# Week 6 Arithmetic Operations with Signed Numbers

#### 9- Arithmetic Operations with Signed Numbers

- Addition
- Subtraction
- Multiplication
- Division

#### 9-1 Addition:

- Addend
- Augend
- Sum

Add the two numbers and discard any final carry bit as long as 2<sup>nd</sup> complement is used for negative numbers.

#### Four conditions for adding numbers:

- Both numbers are positive.
- A positive number that is larger than a negative number.
- A negative number that is larger than a positive number.

Examples:

$$\begin{array}{r}
00010000 & 16 \\
-11101000 & \pm -24 \\
11111000 & -8
\end{array}$$

$$\begin{array}{r} - 00000111 & - 7 \\ 00000100 & - 4 \\ \hline 00001011 & 11 \end{array}$$

$$\begin{array}{rrr}
 & 11111011 & -5 \\
 & + 11110111 & + -9 \\
\hline
 & 1 & 11110010 & -14
\end{array}$$

```
| 125
| +58
| + 00111010 | -58
| 10110111 | 183
| Sign incorrect | | Magnitude incorrect |
```

#### 9-2 Subtraction:

### Subtraction is addition with the sign of the subtrahend changed.

- 1. Take the 2<sup>nd</sup> complement of the subtrahend, then add it to the minuend.
- 2. Discard any final carry bit

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Perform each of the following subtractions of the signed numbers:
                 (a) 00001000 - 00000011
                                             (b) 00001100 - 11110111
                 (c) 11100111 - 00010011 (d) 10001000 - 11100010
       Solution
                 Like in other examples, the equivalent decimal subtractions are given for reference.
                 (a) In this case, 8-3=8+(-3)=5.
                                              00001000
                                                           Minuend (+8)
                                           + 11111101
                                                           2's complement of subtrahend (-3)
                       Discard carry — 1 00000101
                                                           Difference (+5)
                 (b) In this case, 12 - (-9) = 12 + 9 = 21.
                                    00001100
                                              Minuend (+12)
                                                 2's complement of subtrahend (+9)
                                 + 00001001
                                                 Difference (+21)
                                    00010101
                 (c) In this case, -25 - (+19) = -25 + (-19) = -44.
                                              11100111
                                                        Minuend (-25)
                                           + 11101101
                                                        2's complement of subtrahend (-19)
                       Discard carry \longrightarrow 1 11010100
                                                        Difference (-44)
                 (d) In this case, -120 - (-30) = -120 + 30 = -90.
                                    10001000
                                                 Minuend (-120)
                                 + 00011110
                                                 2's complement of subtrahend (+30)
                                    10100110
                                                 Difference (-90)
Related Problem
                 Subtract 01000111 from 01011000.
```

#### 9-3 Multiplication:

8 Multiplicand

× 3 Multiplier

24 Product

There are two methods for multiplication:

- Direct addition
- Partial products
- 1. Determine if the signs of the multiplicand and multiplier are the same or different . Therefore, determine the sign of the product
- 2. Change any negative number to true form (un-complemented)
- 3. Generate the partial products, shift each successive partial product one bit to the left.
- 4. Add each successive partial products to the sum of previous partial product.
- 5. If the sign bit that was determined in step -1- is negative, take the 2<sup>nd</sup> complement of the product.

## Multiplication (cont.)

Multiply the signed binary numbers: 01010011 (multiplicand) and 11000101 (multiplier).

#### Solution

- **Step 1.** The sign bit of the multiplicand is 0 and the sign bit of the multiplier is 1. The sign bit of the product will be 1 (negative).
- Step 2. Take the 2's complement of the multiplier to put it in true form.

#### $11000101 \longrightarrow 00111011$

Steps 3 and 4. The multiplication proceeds as follows. Notice that only the magnitude bits are used in these steps.

1010011	Multiplicand
$\times 0111011$	Multiplier
1010011	1st partial product
+ 1010011	2nd partial product
11111001	Sum of 1st and 2nd
+ 0000000	3rd partial product
011111001	Sum
+ 1010011	4th partial product
1110010001	Sum
+1010011	5th partial product
100011000001	Sum
+_1010011	6th partial product
1001100100001	Sum
+ 0000000	7th partial product
1001100100001	Final product
1001100100001	Final product

**Step 5.** Since the sign of the product is a 1 as determined in step 1, take the 2's complement of the product.

 $1001100100001 \longrightarrow 0110011011111$ 

Attach the sign bit

 $\rightarrow$ 1 0110011011111

#### 9-4 Division of signed numbers:

Division is equivalent to subtracting the divisor from the dividend a number of times equal to the quotient.

- The parts of a division operation are:
   Dividend , Divisor , Quotient
- If the signs are the same, the quotient is positive.
- If the signs are different, the quotient is negative.
- Determine if the signs of the dividend and divisor are the same or different
- 2. Subtract the divisor from the dividend using 2<sup>nd</sup> complement addition and add 1 to the quotient
- 3. Subtract the divisor from the partial remainder and add 1 to quotient.
- 4. If the result positive, then repeat
- 5. If the result zero or negative, then the division is complete

#### 9-4 Division (cont.):

Divide: 01100100 by

00011001

#### Step 1:

the signs of both numbers are positive, so the quotient will be positive. The quotient is initially zero. 00000000 Step 2. Subtract the divisor from the dividend using 2's complement addition (remen ber that final carries are discarded).

01100100 Dividend + 11100111 2's complement of divisor 01001011 Positive 1st partial remainder

Add 1 to quotient: 00000000 + 00000001 = 00000001.

Step 3. Subtract the divisor from the 1st partial remainder using 2's complement add tion.

1 1st partial remainder + 11100111 2's complement of divisor 00110010 Positive 2nd partial remainder

Add 1 to quotient: 00000001 + 00000001 = 00000010.

Step 4. Subtract the divisor from the 2nd partial remainder using 2's complement addition.

00110010 2nd partial remainder
+ 11100111 2's complement of divisor
00011001 Positive 3rd partial remainder

Add 1 to quotient: 00000010 + 00000001 = 00000011.

Step 5. Subtract the divisor from the 3rd partial remainder using 2's complement addition.

00011001 3rd partial remainder + 11100111 2's complement of divisor 00000000 Zero remainder

Add 1 to quotient: 00000011 + 00000001 = 00000100 (final quotient). The process is complete.

#### 10- Hexadecimal Numbers

The hexadecimal number system has 16 digits.

These are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

The hexadecimal system has the base = 16

DECIMAL	BINARY	HEXADECIMAL
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	В
12	1100	C
13	1101	D
14	1110	Е
15	1111	F

#### Binary to hexadecimal

#### Method:

- Break the binary number into 4-bit groups starting at the rightmost bit, Then:
- Replace each 4-bit group with the equivalent hexadecimal symbol.

Convert the following binary numbers to hexadecimal:

(a) 11001010010101111 (b) 1111111000101101001

Solution

(a)  $\underbrace{11001010010101111}_{C A 5 7} = CA57_{16}$ 

**(b)** 0011111110001011010013 F 1 6 9 = 3F169<sub>16</sub>

Two zeros have been added in part (b) to complete a 4-bit group at the left.

#### **Hexadecimal to Binary**

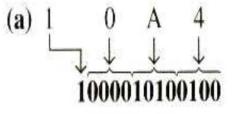
#### Method:

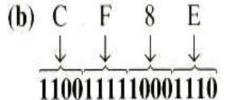
Replace each hexadecimal symbol with the appropriate four bits.

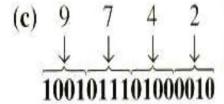
Determine the binary numbers for the following hexadecimal numbers:

- (a)  $10A4_{16}$  (b)  $CF8E_{16}$  (c)  $9742_{16}$

Solution







In part (a), the MSB is understood to have three zeros preceding it, thus forming a 4bit group.

#### **Hexadecimal to Decimal**

#### Method

Convert the hexadecimal to binary then convert from binary to decimal.

Convert the following hexadecimal numbers to decimal:

(a) 
$$1C_{16}$$
 (b)  $A85_{16}$ 

Solution Remember, convert the hexadecimal number to binary first, then to decimal.

(a) 
$$1 \quad C$$
  
 $0001\overline{1100} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = 28_{10}$ 

(b) A 8 5   

$$\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$$
 $101010000101 = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = 2693_{10}$ 

#### Decimal to hexadecimal

#### Method:

Repeated division of a decimal number by 16.

Convert the decimal number 650 to hexadecimal by repeated division by 16.

#### Solution

Hexadecimal remainder

$$\frac{650}{16} = 40.625 \rightarrow 0.625 \times 16 = 10 = A$$

$$\frac{40}{16} = 2.5 \longrightarrow 0.5 \times 16 = 8 = 8$$

$$\frac{2}{16} = 0.125 \longrightarrow 0.125 \times 16 = 2 = 2$$
Stop when whole number quotient is zero.

Stop when whole number  $\frac{2}{40} = \frac{8}{40} =$