Week 6

Arithmetic Operations with Signed Numbers

9- Arithmetic Operations with Signed Numbers

- Addition
- Subtraction
- Multiplication
- Division

9-1 Addition :

- Addend
- Augend
- Sum

Add the two numbers and discard any final carry bit as long as 2nd complement is used for negative numbers.

Four conditions for adding numbers:

- Both numbers are positive.
- A positive number that is larger than a negative number.
- A negative number that is larger than a positive number.
- Both numbers are negative.

Examples:

```
  00001111      7
+ 00000100    + 4
  00001011     11
```

```
  00010000      16
– 11101000    + −24
  11111000      −8
```

```
                          00001111     15
                        + 11111010   + −6
Discard carry ——→  1      00001001      9
```

```
                          11111011     −5
                        + 11110111   + −9
Discard carry ——→  1      11110010    −14
```

```
                                          125
                             01111101     +58
                           + 00111010    − 58
                             10110111     183
Sign incorrect ————————————————┘
Magnitude incorrect ——————————————┘
```

9-2 Subtraction :

Subtraction is addition with the sign of the subtrahend changed.

1. Take the 2nd complement of the subtrahend , then add it to the minuend.
2. Discard any final carry bit

Perform each of the following subtractions of the signed numbers:

(a) 00001000 − 00000011 **(b)** 00001100 − 11110111

(c) 11100111 − 00010011 **(d)** 10001000 − 11100010

Solution Like in other examples, the equivalent decimal subtractions are given for reference.

(a) In this case, 8 − 3 = 8 + (−3) = 5.

	00001000	Minuend (+8)
	+ 11111101	2's complement of subtrahend (−3)
Discard carry ⟶	**1 00000101**	Difference (+5)

(b) In this case, 12 − (−9) = 12 + 9 = 21.

00001100	Minuend (+12)
+ 00001001	2's complement of subtrahend (+9)
00010101	Difference (+21)

(c) In this case, −25 − (+19) = −25 + (−19) = −44.

	11100111	Minuend (−25)
	+ 11101101	2's complement of subtrahend (−19)
Discard carry ⟶	**1 11010100**	Difference (−44)

(d) In this case, −120 − (−30) = −120 + 30 = −90.

10001000	Minuend (−120)
+ 00011110	2's complement of subtrahend (+30)
10100110	Difference (−90)

Related Problem Subtract 01000111 from 01011000.

9-3 Multiplication :

8	Multiplicand
× 3	Multiplier
24	Product

There are two methods for multiplication:

- Direct addition
- Partial products

1. **Determine if the signs of the multiplicand and multiplier are the same or different . Therefore, determine the sign of the product**
2. **Change any negative number to true form (un-complemented)**
3. **Generate the partial products , shift each successive partial product one bit to the left.**
4. **Add each successive partial products to the sum of previous partial product.**
5. **If the sign bit that was determined in step -1- is negative , take the 2nd complement of the product.**

Multiplication (cont.)

Multiply the signed binary numbers: 01010011 (multiplicand) and 11000101 (multiplier).

Solution

Step 1. The sign bit of the multiplicand is 0 and the sign bit of the multiplier is 1. The sign bit of the product will be 1 (negative).

Step 2. Take the 2's complement of the multiplier to put it in true form.

$$11000101 \longrightarrow 00111011$$

Steps 3 and 4. The multiplication proceeds as follows. Notice that only the magnitude bits are used in these steps.

1010011	Multiplicand
× 0111011	Multiplier
1010011	1st partial product
+ 1010011	2nd partial product
11111001	Sum of 1st and 2nd
+ 0000000	3rd partial product
011111001	Sum
+ 1010011	4th partial product
1110010001	Sum
+ 1010011	5th partial product
100011000001	Sum
+ 1010011	6th partial product
1001100100001	Sum
+ 0000000	7th partial product
1001100100001	Final product

Step 5. Since the sign of the product is a 1 as determined in step 1, take the 2's complement of the product.

$$1001100100001 \longrightarrow 0110011011111$$

Attach the sign bit $\longrightarrow$ **1 0110011011111**

9-4 Division of signed numbers:

Division is equivalent to subtracting the divisor from the dividend a number of times equal to the quotient.

- The parts of a division operation are:

  Dividend , Divisor , Quotient

- **If the signs are the same, the quotient is positive.**
- **If the signs are different, the quotient is negative.**

1. **Determine if the signs of the dividend and divisor are the same or different**
2. **Subtract the divisor from the dividend using 2nd complement addition and add 1 to the quotient**
3. **Subtract the divisor from the partial remainder and add 1 to quotient.**
4. **If the result positive , then repeat**
5. **If the result zero or negative , then the division is complete**

9-4 Division (cont.) :

Divide : 01100100 by 00011001

Step 1 :
the signs of both numbers are positive, so the quotient will be positive. The quotient is initially zero. 00000000

Step 2. Subtract the divisor from the dividend using 2's complement addition (remember that final carries are discarded).

01100100	Dividend
+ 11100111	2's complement of divisor
01001011	Positive 1st partial remainder

Add 1 to quotient: 00000000 + 00000001 = 00000001.

Step 3. Subtract the divisor from the 1st partial remainder using 2's complement addition.

01001011	1st partial remainder
+ 11100111	2's complement of divisor
00110010	Positive 2nd partial remainder

Add 1 to quotient: 00000001 + 00000001 = 00000010.

Step 4. Subtract the divisor from the 2nd partial remainder using 2's complement addition.

00110010	2nd partial remainder
+ 11100111	2's complement of divisor
00011001	Positive 3rd partial remainder

Add 1 to quotient: 00000010 + 00000001 = 00000011.

Step 5. Subtract the divisor from the 3rd partial remainder using 2's complement addition.

00011001	3rd partial remainder
+ 11100111	2's complement of divisor
00000000	Zero remainder

Add 1 to quotient: 00000011 + 00000001 = **00000100** (final quotient). The process is complete.

10- Hexadecimal Numbers

The hexadecimal number system has 16 digits.

These are : 0 , 1, 2 , 3, 4 , 5 , 6, 7 , 8 , 9, A , B , C , D , E , F

The hexadecimal system has the base = 16

DECIMAL	BINARY	HEXADECIMAL
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary to hexadecimal

Method:

- **Break the binary number into 4-bit groups starting at the right-most bit , Then:**
- **Replace each 4-bit group with the equivalent hexadecimal symbol.**

Convert the following binary numbers to hexadecimal:

(a) 1100101001010111 **(b)** 111111000101101001

Solution

(a) 1100101001010111

C A 5 7 = $\mathbf{CA57_{16}}$

(b) 00111111000101101001

3 F 1 6 9 = $\mathbf{3F169_{16}}$

Two zeros have been added in part (b) to complete a 4-bit group at the left.

Hexadecimal to Binary

Method :

Replace each hexadecimal symbol with the appropriate four bits.

Determine the binary numbers for the following hexadecimal numbers:

(a) $10A4_{16}$ (b) $CF8E_{16}$ (c) 9742_{16}

Solution

(a) 1 0 A 4 → 1000010100100

(b) C F 8 E → 1100111110001110

(c) 9 7 4 2 → 1001011101000010

In part (a), the MSB is understood to have three zeros preceding it, thus forming a 4-bit group.

Hexadecimal to Decimal

Method

Convert the hexadecimal to binary then convert from binary to decimal.

Convert the following hexadecimal numbers to decimal:

(a) $1C_{16}$ **(b)** $A85_{16}$

Solution Remember, convert the hexadecimal number to binary first, then to decimal.

(a)

$$\overbrace{0001}^{1}\overbrace{1100}^{C} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = \mathbf{28_{10}}$$

(b)

$$\overbrace{1010}^{A}\overbrace{1000}^{8}\overbrace{0101}^{5} = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = \mathbf{2693_{10}}$$

Decimal to hexadecimal

Method :

Repeated division of a decimal number by 16 .

Convert the decimal number 650 to hexadecimal by repeated division by 16.

Solution

	Hexadecimal remainder
$\frac{650}{16} = 40.625 \rightarrow 0.625 \times 16 = 10 =$	A
$\frac{40}{16} = 2.5 \rightarrow 0.5 \times 16 = 8 =$	8
$\frac{2}{16} = 0.125 \rightarrow 0.125 \times 16 = 2 =$	2

Stop when whole number quotient is zero.

2 8 A Hexadecimal numbe

MSD LSD