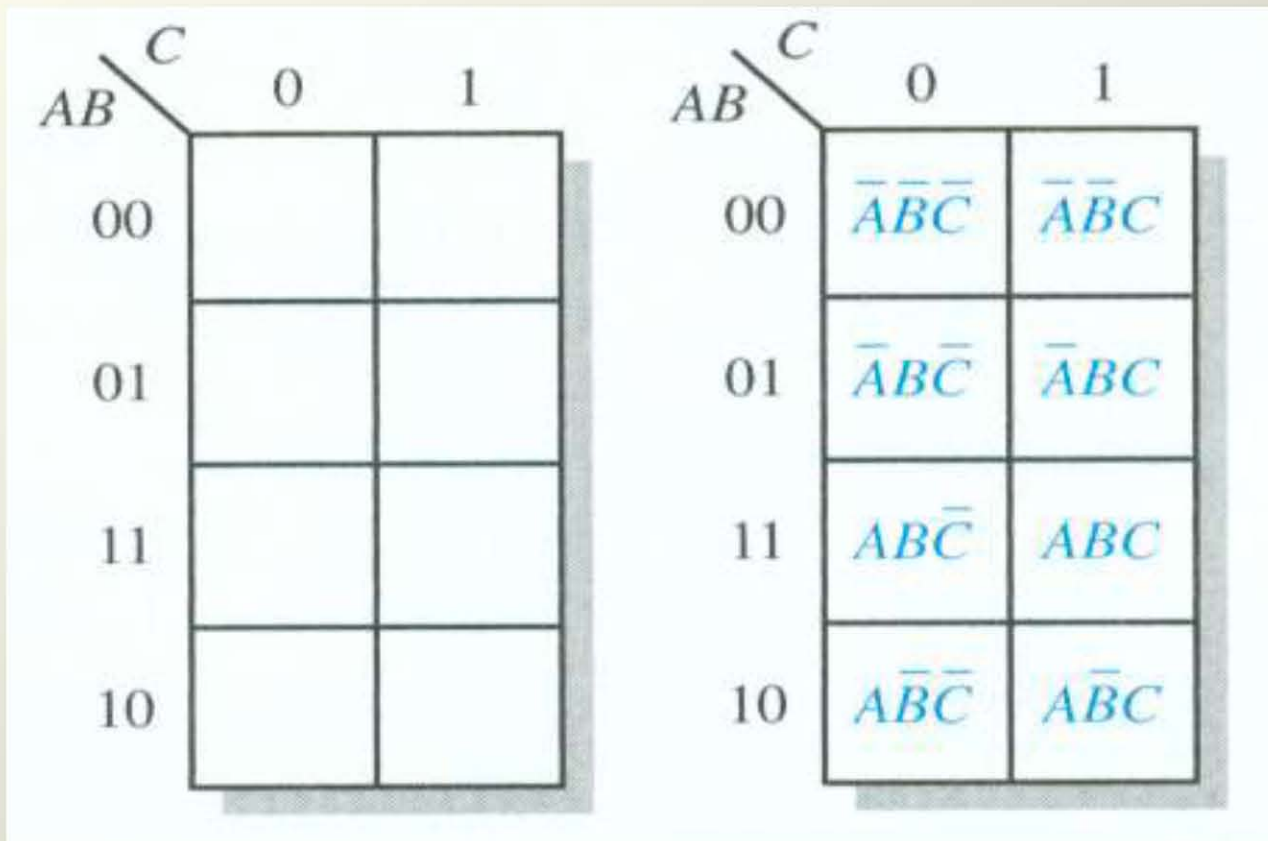
$$\begin{aligned}000 &\longrightarrow A + B + C \\001 &\longrightarrow A + B + \bar{C} \\010 &\longrightarrow A + \bar{B} + C \\101 &\longrightarrow \bar{A} + B + \bar{C}\end{aligned}$$

The resulting standard POS expression for the output X is

$$X = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + \bar{C})$$

## 8- The Karnaugh Map

The Karnaugh map provides a systematic method for simplifying Boolean expressions and if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression.



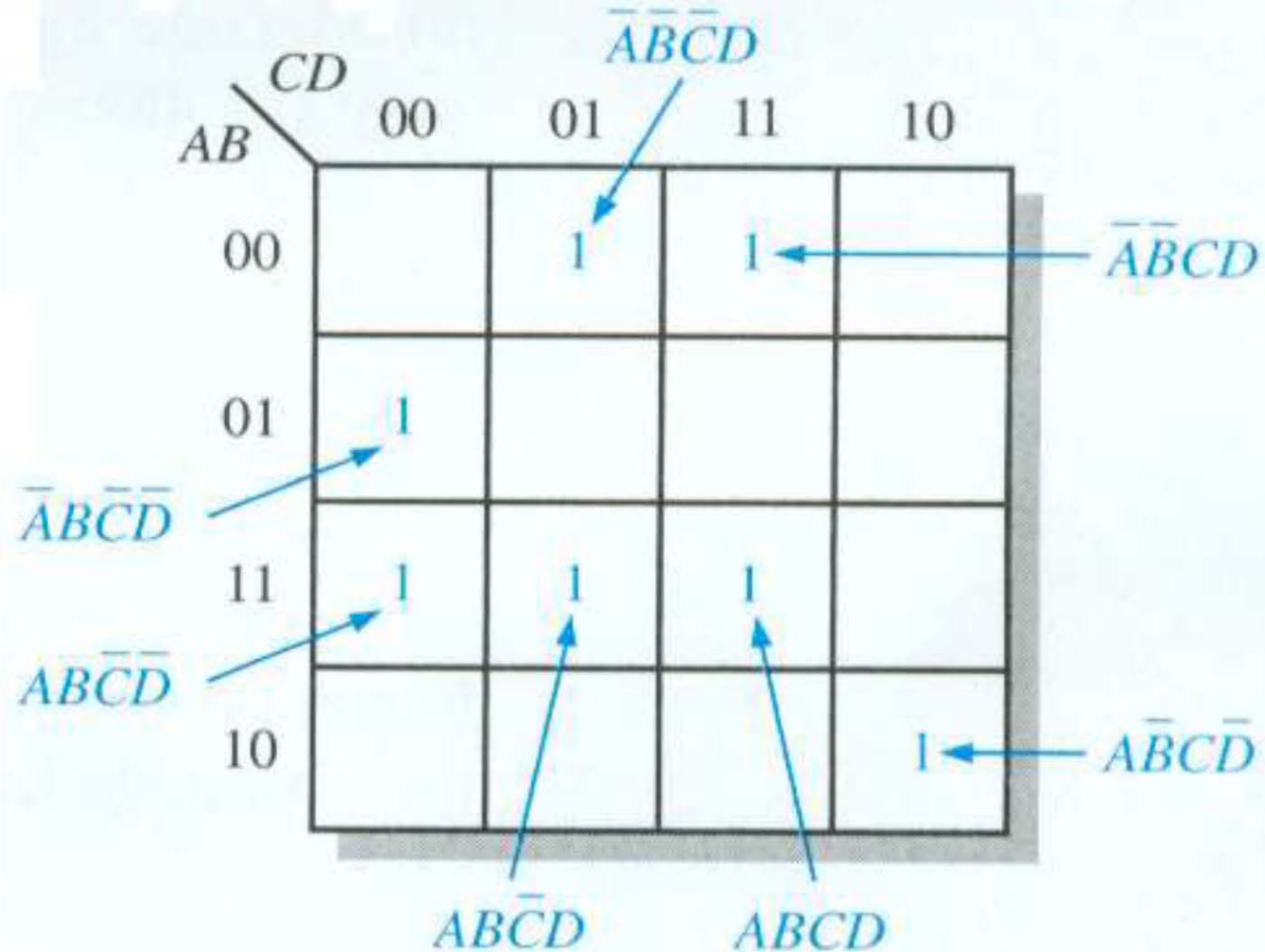
3-Variable Karnaugh Map

		<i>CD</i>			
	<i>AB</i>	00	01	11	10
00					
01					
11					
10					

		<i>CD</i>			
	<i>AB</i>	00	01	11	10
00		$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}\overline{B}C\overline{D}$	$\overline{A}\overline{B}CD$	$\overline{A}\overline{B}C\overline{D}$
01		$\overline{A}B\overline{C}\overline{D}$	$\overline{A}BC\overline{D}$	$\overline{A}BCD$	$\overline{A}B\overline{C}\overline{D}$
11		$A\overline{B}\overline{C}\overline{D}$	$A\overline{B}C\overline{D}$	$ABCD$	$AB\overline{C}\overline{D}$
10		$A\overline{B}\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$	$A\overline{B}C\overline{D}$

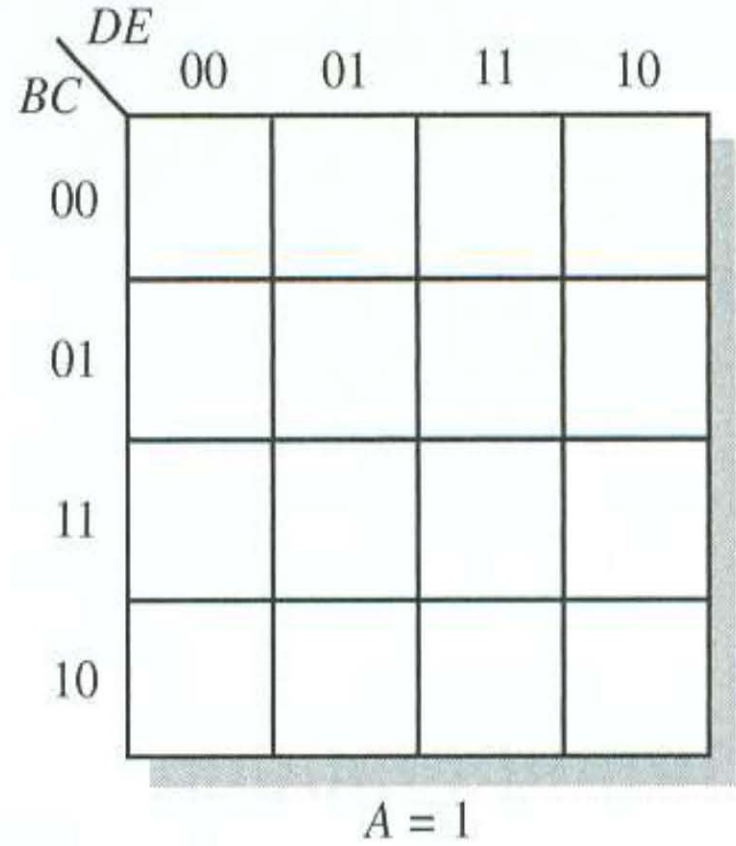
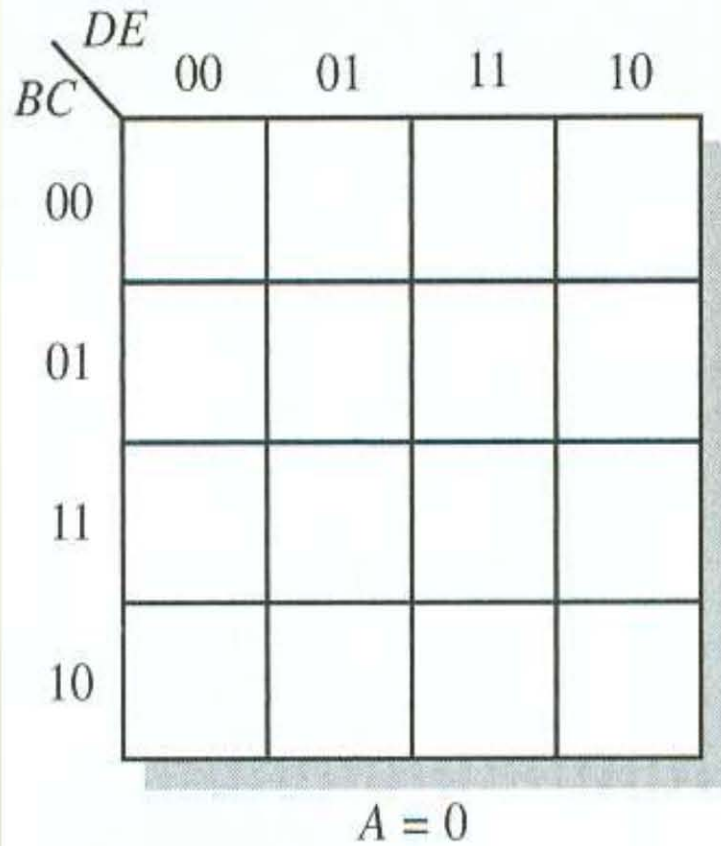
4-Variable Karnaugh Map

$$\bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABCD + A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D}$$



4-Variable Example

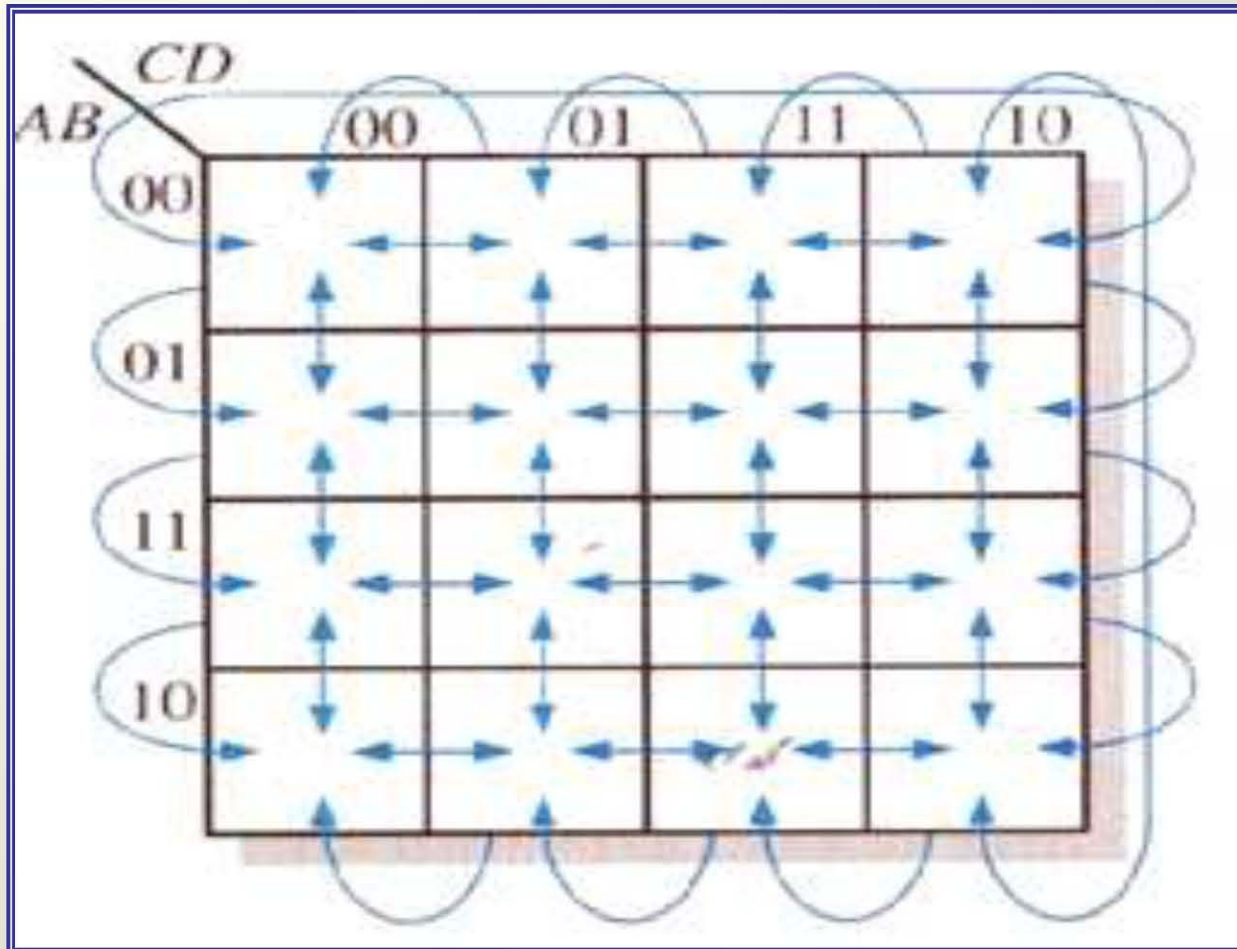
## The Karnaugh Map



5-Variable Karnaugh Mapping



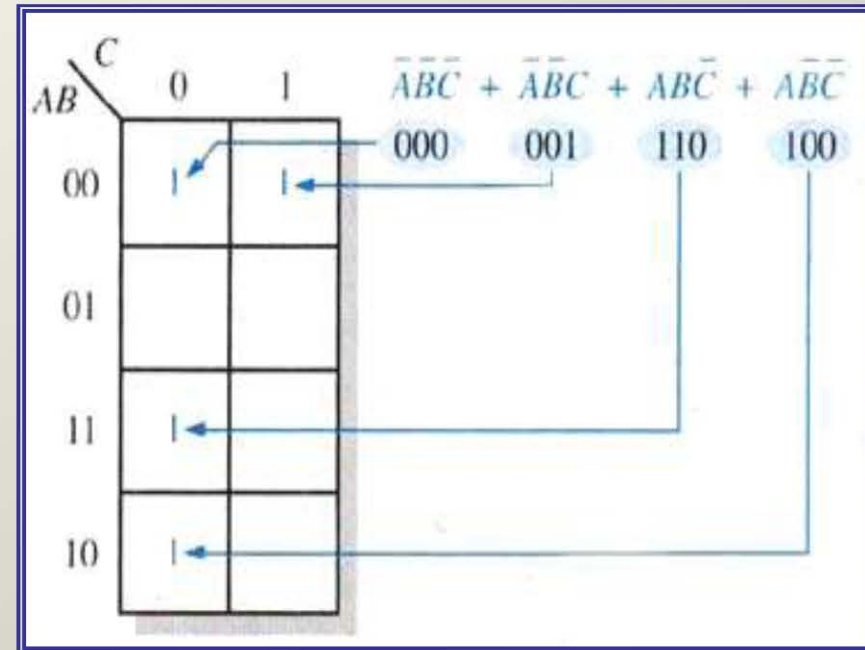
## *Cell Adjacency :*



## 9- Karnaugh Map SOP Minimization

For an SOP expression, a “1” is placed on the Karnaugh map for each product term in the expression. Each “1” is placed in a cell corresponding to the value of a product term. So:-

**Step 1:** Determine the binary values of each product term in the SOP expression.



**Step2 :** as each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same values as the product sum.

## Karnaugh Map SOP Minimization

Example :-Map the standard SOP expression on a Karnaugh map

$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

001	010	110	111
-----	-----	-----	-----

		C		
		0	1	
AB	00		1	$\bar{A}\bar{B}C$
	01	1		$\bar{A}B\bar{C}$
	11	1	1	$A\bar{B}\bar{C}$
	10			$ABC$

## Karnaugh Map SOP Minimization

**Example :-** Map the following not standard SOP .

$$\bar{A} + A\bar{B} + ABC\bar{C}$$

**Solution:-** The SOP is not a standard form because each product term does not have three variables.

$\bar{A}$	+	$A\bar{B}$	+	$ABC\bar{C}$
000		100		110
001		101		
010				
011				

		$C$	
	$AB$	0	1
00		1	1
01		1	1
11		1	
10		1	1