الكورس الاول-حلقات

The Ideals

المحاضرة ١

Def: Let $(S,+,.)$ be a subring of $(R,+,.)$ We say that S an ideal of R if.

1. $a-b\in S\ \forall s,b\in S$
2. $a.r\in S\ \ \forall a\in S, r\in R$

Ex: $(Z_{12},+_{12},._{12})$. let $S=\{0,3,6,9\}$. show that S is in Z_{12}

a.r	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	3	6	9	0	3	6	9	0	3	6	9
6	0	6	0	6	0	6	0	6	0	6	0	6
9	0	9	6	3	0	9	6	3	0	9	6	3

a-b	0	3	6	9
0	0	3	6	9
3	9	0	3	6
6	6	9	0	3
9	3	6	9	0

Remark: The zero of the ring R belong to ideal I of R. because if $x\in I\longrightarrow 0.x=0\in I$

صفر الحلقة R ينتمي الى المثالي I للحلقة R لأنه

Ex: $I_1=\{0,2,4\}$ is an ideal of Z_6

Ex: $I_1=\{0,3\}$ is an ideal of Z_6

Ex: Prove that $A=\{nr, r\in Z\}=nZ$ is a ideal of Z Solution:

Let $a=nr$, $b=kr : a,b\in A$

$a-b=nr-kr=(n-k)r\Rightarrow(n-k)r\in A\Longrightarrow a-b\in A$

$a.b=(nr).(kr)=(nk)r\in A\Rightarrow a.b\in A$ A is a ideal of Z

Def: We say that for any ideal deference R is proper ideal.

نقول عن المثالي يختلف عن R بأنه مثالي فعلي. ويكون مثالي غير فعلي) تافه (اذا كان هذا المثالي يساوي الحلقة R او اذا كان 0

Def: We say that the ring which no proper ideal not zero is simple ring.

نقول عن الحلقة التي لا تحتوي على مثالي فعلي صفري بأنها بسيطة

Q // In (Z , + .) find (4) ∩ (6)

Solution:

$(4)=\{\dots, -8, -4, 0, 4, 8\ 12, \dots\}$

$(6)=\{\dots\dots, -12, -6, 0, 6, 12, \dots\}$

$(4)\cap(6)=\{\dots, -24, -12, 0, 12, 24, \dots\}=(12)$

Find (4) ∩ (6) ∩ (10)

Ex: Let Z_6 be a ring find tow ideals of Z_6 Solution:

$I_1 = \{0,2,4\}$, $I_2 = \{0,3\}$

المحاضرة ٢

Def: Let R be a ring and I an ideal of R if I subgroup of commutative group (+), then R/I . Is called quotient group of R.

Def: Let R be a ring and I an ideal of R. then $\frac{R}{I}$ is called quotient ring of R.

$$\therefore \frac{R}{I} = \{ a + I : a \in R \} \ni a + I = a + c ; i \in I$$

Remark:

1. (a + I) + (b + I) = (a + b) + I
   (a + I)(b + I) = ab + I
2. The identity with (+) is 0 + I
3. The inverse with (+) is -a + I

Not: (a + I) + (-a + I) = (a + (-a)) + I = 0 + I

Ex: $\frac{Z}{Zn} = 0 + nZ , 1 + nZ , 2 + nZ , \dots \dots (n-1) + nZ$

Such that nZ is an ideal

Theorem: Prove that $(\frac{R}{I}, + , .)$ Is a ring.

Proof: T.P $(\frac{R}{I} , +)$ is commutive group:

$+$

$\forall x, y \in \frac{R}{I} \rightarrow x = a + I \ and \ y = b + I$

Now: $x + y = (a + I) + (b + I) = (a + b) + I \in \frac{R}{I}$,Then is closed

$\forall x, y, z \in \frac{R}{I} \rightarrow x = a + I \ , y = b + I \ \ and \ z = c + I$

$x + (y + z) = (a + I) + [(b + I) + (c + I)]$

$= (a + I) + [(b + c) + I]$

$= [(a + (b + c)) + I\ ]$

$= [((a + b) + c) + I] = [(a + b) + I] + (c + I)$

$= [(a + I) + (b + I)] + (c + I)$

$= (x + y) + z$

$\therefore$ associative

$e = 0 + I \in \frac{R}{I}$ because

$x + (0 + I) = (x + 0) + I = x + I$

∴ e = 0 + I is identity

$(a + I) + (-a + I) = (a + (-a) + I) = (a - a) + I = 0 + I$

$-a + I$ is invers

∴ group

$(a + I) + (b + I) = (a + b) + I = (b + a) + I = (b + I) + (a + I)$ ∴ commutive group

T.P $(\frac{R}{+}, .)$ is semi group

$\forall\ x, y \in \frac{R}{I} \rightarrow x = a.I\ \ and\ \ y = b.I$

Now: $x.y = (a.I) + (b.I) = (a.b) + I \in \frac{R}{I}$,Then is closed

$\forall\ x, y, z \in \frac{R}{I} \rightarrow x = a.I\ , y = b.I\ \ and\ \ z = c.I$

$x.(y.z) = (a.I).[(b.I).(c.I)]$

$= (a.I).[(b.c).I]$

$= [(a.(b.c)).I\ ]$

$= [((a.b).c).I] = [(a.b).I].(c.I)$

$= [(a.I).(b.I)].(c.I)$

$= (x.y).z$

∴ semi group

$\forall\ x, y, z \in \frac{R}{I} \rightarrow$

$(x + I)[(y + I) + (z + I)] = (x + I)[(y + z) + I]$

$= x(y + z) + I$

$= (xy + xz) + I$

$$= (x + I)(y + I) + (x + I)(z + I)$$

$\therefore$ Distributive $\therefore \left(\frac{R}{I}, +, .\right)$ Is Ring

المحاضرة ٣

Ex: consider $\left(\frac{Z}{(3)}, +, .\right)$

$\frac{Z}{(3)} = \{0 + (3), 1 + (3), 3 + (3)\}$

$= \{\{\dots - 6, -3, 0, 3, 6, \dots\}, \{\dots, -5, -2, 1, 4, 7, \dots\}, \{\dots, -4, -1, 2, 5, 8, \dots\}\}$

$= \{[0]_{mod\ 3}, [1]_{mod\ 3}, [2]_{mod\ 3}\} = Z_3$

LUCHUER 12

In general:

$\left(\frac{Z}{n}\right) = Z_n\ ; n \in \{1,2,3,\dots\}$

Def: Let R, S be two rings. A function $f: R \to S$ is called ring homomorphism if.

1. $f(a + b) = f(a) + f(b) \quad \forall a, b \in R$
2. $f(a.b) = f(a).f(b)$

Q: R ring. R commutative $\Leftrightarrow (a + b)^2 = a^2 + 2ab + b^2 \ \forall a, b \in R$ Solution:

Let R be a comm.

T.P : $(a + b)^2 = a^2 + 2ab + b^2 \ \forall a, b \in R$

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$

$$a^2 + 2ab + b^2$$

Let: $(a + b)^2 = a^2 + 2ab + b^2$

We get $ab + ba = 2ab = ab + ab$

$\therefore ab + ba = ab + ab$

$\therefore ba = ab$

$\therefore$ comm

Theorem: let R be an integral domain. If

1. $a^2 = 1 \to a \in \{+1, -1\}$
2. $(P(X), \Delta, \cap)$ is a ring. Show that it is contains divisors of zero.

Proof:

1. $a^2 = 1 \to a^2 - 1 = 0 \ (a-1)(a+1) = 0$
   $a = 1 \ \ OR \ a = -1$

2. $\emptyset \neq A \subset X \, , \forall A \in P(X)$
   $A \, \Delta \, A = (A \cup A) - (A \cap A)$
   $= A - A = \emptyset$

المحاضرة ٤

$\therefore \exists$ zero divisors.

Ex: $(Z_{12}, +_{12}, ._{12})$. prove that there exist an ideal of $(Z_{12}, +_{12}, ._{12})$ has zero divisors (H.W) We know conditional of ideal are both $a - b \in S \ and \ a.r \in S \ \forall a \in S \, , r \in R$ s.t S is subring

a.r	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	3	6	9	0	3	6	9	0	3	6	9
6	0	6	0	6	0	6	0	6	0	6	0	6
9	0	9	6	3	0	9	6	3	0	9	6	3

a-b	0	3	6	9
0	0	3	6	9
3	9	0	3	6
6	6	9	0	3
9	3	6	9	0

We can say that $S = \{0,3,6,9\}$ is IDEAL And has zero divisors: $(6 \times 8) mod \ 12 = 0$

Remark: Let S be a subring of R. If S has 1 of the mother R, Then S an integral domain.

Ex: $h: Z \to Z_n \ni h(m) = [m]_{mod \ n}$ show that h is homeomorphism.

Solution:

Clearly h is a function. Why??

1. $h(m_1 + m_2) = [m_1 + m_2]_{\ mod \ n}$

   $= [m_1]_{mod \ n} +_{mod \ n} [\ m]_{mod \ n} \ \forall m_1, m_2 \in Z$

2. $h(m_1 . m_2) = [m_1 . m_2]_{\ mod \ n}$

   $= [m_1]_{mod \ n} ._{mod \ n} [\ m]_{mod \ n} \ \forall m_1, m_2 \in Z \ \ h(m_1). h(m_2)$

$\therefore$ h is a homomorphism.

Def: Let h: $R \to S$ be a ring **homomorphism.** If h is onto and one-one then h is called **isomorphism**.

Ex: let R be a ring and $h: R \to R$ defined as: $h(r) = r \ \forall r \in R$ (i.e: h is identity mapping). Then h is isomorphism Solution:

We must prove that:

1. h homomorphism
2. h on to
3. one – one

1. $h(r_1 + r_2) = r_1 + r_2 = h(r_1) + h(r_2)$ $h(r_1 . r_2) = r_1 . r_2 = h(r_1) . h(r_2)$

   $\therefore$ h homomorphism

2. $\forall\ x \in R, \exists y \in R \ni h(y) = x$ (Def: Onto)

   $\forall\ x \in R, \exists y \in R \ni h(y) = x$

   $\therefore$ h onto

3. $Let\ h(x) = h(y) \to x = y$

$\therefore$ h one – one

$\therefore$ h isomorphism

LUCHUER 13

Remarks:

1. Let R be an integral domain with 1. If S is a subring of R with 1, then S is an integral domain
2. Let $h: R \to S$ be a homomorphism. (ring homo). Then :
   - $h(O_R) = O_s$
   - $h(-a) = -h(a)$
   - $h(1_R) = 1_S$
   - $h(a^{-1}) = (h(a))^{-1}$

المحاضرة ٥

Kernal of homomorphism

نواةالتشاكل

Def: Let $h: R \to S$ be a ring homo. Then kernal of h is defined as:

$$Ker(h) = \{x \in R : h(x) = 0\}$$

Q: prove that $Ker(h)$ is an ideal in R.

Solution:

$h(0) = 0 \to 0 \in Ker(h)$

$\therefore Ker(h) \neq \emptyset$.

- الان الشرط الأول من المثالي:

1- $\forall\, x, y \in Ker(h) \to x - y \in Ker(h)$

والدليل على ذلك

$h(x - y) = h(x) + h(-y)$ حسب تعريف hom

$= h(x) + (-h(y))$

$= h(x) - h(y)$

$\overline{\in Ker(h)}$ $\overline{\in Ker(h)}$

$\therefore x - y \in \ker(h)$ why??

- الان الشرط الثاني من المثالي:

2- $\forall\ x \in Ker(h), y \in R \to x.\, y \in Ker(h)$ and $y.\, x \in Ker\ (h)$

والدليل على ذلك: $h(x.\, y) = h(x).\, h(y)$

$= 0 \,.\, h(y)$

$= 0 \in Ker(h)$

Def: Let R,S be two rings. We say that R embed in S if :

$$R \cong S' \ni S' \text{ subring of } S$$

Not: ≅ means ∃ $h: R \to S \ni h$ is 1 – 1 and homomorphism.

Theorem: Any ring can be embedded in a ring with 1.

Proof:

Let R be any ring.

Let $R \times Z = \{(r, n)\ ; r \in R\ , n \in Z\}$ Let $(R \times Z,$ $+ , .)$ Be a ring with $1 = (0,1)$ $h: R \to R \times Z \ni$

$h(r) = (r, 0)\ \forall r \in R$

$h(r_1 + r_2) = (r_1 + r_2, 0)$

$= (r_1, 0) + (r_2, 0) \to h(r_1) + h(r_2)$

الان $_2 h(r_1 . r_2) = (r_1 . r_2, 0) = (r_1, 0).(r_2, 0) = h(r_1).h(r)$

اكتملت شروط التشاكل باق 1 -1

$h(r_1) = h(r_2) \to (r_1, 0) = (r_2, 0) \to r_1 = r_2$

وعليه $\therefore h\ is\ 1 - 1$

اصبح لدين ا

1-1 + hom

المحاضرة ٦

Natural homomorphism

Def: Let R be a ring and I be an ideal in R. Then natural homo. Is defined $Nat_I: R \to {}^{R}{}_{I}$

$Nat_I(x) = x + I\ \forall x \in R$

كيف نتأكد انه تشاكل ؟ ؟

1- $Nat_I(x + y) = (x + y) + I$

$= (x + I) + (y + I)$

$= Nat_I(x) + Nat_I(y)$

2- $Nat_I(x. y) = (x. y) + I$

$= (x + I).(y + I)$
$= Nat_I(x).Nat_I(y)$

∴ شروط التشاكل تحقق ت

Remark: Nat_I is onto homo. تشاكل شامل

$$\forall\, x+I \in \frac{R}{I}\ ;\ \exists\, x \in R :\ Nat_I(x) = x + I$$

∴ onto

LUCHUER 14

Theorem: Let $h: R \to S$. Be a ring homo. Then h is 1-1 ⇔ Ker(h) = {0}

Proof: ⇒
Suppose that h is 1-1
T.P Ker(h) = { 0 }
Let $x \in Ker(h) \to h(x) = 0$
$\therefore x = 0 \Rightarrow Ker(h) = 0$ ⇐
Let $Ker(h) = 0$
Suppose h(x) = h(y) $\forall x, y \in R$
$h(x) - h(y) = 0$

$\Longrightarrow h(x - y) = 0 \Longrightarrow x - y \in Ker(h)$
$\therefore x - y = 0 \Longrightarrow x = y$
∴ h is 1-1

Theorem: Let $h: R \to S$ be a ring homo and onto. then $\frac{R}{Ker(h)} \cong S$

Proof:

We must prove that homo & 1-1 & onto??? Homo: $f(x + Ker(h) + yKer(h))$

$= f((x + y) + Ker(h))$
$= (f(x + y)) + Ker(h)$
$= f(x + Ker(h)) + f(y + Ker(h))$
Also, $f\ ((xKer(h))\ .(y + Ker(h)))$
$= f(x.y) + Ker(h)$
$= f(x.Ker(h))\ .f(y\,.Ker(h))$
∴ Homom Onto: $\forall y \in S\ ;\ \exists x + Ker(h) \in \frac{R}{K}$

$\therefore y = f(x + Ker(h))$

$\therefore$ onto

$1 - 1$: Let $f(x + Ker(h)) = f(y + Ker(h))$

$f(x) = f(y)$

$f(x) - f(y) = 0$

$f(x - y) = 0$

$x - y = 0$

$\therefore x = y$

المحاضرة ٧

Q: Let $f: z \to z$ be a homo. Then $f = 0$ *or* $f = I_z$ Proof:

Suppose that $f \neq 0$ $\therefore$ T.P

$f = I_z$??

$f(n) = f(\underbrace{1 + 1 + \cdots + 1}_{n\text{ - times}})$

$= \underbrace{f(1) + f(1) + \cdots + f(1)}_{n\text{ - times}}$

Also:

$f(-n) = f(\underbrace{-1 - 1 - 1 - \cdots - 1}_{n\text{ - times}})$

$= n\, f(-1) = -nf(1)$. There

$f(n) = nf(1)$ ______________________(1)

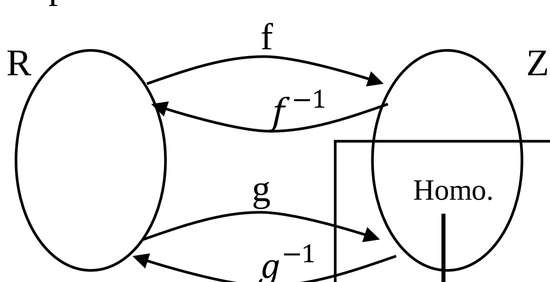
$f(n) = f(n.1) = f(n)\,.f(1)$ _________(2)

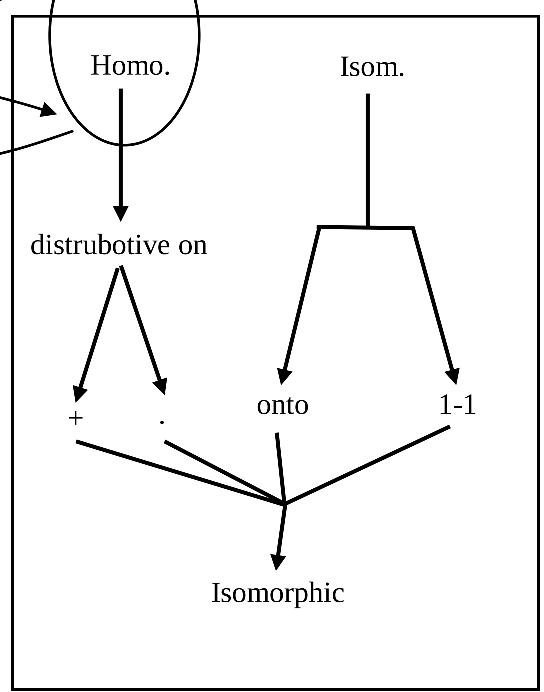
$\therefore n\,f(1) = f(n)\,f(1) \Longrightarrow f(n) = n \Longrightarrow f = I_Z$ Q: Let R be a ring. Then there exists at most one isomorphism $h: R \to Z$ Proof:

$fog^{-1}: Z \to Z$

$\therefore fog^{-1} = i_Z$

$\therefore f = g$ Whey





Theorem: Let R be a ring. Then any onto homo. from R to Z can be determine by its kernel

Proof:

Suppose that $R \overset{f}{\underset{g}{\to}} Z$

Suppose that f, g are two onto homos.

$\frac{R}{Ker(f)} \cong Z$

R

$$\frac{\quad}{\cancel{Ker(g)}} \cong Z$$

$$\frac{R}{Ker(f)} \cong \frac{R}{Ker(g)}$$

$$\therefore Ker(f) = Ker(g)$$

$h_1 : \frac{R}{Ker(f)} \to Z$ isomorphism

$h_2 : \frac{R}{Ker(f)} \to Z$ isomorphism

$\because h_1 = h_2$ Whey ??

$$\therefore Ker(f) = Ker(g)$$

المحاضرة ٨

Fields

الحقول

Def: Let $(R, +, .)$ be a commutative ring with identity, So $R^{-1} = R - \{0\}$

معنى اخر :

Any non-Zero element in R has an inverse. Then we say that $(R, +, .)$ Is filed

((كل عنصر في R غير صفري له نظير عندئذ R تكون حقلا))

F $\xrightarrow{\text{يعطي}}$ R

Remark: Every field is a ring

Theorem: Every field is a integral domain. But the converse is not true.

Proof:

Let $(F, +, .)$ Be a field. We need to prove that F has non-Zero divisors??

$a \,.\, b = 0$ for some $a, b \in F$ $a \neq 0$

$a^{-1}(a. b) = 0 \to a^{-1}(a. b = 0)$

$\to (a^{-1}. a). b = 0$

$e. b = 0$

$b = 0$

وهذا يخالف شروط وجود القاسم الصفري

$\therefore$ F has non-zero divisors $\therefore$ F integral domain.

الان نبين العكس غير صحيح

بمعنى F ساحة تامة ولكن ليس حقل وعليه يجب إعطاء مثال

Ex: $(Z, + , .)$ Is integral domain because has nonzero divisors but $(Z, + , .)$ not filed

$2 \in Z^{-1} \rightarrow$ 2 ليس له نظير في z

سؤال من له نظير في Z ؟؟؟

Ex: $(R^*, + , .)$ Is a field why?
Ex: $(Q , + , .)$ Is a field why?
Ex: $(Z , + , .)$ Is not a field why?

Remark: If $F = \{a + b\sqrt{3} : a, b \in Q\}$ then

$I_+ = 0 + 0\sqrt{3}$ محايد جمعي

$I_. = 1 + 0\sqrt{3}$ محايد ضربي

Q: prove that if $(F, + , .)$ Is a filed and $a. b = 0 \ \forall a, b \in F$, then $a = 0 \ or \ b = 0$ Proof:

Let $a. b = 0 \ \forall a, b \in F$

Let $a \neq 0 \rightarrow \exists \ a^{-1} \in F$ why??

$\therefore a. a^{-1} = a^{-1}. a = 1$

الان نضرب طرفي المعادلة $a. b = 0$ بـ a^{-1}

$a^{-1}(a. b) = a^{-1}. 0$

$(a^{-1}. a). b = 0$

$e. b = 0$

$b = 0$

ملاحظة: قلنا سابقا في النظرية ان كل حقل هو ساحة تامة ولكن العكس غير صحيح وقدمنا برهان ومثال على العكس غير صحيح. الان نستطيع ان نقول ان العكس صحيح في حالة الساحة التامة تكون منتهية.

Theorem: any finite integral domain $(R , + , .)$ Is a field.

Proof:

Suppose that $R = \{a_1, a_2, ... , a_n\} \ \forall a \in R \ni a \neq 0 \rightarrow a. a_1 , a. a_2 , ... , a. a_n$

وهذا يعني n من العناصر المختلفة.

واذا كان غير ذلك $\Leftarrow$

$\exists \ a_i , a_j \in R \ni a. a_i = a. a_j$

$\Rightarrow a_i = a_j$

$\because 1 \in R \rightarrow a. a_i = 1 \ \rightarrow a^{-1} = a_i$ why?

$\therefore R$ field.

Theorem: Let R be a commutative ring with 1. Then R is field $\Leftrightarrow$ R has no non trivial ideal.

Proof:

$\Longrightarrow$ Let R be a field and I ideal of $R \ni I \neq 0$ and $I \neq R$

$\therefore 0 \subset I \subset R$

$\exists\, x \neq 0 \ni x \in I \subset R$

$\therefore \in x\ R$. But R field $\Rightarrow x_{-1}$ exists

$\therefore 1 = x^{-1} .\ x \in I \Rightarrow I = R \rightarrow$ C!

$\therefore I = 0$ and $I = R$.

$\Longleftarrow$ Let R has no non trivial ideals

Let $x \in R - \{0\} \Rightarrow x \in R^{-1}$

Consider the ideal (x)

$(x) \neq 0 \Rightarrow (x) = R$

$1 \in (x) = \{rx : r \in R\}$

$1 = r'x \Rightarrow r' = x^{-1} \ni r' \in R$

$\therefore$ R field

Theorem: Let $f: F_1 \rightarrow F_2$ be a field home. (onto). Then either $f = 0$ or $F_1 \cong F_2$

Proof:

Consider $Ker(f) \subseteq F_1$ ideal

$\Longrightarrow Ker(f) = 0$ or $Ker(f) = F_1$

$\therefore f$ one – one

$\therefore f$ isomorphism (why?)

$\therefore F_1 \cong F_2$

Def: Let F be a field and $\emptyset \neq K \subseteq F$ and $(K, +, .)$ is also field. We say K is subfield of F Or: F is an extension field of K

Ex: Q is subfield of R

Ex: C is an extension field of both R and Q

Remark: Let $(K, +, .)$ be a subfield of $(F, +, .) \Leftrightarrow$

1. $(K, +)$ subgroup of $(F, +)$
   $\forall \in a, bK \rightarrow a - b \in K$.
2. $(K - \{0\}, .)$ Subgroup of $(F - \{0\}, .)$
   $\equiv \forall \in a, b\ K ; b \neq 0 \rightarrow a.b^{-1} \in K$

LUC (3)

Theorem: Let R be an integral domain and it is a subring of the field F. Then $F' = \{a.\, b^{-1}: a, b \in R\}$ is a subfield of F and F' is the smallest one containing R $(R \subset F')$

Proof:

1. $\forall x, y \in F' \rightarrow x - y \in F'$
   Let $x \in F' \rightarrow x = a_1 \,.\, b_1^{-1}$ and $y \in F' \rightarrow y = a_2 \,.\, b_2^{-1}$
   Now: $x - y = a_1 \,.\, b_1^{-1} - a_2 \,.\, b_2^{-1} = (a_1 b_2 - a_2 b_1) \,.\, (b_1 b_2)^{-1} \in F'$
2. $\forall x, y \in F' \rightarrow x.\, y^{-1} \in F'$
   Now: $x \,.\, y^{-1} = (a_1\, b_1^{-1}) \,.\, (a_2 b_2^{-1})^{-1} = (a_1 . b_1^{-1}) \,.\, (b_2 a_2^{-1})$
   $= (a_1 \,.\, b_2) \,.\, (b_1 \,.\, a_2)^{-1} \in F'$

Let F'' subfield of F'

$\therefore a \,.\, b^{-1}, \in F''$

$\therefore F' \subseteq F''$

LUC (4)

We have three types of ideals:

المحاضرة ١٠

Minimum ideal

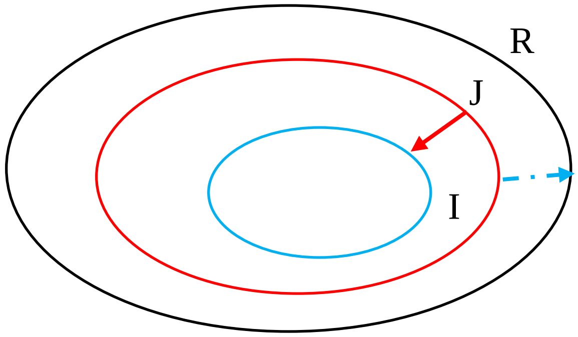
Def: Let R be a ring and I an ideal of R then I is minimum ideal if

1. $1 - I \neq 0$
2. $\exists J$ ideal of R $\ni 0 \subseteq J \subseteq I \implies 0 = J \text{ or } I = J$

المحاضرة
١١

Maximal ideal

Def: Let R be a ring and I be an ideal of R we say I is maximal ideal of R if there exists J is a proper ideal of R such that $I \subset J \subseteq R \Rightarrow J = R \text{ or } I = J$



Remark: The ring may contain more than one Maximal ideal.

متى يقال للمثالي الفعلي انه مثالي اعظم؟

١. المثالي لا يساوي مجموعة الصف ر
٢. المثالي لا يساوي الحلق ة
٣. لا يوجد أي مثالي فعلي في الحلقة يحتوي هذا المثال ي

هو)المثا ي ل الفع ي ل الأعظم(يحوي المثاليات ولا احد يحتوي ه

(i.e.) is Maximal of R if $I \neq R$ and if $\exists$ an ideal

$(M, +, .)$ in a ring R, s.t $J \subset M \subseteq R$ then $M = R$ **Example**: Determine

the Maximal ideals in the ring $(Z_{12}, +_{12}, ._{12})$

Solution:

اولاً: نجد كل المثاليات الفعلية في الحلقة والتي لا تساوي الصفر ولا الحلقة لان)0(والحلقة) مثاليات تاف ه

ونجد المثاليات الفعلية لحلقة الاعداد الصحيحة ذات القياس من خلال إيجاد تحليل القياس n الى اعداه الأولي ة

$12 = 2(6) = 4(3)$

ثم نستخدم قانون مولد تلك الاعداد)المولد هو مضاعفات العدد قياس n)

1. $I_1 = (\bar{2}) = (\{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}, \bar{10}\}, +_{12}, ._{12})$
2. $I_2 = (\bar{3}) = (\{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}, +_{12}, ._{12})$
3. $I_3 = (\bar{4}) = (\{\bar{0}, \bar{4}, \bar{8}\}, +_{12}, ._{12})$
4. $I_4 = (\bar{6}) = (\{\bar{0}, \bar{6}\}, +_{12}, ._{12})$

$\Rightarrow I_1 = (\bar{2})$ *and* $I_2 = (\bar{3})$ are Maximal ideal in ring Z_{12} , Since $\nexists$ proper ideal of ring Z_{12} containing $I_1 = (\bar{2})$ *and* $I_2 = (\bar{3})$. But $I_3 = (\bar{4})$ and $I_4 = (\bar{6})$ is not maximal ideal in ring Z_{12}. Since $(\bar{4}) \subset (\bar{2})$ and $(\bar{6}) \subset (\bar{3})$

H.W: Determine the Maximal ideals in the ring $(Z_{20}, +_{20}, \cdot_{20})$ Solution: