Heat Transfer I

" Heat Transfer Mechanisms"

Lecture No. (1)

Chapter One

Modes of Heat transfer

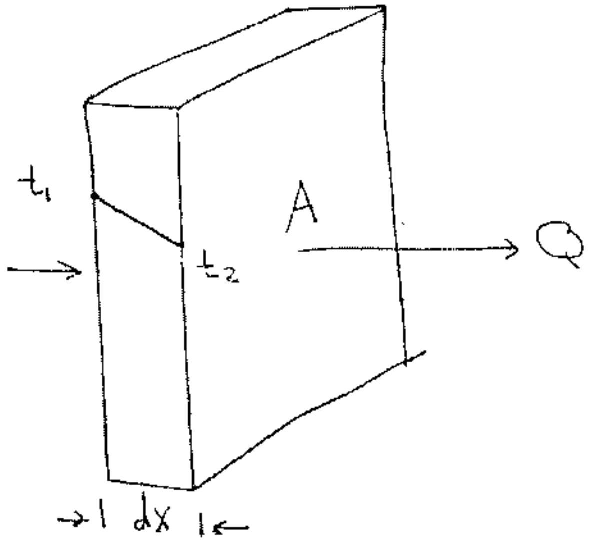
- **Conduction** التوصيل
  - "Solids"
  - "liquid"
  - "Gases"
- **Convection** الحمل
  - "Liquid & Gas"
- **Radiation** الإشعاع
  - → vaccum
  - → Liquid & Gas

(ملحوظة) انتقال الحرارة يتم من الوسط الأعلى حرارة الى الأقل حرارة

* Conduction :-

انتقال الحرارة من الجزيئات الأعلى طاقة إلى أقل طاقة نتيجة الاصطدام بينها

اتجاه انتقال الحرارة "Q" يكون من الأعلى إلى الأقل في درجة الحرارة



إذا كان $t_1 = t_2$ ← $Q=0$ لا يحدث انتقال لدرجة الحرارة

$$Q \propto \Delta t$$

$$Q \propto \frac{1}{dx}$$

$$Q \propto A$$ ← المساحة العمودية على اتجاه انتقال حرارة

$$\rightarrow Q = -KA\frac{\Delta t}{dx}$$

* Fourier's Law :-

$$Q = -K A \frac{dt}{dX}$$

Q :. Heat Rate (watt) معدل انتقال الحرارة

A :. Area Prependicular to Heat Rate (m^2)
المساحة العمودية على اتجاه انتقال الحرارة

Δt :. (°C) / (K) فرق درجات الحرارة

ΔX :. (m) المسافة بين سطحي انتقال الحرارة

K :. thermal Conductivity (w/m. K) الحرارية الموصلية

thermal Conductivity :-
معدل انتقال الحرارة خلال وحدة الأطوال لكل وحدة المساحات لكل فرق في درجات الحرارة

Cp [Kj /Kg. K]	K [w/m. K]
Heat Capacity السعة الحرارية	thermal Conductivity الحرارية الموصلية
تقيس قدرة المادة على السعة تخزين الحرارة بها.	تقيس مدى قدرة المادة على انتقال الحرارة منها "خلالها".

مثال :- الماء قادرة على تخزين الحرارة خلالها بصورة جيدة بينما الحرارية الموصلية بها ضعيفة

* Heat Flux :-

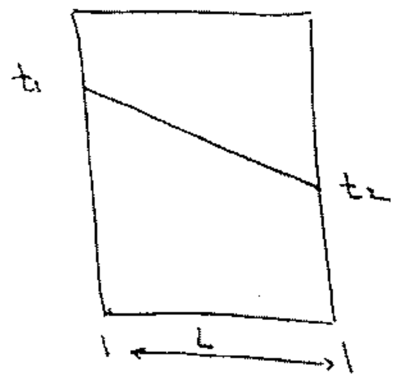
$$q_x = \frac{Q}{A} = -K \frac{\Delta t}{\Delta x} \quad W/m^2$$

q: معدل انتقال الحرارة خلال وحدة المساحات

$\frac{\Delta t}{\Delta x}$ → Temperature gradient

* الإشارة السالبة لتوضيح أن الحرارة تنتقل من الوسط الأعلى في درجة الحرارة إلى الأقل *

$$q = -K \frac{t_2 - t_1}{L} \qquad \boxed{t_2 < t_1}$$



تعتبر الموصلية الحرارية إحدى خواص المادة المميزة لها وهي التي تحدد شكل وطريقة انتقال الحرارة بالتوصيل خلال المادة.

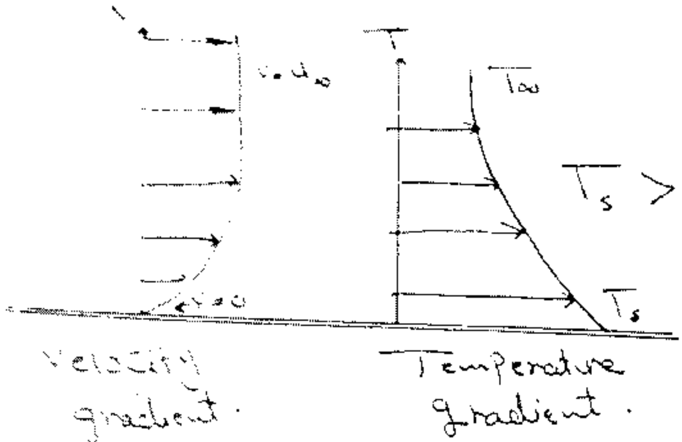
$$\Rightarrow K_{Gases} < K_{Insulator} < K_{liquids} < K_{Metalic\ Solids} < K_{Metalic\ Alloys} < K_{Pure\ Metals}$$

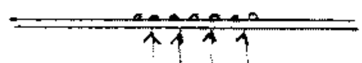
لذلك عندما نريد الإحساس بالدفء يكون الجاكيت ذات طبقات حيث يكون بينها طبقات هواء عازل جيد للحرارة.

"الحمل" - Convection heat transfer -

* يحدث عندما تنتقل الحرارة من سطح صلب إلى مائع ملاصق له في حالة حركة ومن وجود اختلاف درجات الحرارة بين السطح والمائع.

* يحدث أيضاً انتقال حرارة بين السطح و جزيئات المائع الملاصقة له عن طريقة التوصيل





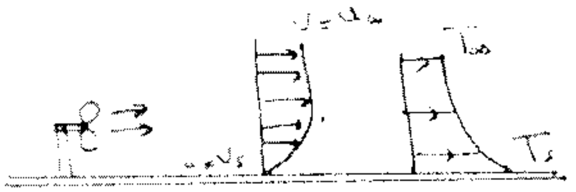
← انتقال حرارة بالتوصيل بين جزيئات المائع ذات الجزء حيث يلتصق.
← بعدها يحدث الحمل في الطبقات الأعلى.

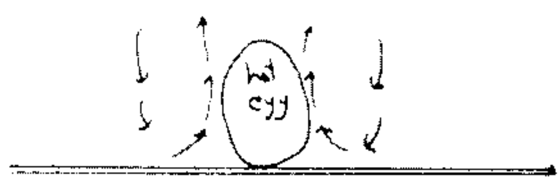
Forced Convection
الحمل القسري

* يحدث عندما تكون حركة المائع نتيجة لقوة خارجية Pump, Fan

Natural Convection
الحمل الطبيعي

* يحدث عندما تكون حركة المائع نتيجة اختلاف الكثافة بين طبقات المائع.





* ملحوظة: في غياب الحركة لا يمكن أن يكون انتقال الحرارة بالحمل ويكون بالتوصيل.

$= 0$ m/sec $\rightarrow h = 0 \rightarrow Q_{conv} = 0$

* Newton's law :-

$$Q = h\,A\,\Delta t$$

Q :- (watt) معدل انتقال الحرارة

h :- [W/m²K] معامل انتقال الحرارة بالحمل

A :- (m²) مساحة السطح المعرض للمائع والتي يحدث فيها انتقال الحرارة

Δt :- [K] فرق درجات الحرارة بين المائع والسطح الصلب

* يتوقف معامل انتقال الحرارة على كثير من العوامل سيتم دراستها فيما بعد

(h) Convective heat transfer coefficient →

- geometry → Fluid pro
- smooth → Pressure
- Roughness → Convection type
- velocity → etc

* لا يُعتبر معامل انتقال الحرارة بالحمل (h) خاصية من خواص المائع بينما تُعتبر الموصلية الحرارية (K) خاصية من خواص المادة وإن كانت تتغير بتغير درجة الحرارة والضغط.

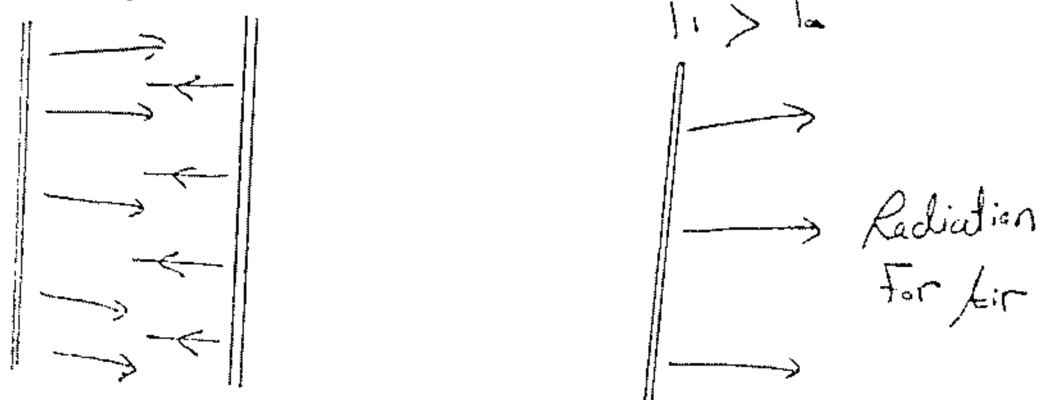
$$h_{Boiling\ \&\ Condensation} > h_{Forced,\ Liquid} > h_{natural,\ Liquid} > h_{Forced,\ gas} > h_{natural\ gas}.$$

h :- Heat transfer Coefficient by convection [W/m²K]

* Radiation heat transfer :- الإشعاع

* يحدث هذا النوع من انتقال الحرارة بين الأسطح المتقابلة والتي تكون ذات درجات حرارة مختلفة *

* يختلف هذا النوع حيث إنه لا يتطلب وسط ينتقل خلاله بل من الممكن الانتقال بالفراغ



الإشعاع بين الأجسام الصلبة — الإشعاع بين سطح ساخن والوسط المحيط

* Stefan Boltzmann law :-

$$Q = \sigma A T^4$$

- Q ← أقصى انتقال حرارة ممكن حدوث لسطح جسم أسود
- σ ← Stefan Boltzmann's Constant, $\sigma = 56.697 \times 10^{-9}\ W/m^2 \cdot K^4$
- A ← مساحة السطح المشع
- T ← درجة حرارة السطح المشع وتكون بالكلفن

$$Q = \sigma A_1 (T_1^4 - T_2^4)$$ ← كلفن

أقصى إشعاع من الجسم الأسود رقم ① درجة حرارته T_1 الى الجسم الأسود T_2.

$$Q = \sigma A_2 (T_2^4 - T_1^4)$$ ← كلفن

أقصى إشعاع من الجسم الأسود رقم ② درجة حرارته T_2 الى الجسم الأسود T_1

For real bodies :

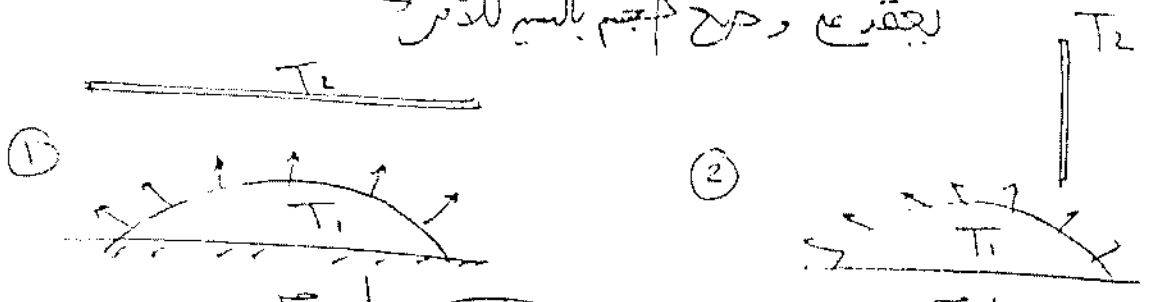
$$Q = \sigma A_1 \varepsilon_1 F_{12} (T_1^4 - T_2^4)$$

حيث

ε_1 : emissivity الانبعاثية

F_{12} : Factor shape

← يعتمد على وضع الجسم بالنسبة للآخر



$F_{12}|_1$ $F_{12}|_2$

$$F_{12}|_1 \neq F_{12}|_2$$

← جميع المواد (السائلة / الصلبة / الغازية) تشع حرارة ولكن بدرجات مختلفة

← الأجسام الصلبة هي الظاهرة الأكثر تداولاً في حالة الإشعاع

← جميع المواد (السائلة / الصلبة / الغازية) تمتص وتشع وينفذ من خلاله الضوء

← الجسم الأسود هو الأكثر إشعاعاً من الأجسام الأخرى في حالة تساوي درجة الحرارة

$$\varepsilon_{black\ body} = 1$$

Heat Transfer I

" Heat Transfer Mechanisms"
solved problems

Lecture's No. (2)

Ex1// The wall of an industrial furnace is constructed from 0.15 m thick fireclay brick having a thermal conductivity of 1.7 W/m·K. Measurements made during steady-state operation reveal temperatures of [illegible] and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall which is [illegible] by [illegible] on a side?

Solved Problem Ch [1]

ex 1.1 :-

Given

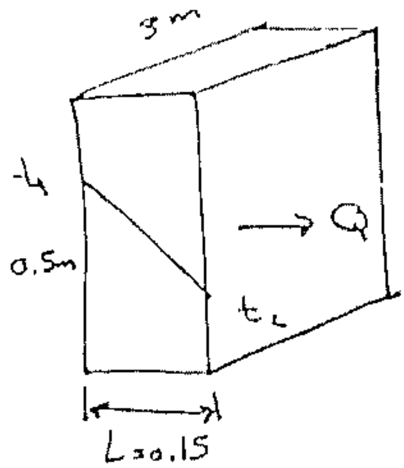
$L = 0.15 \, m$

$K = 1.7 \, W/m.K$

$t_1 = 1400 \, K$

$t_2 = 1150 \, K$

$A = 0.5 \times 3 \, m^2$



Req Q :-

Sol

Heat Rate :-

$$Q = -K A \frac{\Delta t}{L}$$

$$Q = -1.7 * 0.5 * 3 * \frac{1150 - 1400}{0.15} = 4250 \text{ Watt}$$

↑ Required

Heat Flux :-

$$q = Q/A = \frac{4250}{0.5 \times 3} = 2833.33 \, W/m^2$$

[illegible] are separated [illegible]

by a thick [illegible] 15 cm thick [illegible]

[illegible] W/m·K and [illegible] conductivity of [illegible]. Under steady [illegible]

[illegible] is [illegible]

free convection heat [illegible]

characterized by a [illegible]

[illegible]

example 1.2 :-

Given :-

$T_o = 300\,K$

$L = 0.15\,m$

$K = 1.25\,W/m.K$

$\varepsilon = 0.8$

$T_2 = 102 + 273 = 375\,K$

$h_o = 20\,W/m^2 K$

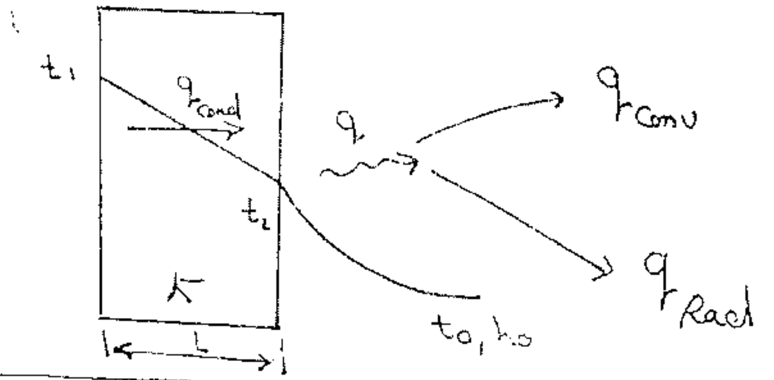
إذا أعطى ε أو قال Black body يبقى فيه انتقال حرارة بالإشعاع

$\varepsilon|_{Black\ body} = 1$

Req :-

$t_i = ??$

الحرارة التي بيفقدها الجسم بالتوصيل تنطلق في الهواء عن طريق الحمل والإشعاع



$$q_{Cond} = q_{Rad} + q_{Conv}$$

$$-K\frac{t_2 - t_1}{L} = h_o(T_2 - T_o) + \varepsilon\sigma(T_2^4 - T_o^4)$$

$$-1.25 * \frac{375 - T_1}{0.15} = 20(375 - 300) + 0.8 * 5.67 \times 10^{-9} * (375^4 - 300^4)$$

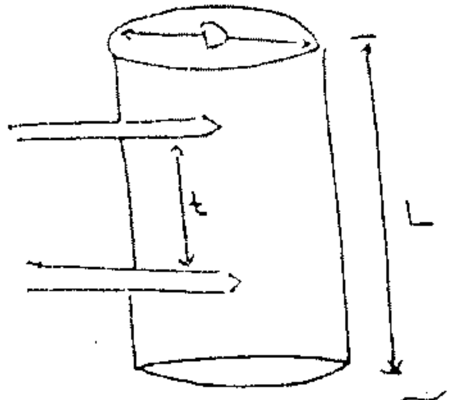
$$T_1 = 618.55\,K$$

$$t_1 = 345.55\,°C$$

[illegible] maetters [illegible]

Pro 1.20 :-

D = 4 cm
L = 7 cm
t = 3 cm
I = 0.6 Amp
V = 110 Volt
Δt = 10°C



It's Real experiment to determine K

Req. K

Sol

$$Q = IV = 0.6 \times 110 \text{ watt}$$

$$Q = K A \frac{\Delta t}{L}$$ → $\frac{\pi}{4} D^2$ "Cross Section"

$$0.6 \times 110 = K \times \frac{\pi}{4} \times 0.04^2 \times \frac{10}{0.07}$$

$$\boxed{K = 157.56 \text{ W/m.K}}$$

Note $\boxed{Q_{elec} = IU = I^2 R}$ ← حفظ

R :- electric Resistance المقاومة الكهربية

حفظ ← $\boxed{R = \rho \frac{L}{A}}$ → Length طول السلك (m)

ρ → المقاومة النوعية (Ω.m)

A → Cross Section Area (m²) مساحة المقطع (م²)

* E.

Giv

(11)

Pro 1.30

Given

$A = 0.006 \times 3 = 0.018\,m^2$

$\varepsilon = 0.85$

$Q = 1600$ watt

$T_2 = 25 + 273 = 298\,K$

Re

ن سرية

Conve

Req

① T_1

② Radiation Heat transfer of Black Body @ 1000 K

③ Temperature of Black Body Radiate as Sun Radiation. $q_{sun} = 1350\,w/m^2$

Sol

* $Q = \varepsilon A \sigma (T_1^4 - T_2^4)$

$1600 = 0.85 * 0.006 * 3 * 56.7 \times 10^{-9} (T_1^4 - 298^4)$

Get $T_1 = 1166\,K$

* $Q_{Black} = A \sigma T^4 = 56.7 \times 10^{-9} \times 1000^4 \times 0.018$

$= 1020.6$ watt

* $q_{sun} = \sigma T_{Black}^4 \rightarrow 1350 = 56.7 \times 10^{-9} T_{Black}^4$

$T_{black} = 392$ Kelwin

Pro 1.24

Given :-

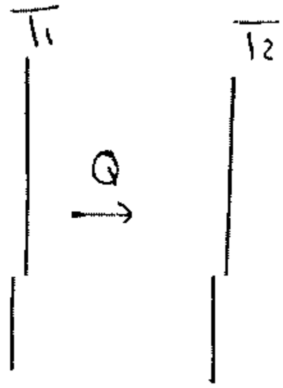
$\varepsilon = 1$

$T_1 = 1073 K$

$T_2 = 523 K$

time = 3600 sec

$A = 1 m^2$



Req Q & Energy

Sol

$$Q = \varepsilon \sigma A (T_1^4 - T_2^4)$$

$$Q = 1 * 56.7 \times 10^{-9} * (1073^4 - 523^4) = 70916.97 \text{ watt}$$

$$Energy = Q * time = 70916.97 * 3600 = 255301 KJ$$

Energy = 255301 KJ

* Example 1.3 :-

Given :-

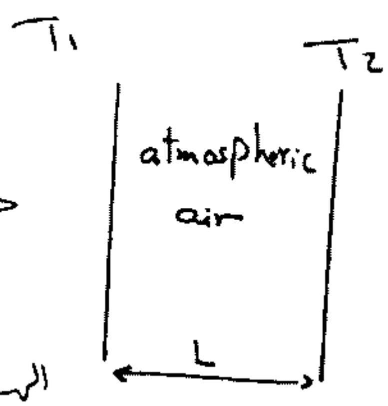
$T_1 = 300\,K$

$T_2 = 200\,K$

$L = 1\,cm$

$\varepsilon = 1$ $\quad K_{air} = 0.0223\ W/m.K$

Req q $[W/m^2]$



@ Atmospheric air

خد بالك

المسافة القائمة للهواء صغيرة جداً لذلك تكون سرعة
الهواء صفر $V_{air} = 0$ لذلك لا يوجد Convection

to get K @ $T_{av} = \dfrac{300 + 200}{2} = 250\,K$

From table 9 @ $t = 250 - 273 = -23\,°C$

get $K|_{air} = 0.0223\ W/m.K$

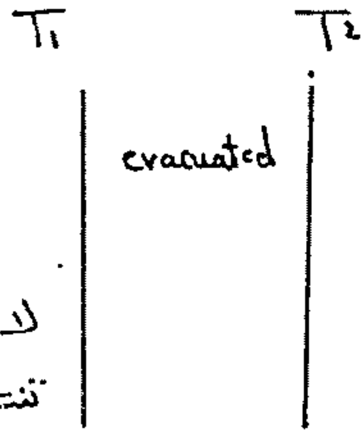
$$q = q_{Cond} + q_{Rad}$$

$$q = \sigma\varepsilon\left(T_1^4 - T_2^4\right) + K_{air}\frac{T_1 - T_2}{L}$$

..... تكمل حساب

$$q = 591\ W/m^2$$

(b) Evacuated :-



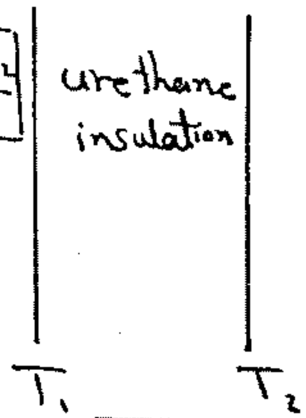
$$q = q_{Rad}$$

لا يوجد وسط لانتقال

تنتقل الحرارة بالإشعاع فقط

$$q = 368 \ w/m^2$$

(c) Urethane insulation $K = 0.026 \ w/m.°c$



$$q = q_{Cond} = K_{Ins} \frac{t_1 - t_2}{L}$$

× تنتقل الحرارة فقط بالتوصيل في حالة العازل

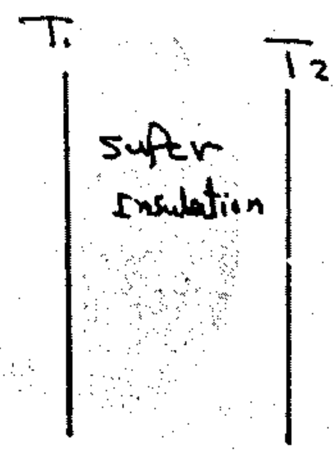
ممكن تنتقل الحرارة بالإشعاع ولكن يكون صغيراً جداً (خد بالك)

لذلك تهمل.

$$q = 260 \ w/m^2$$

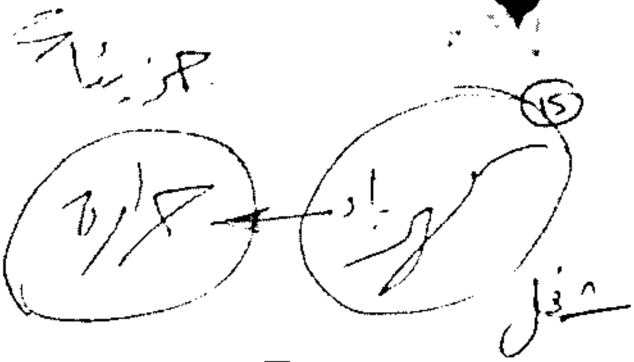
(d) Super_ insulation $K = 0.00002 \ w/m.°c$

$$q = q_{Cond} = K_{Sup\ Ins} \frac{t_1 - t_2}{L}$$



$$q = 0.2 \ w/m^2$$

Pro:-



Given:-

$L = 0.5\,m$

$Q_{elec} = 800$ watt

$D_{Rod} = 0.005\,m$

$t_{surface\,Rod} = 120°C$

$\dot{m}_{water} = 60\,Kg$

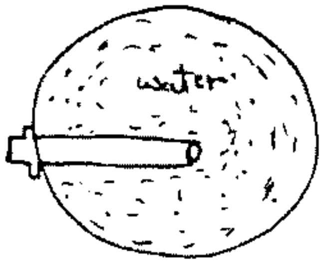
$t_{w_i} = 20°C \qquad t_{w_o} = 80°C$

Req:-

→ $h_{initial}$

→ h_{end}

→ time



Sol :-

$$time * Q = m_w \, cp_w \, (t_{w_o} - t_{w_i})$$

(cp_w in j / Kg.K)

$$time \times 800 = 60 * 4.18 \, (80 - 20) \times 1000$$

Req ① → $time = 18810$ sec $= 5.225$ hour

to get $h_{initial}$ & h_{end}

$$Q_{elec} = h_{initial} \, A_{surface} \, (t_{surface\,Rod} - t_{w_i})$$

$$= h_{end} \, A_{surface} \, (t_{surface\,Rod} - t_{w_o})$$

$$800 = h_{initial} * \pi * 0.005 * 0.5 \times (120 - 20) = h_{end} * \pi \times 0.5 \times 0.005 \times (120 - 80)$$

$* \; h_{initial} = 1018.59 \; w/m^2.K$

$* \; h_{end} = 2546.47 \; w/m^2.K$

Pro :- 1.79

Given $D_o = 20\ cm$

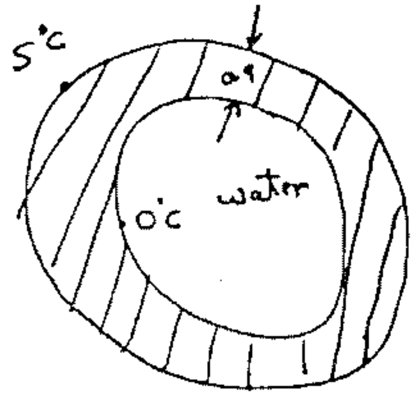
$L = 0.4\ cm$

$t_{ice,w} = 0°C$

$t_o = 5°C$

$h_{fg} = 333.7\ KJ/kg$

Req Q, $\dot{m}_{melt}$

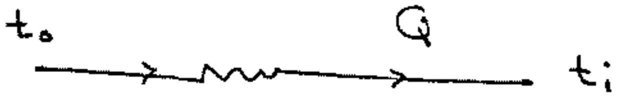


Sol :-

$K_{iron} = 80.2\ W/m.C$

$r_1 = 9.6\ cm$

$r_2 = 10\ cm$



$$R_{th} = \frac{r_2 - r_1}{4\pi r_1 r_2 K}$$

$$R_{th} = \frac{(10 - 9.6)\times 10^{+2}}{4\ \pi * 9.6 * 10 * 80.2} = 4.134\times 10^{-4}$$

$$Q = \frac{t_o - t_i}{R_{th}} = \frac{5 - 0}{4.134\times 10^{-4}} = 12{,}09{,}387\ KW$$

$$\dot{m}_{melt} = Q/h_{fg} = \frac{12.09\ KW}{333.7} = 0.0362\ Kg/sec$$

to be Continue. 84, 1.7

Pro :- 1.63

Given :-

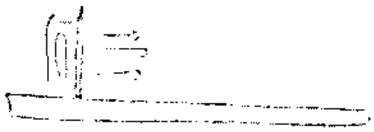
$Q_{elec} = 1000$ watt

$t_\infty = 20°C$

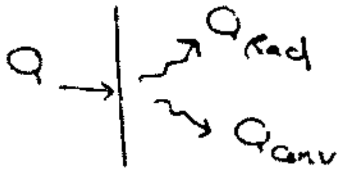
$h = 35\ w/m^2$

$\varepsilon = 0.6$

$A_{sur} = 0.02 m^2$



Req Temp of the Base of the iron



$$Q_{elec} = Q_{Rad} + Q_{conv}$$

$$1000 = \varepsilon \sigma A (T_{Base}^4 - T_\infty^4) + hA(T_{Base} - T_\infty)$$

$$1000 = 0.6 * 0.02 * 56.7 \times 10^{-9} (T_{Base}^4 - 293^4) + 35 * 0.02 (T_{Base} - 293)$$

get $T_{Base} = 946.9K$

Pro :- 1.65

Given :-

$K = 237\ w/m.C$

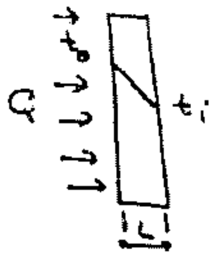
$D = 20\ cm$

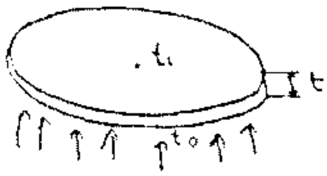
$t = 0.4\ cm$

$Q = 800$ watt

$t_i = 105°C$

Sol :- $t_o = ??$





$$Q = KA \frac{t_o - t_i}{L}$$

$$800 = 237 * \frac{\pi}{4} * 0.2^2 * \frac{t_o - 105}{0.004}$$

$$t_o = 105.43°C$$

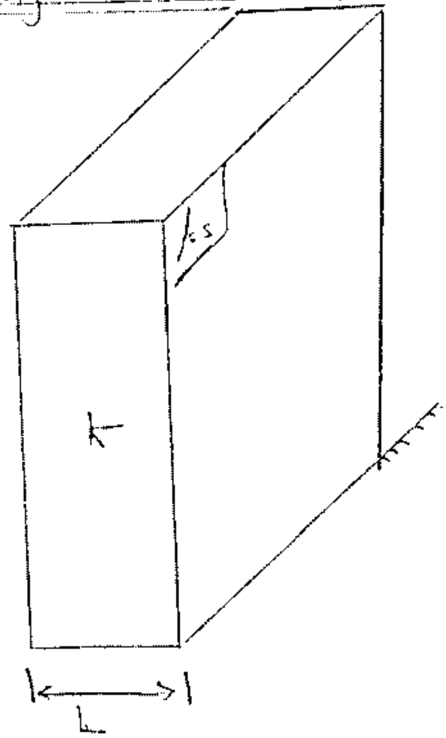
Heat Transfer I

" One - Dimensional Steady state Conduction "

Lecture No. (3)

Ch3 :- One Dimensional steady-state Conduction

* For Cartizian wall :-



* Conduction Thermal Resistance

$$R_{th} = L / kA \quad [°C/W]$$

* Convection Thermal Resistance

$$R = \frac{1}{h A} \quad [°C/W]$$

h_{cn} → Convective heat transfer coefficient $W/m^2.K$

* Radiation Thermal Resistance

$$R = \frac{1}{h_{Rad} A_s} \quad [°C/W]$$

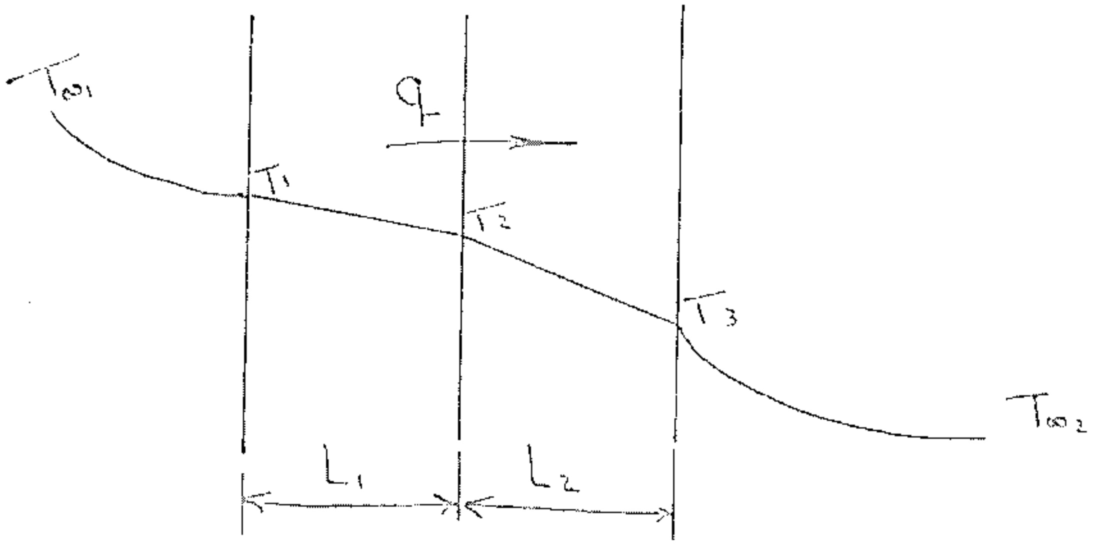
$$h_{Rad} = \sigma \varepsilon \left(T_s^2 + T_{sur}^2\right)\left(T_s + T_{sur}\right)$$

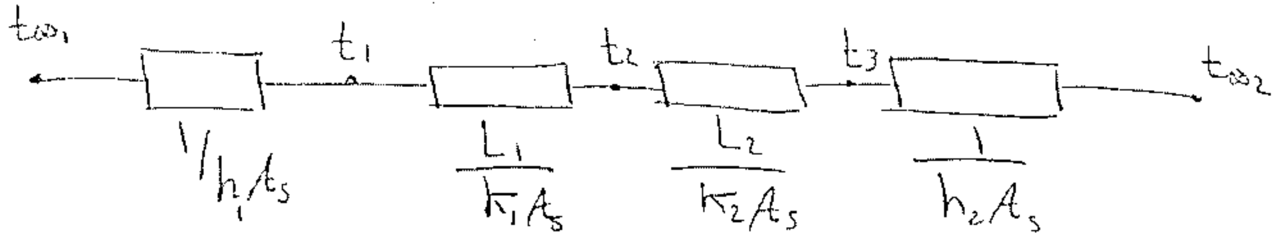
Overall heat transfer Coefficient (U)

$$Q = \frac{\Delta t}{R_{tot}} = U A \Delta t$$

Overall heat transfer Coefficient →

$$U = \frac{1}{A R_{tot}}$$

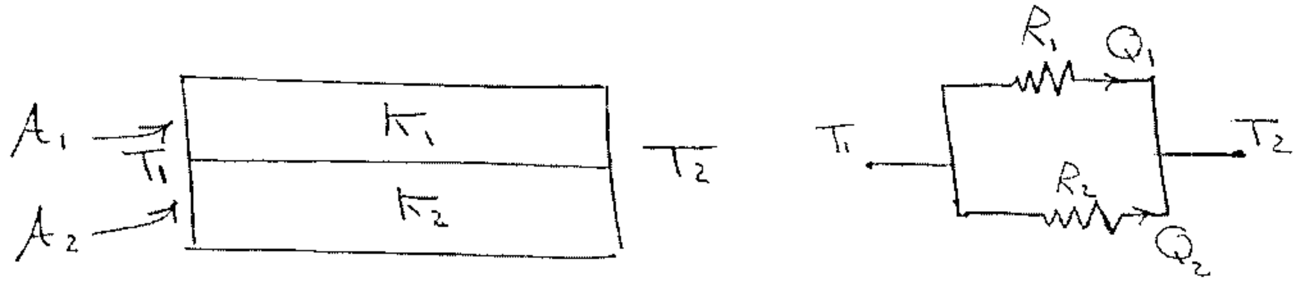




$$R_{tot} = \frac{1}{h_1 A_s} + \frac{L_1}{K_1 A_s} + \frac{L_2}{K_2 A_s} + \frac{1}{h_2 A_s} \quad [°C/W]$$

$$U = \frac{1}{A R_{tot}} = \frac{1}{\frac{1}{h_1} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + 1/h_2} \quad [W/m^2.C]$$

* Some examples :-

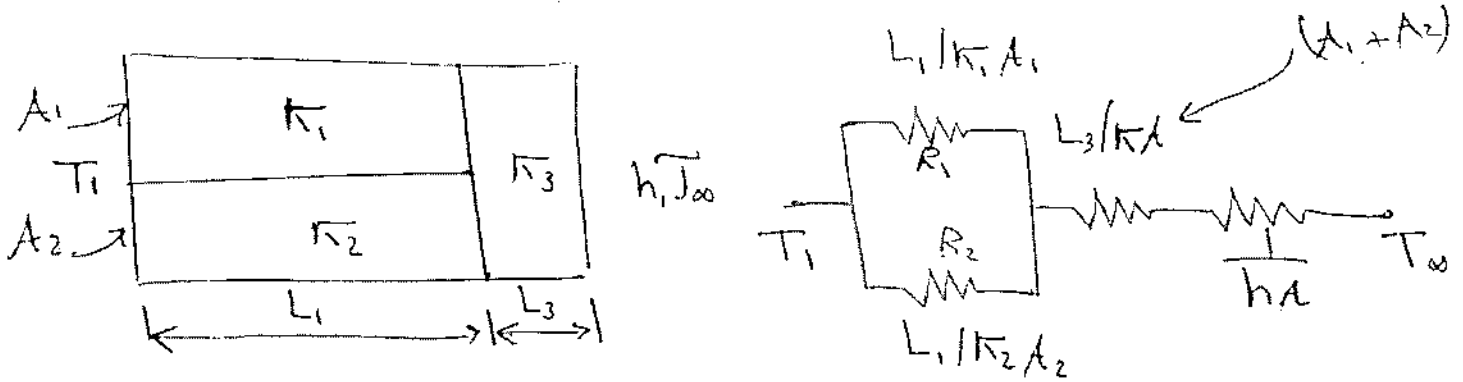


$$Q = Q_1 + Q_2 = \frac{\Delta t}{R_{tot}}$$

$$Q = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

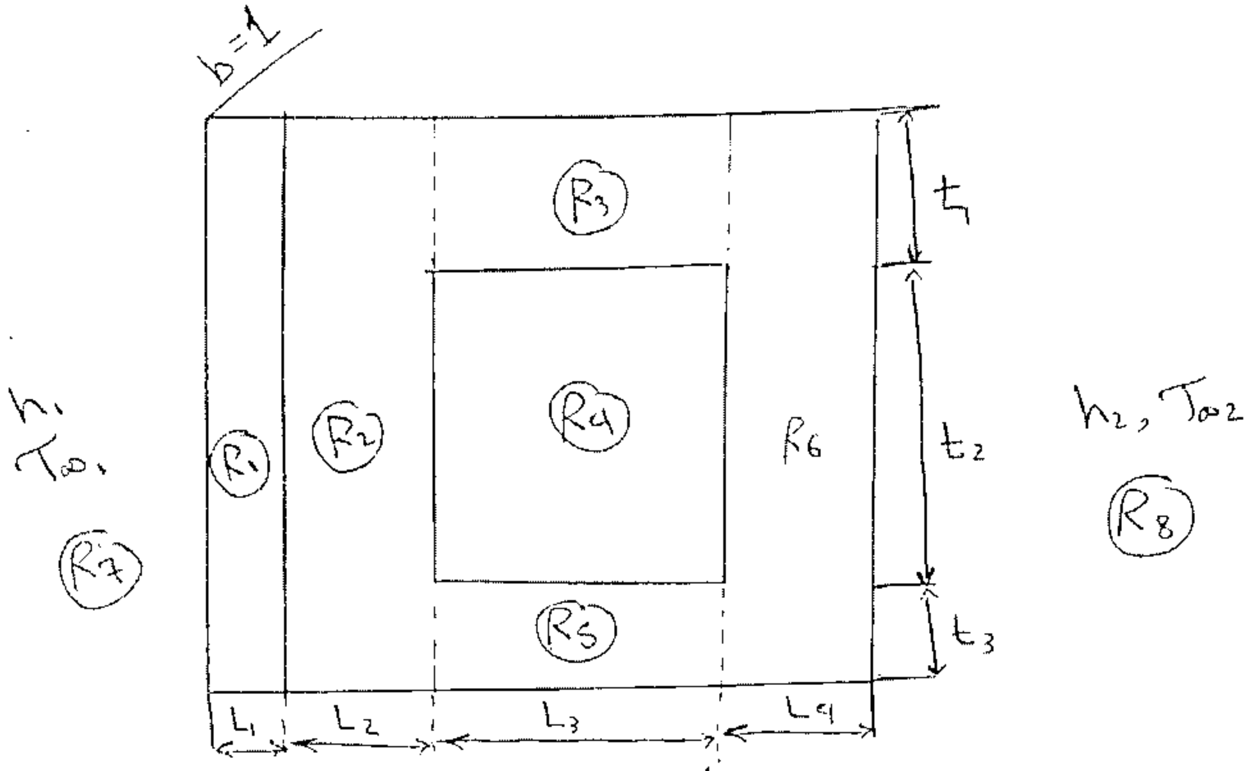
$$R_{tot} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

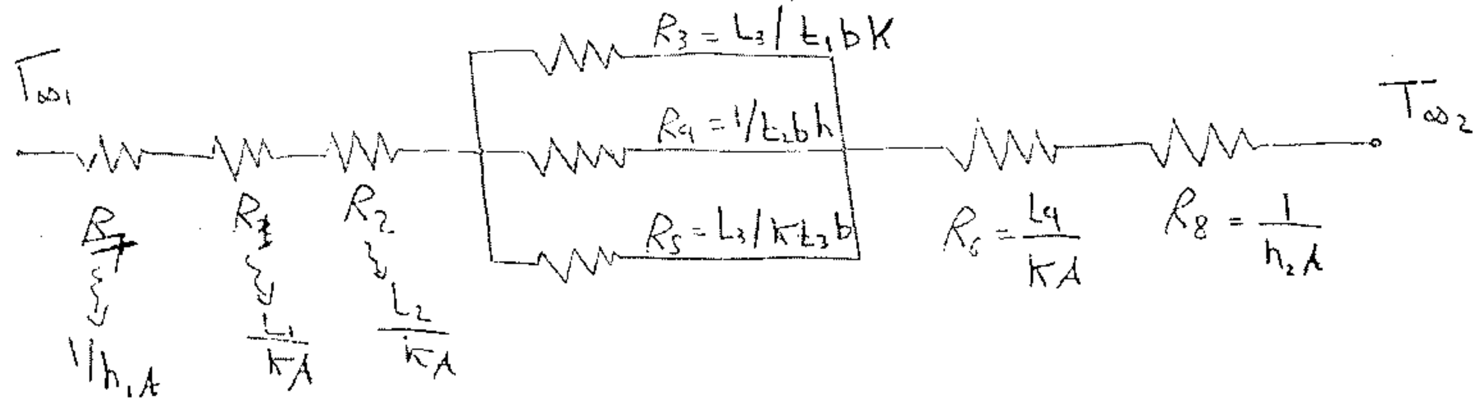
$$\boxed{R_{tot} = \frac{R_1 R_2}{R_1 + R_2}} \longrightarrow$$ مقاومة التوازي



$$\boxed{R_{tot} = \frac{R_1 R_2}{R_1 + R_2} + \frac{L_3}{KA} + \frac{1}{hA}}$$

$$\boxed{Q = \frac{t_1 - t_\infty}{R_{tot}}} \qquad \boxed{U = \frac{1}{A R_{tot}}}$$





$$R_{tot} = R_7 + R_1 + R_2 + R_6 + R_8 + \left(\frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} \right)$$

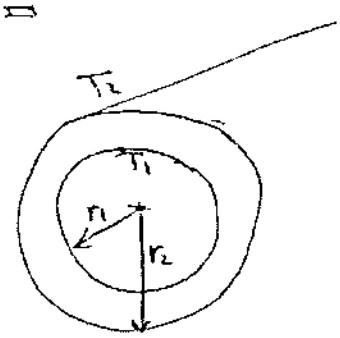
$$Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{tot}} = U A \Delta t$$

$$U = \frac{1}{A R_{tot}} \quad w/m^2.C$$

overall heat transfer coefficient

Cylinderical wall :-

* Conduction thermal Resistance :-



$$Q_{cond,cyl} = -kA\frac{dT}{dr}$$

$$\int_{r_1}^{r_2} \frac{Q_{cond,cyl}}{A} dr = -\int_{T_1}^{T_2} k \, dt$$

$$\int_{r_1}^{r_2} \frac{Q_{cond,cyl}}{2\pi r L} dr = -\int_{T_1}^{T_2} k \, dt$$

$$\frac{Q_{cond,cyl}}{2\pi L} \ln\frac{r_2}{r_1} = k\,(T_1 - T_2)$$

$$Q_{cond,cyl} = \frac{T_1 - T_2}{\dfrac{\ln r_2/r_1}{2\pi k L}}$$

$$\boxed{R_{th,cond,cy} = \frac{\ln r_2/r_1}{2\pi k L}}$$

* Convection Thermal Resistance :-

$$R_{th,\,cyl,\,conv} = \frac{1}{h A_s} \qquad A_s = 2\pi r L$$

حسب المسألة أختار r_1 أو r_2

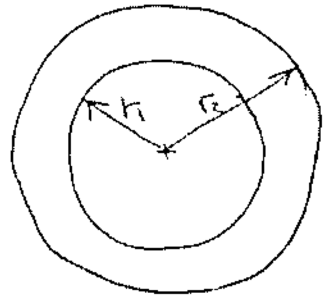
* Radiation thermal Resistance :-

$$R_{th,\,cyl,\,rad} = \frac{1}{h_R A_s}$$

$$A_s = 2\pi r L$$

$$h_R = \sigma \varepsilon (T_s^2 + T_{surr}^2)(T_s + T_{surr})$$

• For spherical coordinates:-



$$Q = -k A \frac{dt}{dr}$$

$$Q = -k\, 4\pi r^2 \frac{dt}{dr}$$

$$Q \int_{r_1}^{r_2} \frac{1}{r^2}\, dr = + \int_{t_1}^{t_2} -4\pi k\, dt$$

$$\int r^{-2} dr = \frac{r^{-2+1}}{-1} = -\frac{1}{r}\Big]_{r_1}^{r_2}$$

$$Q\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi k (t_1 - t_2)$$

$$Q\left[\frac{r_2 - r_1}{r_2 r_1}\right] = 4\pi k (t_1 - t_2)$$

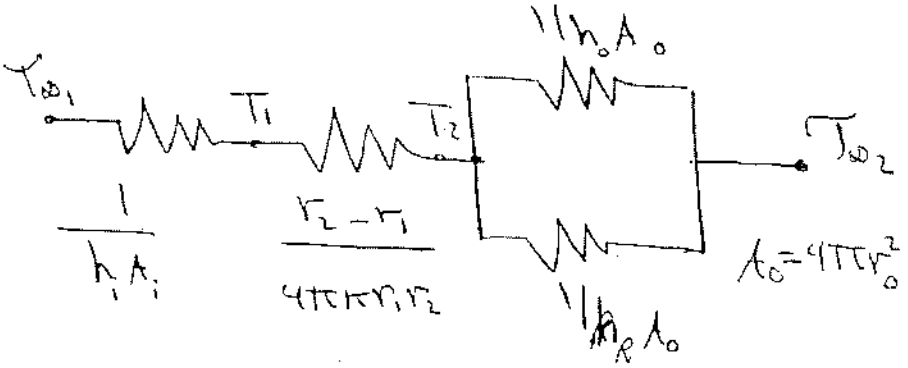
$$Q = \frac{t_1 - t_2}{\dfrac{r_2 - r_1}{4\pi k r_1 r_2}}$$

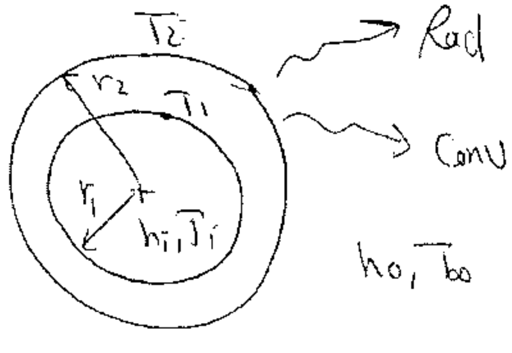
$$R_{th,\,cond,\,sph} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

$$R_{conv,\,th,\,sph} = \frac{1}{h A_s} \qquad A_s = 4\pi r^2$$

$$R_{Rad,\,th,\,sph} = \frac{1}{h_R A_s} \qquad A_s = 4\pi r^2$$
$$h_R = \varepsilon\sigma (T_s^2 + T_{surr}^2)(T_s + T_{surr})$$

* ~~~ . ~~~ . ~~~ *





$$R_{tot} = \frac{1}{h_i A_i} + \frac{r_2 - r_1}{4\pi r_2 r_1 k} + \frac{1}{\frac{1}{h_o A_o} + \frac{1}{h_R A_o}}$$

$$Q = \frac{T_{\infty_1} - T_{\infty_2}}{R_{tot}} \qquad \text{Get } Q = \_$$

$$Q = \frac{T_{\infty_1} - T_1}{1/h_i A_i} \qquad \text{Get } T_1 = \_$$

$$Q = \frac{T_2 - T_{\infty 2}}{1/h_o A_o + \frac{1}{h_R A_o}} \qquad \text{Get } T_2 = \_$$

Note

for cylinder, sphere

$$Q = \frac{\Delta t}{R_{tot}} = U_i A_i \Delta t = U_o A_o \Delta t$$

$$\boxed{U_i A_i = U_o A_o}$$

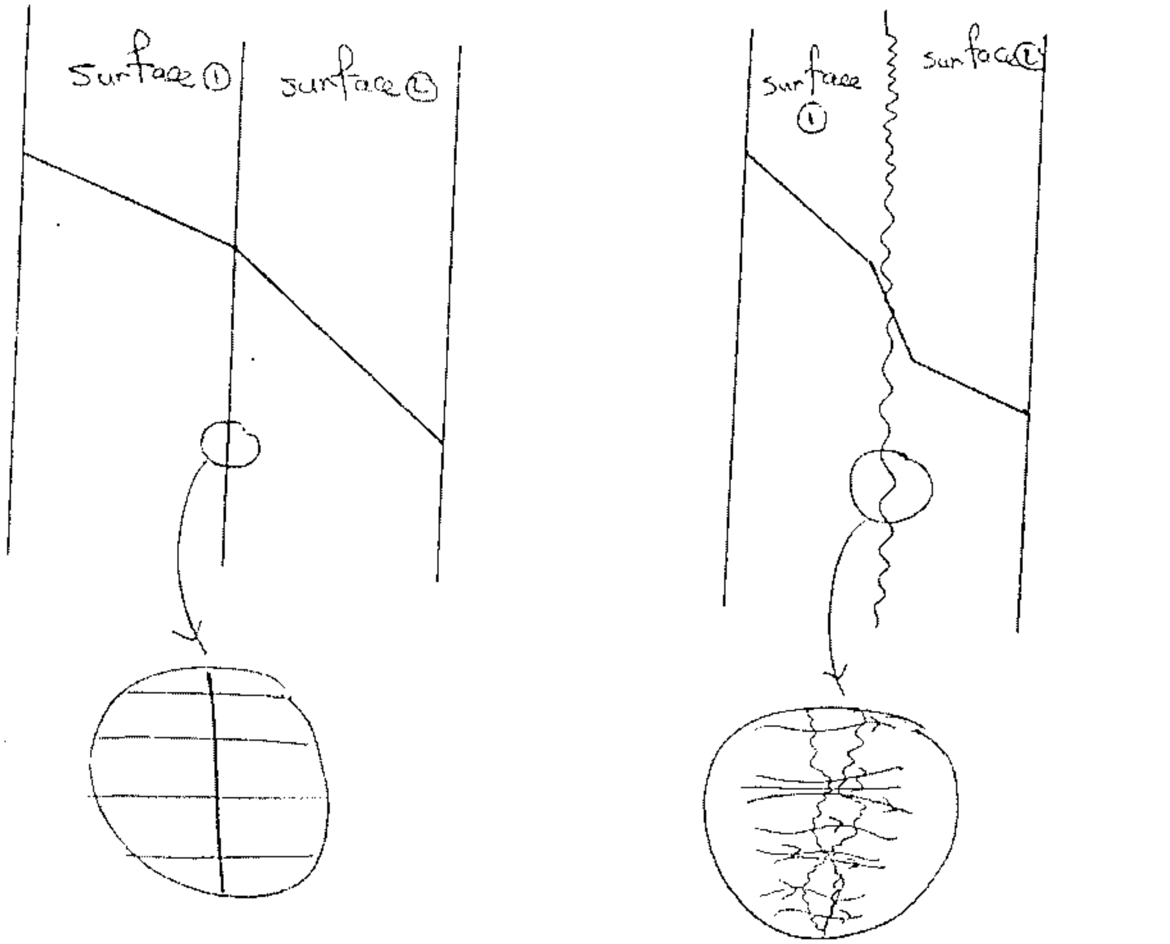
U_i → * Overall Coefficient Based on Inner Area.

A_i → inner Area.

U_o → Overall heat transfer Coefficient Based on outer Area.

A_o → outer Area

Thermal Contact Resistance :-



a) Ideal thermal Contact
التقاء مثالي لسطحين

b) Actual thermal Contact
التقاء حقيقي لسطحين

عند التقاء سطحين معًا يكون هناك فجوة بينهما نتيجة لتشطيب الأسطح وهذه الفجوة تسبب مقاومة لانتقال الحرارة .

* Thermal contact Resistance :- $R_c = 1/h_c = \dfrac{\Delta T_{interface}}{Q/A}$

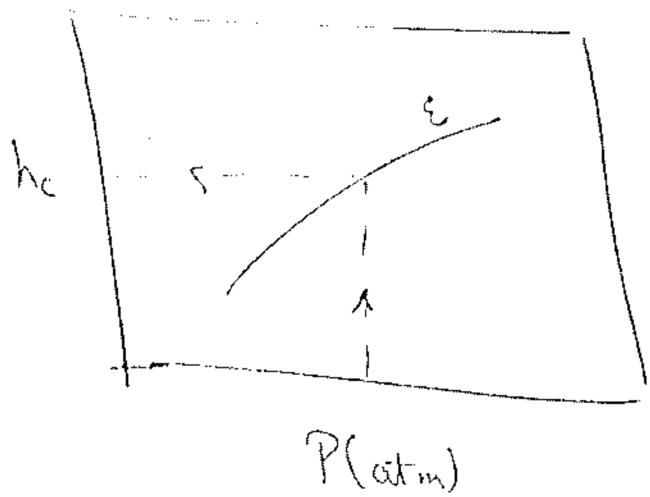
depend on
- → surface Roughness
- → material Properties
- → temperature & Pressure at the interface
- → Fluid trapped at the interface

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{interface}}{Q/A}$$

R_c → Thermal Contact Resistance

h_c → thermal Contact Conductance

نستخدم هذه المقاومة (R_c) عندما تكون باقي المقاومات
الحرارية صغيرة مقارنة بها وبالتالي تكون عامل مؤثر في انتقال
الحرارة.



Given $P(atm) = ٤$

Get R_c

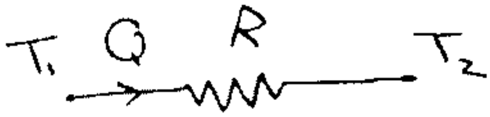
Heat Transfer I

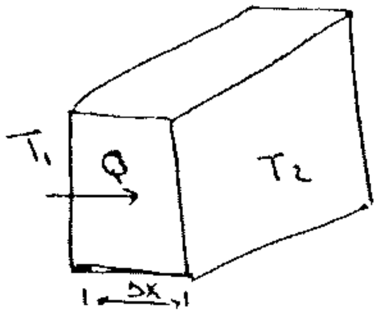
"Conduction Heat Transfer in Multilayered walls"

Lecture No. (4)

Solved Problem ch[1]

* Conduction Heat transfer :-



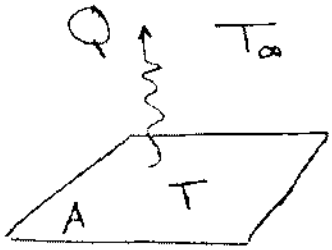


$$\longrightarrow \quad Q = \frac{\Delta t}{R}$$

$$Q = k A \frac{\Delta t}{\Delta x} \quad \rightleftharpoons \quad R = \frac{\Delta x}{k A}$$

$$\boxed{\text{Conduction Thermal Resistance} = R_{Cond} = \frac{L}{k A}} \quad [k/w]$$

* Convection Heat transfer :-



$$Q = h A (T - T_\infty) \rightleftharpoons Q = \frac{T - T_\infty}{R}$$

$$\boxed{\text{Convection Thermal Resistance} = \frac{1}{h A}}$$

النتيجة

* $R\,[k/w]$ * $h\,[w/m^2.k]$ * $k\,[w/m.k]$

Radiation Heat transfer :-

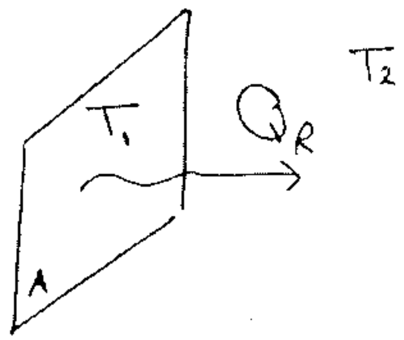
$$Q = \varepsilon A \sigma (T_1^4 - T_2^4)$$

$$Q = \varepsilon A \sigma (T_1^2 - T_2^2)(T_1^2 + T_2^2)$$

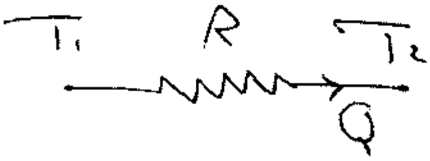
$$Q = \varepsilon A \sigma (T_1 + T_2)(T_1^2 + T_2^2)(T_1 - T_2)$$

let $h_{Rad} = \varepsilon \sigma (T_1 + T_2)(T_1^2 + T_2^2)$

Then $\boxed{Q = h_{Rad} A (T_1 - T_2)}$



$\Rightarrow$

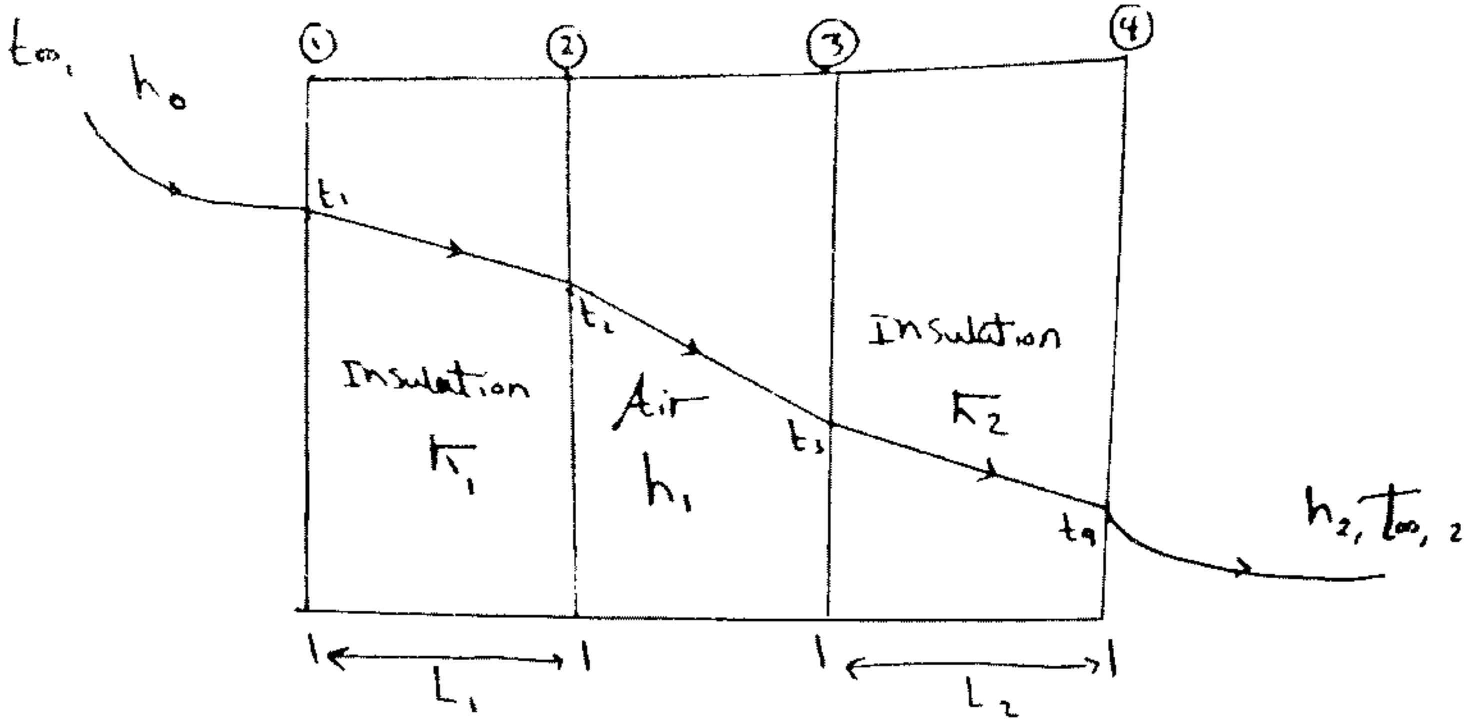


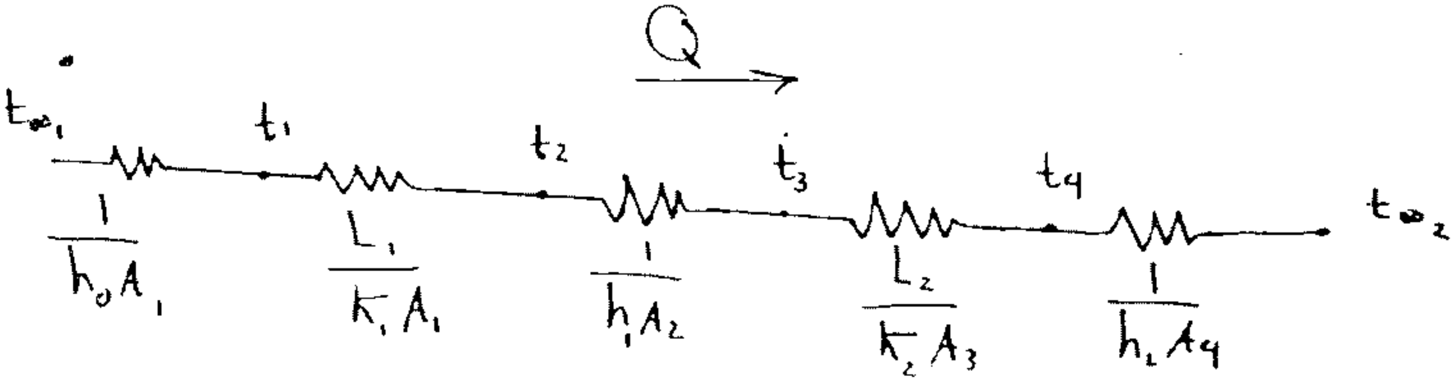
$$Q = \frac{T_1 - T_2}{R}$$

$$Q = h_{Rad} A (T_1 - T_2)$$

$$R = \frac{1}{h_R A}$$

$$\boxed{\text{Radiation Heat transfer Resistance} = \frac{1}{h_R A}} \quad [K/W]$$





$$R_{tot} = \frac{1}{h_o A_1} + \frac{L_1}{k_1 A_1} + \frac{1}{h_1 A_2} + \frac{L_2}{k_2 A_3} + \frac{1}{h_2 A_4} \quad [K/W]$$

$$\boxed{Q = \frac{-t_{\infty_2} + t_{\infty_1}}{R_{tot}}}$$

Get $Q = \checkmark$

Intermediate temperature t_2 & t_3 & t_4, t_1

$$Q = \frac{t_{\infty_1} - t_1}{\frac{1}{h_o A_1}} = \frac{t_{\infty_1} - t_2}{\frac{1}{h_o A_1} + \frac{L}{k A_1}} = \frac{t_3 - t_{\infty,2}}{\frac{L_2}{k A} + \frac{1}{h_2 A_4}} = \frac{t_4 - t_{\infty,2}}{\frac{1}{h_2 A_4}}$$

Get t_1 & t_2 & t_3 & t_4 (Note) $t_1 > t_2 > t_3 > t_4$

Pro 1.4

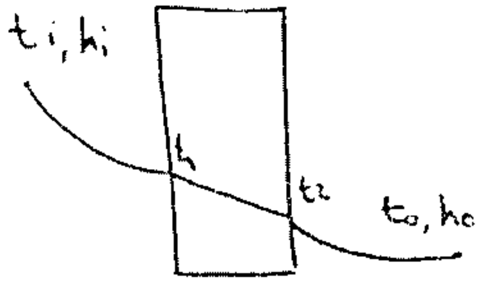
Given

$A = 0.18\,m^2$ $L = 0.016\,m$

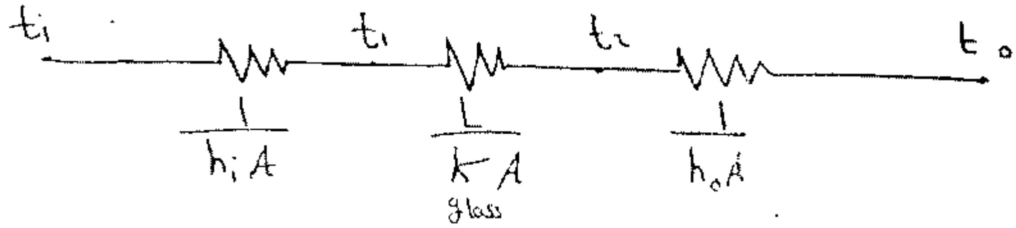
$t_i = 20°C$ $h_i = 10\,W/m^2K$

$t_o = -20°C$ $h_o = 100\,W/m^2K$

$K_{glass}\big|_{@t=0°C} = 0.78\,W/mK$



Req :- $Q = ??$



$$R = \frac{1}{h_i A} + \frac{L}{K_{glass} A} + \frac{1}{h_o A} = \frac{1}{10 \times 0.18} + \frac{0.016}{0.78 \times 0.18} + \frac{1}{0.18 \times 100}$$

$$\boxed{R = 0.725\,K/W}$$

$$Q = \frac{t_i - t_o}{R_{th}} = \frac{20 + 20}{0.725} = 55.16 \text{ watt}$$

$$Q = \frac{t_i - t_1}{\frac{1}{h_i A}} = \frac{t_2 - t_o}{1/h_o A}$$

Get $t_1 = -$ °C

$t_2 = -$ °C

Problems :-

Pro 1.2 :-

Given :-

$A = \frac{60 \times 30}{10^4} = 0.18\ m^2$ $\qquad t_1 = 20^\circ C$

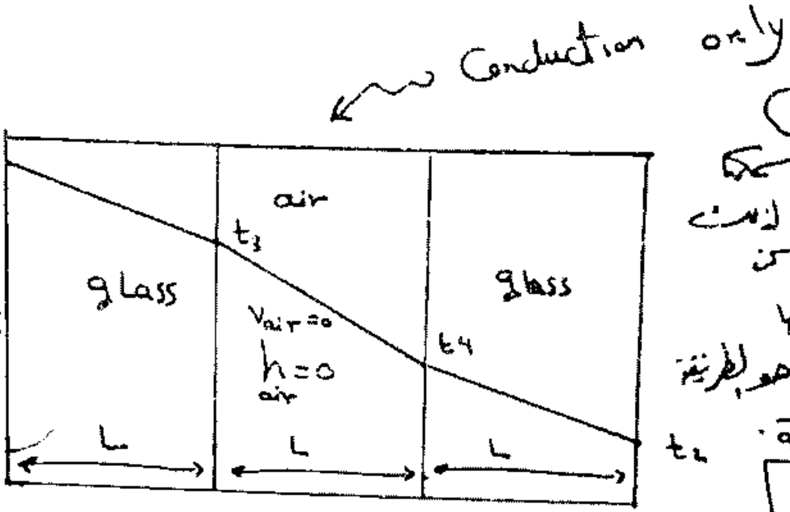
$L = 8\ mm = 0.008\ m$ $\qquad t_2 = -20^\circ C$

Req :- Q ?

@ $t_{av} = \frac{20 - 20}{2} = 0^\circ C$

get $K|_{air} = 0.0249\ w/m\,K$

$K|_{glass} = 0.78$



طبقة الهواء ساكنة صغيرة جداً لذلك فإن الهواء ساكن

$V_{air} = 0$

لذلك التوصيل هو الطريقة التي تنتقل بها الحرارة.

$h = 0$

$$\frac{L}{K_g A} \qquad \frac{L}{K_{air} A} \qquad \frac{L}{K_g A}$$

$$R_{th} = \frac{L}{K_g A} + \frac{L}{K_{air} A} + \frac{L}{K_g A} = \frac{0.008 \times 2}{0.78 \times 0.18} + \frac{0.008}{0.0249 \times 0.18} = 1.935\ K/W$$

$$Q = \frac{\Delta t}{R_{th}} = \frac{20 + 20}{1.935} = 20.66\ watt$$

لإيجاد t_3 و t_4

$$Q = \frac{t_1 - t_3}{L / K_g A} = \frac{t_1 - t_4}{\frac{L}{K_g A} + \frac{L}{K_{air} A}}$$

get t_3 & t_4

heat

Pro 1.5

* **Given :-**

$A = 0.18\,m^2$

$L = 0.008\,m$

$t_i = 20°C \quad h_i = 10\,W/m^2K$

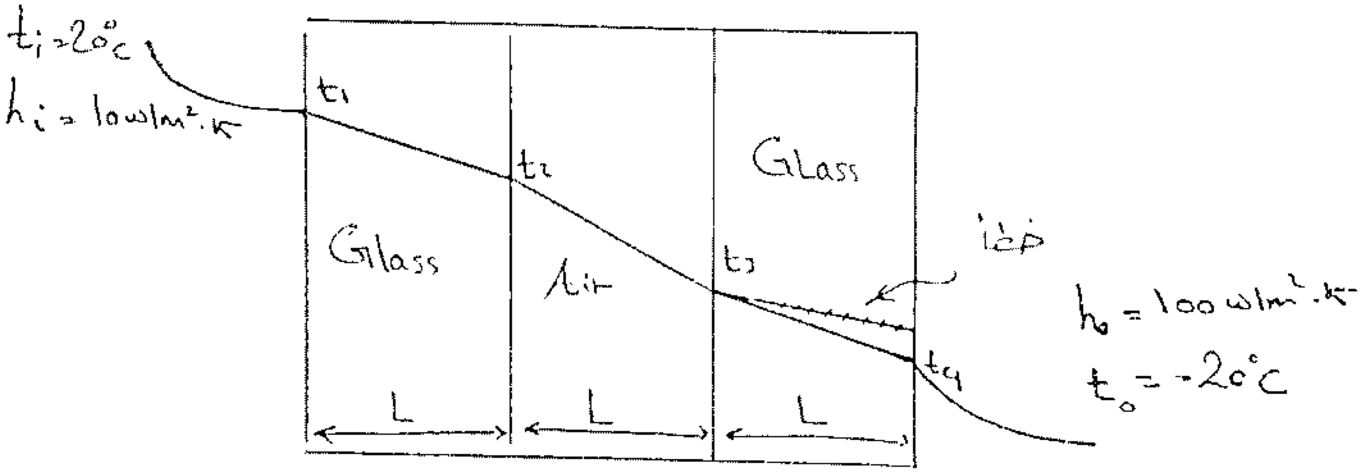
$t_o = -20°C \quad h_o = 100\,W/m^2K$

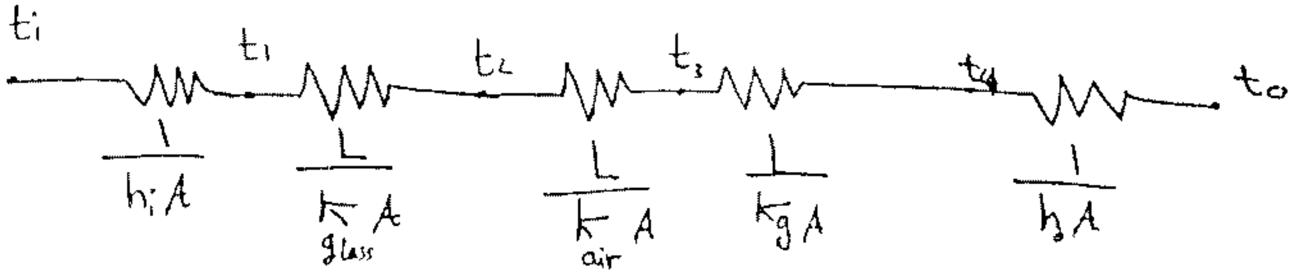
@ $t_{av} = \frac{20-20}{2} = 0°C$

Get $k_{glass} = 0.78\,W/m.K$

$k_{air} = 0.0223\,W/m.K$

Req $Q = ??$





$$R_{th} = \frac{1}{h_i A} + \frac{L}{k_{glass} A} + \frac{L}{k_{air} A} + \frac{L}{k_g A} + \frac{1}{h_o A}$$

$$Q = \frac{t_i - t_o}{R_{th}} = \quad \text{watt}$$

To get intermediate Temperature.

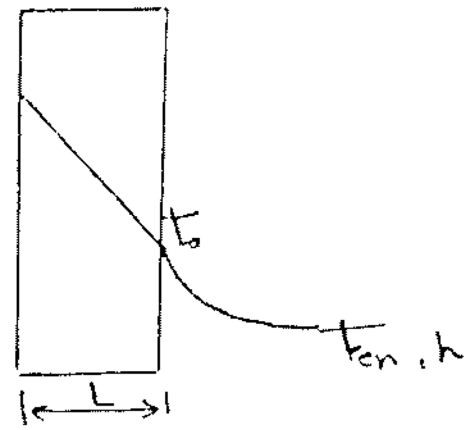
$$Q = \frac{t_i - t_1}{\frac{1}{h_i A}} = \frac{t_1 - t_2}{\frac{L}{K_g A}} = \frac{t_2 - t_3}{\frac{L}{K_{air} A}} = \frac{t_3 - t_4}{\frac{L}{K_g A}}$$

Get $t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4$

Pro 1.32

Given

$T_{envirom ent} = T_{en} = 311 K$



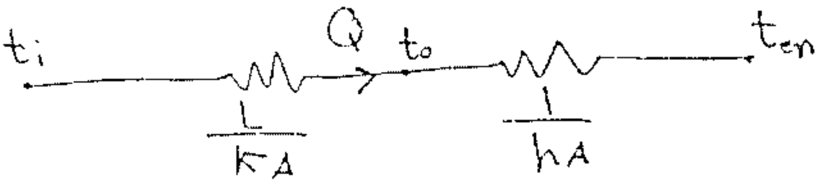
$L = 25 \text{ mm}$

$K = 1.4 \text{ w/m.°c}$

$T_i = 588 K$

$T_o = 314 K$

Req * Convective Heat transfer Coefficient [h] :-



Q :-

$$Q = \frac{t_i - t_o}{\frac{L}{KA}} = \frac{t_o - t_{en}}{1/hA}$$

$$\frac{588 - 314}{\frac{0.025}{1.4}} = \frac{314 - 311}{1/h}$$

$h = 5114.6 \text{ w/m}^2 K$

Required

Pro 1.10

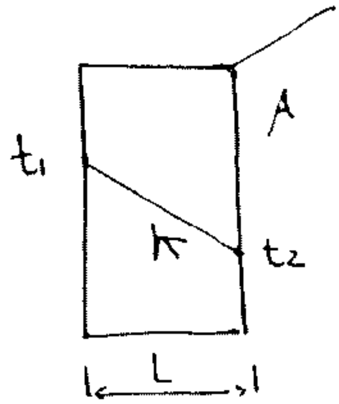
* Given :-

$Q = 3000$ watt

$A = 1\ m^2$

$L = 0.025\ m$

$K = 0.2\ w/m.C$



Req Δt

$$Q = -KA\frac{\Delta t}{L}$$

$$3000 = -0.2 * 1 * \frac{\Delta t}{0.025}$$

$$\boxed{\Delta t = -375\ ^\circ C}$$

Pro 1.18 :-

Given:-

$L = 0.005\,m$ $t_i = 10°C$

$A = 4\,m^2$ $t_o = 3°C$

$K = 0.78\,w/m.K$

Req :- Energy transfered in 5 hours

$$Q = -KA\frac{t_o - t_i}{L} = \frac{10-3}{0.005} \times 0.78 \times 4$$

$$Q = 4368 \text{ watt}$$

Energy transfered in 5 hours = Q * time (sec)

* Energy = 4368 x 5 x 60 x 60 = 78624 KJ

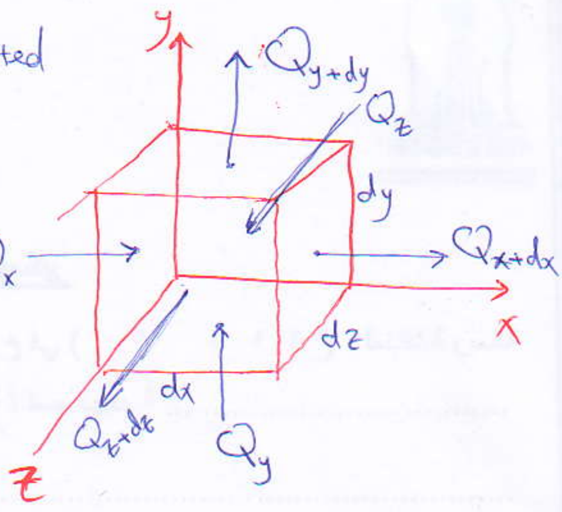
Heat Transfer I

"Heat Conduction Equation in Large Plane Wall, Long cylinder and sphere"

Lecture No. (5)

* Heat Transfer Conduction Equations in plane wall

energy storage = The difference of heat transfered + Heat generated



$$m \cdot c_p \frac{dT}{dt} = \left[(Q_x - Q_{x+dx}) + (Q_y - Q_{y+dy}) + (Q_z - Q_{z+dz})\right] + \dot{q} * dx\,dy\,dz$$

$m = \rho(dx\,dy\,dz)$, $Q_x = -K\,dy\,dz\,\frac{dT}{dx}$

$$Q_y = -K\,dx\,dz\,\frac{dT}{dy}$$

$$Q_z = -K\,dx\,dy\,\frac{dT}{dz}$$

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x}\cdot dx = -K\,dy\,dz\,\frac{dT}{dx} - K\,dy\,dz\,\frac{d^2T}{dx^2}\cdot dx$$

$$\Rightarrow (Q_x - Q_{x+dx}) = K\,dx\,dy\,dz\,\frac{d^2T}{dx^2}$$

$$Q_{y+dy} = Q_y + \frac{\partial}{\partial y} Q_y \cdot dy = -K\,dx\,dz\,\frac{dT}{dy} - K\,dx\,dz\,\frac{d^2T}{dy^2}\cdot dy$$

$$\Rightarrow (Q_y - Q_{y+dy}) = K\,dx\,dy\,dz\,\frac{d^2T}{dy^2}$$

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z} Q_z\,dz = -K\,dx\,dy\,\frac{dT}{dz} - K\,dx\,dy\,\frac{d^2T}{dz^2}\cdot dz$$

$$\Rightarrow (Q_z - Q_{z+dz}) = K\,dx\,dy\,dz\,\frac{d^2T}{dz^2}$$

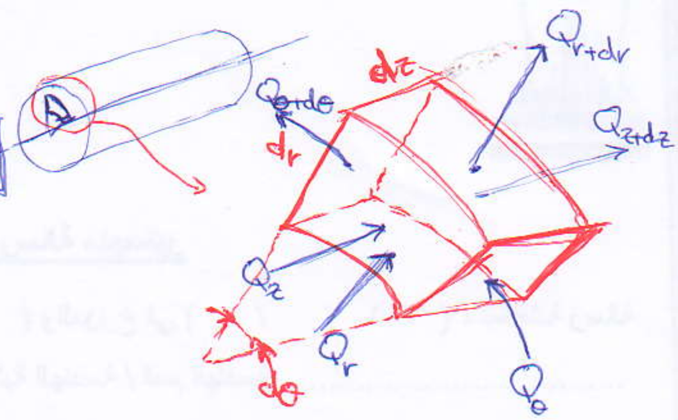
$$\rho\,c_p\,\frac{dT}{dt} * dx\,dy\,dz = K\,dx\,dy\,dz\left[\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2}\right] + \dot{q} * dx\,dy\,dz$$

$$\Rightarrow \frac{\rho\,c_p}{K}\,\frac{dT}{dt} = \boxed{\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{dT}{dt}} \quad (\div\,K\,dx\,dy\,dz)$$

$$\frac{\rho\,c_p}{K} = \frac{1}{\alpha}$$

* Heat Transfer Conduction Equations in cylinder

energy storage = The difference in heat transfered + Heat generated



$$m \cdot c_p \frac{dT}{dt} = \left[(Q_r - Q_{r+dr}) + (Q_\theta - Q_{\theta+d\theta}) + (Q_z - Q_{z+dz})\right] + \dot{q} * r\,dr\,d\theta \cdot dz$$

$$m = \rho * (r\,dr\,d\theta\,dz)$$

$$Q_r = -K\, r\, d\theta\, dz \frac{dT}{dr}$$

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r} Q_r \cdot dr$$

$$\Rightarrow (Q_r - Q_{r+dr}) = -\frac{\partial}{\partial r} Q_r \cdot dr = K\, dr\, d\theta\, dz \frac{\partial}{\partial r}\left(r \frac{dT}{dr}\right)$$

$$Q_\theta = -K\, dr\, dz \frac{dT}{r\,d\theta}$$

$$Q_{\theta+d\theta} = Q_\theta + \frac{\partial}{r\,\partial\theta} Q_\theta \cdot r\,d\theta$$

$$\frac{1}{r} K\, dr\, dz \frac{1}{r} \frac{d^2T}{d\theta^2} \cdot r\,d\theta$$

$$\Rightarrow (Q_\theta - Q_{\theta+d\theta}) = -\frac{\partial}{r\,\partial\theta} Q_\theta \cdot r\,d\theta = K\, r\, dr\, d\theta\, dz \frac{d^2T}{r^2 d\theta^2}$$

$$Q_z = -K\, r\, dr\, d\theta \frac{dT}{dz}$$

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z} Q_z \cdot dz$$

$$\Rightarrow (Q_z - Q_{z+dz}) = -\frac{\partial}{\partial z} Q_z \cdot dz = K\, r\, dr\, d\theta\, dz \frac{d^2T}{dz^2}$$

$$\Rightarrow \rho\, c_p \frac{dT}{dt} r\,dr\,d\theta\,dz = K\, r\,dr\,d\theta\,dz \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) + \frac{1}{r^2}\frac{d^2T}{d\theta^2} + \frac{d^2T}{dz^2}\right] + \dot{q} \cdot r\,dr\,d\theta\,dz$$

$(\div K\, r\,dr\,d\theta\,dz)$

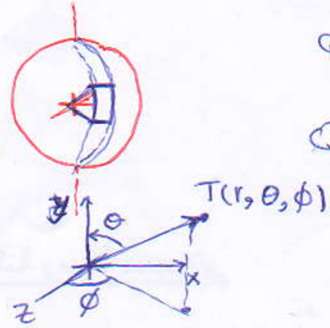
$$\Rightarrow \boxed{\frac{1}{\alpha}\frac{dT}{dt} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) + \frac{1}{r^2}\frac{d^2T}{d\theta^2} + \frac{d^2T}{dz^2}\right] + \frac{\dot{q}}{K}}$$

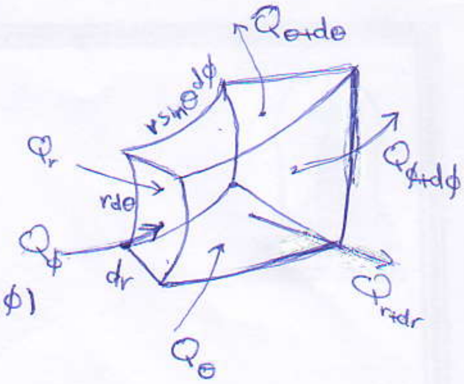
$$\frac{1}{\alpha} = \frac{\rho\, c_p}{K}$$

note: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) = \frac{1}{r}\left[r\frac{d^2T}{dr^2} + \frac{dT}{dr}\right] = \left[\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr}\right]$

Heat Transfer Conduction Equations in sphere

energy Storage = The difference of heat transfered + Heat generated





$$m \cdot c_p \frac{dT}{dt} = \left[(Q_r - Q_{r+dr}) + (Q_\theta - Q_{\theta+d\theta}) + (Q_\phi - Q_{\phi+d\phi})\right] + \dot{q}(dr\, r d\theta\, r\sin\theta d\phi)$$

$$m = \rho \cdot (r^2 \sin\theta\, dr\, d\theta\, d\phi)$$

$$Q_r = -k\, r^2 \sin\theta\, d\theta\, d\phi \frac{dT}{dr}$$

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r} Q_r \cdot dr$$

$$\Rightarrow (Q_r - Q_{r+dr}) = -\frac{\partial}{\partial r} Q_r \cdot dr = K \sin\theta\, dr\, d\theta\, d\phi \left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)\right]$$

$$Q_\theta = -K\, dr \cdot r\sin\theta\, d\phi \frac{dT}{r d\theta}$$

$$Q_{\theta+d\theta} = Q_\theta + \frac{\partial}{r\partial\theta} Q_\theta \cdot r d\theta$$

$$\Rightarrow (Q_\theta - Q_{\theta+d\theta}) = -\frac{\partial}{r\partial\theta} Q_\theta \cdot r d\theta = K\, dr\, d\phi \left[\frac{\partial}{\partial\theta}\left(\sin\theta \frac{dT}{r d\theta}\right)\right] \cdot r d\theta$$

$$= K\, dr\, d\theta\, d\phi \left[\frac{\partial}{\partial\theta}\left(\sin\theta \frac{dT}{d\theta}\right)\right]$$

$$Q_\phi = -K\, r dr\, d\theta \frac{dT}{r\sin\theta\, d\phi}$$

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r\sin\theta\,\partial\phi} Q_\phi \cdot r\sin\theta\, d\phi$$

$$\Rightarrow (Q_\phi - Q_{\phi+d\phi}) = -\frac{\partial}{r\sin\theta\,\partial\phi} Q_\phi \cdot r\sin\theta\, d\phi$$

$$= K\, r\, dr\, d\theta \frac{1}{r\sin\theta} \cdot \frac{1}{r\sin\theta} \frac{d^2T}{d\phi^2} \cdot r\sin\theta\, d\phi$$

$$= K\, r^2 \sin\theta\, dr\, d\theta\, d\phi \frac{1}{r^2\sin^2\theta} \frac{d^2T}{d\phi^2}$$

$$\Rightarrow \rho\, c_p \frac{dT}{dt} \cdot (r^2 \sin\theta\, dr\, d\theta\, d\phi) = K(r^2\sin\theta\, dr\, d\theta\, d\phi)\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{dT}{dr}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{dT}{d\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{d^2T}{d\phi^2}\right] + \dot{q}(r^2\sin\theta\, dr\, d\theta\, d\phi)$$

$$\div (K\, r^2 \sin\theta\, dr\, d\theta\, d\phi)$$

$$\Rightarrow \boxed{\frac{1}{\alpha}\frac{dT}{dt} = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dT}{dr}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{dT}{d\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{d^2T}{d\phi^2}\right] + \frac{\dot{q}}{K}}$$

* Types of conduction heat transfer equations.

* Steady state ($dT/dt \neq 0$) (called The Poisson Equation)

$$\boxed{\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} + \frac{\dot{q}}{K} = 0}$$

* Transient, no heat generation ($\dot{q} = 0$) (called Diffusion Equation)

$$\boxed{\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} = \frac{1}{\alpha}\frac{dT}{dt}}$$

* Steady state, no heat generation ($\frac{dT}{dt} = 0$, $\dot{q} = 0$) (called Laplace Equation)

$$\boxed{\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} = 0}$$

if steady state, no heat generation, one dimension :

$$\Rightarrow \frac{d^2T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = c_1 \quad , \quad \boxed{T(x) = c_1 x + c_2}$$

to find the temperature distribution we must find the constants c_1 & c_2 from the Boundary conditions

Boundary Conditions Types :

1 - Constant surface Temperature

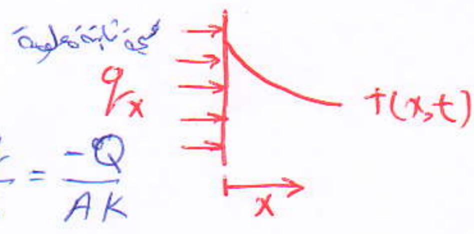
$$T(0,t) = T_s$$

$$\text{at } x = 0 \Rightarrow T = T_1$$

2 - Constant surface heat flux q = constant عند السطح

$$\text{at } x = 0 \quad q = q_x \quad , \quad q = \frac{Q}{A}$$

$$\Rightarrow -K \left.\frac{dT}{dx}\right|_{x=0} = q_x \quad , \quad \left.\frac{dT}{dx}\right|_{x=0} = \frac{-q}{K} = \frac{-Q}{AK}$$

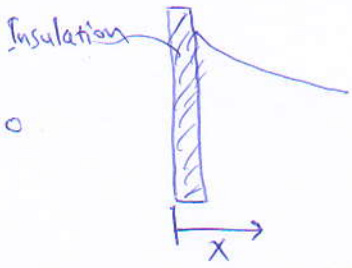


* special cases

A) Insulated surface (adiabatic surface)

at $x=0$ $\quad q_x = 0 \Rightarrow -k \left.\frac{dT}{dx}\right|_{x=0} = 0$

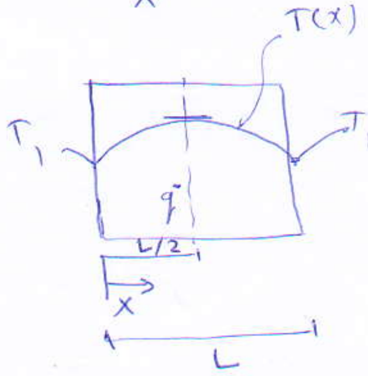
$$\Rightarrow \left.\frac{dT}{dx}\right|_{x=0} = 0$$



B) Thermal symmetry

at symmetrical axis $(x = L/2)$

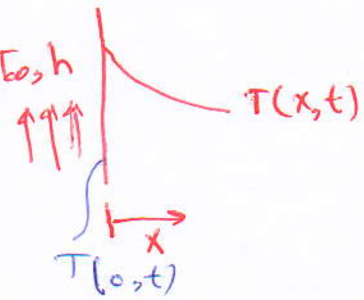
$$\left.\frac{dT}{dx}\right|_{x=L/2} = 0$$



3- Convection Boundary condition

at $x=0$, $Q_{cond.} = Q_{conv.}$ or $q_{cond.} = q_{conv.}$ $(A = \text{constant})$

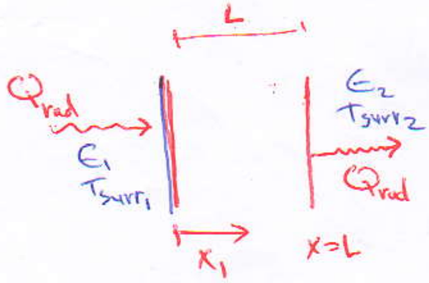
$$-k \left.\frac{dT}{dx}\right|_{x=0} = h\left(T_\infty - T_{(0,t)}\right)$$



4- Radiation Boundary Condition

at $x=0$, $Q_{cond} = Q_{rad.}$ or $q_{cond} = q_{Rad}$

$$\Rightarrow -k \left.\frac{dT}{dx}\right|_{x=0} = \sigma \epsilon_1 \left[T_{surr_1}^4 - T_{(0,t)}^4\right]$$



at $x=L$ $\quad q_{cond} = q_{rad}$

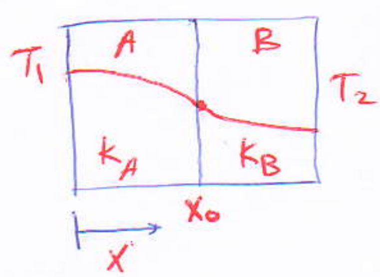
$$-k \left.\frac{dT}{dx}\right|_{x=L} = \sigma \epsilon_2 \left(T_{(L,t)}^4 - T_{surr_2}^4\right)$$

5- Interface Boundary condition

at $x = x_0 \Rightarrow T_A(x_0,t) = T_B(x_0,t)$

$$\Rightarrow q_{cond})_A = q_{cond})_B$$

$$\Rightarrow -k_A \left.\frac{dT}{dx}\right|_{x=x_0} = -k_B \left.\frac{dT}{dx}\right|_{x=x_0}$$



ex 2.12

$1\text{-D},\ SS,\ \dot{q}=0$ $\qquad h = 3000\ W/m^2.K$

$Q = 850\ W,\ A = \pi D^2/4 = 0.031429\ m^2$ $\qquad t_\infty = 100$

$\qquad k = 100$

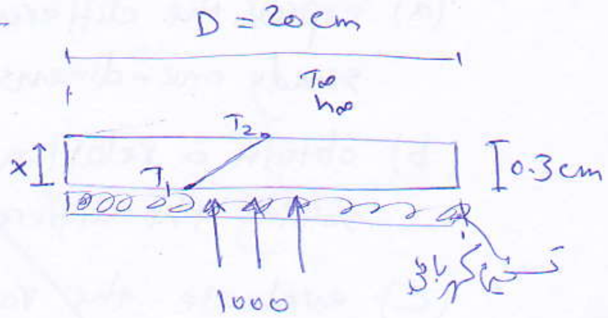
$\frac{d^2T}{dx^2} = 0,\quad \frac{dT}{dx} = C_1,\quad T = C_1 x + C_2$

B.Cs

① at $x = 0$ $\quad Q = 850\,W$ & $\boxed{T_1 = C_2}$ $\qquad$ Pan

$\Rightarrow -KA \frac{dT}{dx}\Big|_{x=0} = 850,\quad \frac{dT}{dx}\Big|_{x=0} = \frac{-850}{KA}$

$\Rightarrow \boxed{C_1 = \frac{-850}{KA}} = -270.45$



② at $x = L$ $\quad T = T_2$

$T_2 = C_1 L + C_2,\quad T_2 = ?$ مجهول

at $x = L,\quad q_{cond} = q_{conv.}$

$-KA \frac{dT}{dx}\Big|_{x=L} = hA(T_2 - T_\infty) = 850 \Rightarrow -kC_1 = h(T_2 - 100)$

$\Rightarrow$ ~~$\frac{850}{A}$~~

$\Rightarrow T = \frac{-850}{KA} x + T_1$

$\frac{850}{A} = \frac{k(T_1 - T_2)}{L} = h(T_2 - T_\infty) \Rightarrow$ $\boxed{T_2 = 109.01\,^\circ C,\ T_1 = \text{~~190.15~~ } 109.827\,^\circ C}$

$T_1 = \frac{L}{KA} 850 + T_2$ — ① $\qquad \frac{850}{Ah} + T_\infty = T_2$ — ②

$\Rightarrow T_1 = \underbrace{\frac{L}{KA} 850}_{0.811} + \underbrace{\frac{850}{Ah}}_{9.015} + \underbrace{T_\infty}_{100} =$ ~~190.15~~

$\Rightarrow \boxed{C_2 = 109.827}$

$\Rightarrow \boxed{T = -270.45\,X + 109.827}$

for $k = 100$
$C_2 = 109.827$

for $k = 17$
$C_2 = 113.788$
$C_1 = -1590.9$

$\boxed{T_{steel} = -1590.9\,X + 113.788}$

ex. 2.12 /

مقلاة من الفولاذ تستخدم لغلي الماء تسخن بواسطة سخان كهربائي اسفلها سعته 1000 واط
وكفاءة التوصيل 85% قطر المقلاة 20 cm وسمكها 0.3 cm معامل انتقال الحرارة بالحمل للماء 3000
ودرجة حرارة الماء 100°C على فرض انتقال الحرارة احادي البعد بالتوصيل والحالة المستقرة مع الزمن ولا وجود للتوليد الحراري
اكتب معادلة توزيع الحرارة ضمن قاعدة تلك المقلاة . $K = 100\ W/m.K$

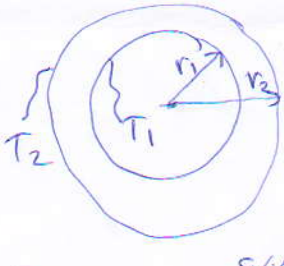
$K_{\text{للفولاذ}} = 17$

كرة

ex. ch3 16ell

Consider a spherical container of inner radius $r_1 = 8$ cm, outer radius $r_2 = 10$ cm and $k = 45$ W/m.°C $T_1 = 200$ °C, $T_2 = 80$ °C. obtaine a general relation for temperature distribution inside the shell under steady conditions, no heat genration, one dimensional heat conduction. Also determine the rate of heat loss from the container.

Solu.



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dT}{dr}\right) = 0 \Rightarrow \frac{\partial}{\partial r}\left(r^2\frac{dT}{dr}\right) = 0$$

$$\Rightarrow r^2\frac{dT}{dr} = c_1 \Rightarrow \frac{dT}{dr} = \frac{c_1}{r^2} = c_1 r^{-2}$$

$$\Rightarrow T_{(r)} = -\frac{c_1}{r} + c_2$$

B.Cs. at $r = r_1$, $T = T_1 \Rightarrow T_1 = -\frac{c_1}{r_1} + c_2$ — ①

at $r = r_2$, $T = T_2 \Rightarrow T_2 = -\frac{c_1}{r_2} + c_2$ — ②

طرح ① من ②

$$\Rightarrow T_1 - T_2 = -c_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = c_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = c_1\left(\frac{r_1 - r_2}{r_2 r_1}\right)$$

$$\Rightarrow \boxed{c_1 = (T_1 - T_2)\frac{r_1 r_2}{r_1 - r_2}}$$

نعوض قيمة c_1 في معادلة ①

$$T_1 = (T_2 - T_1)\frac{r_2}{r_1 - r_2} + c_2 \Rightarrow \boxed{c_2 = T_1 + (T_1 - T_2)\frac{r_2}{r_1 - r_2}}$$

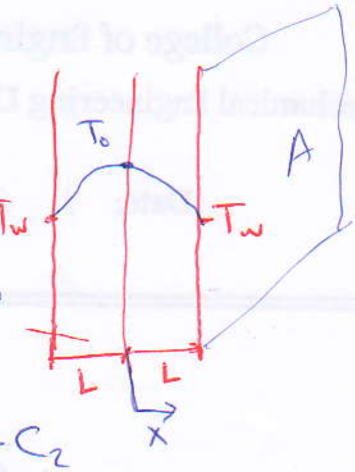
$$\boxed{T = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2}{(r_1 - r_2)}(T_1 - T_2) + T_1}$$

$$Q_{sphere} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{c_1}{r^2} = -4\pi k c_1$$

$$= 4\pi k (T_2 - T_1)\frac{r_1 r_2}{r_1 - r_2} = 4 * \pi * 45 (80 - 200)\frac{(0.08 \times 0.1)}{(0.08 - 0.1)}$$

$$\Rightarrow \boxed{Q_{sphere} = 27.143 \text{ kw}}$$

* One Dimensional Heat Conduction in plane with heat generation



$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{K} = 0$$

$T = T_w$ at $x = L$, $x = -L$

$T = T_0$ at $x = 0$

$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{K}$$

$$\frac{dT}{dx} = -\frac{\dot{q}}{K}x + C_1 \quad , \quad T = \frac{-\dot{q}x^2}{2K} + C_1 x + C_2$$

at $x = 0$ $T = T_0$ $\Longrightarrow$ $\boxed{T_0 = C_2}$

$$T_w = \left[\frac{-\dot{q}L^2}{2K} + C_1 L + T_0 = \frac{-\dot{q}L^2}{2K} - C_1 L + T_0\right]$$

$2C_1 L = 0 \Longrightarrow \boxed{C_1 = 0}$

$$\Longrightarrow T = \frac{-\dot{q}x^2}{2K} + T_0 \qquad T - T_0 = \frac{-\dot{q}x^2}{2K} \quad \text{——} ①$$

~~at $x = 0$ ⇒ $T = T_0$ ⇒ $T_0 = \frac{\dot{q}L^2}{4K}$~~

at $x = L$ $T = T_w$

$$T_w = -\frac{\dot{q}L^2}{2K} + T_0 \Longrightarrow T_w - T_0 = \frac{-\dot{q}L^2}{2K} \quad \text{——} ②$$

نقسم معادلة ① على ② لحل معادلة توزيع الحرارة بالصيغة اللابعدية

$$\Longrightarrow \frac{T - T_0}{T_w - T_0} = \frac{x^2}{L^2} \qquad \frac{T - T_0}{T_w - T_0} = \left(\frac{x}{L}\right)^2 \quad \text{——} ③$$

الحرارة المتولدة من $\dot{q}$ تساوي الحرارة الخارجة من سطحي الجدار

$$2\left[-kA\left.\frac{dT}{dx}\right)_{x=L}\right] = \dot{q}\underbrace{A2L}_{\text{الحجم}}$$

نستخرج قيمة $\left.\frac{dT}{dx}\right)_{x=L}$ من اشتقاق معادلة ③

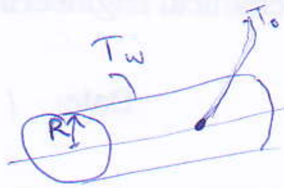
$$\left.\frac{dT}{dx}\right)_{x=L} = (T_w - T_0)\left(\frac{2x}{L^2}\right)_{x=L} = (T_w - T_0)\frac{2}{L}$$

$$\Rightarrow -2KA(T_w - T_0)\frac{2}{L} = \dot{q}A2L \;, \Rightarrow \boxed{T_0 = \frac{\dot{q}L^2}{2K} + T_w}$$

$$\frac{T - T_w}{T_0 - T_w} = 1 - \frac{x^2}{L^2}$$

* One-Dimensional Heat Conduction in cylinder with heat generation Sources

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}}{K} = 0 \quad \text{--- (1)}$$



B.Cs

1st B.C: $\boxed{T = T_w}$ at $r = R$

heat generated = heat lost at surface (2nd B.C)

$$\underbrace{\dot{q}\,\pi R^2 L}_{\text{الحجم}} = -K \underbrace{2\pi R L}_{\text{المساحة السطحية}} \left.\frac{dT}{dr}\right)_{r=R} \Rightarrow \boxed{\frac{dT}{dR} = \frac{-\dot{q}R}{2K}} \quad \text{at } r=R$$

$$\frac{dT}{dr} = 0 \quad \text{at} \quad r = 0$$

نعيد صياغة المعادلة ①

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = \frac{-\dot{q}r}{K} \quad \text{--- (2)}$$

نلاحظ ان

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = \frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

نكامل معادلة ② مرتين لإيجاد معادلة توزيع درجة الحرارة T

$$\Rightarrow r\frac{dT}{dr} = \frac{-\dot{q}r^2}{2K} + C_1 \Rightarrow \frac{dT}{dr} = \frac{-\dot{q}r}{2K} + \frac{C_1}{r} \quad \text{--- (3)}$$

$$\Rightarrow T = \frac{-\dot{q}r^2}{4K} + c_1 \ln r + C_2, \quad \boxed{T = \frac{-\dot{q}r^2}{4K} + c_1 \ln r + C_2}$$

نطبق الشروط الحدية لإيجاد ثوابت C_2 & C_1

نعوض الشرط الثاني في معادلة ③

$$\Rightarrow \frac{-\dot{q}R}{2K} = \frac{-\dot{q}r}{2K} + \frac{C_1}{R} \Rightarrow \boxed{C_1 = 0}$$

نعوض الشرط الاول في معادلة T

$$T_w = \frac{-\dot{q}R^2}{4K} + C_2 \Rightarrow \boxed{C_2 = T_w + \frac{\dot{q}R^2}{4K}}$$

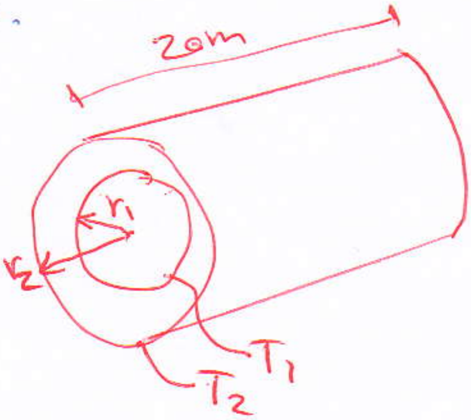
$$T = \frac{-\dot{q}r^2}{4K} + T_w + \frac{\dot{q}R^2}{4K} \Rightarrow T = T_w + \frac{\dot{q}}{4K}(R^2 - r^2)$$

$$\text{at } r = 0 \quad T = T_o \Rightarrow T_o = T_w + \frac{\dot{q}R^2}{4K}$$

درجة حرارة محور الاسطوانة (at r=0)

ex. ch3 ص150   اسطوانة

Consider a steam pipe of length L = 20 m, inner radius (r_1 = 6 cm), outer radius (8 cm = r_2) & thermal conductivity K = 20 W/m·°C, as shown in figure. The inner and outer surfaces of the pipe are maintained at average temperatures of T_1 = 150°C and T_2 = 60°C respectively, obtain a general relation for the temperature distribution inside the pipe under steady conditions and no heat generation, Determine the rate of heat loss from the steam through the pipe.



Soln.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) = 0 \Rightarrow \frac{\partial}{\partial r}\left(r\frac{dT}{dr}\right) = 0$$

$$\Rightarrow r\frac{dT}{dr} = c_1 , \quad \frac{dT}{dr} = \frac{c_1}{r}$$

$$\Rightarrow \boxed{T = c_1 \ln r + c_2} \text{ --- } (*)$$

B.Cs.

at $r = r_1$, $T = T_1 = 150°C$

$$\Rightarrow T_1 = c_1 \ln r_1 + c_2 \text{ --- } (1)$$

at $r = r_2$, $T = T_2 = 60°C$

$$T_2 = c_1 \ln r_2 + c_2 \text{ --- } (2)$$

نطرح معادلة ② من ①

$$\mp T_1 = \mp c_1 \ln r_1 \mp c_2$$

$$T_2 - T_1 = c_1(\ln r_2 - \ln r_1) = c_1 \ln(r_2/r_1) \Rightarrow \boxed{c_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)}}$$

نعوض بقيمة c_1 في معادلة ① لإيجاد قيمة c_2

$$T_1 = \left(\frac{T_2 - T_1}{\ln(r_2/r_1)}\right) * \ln r_1 + c_2 \Rightarrow \boxed{c_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)} \cdot \ln r_1}$$

نعوض عن قيمة c_1 & c_2 في معادلة (*)

$$\Rightarrow T = \frac{T_2 - T_1}{\ln(r_2/r_1)} \ln r - \frac{T_2 - T_1}{\ln(r_2/r_1)} \ln r_1 + T_1$$

$$= \frac{T_2 - T_1}{\ln(r_2/r_1)}(\ln r - \ln r_1) + T_1 = \boxed{\frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_2 - T_1) + T_1 = T(r)}$$

$$\boxed{T(r) = \frac{\ln(r/r_1)}{\ln(r_2/r_1)}(T_2 - T_1) + T_1}$$

$$\frac{dT}{dr} = \frac{c_1}{r}$$

$$Q_{cylinder} = -KA\frac{dT}{dr} = -K(2\pi \not{r} L)\frac{c_1}{\not{r}}$$

$$= -K(2\pi L)\left[\frac{(T_2 - T_1)}{\ln(r_2/r_1)}\right] = -20(2\pi * 20) * \frac{60 - 150}{\ln(8/6)}$$

$$\Rightarrow \boxed{Q_{cylinder} = 786.266 \text{ kW}}$$

Heat Transfer I

"Heat Transfer from Finned surfaces"

Lecture No. (6)

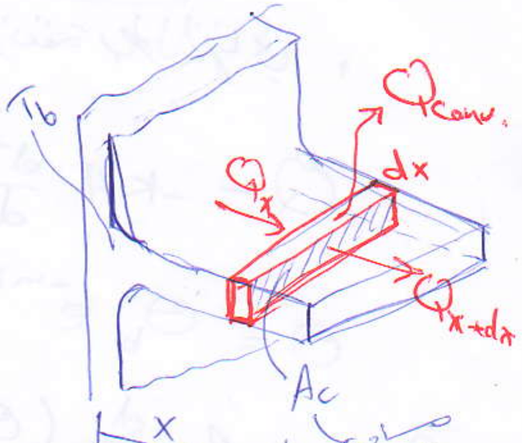
* Fins (extended surfaces)

$$Q_x = Q_{x+dx} + Q_{conv.}$$

$$Q_x - Q_{x+dx} = Q_{conv.}$$

$$Q_x - \left(Q_x + \frac{dQ_x}{dx} \cdot dx\right) = Q_{conv}$$

$$\Rightarrow -\frac{dQ_x}{dx} \cdot dx = Q_{conv.}$$



A_c مساحة المقطع

A_s المساحة السطحية
محيط المقطع الزعنفة * السمك

$A_s = P \cdot dx$
المحيط السمك

$$Q_x = -kA_c \frac{dT}{dx} \Rightarrow -\frac{dQ_x}{dx} = +kA_c \frac{d^2T}{dx^2}$$

$$\Rightarrow kA_c \frac{d^2T}{dx^2} \cdot dx = h A_s (T - T_\infty)$$

$$kA \frac{d^2T}{dx^2} \cdot dx = h P \cdot dx (T - T_\infty)$$

$$\Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

درجة الحرارة الإضافية θ

تحويل الى الصيغة الأخيرة $\theta = T - T_\infty$, $d\theta = dT - 0$, $d^2\theta = d^2T$

$$\Rightarrow \frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0 \qquad \text{let } m^2 = \frac{hP}{kA}, \quad m = \sqrt{\frac{hP}{kA}}$$

حل المعادلة التفاضلية

$$\Rightarrow \boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$

$$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

* يجب عرفة قيم الثوابت C_1 & C_2 من استخدام الشروط الحدية

* الشرط الاول ثابت لجميع الحالات وهو عندما $x = 0$ فان $T = T_b$، $\theta = \theta_b = (T_b - T_\infty)$

$$\Rightarrow \boxed{\theta_b = C_1 + C_2} \quad \text{—— (1)}$$

* الشرط الثاني يطبق عندما x تساوي الطول الكلي للزعنفة L
ويقسم الى عدد من الانواع على حسب الظرف الحراري في نهاية الزعنفة

① النوع الاول (Infinitely long fin) زعنفة طويلة نسبياً بحيث تقترب درجة حرارة نهاية الزعنفة $T_\infty = T(L)$
يعني $L \simeq \infty$, $\theta_L = 0$

$$\Rightarrow \boxed{C_1 e^{\infty} + C_2 e^{-\infty} = 0} \quad \text{—— (2)}$$

$e^{-\infty} = 0 \Rightarrow \boxed{C_1 = 0} \Rightarrow \boxed{C_2 = \theta_b}$

$$\Rightarrow \boxed{\theta = \theta_b \cdot e^{-mx}}$$

$$\frac{\theta}{\theta_b} = \boxed{\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}} \quad , \quad m = \sqrt{\frac{hP}{KA}}$$

* معادلة توزيع درجات الحرارة داخل الزعنفة بطول لا نهائي

لحساب معدل الحرارة المنتقلة من الزعنفة

$$Q = -KA \frac{dT}{dx} = -KA \left.\frac{d\theta}{dx}\right|_{x=0}$$ ← في بداية الزعنفة قبل أن يظهر تأثير $Q_{conv.}$

$$\theta = \theta_b \, e^{-mx}$$

$$Q = -KA \frac{d}{dx}\left(\theta_b \cdot e^{-mx}\right)$$

$$= -KA\,\theta_b \left[e^{-mx} \cdot (-m)\right]$$

$$= \theta_b \, KA \, m \, e^{-mx} = \theta_b \, KA \sqrt{\frac{hP}{KA}} \cdot e^{-mx}$$

$$= \theta_b \cdot \sqrt{PhKA} \cdot e^{-mx}$$

at $x = 0$

$$Q = Q_{Fin} \Rightarrow \boxed{Q_{Fin} = \theta_b \sqrt{PhKA}}$$

الأنواع الأخرى

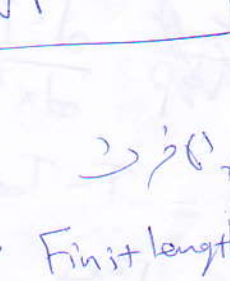
② Finit length طول محدد $(Q_{cond} = Q_{conv.} \text{ at } x = L)$

③ The end of the Fin insulated نهاية الزعنفة معزولة حراريا $\left(-KA \left.\frac{d\theta}{dx}\right|_{x=L} = 0\right)$

---

ex. $K_A = 200 \quad t_A = 75\,°C, \; t_B = 60\,°C, \; t_b = 100\,°C, \; t_\infty = 25\,°C$

Assume very long fins, find $K_B = ?$



$$\frac{\theta}{\theta_0} = e^{-mx}, \quad m = \sqrt{\frac{Ph}{KA}} \Rightarrow \frac{t - t_\infty}{t_b - t_\infty} = e^{-mx}$$

$$\Rightarrow \ln\left(\frac{t_A - t_\infty}{t_b - t_\infty}\right) = -m_A x \quad \text{—— ①}$$

$$\Rightarrow \ln\left(\frac{t_B - t_\infty}{t_b - t_\infty}\right) = -m_b x \quad \text{—— ②}$$

من ① و ②

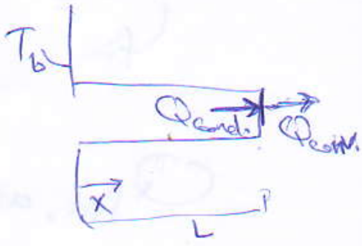
$$\Rightarrow \frac{m_A}{m_B} = 0.532 = \sqrt{\frac{K_B}{K_A}}$$

$$\Rightarrow \boxed{K_B = 56.6 \; w/m \cdot k}$$

Fins

[2] النوع الثاني (Finite length) $\theta_L \neq 0$ $(T_L > T_\infty)$

at $x = L$ $\quad Q_{cond.} = Q_{conv.}$



$$-k\cancel{A}\left.\frac{dT}{dx}\right|_{x=L} = h\cancel{A}(T - T_\infty)\Big|_{x=L}$$

في نهاية الزعنفة مساحة التوصيل تساوي مساحة الحمل

$$\Rightarrow k\left.\frac{d\theta}{dx}\right|_{x=L} = h \cdot \theta\Big|_{x=L}$$

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh(m(L-x)) + \frac{h}{mk}\sinh(m(L-x))}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}}$$

$$\boxed{Q_{Fin} = \sqrt{hpkA}\cdot\left[\frac{\frac{h}{mk} + \tanh(mL)}{1 + \frac{h}{mk}\tanh(mL)}\right]\theta_b}$$

[3] النوع الثالث The end of fin is insulated (Adiabatic) (العزل الحراري لنهاية الزعانف Fins)

at $x = L$ $\quad Q = 0 \Rightarrow -kA\left.\frac{d\theta}{dx}\right|_{x=L} = 0$

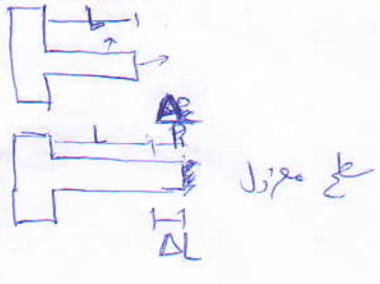
$$\Rightarrow \left.\frac{d\theta}{dx}\right|_{x=L} = 0 \Rightarrow mC_1e^{mL} - mC_2e^{-mL} = 0 \quad (\div m)$$

$$\Rightarrow \boxed{C_1e^{mL} - C_2e^{-mL} = 0} \quad —② \qquad \boxed{C_1 + C_2 = \theta_b} \quad —①$$

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh(m(L-x))}{\cosh(mL)}}$$

$$\boxed{Q_{Fin} = \sqrt{hpkA}\cdot\tanh(mL)\cdot\theta_b}$$

$$\Rightarrow \boxed{L_c = L + \frac{A_c}{p}}$$



معامل تصحيح الطول عند اهمال مساحة نهاية الزعنفة يتم بزيادة الطول الأصلي للزعنفة L الى الطول المصحح L_c (corrected fin length)

مساحة مقطعية $[\underset{\text{التغير بالطول}}{\Delta L} \times \underset{\text{محيط}}{P} = A_c]$ مساحة جانبية $\Leftarrow$ $\boxed{\Delta L = \frac{A_c}{p}}$

* Fin Efficiency (η_{fin})

$$\eta_{fin} = \frac{Q_{fin,\,actual}}{Q_{fin,\,max}}$$

$$Q_{fin,\,actual} = Q_{fin} < \begin{cases} Q_{fin} & \text{لطول الزعنفة غير المحدد} \\ Q_{fin} & \text{لطول محدد} \\ Q_{fin} & \text{(زعنفة معزولة الطرف)} \end{cases}$$

$$Q_{fin,\,max} = h \cdot A_{fin} (T_b - T_\infty)$$

A_{fin} ↓ المساحة الكلية للزعنفة $(P * L)$

θ_b يمثل اكبر مقدار للفرق الحراري المحتمل داخل الزعنفة مع المائع الخارجي

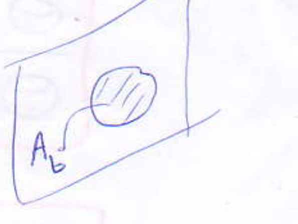
$$\eta_{long\,fin} = \frac{\sqrt{hPKA}\,\theta_b}{h\,A_{fin}\,\theta_b} = \frac{\sqrt{hPKA}}{h(P \cdot L)} = \frac{1}{L}\sqrt{\frac{KA}{hP}} = \frac{1}{mL}$$

$$\eta_{insulated,\,fin} = \frac{\sqrt{hPKA}\,\theta_b \tanh(mL)}{h(PL)\,\theta_b} = \frac{\tanh(mL)}{mL}$$

$$\eta_{finite\ length,\,fin} = \frac{\sqrt{hPKA}\,\theta_b \cdot \left[\frac{(h/mk) + \tanh(mL)}{1 + (h/mk)\tanh(mL)}\right]}{h(PL)\,\theta_b}$$

$$= \frac{1}{mL}\left[\frac{(h/mk) + \tanh(mL)}{1 + (h/mk)\tanh(mL)}\right]$$

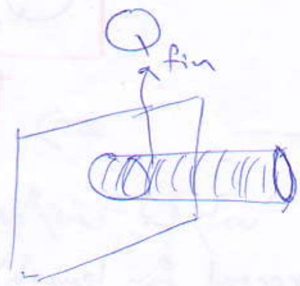
Fin Effectiveness (ϵ_{fin})



$$\epsilon_{fin} = \frac{Q_{fin,\,actual}}{Q_{without\ fin}} = \frac{Q_{fin}}{h\,A_b (T_b - T_\infty)}$$ ← حسب النوع

$$= \frac{\eta_{fin}\,h\,A_{fin}\,(T_b - T_\infty)}{h\,A_b\,(T_b - T_\infty)}$$

($T_b - T_\infty = \theta_b$)



$$\Rightarrow \epsilon_{fin} = \frac{A_{fin}}{A_b}\,\eta_{fin}$$

ex.1 A very long rod of 5 mm diameter has one end maintained at 100 °C. The surface of the rod exposed to ambient air at 25 °C and $h = 100\ W/m^2 \cdot °C$.

1 - Determine the temperature distribution along rods constructed from copper ($k = 398\ W/m \cdot °C$) and stainless steel ($k = 14\ W/m \cdot °C$)

2 - Estimate the rods length in case of infinite length

Solu.

① $q_{fin_{copper}} = \sqrt{hpkA_c}\ \Theta_b = \sqrt{100 * \pi * 0.005 * 398 * (\pi * 0.005^2/4)} * (100 - 25)$

$$\Rightarrow \boxed{q_{fin_{copper}} = 8.31\ W}\ ,\ \boxed{q_{fin_{steel}} = 1.56\ W}$$

② * لإيجاد الطول L الذي يمثل الفاصل بين حالة الزعنفة محددة الطول والزعنفة ذات الطول غير المحدد يتم مساواة كمية الحرارة الناتجة من الزعنفة من حالتين هما الطول غير المحدد وحالة الزعنفة معزولة النهاية.

$$\Rightarrow Q_{fin} = \sqrt{hpkA} \cdot \Theta_b = \sqrt{hpkA} \cdot \Theta_b \tanh(mL)$$

$$\Rightarrow \tanh(mL) \simeq 1$$

* دالة $\tanh(x)$: $-0.99 < x < +0.99$

$$\Rightarrow \tanh(mL) = 0.99$$

$$\Rightarrow mL = 2.647 \Rightarrow L_{copper} = \left(\sqrt{\frac{100 * \pi * 0.005}{398 * \pi * 0.005^2/4}}\right)^{-1} \cdot 2.647$$

$$\Rightarrow \boxed{L_{copper} = 186.7\ mm}$$

$$\Rightarrow L_{steel} = \frac{2.647}{m_{steel}} = \sqrt{\frac{K_{steel} A}{hp}} \cdot 2.647 = \sqrt{\frac{14 * (0.005^2/4)\pi}{100\ (0.005 * \pi)}} \cdot 2.647$$

$$\Rightarrow \boxed{L_{steel} = 35\ mm}$$

ex2 a 6 mm diameter steel rod ($k = 43$) and 0.3 m Long is exposed to air at 38 °C and ($h = 340 \, W/m^2 \cdot °C$). The base temperature of the rod is maintained at 260 °C ? find the heat transfer rate from the rod if:

a – loss heat by convection from its end (finite length)

b – The end of fin is insulated.

Solu.

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{340 * \pi * 0.006}{43 * (\pi * 0.006^2/4)}} \Rightarrow m = 72.6$$

$$\Rightarrow mL = 72.6 * 0.3 = 21.78$$

$$\sqrt{hPkA} = \sqrt{340 * \pi * 0.006 * 43 * \pi * 0.006^2/4} \Rightarrow \sqrt{hPkA} = 0.0883$$

$$h/mK = 0.1089$$

a – $$Q_{fin} = \sqrt{hPkA} \cdot \left[\frac{\frac{h}{mK} + \tanh(mL)}{1 + \frac{h}{mK}\tanh(mL)}\right] \cdot \theta_b$$

$$= 0.0883 \cdot \left[\frac{0.1089 + \tanh(21.78)}{1 + 0.1089 \tanh(21.78)}\right] \cdot (260 - 38)$$

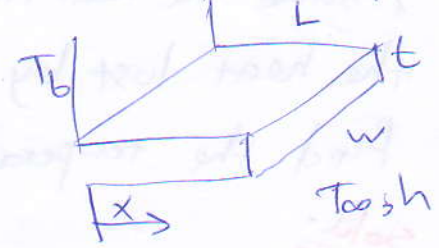
$$\Rightarrow \boxed{Q_{fin} = 19.6 \text{ W}}$$

b – $$Q_{fin} = \sqrt{hPkA} \tanh(mL) \cdot \theta_b$$

$$= 0.0883 * \tanh(21.78) * (260 - 38)$$

$$\Rightarrow \boxed{Q_{fin} = 19.6 \text{ W}}$$

ex3 An Aluminum fin ($k = 200\ W/m.°C$), 3 mm thick and 7.65 cm long protrudes from a wall as shown in figure. The base is maintained at 300°C and ambient temperature is 50°C with ($h = 10\ W/m^2.°C$). Find the heat loss from fin per unit dipth and temperature at 4 cm from long. Neglect the heat transfer from the fin tip.



Solu.

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h*2(w+t)}{k(wt)}} \Rightarrow m = 5.782$$

for insulated tip

$$q = \sqrt{hPKA}\ \theta_b \cdot \tanh(mL)$$

$$= \sqrt{10*2*(1+0.003)*200*(1*0.003)} * (300-50) \tanh(5.782*0.0765)$$

103.892

$$\Rightarrow \boxed{Q_{fin} = 360.4\ W}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh(m(L-x))}{\cosh(mL)} \Rightarrow \frac{T-50}{300-50} = \frac{\cosh(5.782(0.0765-0.04))}{\cosh(5.782*0.0765)}$$

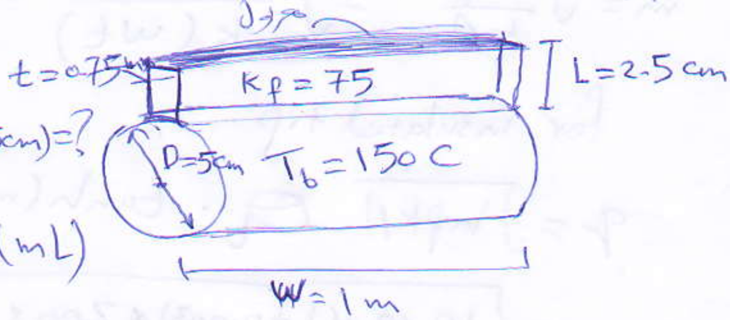
$$= 0.9299$$

$$\Rightarrow \boxed{T(4\ cm) = 282.5\ °C}$$

ex 4 A 12 brass fins ($K = 75\ W/m\cdot°C$) ~~thick~~ 0.75 mm thick are placed on a circular tube with 5 cm outer diameter. The fin is 2.5 cm ~~6.4 mm~~ Long. The tube wall is maintained at 150°C, the environment temperature 40°C, and the convection heat transfer coefficient is $23.3\ W/m^2°C$. Assume the heat transfer from the fins tip is negligable, calculate the heat lost by fins; the heat lost from tube's base and find the temperature of the fin at 1.25 cm far from the base.

solu.

$Q_{fins} = ?$, $Q_{Base} = ?$, $T(1.25cm) = ?$



$$Q_{fin} = \sqrt{PhKA}\cdot\theta_b\cdot\tanh(mL)$$

$$P = 2(w+t) = 2(1+0.00075) = 2.0015\ m$$

$$A = (t*w) = 1*0.00075 = 0.00075\ m^2 \quad , k = 75 \quad , h = 23.3$$

$$\Rightarrow \sqrt{hPKA} = \sqrt{23.3*2.0015*75*0.00075} = 1.62$$

$$\theta_b = (150-40) = 110°C \quad , \quad m = \sqrt{\frac{Ph}{KA}} \Rightarrow m = 28.7935$$

$$\Rightarrow Q_{Fin} = 1.62*110*\tanh(28.7935*0.025)$$
$$= 109.92\ W \quad * 12\ \text{fin}$$

$$\Rightarrow \boxed{Q_{Fins} = 1319\ W}$$

$$Q_{Base} = h\,A_{Base}(T_b - T_\infty) = 23.3\cdot(\pi DL - 12A_c)*(150-40)$$
$$= 23.3(3.14*0.05*1 - (12*0.00075))*110$$

$$\Rightarrow \boxed{Q_{Base} = 379.53\ W}$$

$$\frac{\theta}{\theta_b} = \frac{T-T_\infty}{T_b-T_\infty} = \frac{\cosh(m(L-x))}{\cosh(mL)} \Rightarrow \frac{T-40}{110} = \frac{\cosh(28.7935*0.0125)}{\cosh(28.7935*0.025)}$$

$$\Rightarrow \boxed{T(1.25cm) = 132.25\ °C}$$

Heat Transfer I

"Two - Dimensional steady state Conduction"

Lecture No. (7)

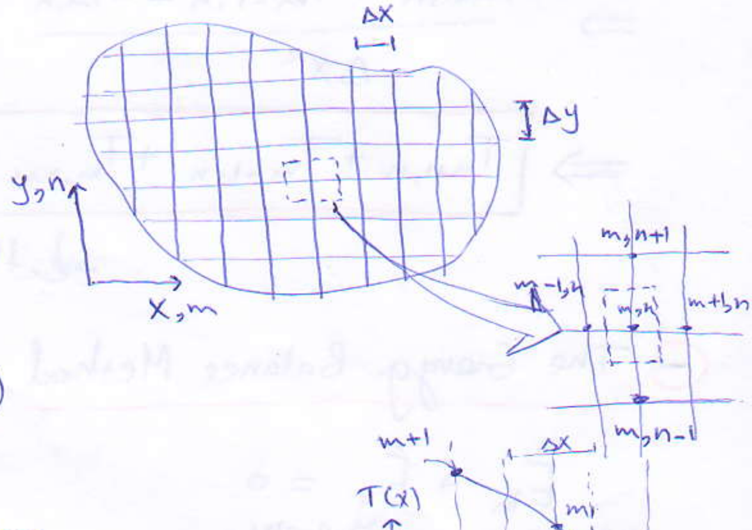
* Two Dimensional steady state conduction

⊛ Numerical Method of Analysis (Finite Difference Equation)

⊖ The Nodal Network

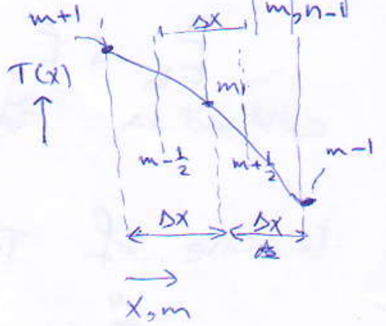
يتم تعريف الحدود التفاضلية في المعادلة العامة

التوصل بدلالة معادلات الفروق المحدودة كما يلي !



$$\frac{\partial T}{\partial x}\Big|_{m-\frac{1}{2},n} \approx \frac{T_{m,n}-T_{m-1,n}}{\Delta X} \quad \text{(Backward)}$$

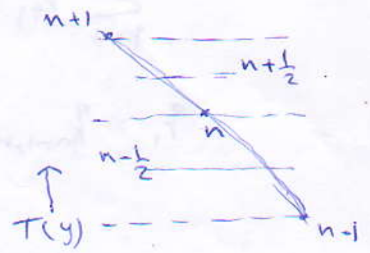
$$\frac{\partial T}{\partial x}\Big|_{m+\frac{1}{2},n} \approx \frac{T_{m+1,n}-T_{m,n}}{\Delta X} \quad \text{(Forward)}$$



$$\frac{\partial^2 T}{\partial x^2}\Big|_{m,n} \approx \frac{\frac{\partial T}{\partial x}\Big|_{m+\frac{1}{2},n} - \frac{\partial T}{\partial x}\Big|_{m+\frac{1}{2},n}}{\Delta X}$$

$$\approx \frac{\left(T_{m+1,n}-T_{m,n}-T_{m,n}+T_{m-1,n}\right)\ast\frac{1}{\Delta X}}{\Delta X}$$

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial x^2}\Big|_{m,n} = \frac{T_{m+1,n}+T_{m-1,n}-2\,T_{m,n}}{\Delta X^2}}$$



$$\frac{\partial T}{\partial y}\Big|_{m,n+\frac{1}{2}} \simeq \frac{T_{m,n+1}-T_{m,n}}{\Delta y}$$

$$\frac{\partial T}{\partial y}\Big|_{m,n-\frac{1}{2}} \simeq \frac{T_{m,n}-T_{m,n-1}}{\Delta y}$$

$$\frac{\partial^2 T}{\partial y^2}\Big|_{m,n} \simeq \frac{\frac{\partial T}{\partial y}\Big|_{m,n+\frac{1}{2}} - \frac{\partial T}{\partial y}\Big|_{m,n-\frac{1}{2}}}{\Delta y} \simeq \frac{\left(T_{m,n+1}-T_{m,n}-T_{m,n}+T_{m,n-1}\right)\ast\frac{1}{\Delta y}}{\Delta y}$$

$$\Rightarrow \boxed{\frac{\partial^2 T}{\partial y^2}\Big|_{m,n} \simeq \frac{T_{m,n+1}+T_{m,n-1}-2\,T_{m,n}}{\Delta y^2}}$$

* for $\Delta X = \Delta y$ & 2D, Steady state, no heat generation

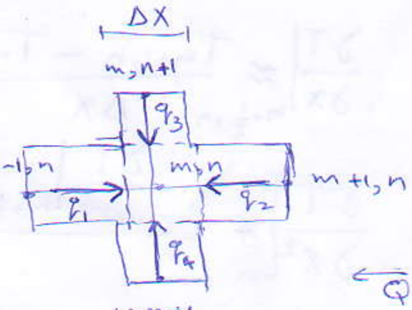
$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} = 0$$

$$\Rightarrow \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{\Delta X^2} + \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{\Delta y^2} = 0 \quad (*\Delta X^2)$$

$$\Rightarrow \boxed{T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0}$$

* لإيجاد درجة الحرارة للعقدة الداخلية بدلالة العقد المحيطة .

⊖ The Energy Balance Method



$$\dot{E}_{in} + \dot{E}_g = 0$$

($\dot{E}_{in}$: معدل الطاقة الداخلة ، $\dot{E}_g$: معدل الطاقة المتولدة)

In case of Two dimensional conduction with heat generation :

$$\sum_{i=1}^{4} q_{(i)\rightarrow(m,n)} + \dot{q}\,(\Delta X \cdot \Delta y \cdot 1) = 0 \qquad [(Z=1)\ \text{العمق}]$$

(الحجم: $\Delta X \cdot \Delta y \cdot 1$)

$$q_1 = q_{(m-1,n)\rightarrow(m,n)} = -K\,(\Delta y \cdot 1)\,\frac{T_{m,n} - T_{m-1,n}}{\Delta X}$$

(مساحة المقطع: $\Delta y \cdot 1$)

$$q_1 = K\,(\Delta y \cdot 1)\,(T_{m-1,n} - T_{m,n}) * (1/\Delta X)$$

$$q_2 = K\,(\Delta y \cdot 1)\,(T_{m+1,n} - T_{m,n}) * (1/\Delta X)$$

$$q_3 = K\,(\Delta X \cdot 1)\,(T_{m,n+1} - T_{m,n}) * (1/\Delta y)$$

$$q_4 = K\,(\Delta X \cdot 1)\,(T_{m,n-1} - T_{m,n}) * (1/\Delta y)$$

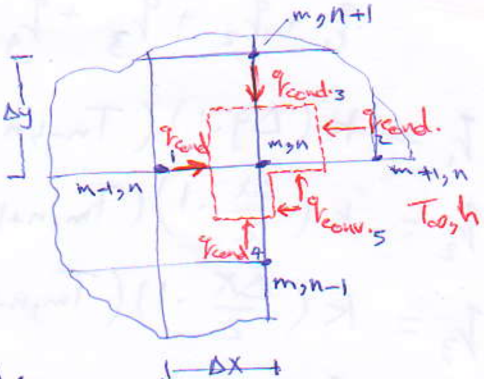
for $\Delta X = \Delta y$

$$\Rightarrow K\,(T_{m-1,n} - T_{m,n} + T_{m+1,n} - T_{m,n} + T_{m,n+1} - T_{m,n} + T_{m,n-1} - T_{m,n}) + \dot{q}\,\Delta X^2 = 0$$

$$\Rightarrow \boxed{T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} + \frac{\dot{q}\,\Delta X^2}{K} = 0}$$

* لإيجاد درجة حرارة العقدة الداخلية مع وجود توليد حراري بدلالة النقاط المحيطة .

- Energy Balance Method (for node exposed to Convection)



$$q_1 = K(\Delta y \cdot 1)(T_{m-1,n} - T_{m,n})(1/\Delta X)$$

$$q_2 = K(\tfrac{\Delta y}{2} \cdot 1)(T_{m+1,n} - T_{m,n})(1/\Delta X)$$

$$q_3 = K(\Delta X \cdot 1)(T_{m,n+1} - T_{m,n})(1/\Delta y)$$

$$q_4 = K(\tfrac{\Delta X}{2} \cdot 1)(T_{m,n-1} - T_{m,n})(1/\Delta y)$$

$$q_{conv.} = h(\tfrac{\Delta X}{2} \cdot 1)(T_\infty - T_{m,n}) + h(\tfrac{\Delta y}{2} \cdot 1)(T_\infty - T_{m,n})$$

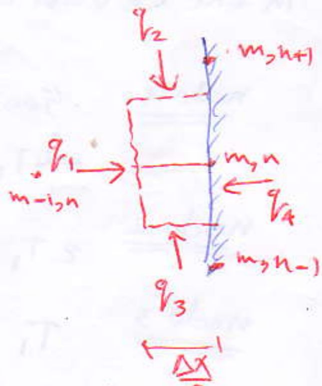
$$q_1 + q_2 + q_3 + q_4 + q_{conv.} = 0 \quad \& \quad (\Delta X = \Delta y)$$

$$K(T_{m-1,n} - T_{m,n}) + \frac{K}{2}(T_{m+1,n} - T_{m,n}) + K(T_{m,n+1} - T_{m,n}) + \frac{K}{2}(T_{m,n-1} - T_{m,n}) + (T_\infty - T_{m,n})(h\Delta X) = 0 \quad (\div K)$$

$$\Rightarrow T_{m-1,n} + T_{m,n+1} - 2T_{m,n} + \frac{1}{2}T_{m+1,n} + \frac{1}{2}T_{m,n-1} - T_{m,n} + \frac{h\Delta X}{K}T_\infty - \frac{h\Delta X}{K}T_{m,n} = 0$$

$$\Rightarrow \boxed{T_{m-1,n} + T_{m,n+1} + \frac{1}{2}(T_{m+1,n} + T_{m,n-1}) + \frac{h\Delta X}{K}T_\infty - \left(3 + \frac{h\Delta X}{K}\right)T_{m,n} = 0}$$

ex1 using the energy balance method drive the finite difference equation for the m,n nodal point located on plane, insulated surface of a medium with uniform heat generation. **Sol.:**



$$q_1 + q_2 + q_3 + \underset{zero}{q_4} + \dot{q}(\tfrac{\Delta X}{2} \cdot \Delta y \cdot 1) = 0$$

$$q_1 = K(\Delta y \cdot 1)(T_{m-1,n} - T_{m,n})(1/\Delta X)$$

$$q_2 = K(\tfrac{\Delta X}{2} \cdot 1)(T_{m,n+1} - T_{m,n})(1/\Delta y)$$

$$q_3 = K(\tfrac{\Delta X}{2} \cdot 1)(T_{m,n-1} - T_{m,n})(1/\Delta y) \quad , q_4 = 0 \text{ (insulated)}$$

for $\Delta X = \Delta y$, $(\div K)$

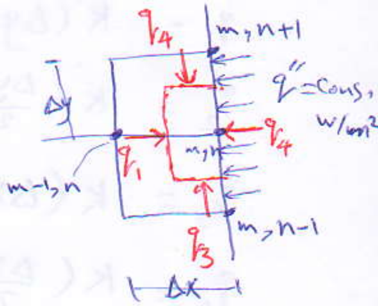
$$\Rightarrow T_{m-1,n} - T_{m,n} + \frac{1}{2}(T_{m,n+1} + T_{m,n-1}) - T_{m,n} + \frac{\dot{q}\Delta X^2}{2K} = 0 \quad (*2)$$

$$\Rightarrow \boxed{2T_{m-1,n} + T_{m,n+1} + T_{m,n-1} + \frac{\dot{q}\Delta X^2}{K} - 4T_{m,n} = 0}$$

ex 2 case 5 Node at a plane surface with uniform heat flux.

Soln

$q_1 + q_2 + q_3 + q_4 = 0$



$q_1 = k(\Delta y \cdot 1)(T_{m-1,n} - T_{m,n})(1/\Delta x)$

$q_2 = k(\frac{\Delta x}{2} \cdot 1)(T_{m,n+1} - T_{m,n})(1/\Delta y)$

$q_3 = k(\frac{\Delta x}{2} \cdot 1)(T_{m,n-1} - T_{m,n})(1/\Delta y)$

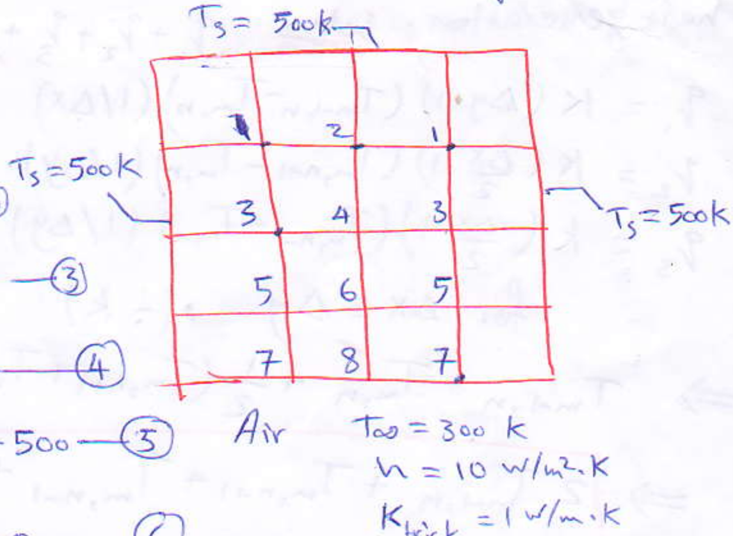
$q_4 = q'' \cdot (\Delta y \cdot 1)$

$\Delta x = \Delta y$ & $(\div k)$

$T_{m-1,n} - T_{m,n} + \frac{1}{2}(T_{m,n+1} + T_{m,n-1}) - T_{m,n} + \frac{q'' \cdot \Delta x}{k} = 0 \quad (*2)$

$\Longrightarrow \boxed{(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2q'' \cdot \Delta x}{k} - 4T_{m,n} = 0}$

ex 3 A large industrial furnace is supported on a long column of fireclay brick, which is 1 m by 1 m on a side. During steady state operation, installation in such that three surfaces of the column are maintained at 500 K while the remaining surface is exposed to an airstream for which $T_\infty = 300$ K and $h = 10$ W/m²·K. Using a grid of $\Delta x = \Delta y = 0.25$ m, determine the two dimensional temperature distribution in the column and the heat rate to the airstream per unit length of column.



node 1: $500 + 500 + T_2 + T_3 - 4T_1 = 0$

$\Longrightarrow -4T_1 + T_2 + T_3 = -1000$ — ①

node 2: $2T_1 - 4T_2 + T_4 = -500$ — ②

node 3: $T_1 - 4T_3 + T_4 + T_5 = -500$ — ③

node 4: $T_2 + 2T_3 - 4T_4 + T_6 = 0$ — ④

node 5: $T_3 - 4T_5 + T_6 + T_7 = -500$ — ⑤

node 6: $2T_5 - 4T_6 + T_4 + T_8 = 0$ — ⑥

node 7: $2T_5 - 2\left(\frac{10 * 0.25}{1} + 2\right)T_7 + T_8 + 500 + \frac{2 * 10 * 0.25}{1} * 300 = 0$ (with $-9\,T_7$ written above)

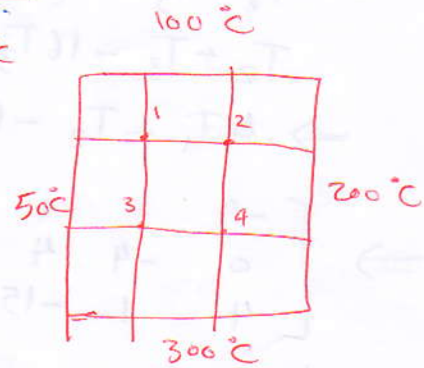
$$\Rightarrow 2T_5 - 9T_7 + T_8 = -2000 \quad (7)$$

node 8: $2T_6 + 2T_7 - 9T_8 = -1500 \quad (8)$

$$\begin{bmatrix} -4 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & -9 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & -9 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{bmatrix} = \begin{bmatrix} -1000 \\ -500 \\ -500 \\ 0 \\ -500 \\ 0 \\ -2000 \\ -1500 \end{bmatrix}$$

---

ex4 find $T_1, T_2, T_3 = ???$ $\quad T_4 = 206.25\,^\circ C$



node 1: $100 + 50 + T_2 + T_3 - 4T_1 = 0$

$$-4T_1 + T_2 + T_3 = -150 \quad (1)$$

node 2: $T_1 + 200 + 100 + 206.25 - 4T_2 = 0$

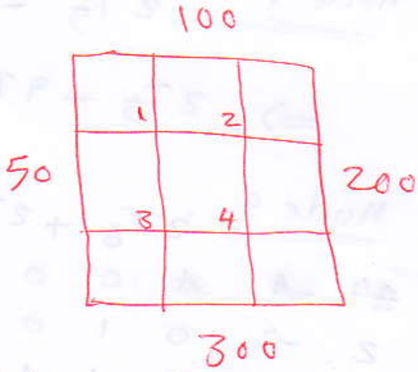
$$T_1 - 4T_2 + 0T_3 = -506.25 \quad (2)$$

node 3: $50 + 206.25 + T_1 + 300 - 4T_3 = 0$

$$T_1 + 0T_2 - 4T_3 = -556.25 \quad (3)$$

$$\Rightarrow T_1 = 118.76\,^\circ C, \quad T_2 = 156.25\,^\circ C, \quad T_3 = 168.75\,^\circ C$$

ex: find $T_1, T_2, T_3, T_4 = ????$



$-4T_1 + T_2 + T_3 = -150$ —— (1)

$T_1 - 4T_2 + T_4 = -300$ —— (2)

$T_1 - 4T_3 + T_4 = -350$ —— (3)

$T_2 + T_3 - 4T_4 = -500$ —— (4)

re arrangement eq (3) $\Rightarrow T_4 = 4T_3 - T_1 - 350$ —— (*)

equ. (*) in (2)

$T_1 - 4T_2 + 4T_3 - T_1 - 350 = -300$

$\Rightarrow -4T_2 + 4T_3 = 50$ —— (2)

equ. (*) in (4)

$T_2 + T_3 - 4(4T_3 - T_1 - 350) = -500$

$T_2 + T_3 - 16T_3 + 4T_1 + 1400 = -500$

$\Rightarrow 4T_1 + T_2 - 15T_3 = -1900$ —— (3)

$$\Rightarrow \begin{bmatrix} -4 & 1 & 1 & -150 \\ 0 & -4 & 4 & 50 \\ 4 & 1 & -15 & -1900 \end{bmatrix} \Rightarrow \begin{matrix} T_1 = 118.75\ ^\circ C \\ T_2 = 156.25\ ^\circ C \\ T_3 = 168.75\ ^\circ C \end{matrix}$$

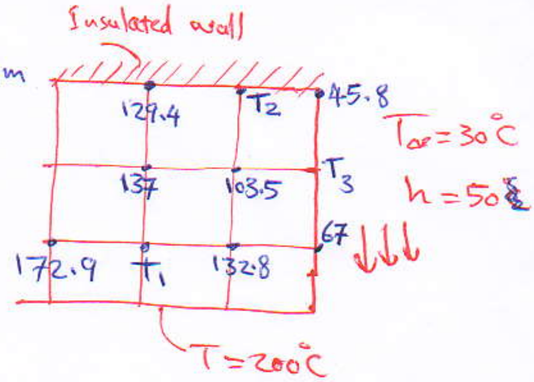
$\Rightarrow T_4 = 206.25\ ^\circ C$

ex 6

find $T_1, T_2, T_3 = ???$

find Q from wall to fluid

$Q_{conv.} = ?$ take $k = 1.5$



Solu.

node 1

$$172.9 + 137 + 132.8 + 200 - 4T_1 = 0$$

$$\Rightarrow \boxed{T_1 = 160.7\,°C}$$

node 2

$$2(103.5) + 129.4 + 45.8 - 4T_2 = 0$$

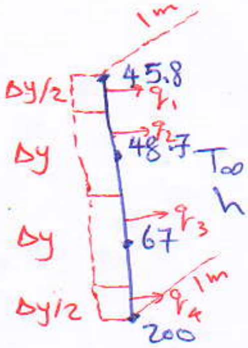
$$\Rightarrow \boxed{T_2 = 95.6\,°C}$$

node 3

$$2(103.5) + 45.8 + 67 + \frac{2 * 50 * 0.1}{1.5} * 30 - 2\left(\frac{50 * 0.1}{1.5} + 2\right)T_3 = 0$$

$$207 + 45.8 + 67 + 200 - 10.67\,T_3 = 0$$

$$\Rightarrow \boxed{T_3 = 48.7\,°C}$$



$$Q_{conv.} = q_1 + q_2 + q_3 + q_3$$

$q_1 = k[(\Delta y/2) * 1](45.8 - 30) =$ ~~1.185~~ $= 39.5$

$q_2 = k(\Delta y \cdot 1)(48.7 - 30) =$ ~~2.805~~ $= 93.5$

$q_3 = k(\Delta y \cdot 1)(67 - 30) =$ ~~5.55~~ $= 185$

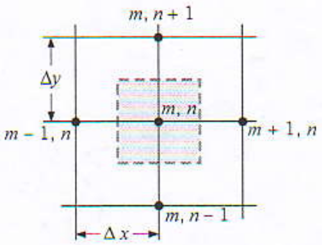
$q_4 = k[(\Delta y/2) * 1](200 - 30) =$ ~~12.75~~ $= 425$

$$\Rightarrow Q_{conv.} = 39.5 + 93.5 + 185 + 425$$

$$\Rightarrow \boxed{Q_{conv.} = 743 \text{ W / m depth}}$$

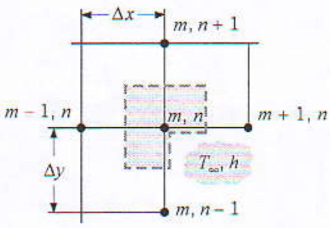
TABLE 4.2 Summary of nodal finite-difference equations

Configuration — **Finite-Difference Equation for $\Delta x = \Delta y$**



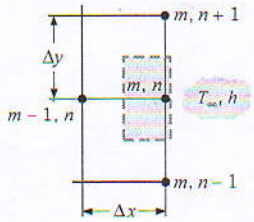
$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0 \tag{4.29}$$

Case 1. Interior node



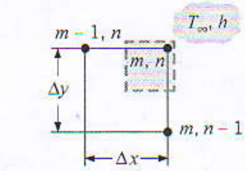
$$2(T_{m-1,n} + T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2\frac{h\,\Delta x}{k}T_\infty - 2\left(3 + \frac{h\,\Delta x}{k}\right)T_{m,n} = 0 \tag{4.41}$$

Case 2. Node at an internal corner with convection



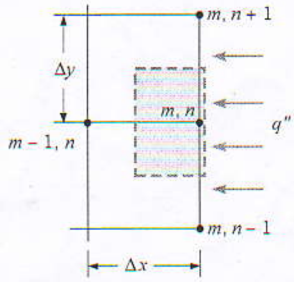
$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\,\Delta x}{k}T_\infty - 2\left(\frac{h\,\Delta x}{k} + 2\right)T_{m,n} = 0 \qquad (4.42)^a$$

Case 3. Node at a plane surface with convection



$$(T_{m,n-1} + T_{m-1,n}) + 2\frac{h\,\Delta x}{k}T_\infty - 2\left(\frac{h\,\Delta x}{k} + 1\right)T_{m,n} = 0 \tag{4.43}$$

Case 4. Node at an external corner with convection



$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2q''\,\Delta x}{k} - 4T_{m,n} = 0 \qquad (4.44)^b$$

Case 5. Node at a plane surface with uniform heat flux

[a,b] To obtain the finite-difference equation for an adiabatic surface (or surface of symmetry), simply set h or q'' equal to zero.

EXAMPLE 4.2

Using the energy balance method, derive the finite-difference equation for the (m, n) nodal point located on a plane, insulated surface of a medium with uniform heat generation.

Heat Transfer I

"Critical Radius of Insulation"

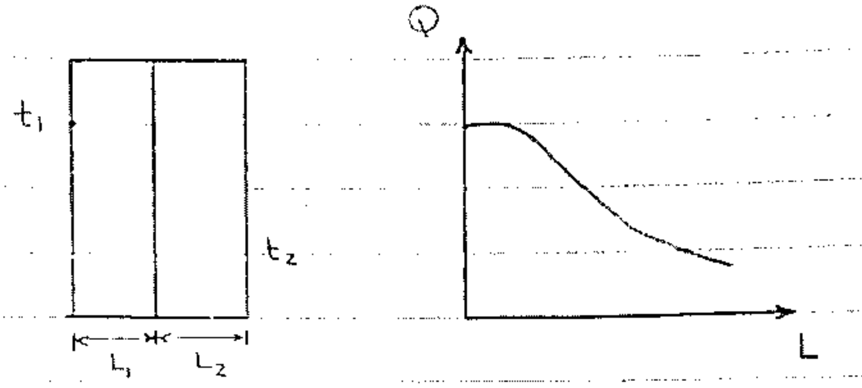
Lecture No. (8)

Critical Radius of Insulation

* For Cartizian wall:-

الزيادة في سمك العازل يؤدي إلى التقليل في معدل تدفق الحرارة في حالة الجدار حيث ثبوت مساحة السطح مع زيادة السمك.

$$Q = \frac{t_1 - t_2}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}}$$



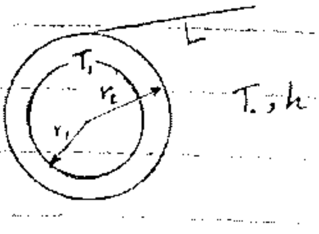
$Q \downarrow \quad L \uparrow \quad$ **A = Const** للجدار ولذلك لا يوجد سمك

---

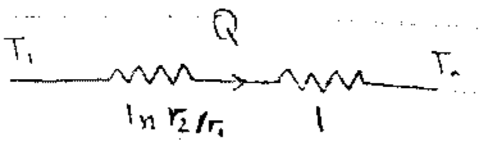
* For cylinderical wall:-

الزيادة في سمك العازل يؤدي إلى الزيادة في مساحة السطح الخارجية وبالتالي فإن زيادة سمك العازل يؤدي إلى زيادة معدل إنتقال الحرارة عن طريق زيادة مساحة السطح الخارجية وتظل هذه الزيادة إلى قيمة معينة للقطر الخارجي بعدها الزيادة في سمك العازل يؤدي إلى تقليل معدل إنتقال الحرارة وتلك القيمة المعينة للقطر الخارجي هي القيمة الحرجة.

$$Q = \frac{T_1 - T_0}{\frac{\ln r_2/r_1}{2\pi k L} + \frac{1}{h(2\pi L) r_2}}$$



$$\frac{\partial Q}{\partial r} = \frac{-\left[\frac{1}{2\pi k L}\cdot\frac{1}{r_2} - \frac{1}{h(2\pi L) r_2^2}\right](t_1 - t_0)}{\left(\frac{\ln r_2/r_1}{\ldots}\right)^2} = 0$$



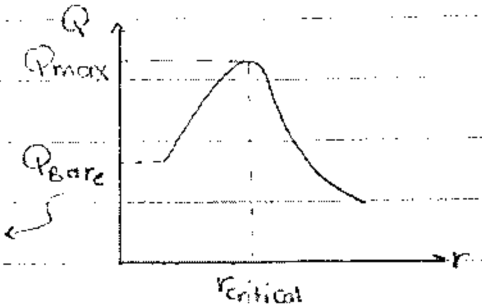
cylinderical

$$dQ_r = -K\, dZ\, r\, d\theta \frac{dt}{dr}$$

$$dQ_{(r+dr)} = -K\, dZ\, (r+dr)\, d\theta \cdot \frac{d}{dr}\left(t + \frac{dt}{dr} dr\right)$$

$$dQ_r - dQ_{(r+dr)} = K\, dZ\, dr\, d\theta \frac{dt}{dr}$$

then $\frac{1}{k} = \frac{1}{h r_2}$



$$r_2 = \frac{k}{h}$$

بدون عازل

$$\therefore r_{critical} = \frac{k}{h}$$ في حالة الإسطوانة

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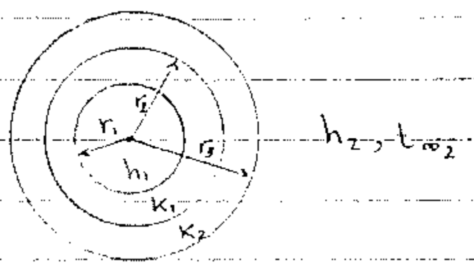
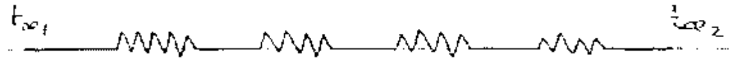
* For Sphere





$$\frac{1}{h_i A_1} \quad \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} \quad \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} \quad \frac{1}{h_o A_o}$$

$$Q = \frac{t_{\infty_1} - t_{\infty_2}}{\frac{1}{h_i A_i} + \frac{r_2 - r_1}{4\pi k_1 r_1 r_2} + \frac{r_3 - r_2}{4\pi k_2 r_2 r_3} + \frac{1}{h_o A_o}}$$

$\frac{\partial Q}{\partial r_3} = 0 \longleftarrow$ To get maximum Heat transfer.

$$r_{critical}\Big|_{sphere} = \frac{2 k_2}{h_2}$$

Conduction Shape Factor

معادلة انتقال الحرارة على أن تكتب في الصورة التالية
وذلك عندما تكون الأجسام
التي ندرسها أكثر تعقيداً .

$$Q = F\,K\,\Delta t$$

$F \rightarrow$ Conduction Shape Factor

« No Heat Generation »

- انتقال الحرارة يكون بالتوصيل

$$\therefore Q = \frac{\Delta t}{R}$$

$$\therefore F = \frac{1}{KR}$$

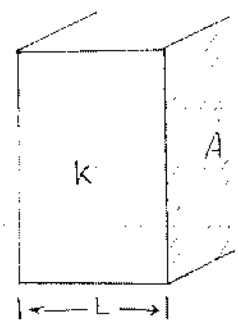
---

* For Plane wall:-

$$Q = K\,A\,\frac{\Delta t}{L}$$

$$R = \frac{L}{KA}$$

$$F = \frac{A}{L}$$



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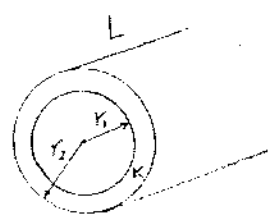
* For Cylinder :-

$$Q = \frac{K\,\Delta t}{\frac{\ln r_2/r_1}{2\pi L}}$$

$$F = \frac{2\pi L}{\ln r_2/r_1}$$



* Sphere

$$F = \frac{4\pi}{(r_2 - r_1)/r_1 r_2}$$

Ex 3.9

Given:

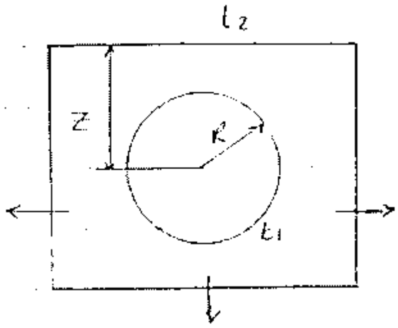
R = 1 m

Z = 4 m

Q = 500 watt

$t_2 = 20$ °C

K = 1 W/m.K



Req: t_1 = ??

Sol

$$F = \frac{4\pi R}{1 + \frac{R}{2Z}} = \frac{4\pi}{1 + \frac{1}{2*4}} = 14.36 \text{ m}$$

$$Q = FK(t_1 - 20)$$

$$500 = 14.35 * 1 * (t_1 - 20)$$

$$t_1 = 54.8 \text{ °C}$$

Effect of variable Conduction :-

$$k = k_o \, (1 + \alpha t)$$

- k → thermal Conductivity
- k_o → at 0°C.
  الحرارية بالموصلية عند
  درجة صفر سيلزيوس
- α = Const.
  - ⊕ Insulating Material
  - ⊖ metalic Conductors

Prove that for plane wall and thermal Conductivity is variable that

$$q_x = \frac{-k_o \left[2 + \alpha (t_2 + t_1)\right](t_2 - t_1)}{2(x_2 - x_1)}$$

Sol :-

$$q_x = -k_o \left[1 + \alpha t\right] \frac{dt}{dx}$$

$$q_x \left[x_2 - x_1\right] = -\int_{t_1}^{t_2} k_o \left[1 + \alpha t\right] dt$$

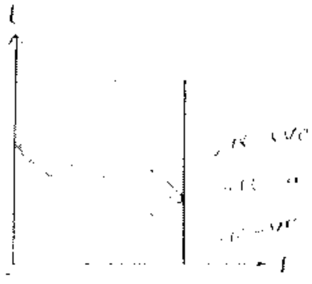
$$q_x \left[x_2 - x_1\right] = -k_o \left[\frac{2t + \alpha t^2}{2}\right]_{t_1}^{t_2}$$

$$q_x \left[x_2 - x_1\right] = -k_o \left[\frac{2(t_2 - t_1) + \alpha (t_2^2 - t_1^2)}{2}\right]$$

$$q_x \left[x_2 - x_1\right] = -\frac{k_o}{2} \left[2 + \alpha (t_2 + t_1)\right](t_2 - t_1)$$

$$q_x = \frac{-k_o \left[2 + \alpha (t_2 + t_1)\right](t_2 - t_1)}{2(x_2 - x_1)}$$

- If α +ve → curve is convex
- If α −ve → curve is concave
- If α = 0 → curve is linear



- For Hollow Cylinders :-

$$k = k_o\,(1+\alpha t)$$

$$Q = -k_o\,(1+\alpha t)\cdot 2\pi r L\cdot\frac{dt}{dr}$$

$$\int_{r_1}^{r_2} Q\,\frac{1}{r}\,dr = \int_{t_1}^{t_2} -k_o\,(1+\alpha t)\,2\pi L\,dt$$

$$Q\,\ln\frac{r_2}{r_1} = -2\pi L k_o\left[(t_2-t_1)+\frac{\alpha}{2}\,(t_2^2-t_1^2)\right]$$

$$\therefore\ Q = \frac{-2\pi L k_o\left[(t_2-t_1)+\frac{\alpha}{2}(t_2^2-t_1^2)\right]}{\ln r_2/r_1}$$

- For Hollow Sphere :-

$$Q = -kA\,\frac{dt}{dr} = -k_o\,(1+\alpha t)\,4\pi r^2\,\frac{dt}{dr}$$

$$\int_{r_1}^{r_2}\frac{Q}{r^2}\,dr = \int_{t_1}^{t_2} -4\pi k_o\,(1+\alpha t)\,dt$$

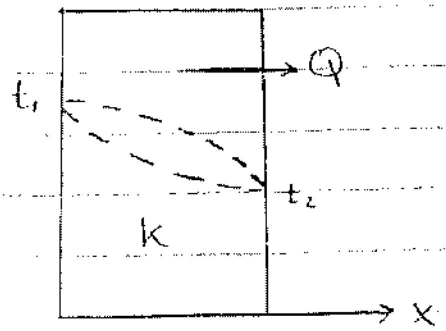
$$Q\left[\frac{-1}{r_2}+\frac{1}{r_1}\right] = -4\pi k_o\left[(t_2-t_1)+\frac{\alpha}{2}\,(t_2^2-t_1^2)\right]$$

$$Q\left[\frac{r_2-r_1}{r_2 r_1}\right] = -4\pi k_o\left[(t_2-t_1)+\frac{\alpha}{2}\,(t_2^2-t_1^2)\right]$$

$$Q = \frac{-4\pi k_o\left[(t_2-t_1)+\frac{\alpha}{2}(t_2^2-t_1^2)\right]}{(r_2-r_1)/r_2 r_1}$$

Pro 3.36

Given $K = K_o\, e^{-x/L}$



Req :- Temperature Distribution

Sol :-

General equation of heat conduction, one dimension

$$\frac{1}{r^n}\frac{d}{dr}\left(r^n k \frac{dt}{dr}\right) + q_v = \rho C_p \frac{dt}{d\tau}$$

(with $q_v = 0$ and $\rho C_p \frac{dt}{d\tau} = 0$)

$n = 0$, $dr = dx$

$$\frac{d}{dx}\left(k \frac{dt}{dx}\right) = 0$$

$$k \frac{dt}{dx} = C_1 \Longrightarrow K_o e^{-x/L} \frac{dt}{dx} = C_1$$

$$\frac{dt}{dx} = \frac{C_1}{K_o} e^{+x/L}$$

$$t(x) = \frac{C_1 L}{K_o} e^{x/L} + C_2$$

Boundary Condition

at $x = 0$ $t = t_1$

$$t_1 = \frac{C_1 L}{K_o} + C_2$$

at $x = L$ $t = t_2$

$$t_2 = \frac{C_1 L}{K_o} e + C_2$$

Get: C_1, C_2

$$t(x) = \frac{C_1 L}{K_0} e^{x/L} + C_2$$

Pro 3.14

$$K = 0{,}03 + 5 * 10^{-6} t^2$$

Sol

$$q = -K \frac{dt}{dx}$$

$$\int_{x_1}^{x_2} q \, dx = \int_{t_1}^{t_2} (-0{,}03 - 5 * 10^{-6} t^2)\, dt$$

$$q(L-0) = -\left[0{,}03\, t + \frac{5*10^{-6}}{3} t^3\right]\Bigg|_{t_1}^{t_2}$$

$$-q(0{,}25) = 0{,}03(40-300) + \frac{5*10^{-6}}{3}(40^3 - 300^3)$$

$$q = 210.7 \text{ watt/m}^2$$

If Required average thermal Conductivity

$$T_{av} = \frac{T_1 + T_2}{2}$$

$$K_{av} = 0{,}03 + 5*10^{-6}\left(\frac{T_1+T_2}{2}\right)^2 = 0{,}1745 \text{ W/m.K}$$

$$q = K_{av} \frac{\Delta t}{L} = 181{,}48 \text{ watt}$$

Ex 3.37

Given

$k_1 = k_2 = 20 \ W/m\text{-}k$

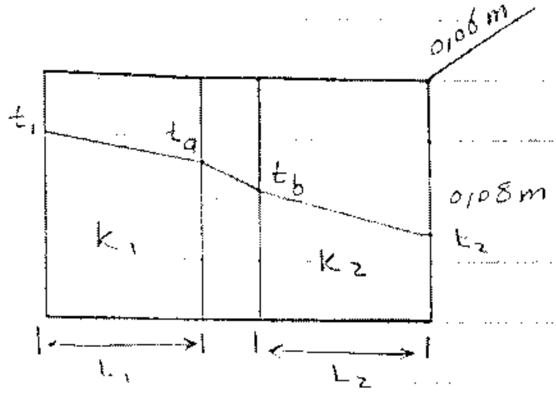
$L_1 = L_2 = 0.01 \ m$

$P = 20 \ atm$

$\mathcal{E} = 0.75 \ \mu m$

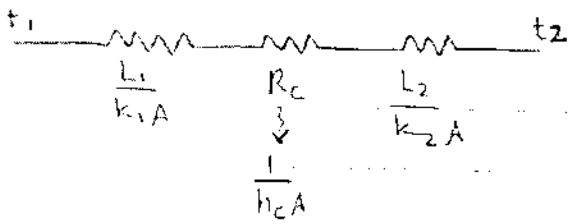
$t_1 = 120 ^\circ C$

$t_2 = 10 ^\circ C$



Req $Q = ??$, $(t_a - t_b)$

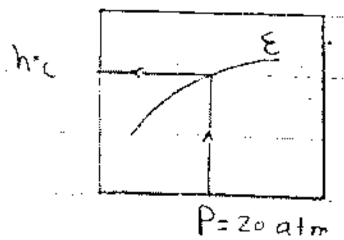
Sol :-



Get $h_c = \checkmark$

$$Q = \frac{t_1 - t_2}{R_1 + R_c + R_2}$$

$$= \frac{t_a - t_b}{R_c}$$



Get $Q = \checkmark$

$t_a - t_b = \checkmark$

Heat Transfer I

"Conduction General Equation for plane Wall , Cylinder and sphere"

Lecture No. (9)

(ch 4)

Conduction General Equation for Plane wall, cylinder sphere for one Dimension :-

$$\frac{1}{r^n}\frac{d}{dr}\left(r^n k \frac{dt}{dr}\right) + q_v = \rho c_p \frac{dt}{d\tau}$$

For Plane wall $n = 0$

For cylinder $n = 1$

For sphere $n = 2$

* دائما حل مسائل يكون steady-state $\rho c_p \frac{dt}{d\tau} = 0.0$

* ويكون في اتجاه واحد one dimensional

* Given :-

$I = 34000$ amp

$L = 1.25$ cm

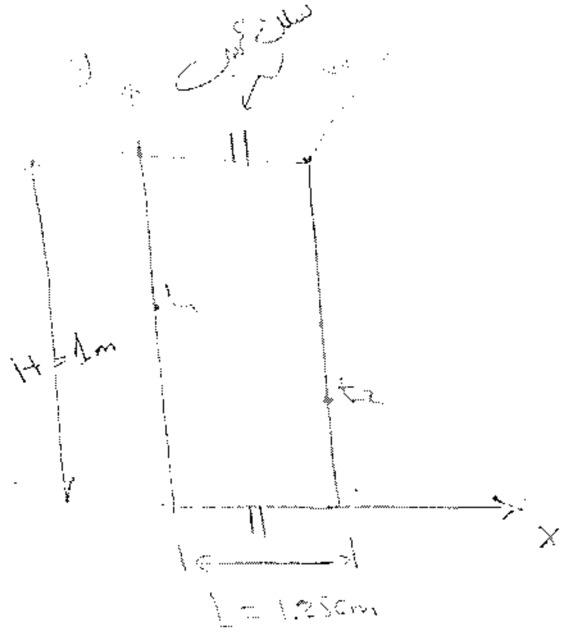
$W = 10$ cm

$t_1 = 95°C$

$t_2 = 80°C$

$\rho = 12 \times 10^{-6}\ \Omega.cm$

$K = 54$ w/m.K



Req

i) $T(x)$

ii) maximum Temp , location

Sol :-

$$R = \frac{\rho . L}{A} = \frac{\rho . H}{A} = \frac{12 \times 10^{-5} \times 10^{-2} \times 1}{1.25 \times 10^{-2} \times 10 \times 10^{-2}} = \quad \Omega$$

$$Q = I^2 R = 34000 \times \quad = \text{watt}$$

$$q_v = \frac{Q}{\text{Volume}} = \frac{\quad}{1.25 \times 10^{-2} \times 10 \times 10^{-2} \times 1} = \quad w/m^3$$

$$\frac{d^2t}{dx^2} + \frac{d^2t}{dy^2} + \frac{d^2t}{dz^2} + \frac{q_0}{k} = \frac{1}{\alpha}\frac{dt}{d\tau}$$

$$\frac{d^2t}{dx^2} + \frac{q_0}{k} = 0.0$$

$$\frac{dt}{dx} = \frac{-q_0}{k}x + C_1$$

$$\boxed{t(x) = \frac{-q_0}{k}\frac{x^2}{2} + C_1 x + C_2}$$

at $x = 0$ $\quad t = t_1 \quad \boxed{t_1 = C_2}$

at $x = L$ $\quad t = t_2 \quad t_2 = \frac{-q_0}{k}\frac{L^2}{2} + C_1 L + t_1$

$$\boxed{C_1 = \frac{t_2 - t_1}{L} + \frac{q_0}{k}\frac{L}{2}}$$

$$\boxed{t(x) = \frac{-q_0}{k}\frac{x^2}{2} + \left[\frac{t_2 - t_1}{L} + \frac{q_0}{k}\frac{L}{2}\right]x + t_1}$$

To get T_{max} $\quad \frac{dt}{dx} = 0.0$

$$\frac{q_0}{k}x = \frac{t_2 - t_1}{L} + \frac{q_0}{k}\frac{L}{2}$$

$$\boxed{x_{max} = \frac{L}{2} + \frac{k}{q_0}\left(\frac{t_2 - t_1}{L}\right)}$$

Get $T_{max} =$

Given:

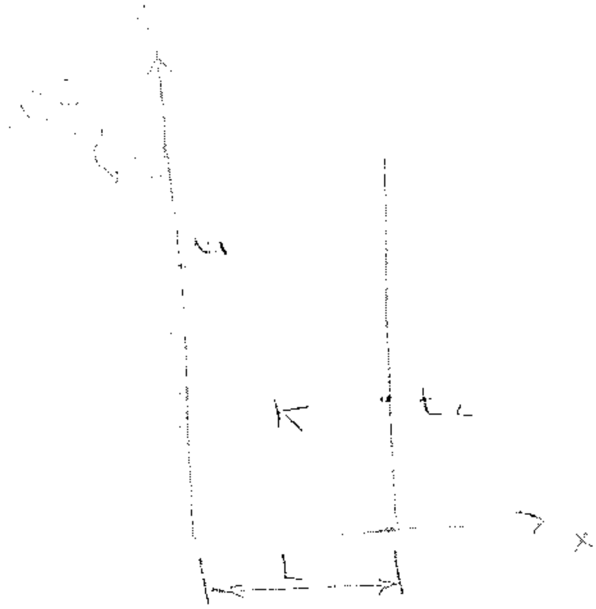
$Q = 800\ watt$

$L = 0.6\ cm$

$A = 160\ cm^2$

$K = 20\ w/mK$

$t_2 = 85^\circ C$



Req:- $T(x)$ & T_{max}

Sol:-

$$\frac{d^2t}{dx^2} = 0.0$$

$$\frac{dt}{dx} = C_1$$

$$t(x) = C_1 x + C_2$$

at $x = 0$

$$q = Q/A = -K \frac{\partial t}{\partial x}\Big|_{x=0}$$

$$C_1 = \frac{-Q}{AK}$$

at $x = L$ $\quad t = t_2$

$$t_2 = \frac{-Q}{AK} L + C_2$$

$$C_2 = t_2 + \frac{Q}{AK} L$$

$$T(x) = \frac{-Q}{AK} x + t_2 + \frac{Q}{AK} L$$

$x = 0$

$t_{max} = t_1 = ...$

[illegible]

[illegible] -

[illegible]

L [illegible] cm

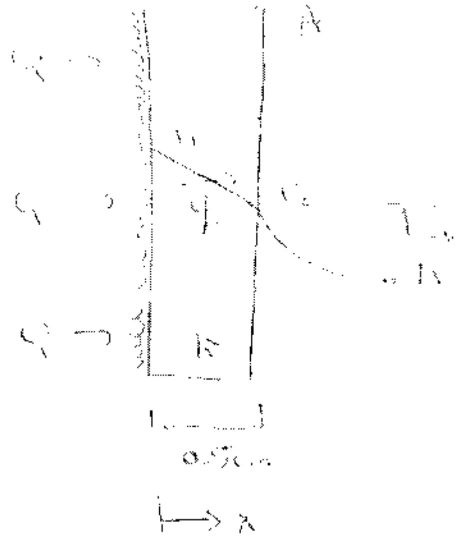
A [illegible]

k [illegible]

T_∞ [illegible]

$h = 3$[illegible] $W/m^2 C$

[illegible] - $T(x) = ??$



[illegible] -

$$q_- = q/A = \frac{[illegible]}{[illegible]} = 4000 \text{ watt}$$

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{dT}{dt}$$

$$dT/dx = C_1$$

$$T(x) = C_1 x + C_2$$

$\to$ at $x = 0$ $\qquad q = -k \left.\frac{dT}{dx}\right|_{x=0}$

$$\boxed{C_1 = \frac{-q}{k}}$$

at $x = L$ $\qquad q_{conv} = q_{cond}$

$$h(T_2 - T_\infty) = -k \left.\frac{dT}{dx}\right|_{x=L}$$

$$h(C_1 L + C_2 - T_\infty) = -k C_1$$

$$\boxed{C_2 = T_\infty + \frac{q}{h} + \frac{qL}{k}}$$

$$T(x) = \frac{-q}{k}x + \frac{qL}{k} + \frac{q}{h} + T_\infty \quad \swarrow \text{Required}$$

$$T(x) = \frac{qL}{k}\left(1 - \frac{x}{L}\right) + q/h + T_\infty$$

5

Given

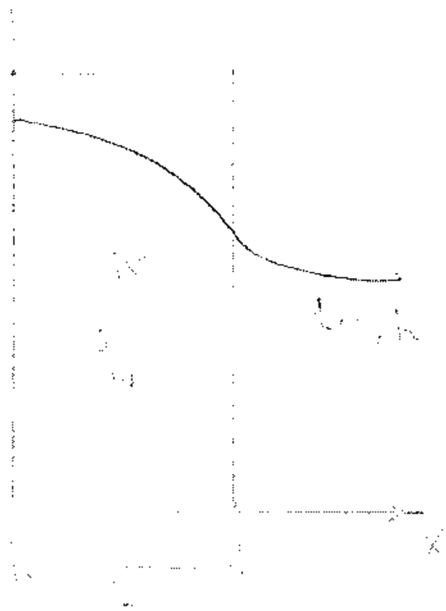
$L = 5\,cm$

$k = 111\ w/m\,°C$

$q_0 = 2\times10^5\ w/m^3$

$t_\infty = 25\,°C$

$h = 44\ w/m^2\,°C$



Req $\quad T_{max}$, T_{min} , $x|_{T_{max}}$, $x|_{T_{min}}$

Sol

$$\frac{d^2t}{dx^2} + \frac{d^2t}{dy^2} + \frac{d^2t}{dz^2} + \frac{q_0}{k} = \frac{1}{\alpha}\frac{dt}{d\tau}$$

$$d^2t/dx^2 = \frac{-q_0}{k}$$

$$\frac{dt}{dx} = \frac{-q_0}{k}x + C_1$$

$$\boxed{t(x) = \frac{-q_0}{k}\frac{x^2}{2} + C_1x + C_2}$$

* at $x = 0$ $\quad \frac{dt}{dx} = 0.0 \quad \boxed{C_1 = 0}$

* at $x = L$ $\quad q = -k\frac{dt}{dx}\Big|_{x=L} = h(t(L) - t_\infty)$

$$-k \left.\frac{dt}{dx}\right|_{x=L} = h\,(t(L) - t_\infty)$$

$$-k\left(\frac{-q_0}{k} L + C_1\right) = h\left[\frac{-q_0}{k}\frac{L^2}{2} + C_1 L + C_2\right] - h t_\infty$$

$$\boxed{C_2 = \frac{q_0 L}{h} + t_\infty + \frac{q_0}{2k} L^2}$$

$$t(x) = \frac{q_0}{2k}\left(L^2 - x^2\right) + t_\infty + q_0 L / h$$

Temp

For maximum location $\frac{dt}{dx} = 0.0$ $\quad X = 0$

$$\boxed{T_{max} = \frac{q_0}{2k} L^2 + t_\infty + \frac{q_0 L}{h}}$$

عن شكل المعادلة

For minimum Location temp at $(X = L)$ ← الحرارة تقل عند $X = L$

$$\boxed{T_{min} = t_\infty + \frac{q_0 L}{h}}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right) + \frac{d^2t}{dz^2} + \frac{1}{r^2}\frac{d^2t}{d\theta^2} + \frac{q_o}{k} = \frac{1}{\alpha}\frac{dt}{d\tau}$$

* Given:-

$D = 5\,cm$

$k = 29.5\ w/m.K$

$q_o = 7\times10^7\ w/m^3$

$t_o = 140°C$

Req:- Temp at Center

Sol:-

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right) = \frac{-q_o}{k}$$

$$\frac{dt}{dr} = \frac{-q_o}{k}\frac{r}{2} + \frac{C_1}{r}$$

$$t(r) = \frac{-q_o}{k}\frac{r^2}{4} + C_1 \ln r + C_2$$

$$t_o = t(R) \qquad t_o = \frac{-q_o}{k}\,\frac{R^2}{4} + C_2$$

$$C_2 = t_o + \frac{q_o}{4k} R^2$$

$$t(r) = \frac{-q_o}{4k} r^2 + t_o + \frac{q_o}{4k} R^2$$

at $r = 0$

$$t\Big|_{Center} = t_o + \frac{q_o}{4k} R^2$$

$t_{Center} = 16$[illegible] °C

Heat Transfer I

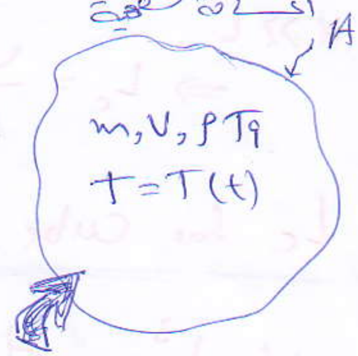
Transient Heat Conduction"

Lecture No. (10)

* Unsteady state Conduction

* Transient Heat Conduction

Lumped heat capacity systems



$\dot{Q}_{conv.} = hA(T_\infty - T(t))$

$T_i = T$ at time = 0
Initial temperature

Heat transfer into the body during dt	=	The increase in the energy of the body during dt

$$\Rightarrow hA(T_\infty - T)\,dt = m\,c_p\,dT$$

$$m = \rho V$$

$$\Rightarrow \rho V c_p\,dT = hA(T_\infty - T)\,dt \qquad (*-1)$$

$$\Rightarrow -\rho V c_p\,dT = hA(T - T_\infty)\,dt$$

$$\boxed{d(T - T_\infty) = dT}$$

$$\Rightarrow \frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA}{\rho V c_p}\,dt$$

Integrating from $t = 0$ & $T = T_i$ to any time t & $T = T(t)$,

$$\ln(T - T_\infty)\Big|_0^t = -\frac{hA}{\rho V c_p}\,t\Big|_0^t$$

$$\Rightarrow \ln(T(t) - T_\infty) - \ln(T_i - T_\infty) = -\frac{hA}{\rho V c_p}\cdot t = \ln\frac{T(t) - T_\infty}{T_i - T_\infty}$$

$$\Rightarrow \boxed{\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho V c_p}\cdot t}}$$

تطبق هذه المعادلة وفق شروط محددة

$\boxed{B_i < 0.1}$

* Criteria for Lumped System Analysis :

* characteristic length (L_c)

$$L_c = \frac{V}{A_s}$$, V = Volume (m^3) , A_s = surface area

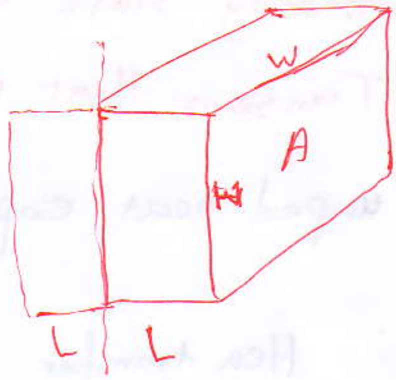
L_c for plane wall :

لنصف جدار ذو سمك L
وذو ارتفاع كبير مقارنة بالسمك

$H \gg L$

$$\Rightarrow L_c = \frac{V}{A} = \frac{L \cdot A}{A} \Rightarrow \boxed{L_c = L}$$

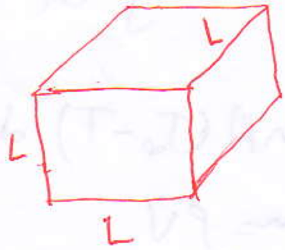
نصف السمك



L_c for Cube body

$$V = L^3 \,, \quad A = 6L^2$$

$$\Rightarrow L_c = \frac{L^3}{6L^2} \Rightarrow \boxed{L_c = \frac{L}{6}}$$



for Long cylinder

$$L \gg r_o \Rightarrow A_s = 2\pi r_o L + 0$$

تهمل مساحة القاعدتين

$$V = \pi r_o^2 L$$

$$\Rightarrow L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} \Rightarrow \boxed{L_c = \frac{r_o}{2}}$$

for cyliner

$$A_s = 2\pi r_o L + 2\pi r_o^2 = 2\pi r_o (r_o + L)$$

$$V = \pi r_o^2 L \Rightarrow L_c = \frac{V}{A} = \frac{\pi r_o^2 L}{2\pi r_o (r_o + L)}$$

$$\Rightarrow \boxed{L_c = \frac{r_o L}{2(r_o + L)}}$$

for sphere

$$A_s = 4\pi r_o^2 \,, \quad V = \frac{4}{3}\pi r_o^3$$

$$L_c = \frac{V}{A} = \frac{\frac{4}{3}\pi r_o^3}{4\pi r_o^2} \Rightarrow \boxed{L_c = \frac{r_o}{3}}$$

* Biot Number (Bi)

$$Bi = \frac{\text{convection heat transfer from the body}}{\text{conduction heat transfer in the body}}$$

$$Bi = \frac{h\,L_c}{K} = \frac{h\,V}{K\,A_s} \quad , \quad Bi \ll 1$$

$Bi < 0.1$ (شرط تطبيق نظرية السعات الحجمية)

* Fourier Number (Fo)

$$Fo = \frac{\text{heat conduction rate}}{\text{rate of thermal energy storage}}$$

$$Fo = \frac{\alpha\, t}{L^2}$$

$$\frac{h A_s t}{\rho V c} = \frac{h t}{\rho c L_c} = \frac{h L_c}{K} \cdot \frac{K}{\rho c} \cdot \frac{t}{L_c^2} = \underbrace{\frac{h L_c}{K}}_{Bi} \cdot \underbrace{\frac{\alpha t}{L_c^2}}_{Fo}$$

$$\frac{\theta}{\theta_i} = e^{-Bi\,Fo} \quad , \quad \theta_i = T_i - T_\infty$$

$$Q = \rho V c\, \theta_i \left(1 - e^{-\frac{hA t}{\rho V c}}\right)$$

Q (Joul)

$\theta_i - \theta_i e^{-\frac{hA t}{\rho c V}} =$

$= \theta_i - \theta$

$= T_i - T_\infty - T + T_\infty$

$= (T_i - T)$

Q = The heat losses for the body in time t

ex1 A piece of aluminum weighting 6 kg and initially at a temperature of 300 °C is suddenly immersed in a fluid at 20 °C. The convection heat transfer coefficient is 58 $w/m^2 \cdot °C$. Taking the aluminum as a sphere having the same weight as that given, estimate the time required to cool the aluminum to 90 °C, using the lumped capacity method analysis ($\rho = 2707$ kg/m^3; $c = 896$ $J/kg \cdot °C$, $k = 237$)

solu.

$$Bi = \frac{hL_c}{K}, \quad L_c = \frac{r_o}{3} \Rightarrow Bi = \frac{58 * 0.0807}{3 * 273} = 0.00683 < 0.1$$

$$m = \rho V = \frac{4}{3}\pi r^3 (2707) = 6 \Rightarrow r_o = 0.0807 \, m$$

$\Rightarrow$ lumped capacity can be used.

$$A = 4\pi r^2 = 0.0822 \, m^2$$

$$\frac{hA}{\rho c V} = \frac{58 (0.0822)}{6 (896)} = 8.87 * 10^{-4}$$

$$\ln \frac{90 - 20}{300 - 20} = -8.87 * 10^{-4} t \Rightarrow \boxed{t = 1563 \, s}$$

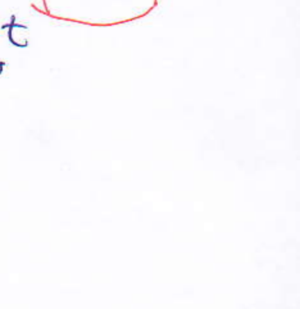
---

ex2 A stainless steel rod ($\rho = 7817$, $c = 460$, $k = 14$) 6.4 mm in diameter is initially at a uniform temperature of 25 °C and is suddenly immersed an a liquid at 150 °C with $h = 120$ $w/m^2 \cdot °C$. Using the Calculate the time necessary for the rod temperature to reach 120 °C

solu.

assume long cylinder with $L = 1 \, m$ $\quad D/L << 1 \Rightarrow L_c \simeq \frac{r_o}{2}$

$$Bi = \frac{hL_c}{K} = \frac{120 * 3.2 * 10^{-2}}{2 * 14} = 0.0137 < 0.1$$



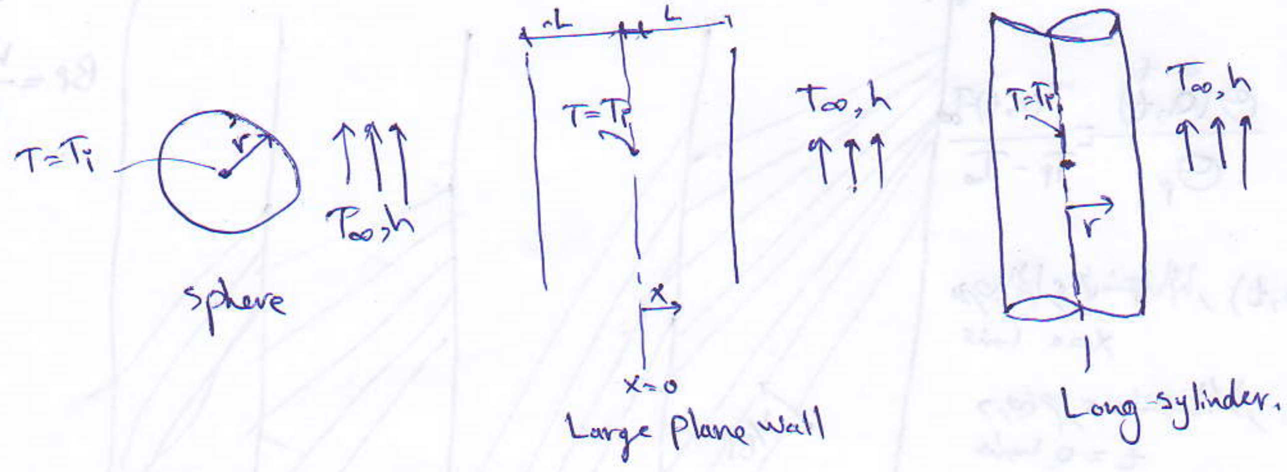
$\therefore$ lumped capacity method can be used.

$$\frac{A}{V} = \frac{1}{L_c} = \frac{4}{D}, \quad \frac{hA}{\rho c V} = \frac{120 * 4}{7817 * 460 * 0.0064} = 0.02086$$

$$\Rightarrow \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho c V} t} \Rightarrow \ln \frac{120 - 25}{150 - 25} = -0.02086 t$$

$$\Rightarrow \boxed{t = 13.16 \, sec}$$

* Transient Heat Conduction in Large Plane walls, Long cylinders & spheres ($T(t,x)$, $T(t,r)$)



Dimensionless temperature : $\Theta(t,x) = \dfrac{T(t,x) - T_\infty}{T_i - T_\infty}$

Dimensionless distanse from the center : $X = \dfrac{x}{L}$

Dimensionless heat transfer coefficient : $Bi = \dfrac{hL}{k}$ (Biot number)

Dimensionless time : $Fo = \dfrac{\alpha t}{L^2}$ (Fourier number).

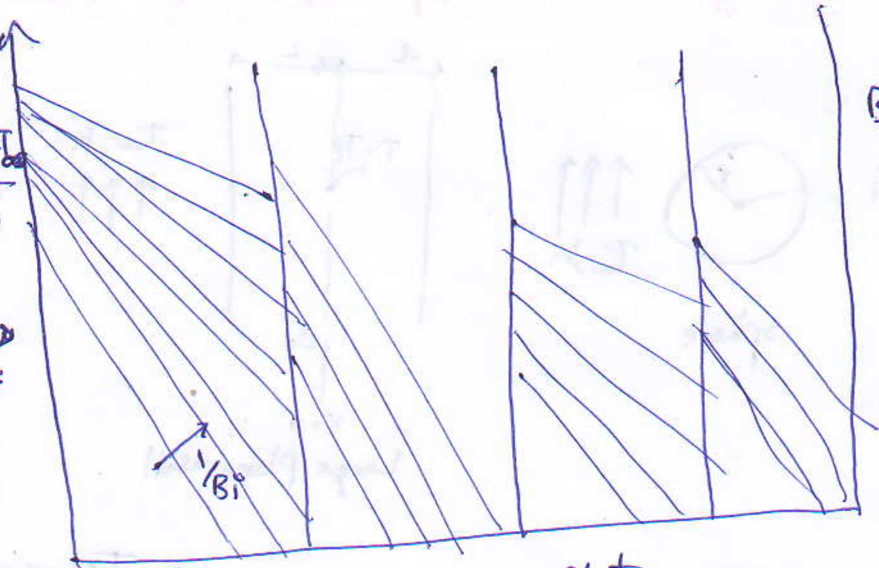
Q_p = represents the initial internal energy content of the body in the refrence to the environment temperature.

$$Q_i = mc\,(T_i - T_\infty) = \rho V C\,(T_p - T_\infty) = \rho V C\,\Theta_p$$ (Jule)

Figure (4-1)a pp. 16 (Large plate with width of 2L)

$$\frac{\Theta(0,t)}{\Theta_i} = \frac{T(0,t)-T_\infty}{T_i - T_\infty}$$

$$Bi = \frac{hL}{K}$$



$T(0,t)$ درجة الحرارة في منتصف الجدار عندما $x=0$

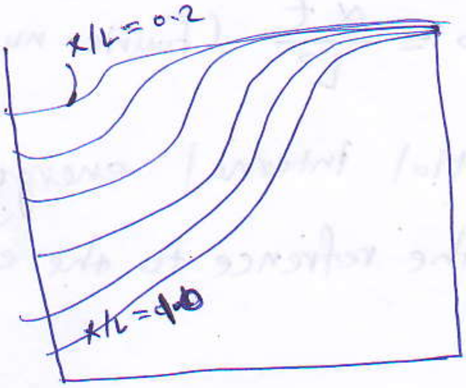
$T_i =$ درجة حرارة منتصف الجدار عندما $t=0$

$$\rightarrow Fo = \frac{\alpha t}{L^2}$$

* المخطط يحتوي على ثلاث قيم تصف النقطة الواحدة لمعرفة احداثيات النقطة يجب معرفة قيمتين على الأقل لمعرفة القيمة المجهولة الثالثة

Figure (4-1) b pp. 17

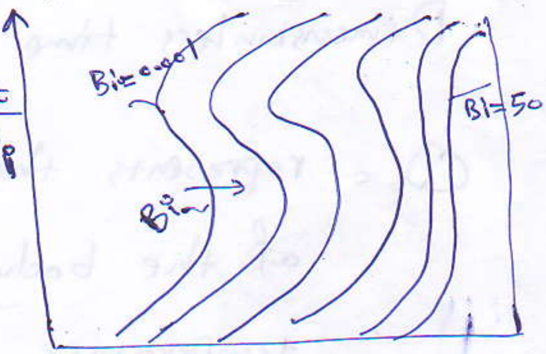
$$\frac{\Theta(x,t)}{\Theta(0,t)} = \frac{T(x,t)-T_\infty}{T(0,t)-T_\infty}$$



$\rightarrow (1/Bi)$

Figure (4-1) c pp. 17

$$\frac{Q_t}{Q_i}$$



$$\overrightarrow{Bi^2 Fo} = \frac{h^2 \alpha t}{K^2}$$

* من المخطط (b) يمكن معرفة درجة الحرارة المتغيرة مع محور x (على بعد x) من مركز الجدار) بمعرفة درجة الحرارة في مركز الجدار عند نفس الزمن

* من المخطط (c) يمكن معرفة مقدار الحرارة المفقودة او المكتسبة مع الزمن Q_t بمعرفة مقدار الطاقة الحرارية الابتدائية Q_i من القانون

$$Q_i = \rho V C \Theta_i$$

* Figure (4-2) a PP. 14 (Long cylinder) to find Θ_t
* figure (4-2) b, c PP. 15 (Long cylinder) to find Θ_r & Q_t
* figure (4-3) a PP. 12 (sphere) to find Θ_t ($T(o,r)$)
* figure (4-3) b, c PP. 13 (sphere) to find Θ_r & Q_t

ex 3 In a fabrication prosess, steel components are formed hot and then quenched in water. Consider a 2 m longe, 0.2 m diameter steel cylinder ($k = 40$, $\alpha = 1 * 10^{-5}\ m^2/s$) initially at 400°C, that is suddenly quenched water at 50°C. If the heat transfer coefficient is 200 W/m².K calculate the following 20 min. after immersion:
(1) the center temperature (2) the surface temperature.
(3) the heat transfer to the water during the initial 20 min.

solu: $Bi = \frac{h L_c}{2k}$, $L_c \simeq \frac{r_o}{2} \Rightarrow Bi = \frac{200 * 0.1}{2 * 40} = 0.25 > 0.1$

we can not ues the lumped capacity method.

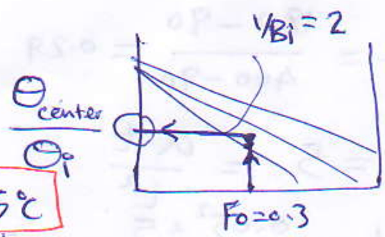
to use charts

$$Fo = \frac{\alpha t}{r_o^2} = \frac{1*10^{-5} * 20 * 60}{(0.1)^2} = 1.2 , \quad Bi = \frac{h r_o}{k} = \frac{200 * 0.1}{40} = 0.5$$

$$Bi^2 Fo = 0.5^2 * 1.2 = 0.3$$

from fig. (4-2) a with $1/Bi = 2$, $Fo = 0.3$

$$\Rightarrow \frac{T(o,t) - T_\infty}{T_i - T_\infty} = 0.35$$

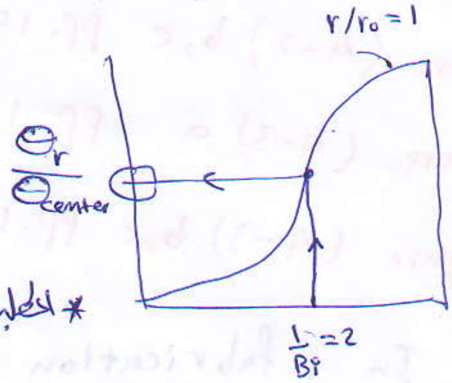


$$\Rightarrow \frac{T(o,t) - 50}{400 - 50} = 0.35 \Rightarrow \boxed{T(o,t) = 172.5°C}$$

* الطلب الأول
درجة الحرارة مركز اسطوانة بعد مرور 20 دقيقة

from fig (4.2) b with curve $r/r_o = 1$ & $\frac{1}{Bi} = 2$

$$\Rightarrow \frac{T(r_o,t) - T_\infty}{T(o,t) - T_\infty} = 0.8 \qquad (T_\infty = 50,\ T(o,t) = 172.5)$$



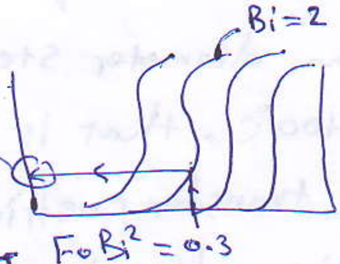
$$\Rightarrow \boxed{T(r_o,t) = 148\ ^\circ C}$$

* المطلب الثاني درجة حرارة السطح الخارجي بعد مرور 20 دقيقة

from fig (4.2) c with $Bi = 0.5$ & $Bi^2 Fo = 0.3$

$$\Rightarrow \frac{Q_t}{Q_i} = 0.61 \ , \ Q_i = \rho C V (T_i - T_\infty)$$

$$, \rho c = \frac{k}{\alpha}$$



$$\Rightarrow Q_i = \frac{k}{\alpha} \pi r_o^2 \cdot L \,(T_i - T_\infty) \qquad L = 2$$

$$\frac{40}{1*10^{-5}} * \pi * 0.1^2 * 2 * (400 - 50) = 8.8*10^7 \text{ J}$$

$$Q_t = 0.61 * Q_i \Rightarrow \boxed{Q_t = 53680 \text{ kJ}}$$

* المطلب الثالث كمية الحرارة المفقودة بعد مرور 20 دقيقة

4.39 H

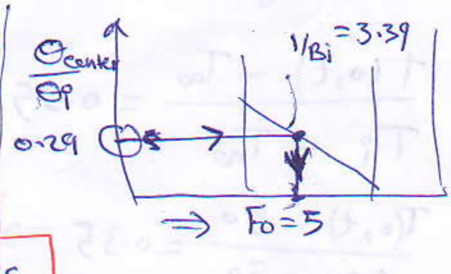
ex4/ A large slab of aluminum ($k = 230$, $\alpha = 8.42*10^{-5}$) has a thickness of 10 cm and is initially uniform in temperature at 400°C. Suddenly it is exposed to a convection environment at 90°C with $h = 1400$ W/m².c. How long dose it take the centerline temperature to drop to 180°C

Solun $T_i = 400\ ^\circ C$, $T_\infty = 90$, $h = 1400$, $T(t) = 180\ ^\circ C$

$L = 5$ cm $\quad \frac{k}{hL} = 3.39 = \frac{1}{Bi} \quad , \ Fo = \frac{\alpha t}{L^2}$

from fig (4.1) a with $\frac{1}{Bi} = 3.39$ & $\frac{\theta}{\theta_i} = 0.29$

$$\Rightarrow \frac{\theta_{center}}{\theta_i} = \frac{180 - 90}{400 - 90} = 0.29 \quad \& \quad \frac{1}{Bi} = 3.39$$



$$\Rightarrow Fo = 5 = \frac{\alpha t}{L^2}$$

$$\Rightarrow t = \frac{0.05^2 * 5}{8.42*10^{-5}} \Rightarrow \boxed{t = 148 \text{ sec}}$$

Heat Transfer I

"Radiation Heat Transfer"

Lecture No. (11)

Radiation Heat Transfer

Introduction:

So far, we have considered the [illegible]uction and conve[illegible] modes of heat transfer, which are related to the [illegible]

[illegible]

Phy ... Ph homsine

... of ... electro ... thermal ... of the type ... wave ... speed of light ... This ... the ... of the ...

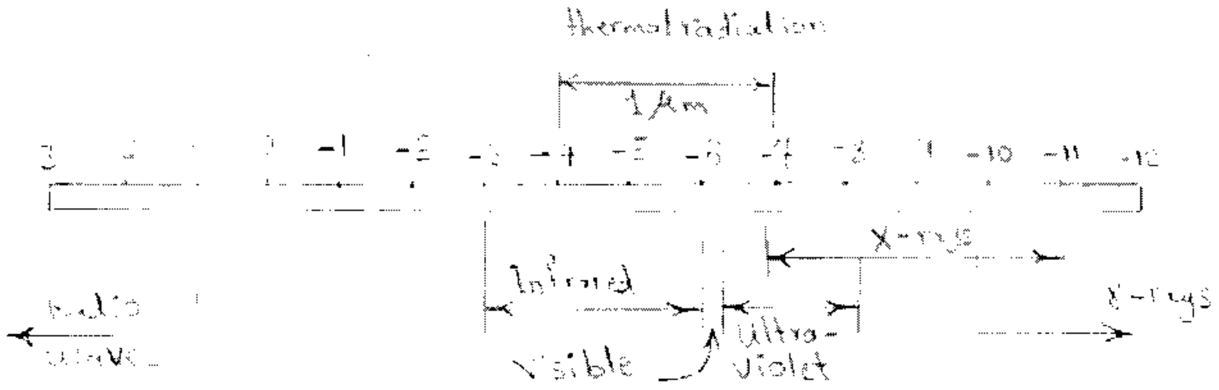
$C = \lambda \nu$ where C : speed of ...

λ : wavelength

ν : frequency

... λ ... micrometers, angstr... (1Å = 10^{-10} m) ... (1 μm = 10^{-6} m).

... electromagnetic spectrum ... figure ... radiation lies in the range ... 1 ... visible light portion ... extending from about ...



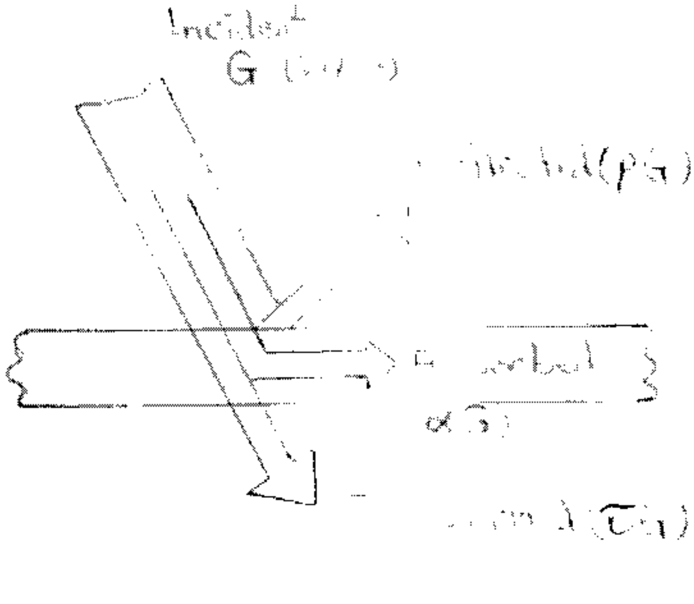
... of ... the ... high ... the ... is proportional to ... temperature ... the ...

$$E_b = \sigma T^4$$

(W/m² or J/...)

$$\rho + \alpha + \tau = 1$$

$$\rho + \alpha = 1$$



Ab[illegible]vity (α) = [illegible] / [illegible] $0 \leq \alpha \leq 1$

[illegible]ity (ρ) = reflected [illegible] / Incident [illegible] $0 \leq \rho \leq 1$

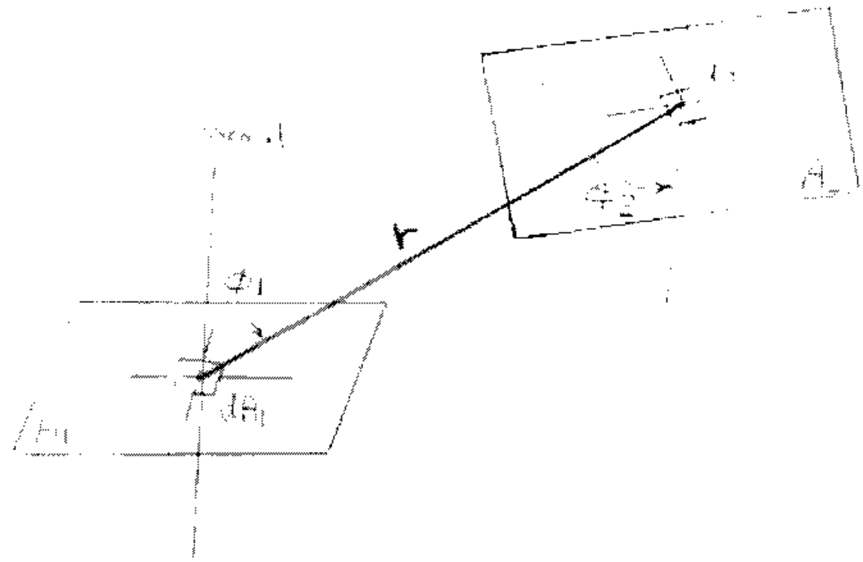
T[illegible]vity (τ) = [illegible] / Incident [illegible] $0 \leq \tau \leq 1$

Em[illegible]

[illegible] temperature [illegible] ϵ [illegible] varies [illegible] ($0 \leq \epsilon \leq 1$). [illegible]

R[illegible] Shape F[illegible]

[illegible] A_1 & A_2 [illegible] energy [illegible] at diff[illegible] temperat[illegible]. The [illegible] deter[illegible] [illegible] problem [illegible] shape [illegible] factor [illegible] as:



[illegible] 2.

[illegible] 1.

[illegible]

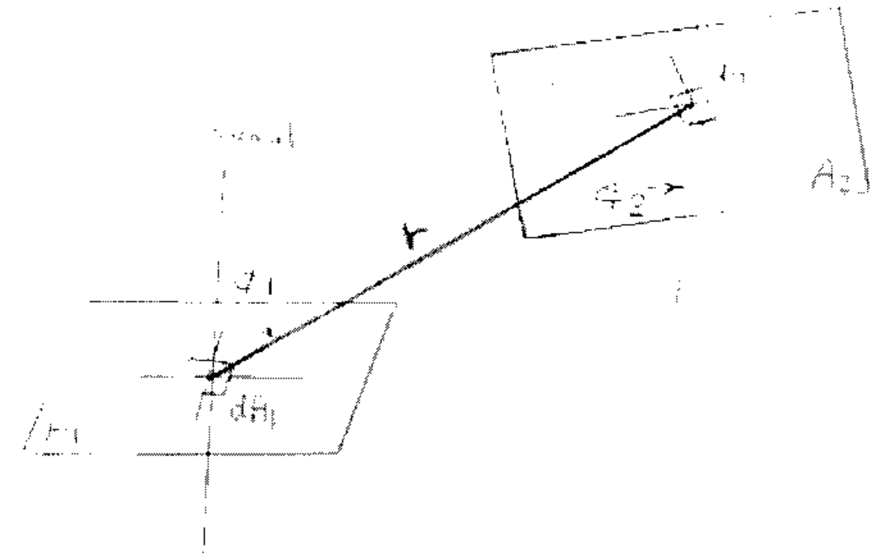
[illegible]

$E_1 A_1 F_{12}$

[illegible]

$E_2 A_2 F_{21}$

[illegible] heat energy [illegible]



[illegible]

[illegible]

[illegible]

[illegible]

$E_{b1} A_1 F_{12}$

[illegible]

$E_{b2} A_2 F_{21}$

[illegible]

$$E_{b1} A_1 F_{12} - E_{b2} A_2 F_{21} = Q_{1-2}$$

[illegible] each [illegible] Q_{1-2} [illegible]
$T_1 = T_2$

$$F_{12} = [illegible]$$

$$A_1 F_{12} = A_2 F_{21} \quad *$$

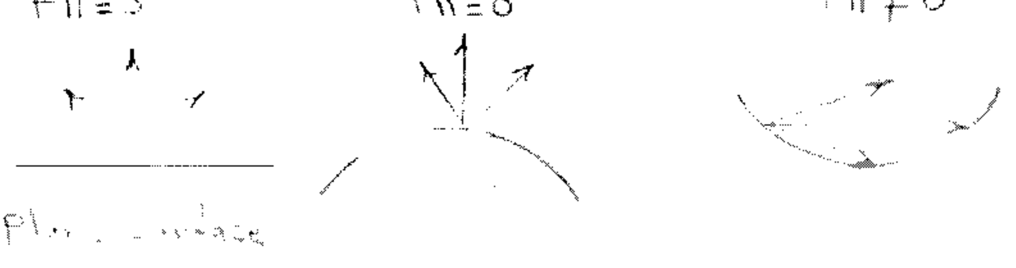
[illegible] exchange [illegible]

$$Q_{1-2} = A_1 F_{12} (E_{b1} - E_{b2}) = A_2 F_{21} (E_{b1} - E_{b2})$$

[illegible]

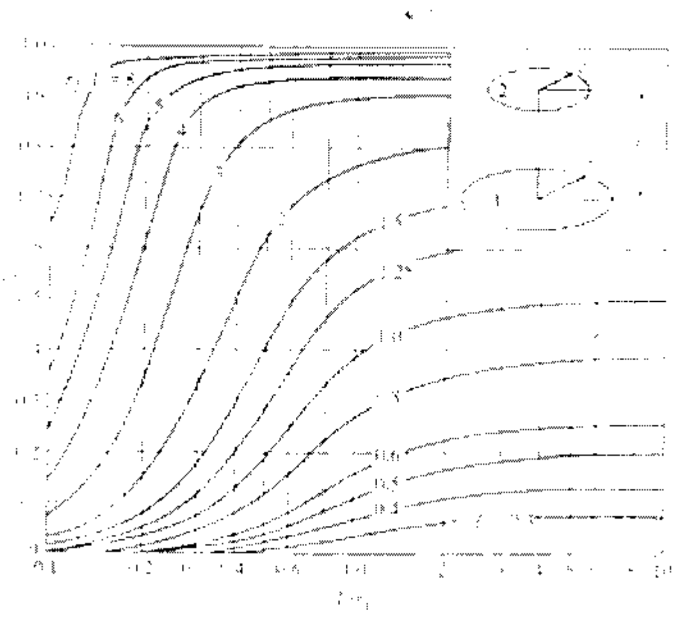
$$A_i F_{ij} = A_j F_{ji}$$

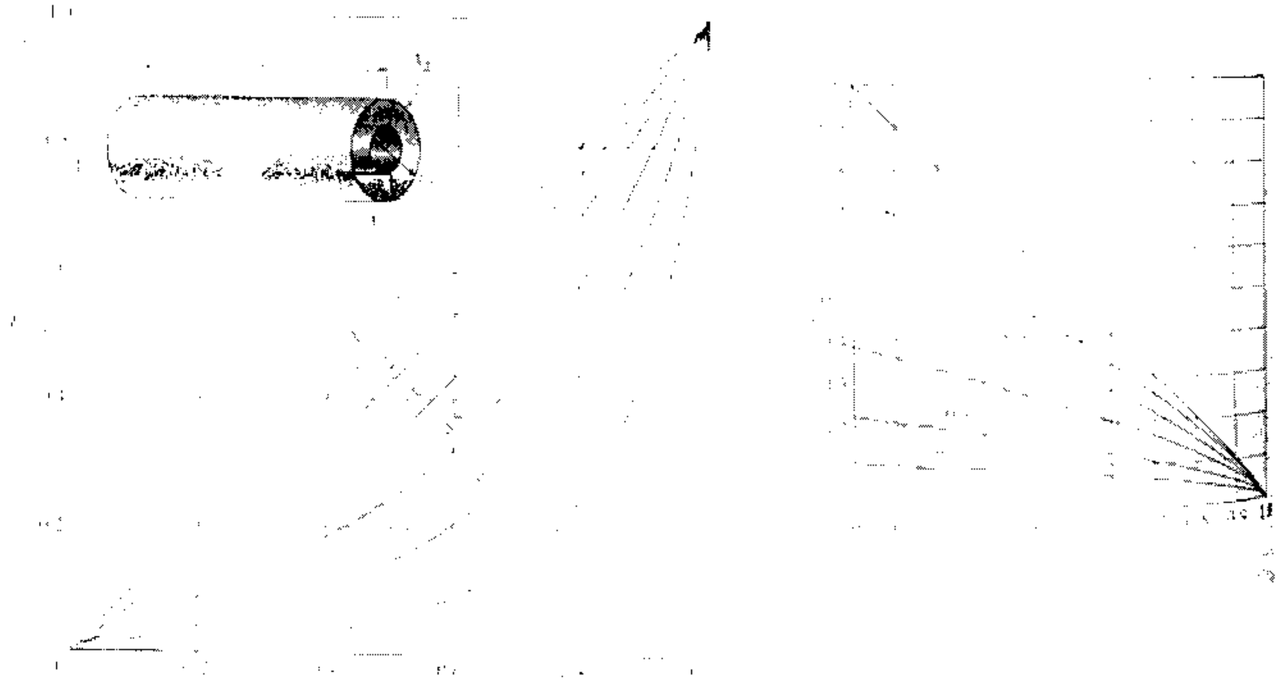
[illegible] plane [illegible]



[illegible] (2D) [illegible]

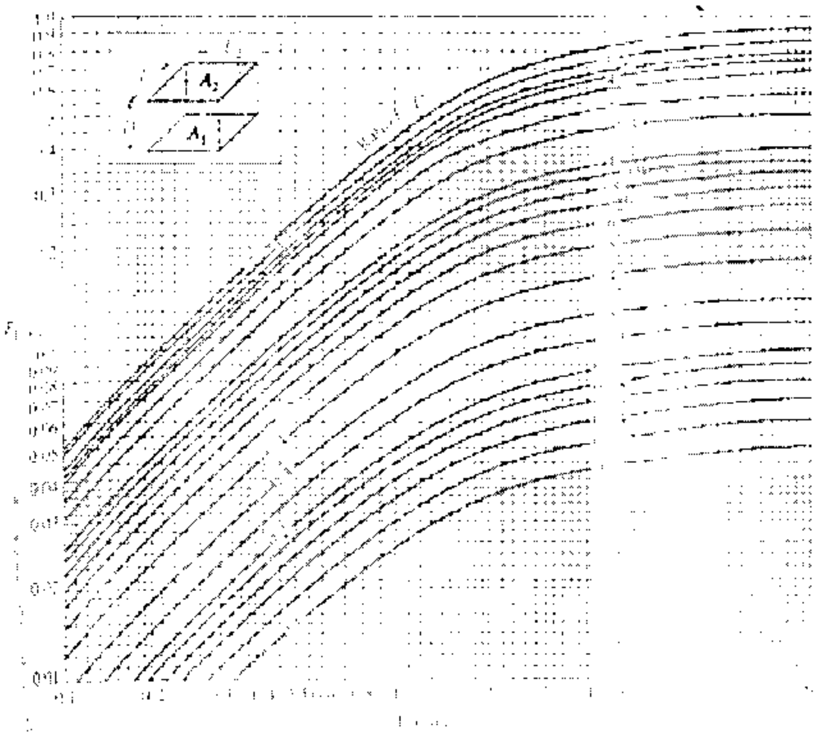
View factor between two coaxial parallel disks.

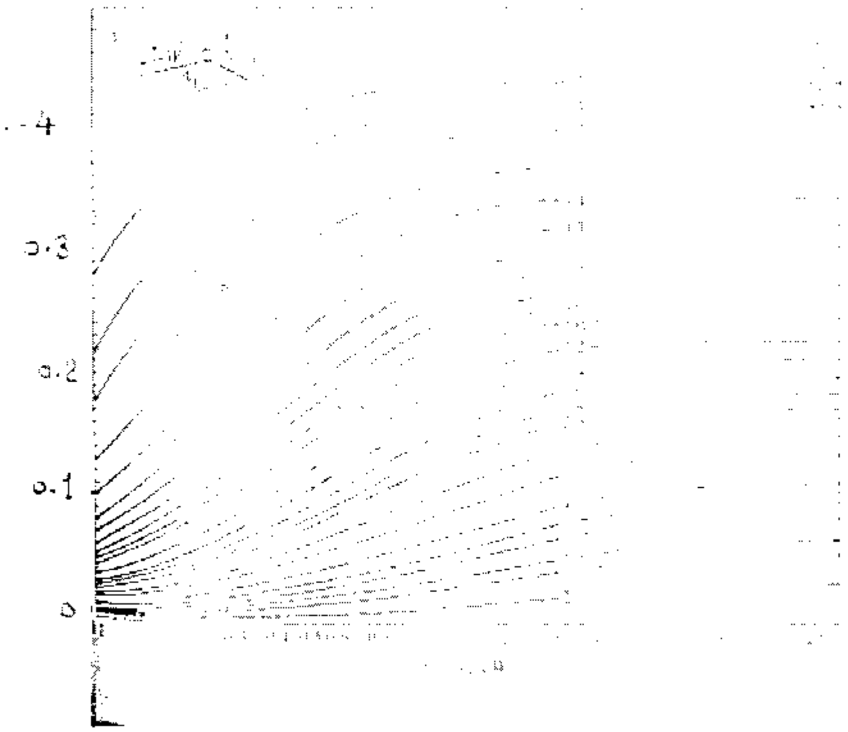




View factors for two concentric cylinders of finite length: [illegible] outer cylinder to inner cylind[illegible] [illegible] to itself

View factor between [illegible] parallel [illegible]

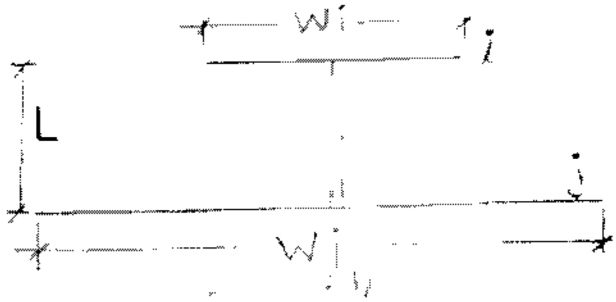




View factor between two perpendicular rectangles with a common edge

1 – [illegible] parallel [illegible]

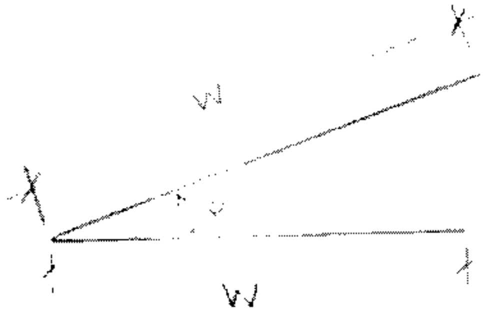
[illegible]



$W_i = [illegible] = \frac{w_i}{L}$

$$F_{ij} = \frac{\left[(W_i + W_j)^2 + 4\right]^{1/2} - \left[(W_j - W_i)^2 + 4\right]^{1/2}}{2 W_i}$$

2 – [illegible] equal width [illegible] common edge.

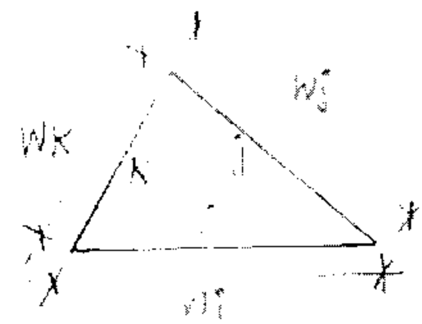


$F_{ij} = [illegible] \sin\left(\frac{\alpha}{2}\right)$

3 – [illegible]

$F_{ij} = \frac{1}{2}\left\{ 1 + [illegible] - \left[1 + [illegible]\right]^{1/2} \right\}$

4 – [illegible]



$F_{ij} = \frac{[illegible] + w_j - w_{[illegible]}}{[illegible]}$

Sh[illegible] F [illegible] Relations

1 – Th[illegible]

[illegible] F_{ij} + F_{ji} [illegible]

[illegible] the [illegible]

$F_{ij} = F_{ji}$ when $A_i = A_j$

$F_{ij} \neq F_{ji}$ when $A_i \neq A_j$

$\therefore A_i F_{ij} = A_j F_{ji}$

[illegible] the reciprocity [illegible]

2 – [illegible] rule

[illegible] the view factor from surface [illegible] all surfaces of the enclosure [illegible] equal to [illegible] This is known as the summation [illegible]

$$\sum_{j=1}^{N} F_{ij} = 1$$

, where N is the number of surfaces of the enclosure.

[illegible]

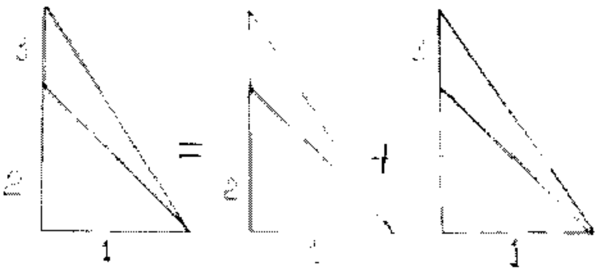
$$\sum_{j=1}^{3} F_{1j} = F_{11} + F_{12} + F_{13} = 1$$

3 – [illegible] rule

[illegible] the paper [illegible] surface 1 [illegible] the combined [illegible] 3 [illegible] of 3 [illegible]

$$F_{1-(2,3)} = F_{1-2} + F_{1-3}$$

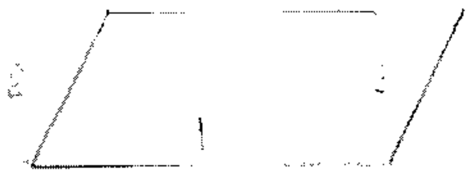
$A_1 F_{1-} \ldots F_{12} + A_1 F_{13}$



$$F = \frac{A\,F + A\,F}{A + A}$$

Fik

Fij = F



Ex

Fu = 0

Fr[illegible]

$$\therefore A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{[illegible]^2}{[illegible]} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

$$F_{21} + F_{22} = 1$$

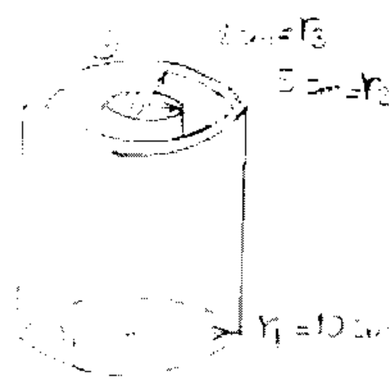
$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

Notes — [illegible] evaluated [illegible] N [illegible]

$$N^2 - [N + \tfrac{1}{2} N (N - 1)] = \tfrac{1}{2} N (N - 1)$$

[illegible] $\tfrac{1}{2} \times 6 (6 - 1) = 15$ [illegible] directly [illegible] applying the [illegible]

2 - [illegible] the fraction of the radiation leaving the base of the [illegible] [illegible] [illegible] [illegible] figure [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $r_1 = 1$[illegible] [illegible] [illegible] [illegible] [illegible] radiation [illegible] [illegible] $r_2 =$ [illegible]



[illegible]

$F_1 = F_{1-} + F_{1-}$ [illegible]

3 [illegible]

[illegible]

[illegible] F_{12} [illegible] F_{13} [illegible]

[illegible] the [illegible] for [illegible]

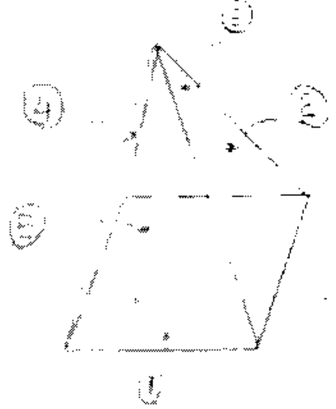
[illegible] [illegible]

$\frac{L}{r_1}$ [illegible] $\frac{r}{L}$ [illegible] $= F_{1-}$ [illegible] $F_{12} =$ [illegible]

$\frac{L}{r_1}$ [illegible] $\frac{r}{L}$ [illegible] F_1 [illegible]

[illegible] $= F_1 - F_{12} =$ [illegible]

3 - [illegible] the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]



$F_{12} + \ldots = F_{1} \ldots$ [illegible]

$\sum_{j=1} F_{1j} = \ldots + F_{12} + \ldots + F_{14} + \ldots = 1$

$F_{11} = 0$ [illegible]

$F_{1} \ldots = F_{1} \ldots 25$

4 - [illegible] find F [illegible]

$F_{11} + F_{12} + \ldots = 1$

$F_{41} = 1$ [illegible]

$F_{14} = F_{12} + F_{13}$

$A_1 F_{14} = A_4 F_{41}$

$A_1 = \ldots$ [illegible] $\ldots 142$

$A_4 = \ldots (\ldots)^{1/2} \ldots$ [illegible]

$A_2 =$ [illegible]

$F_{14} =$ [illegible]

[illegible] $= 0.425$

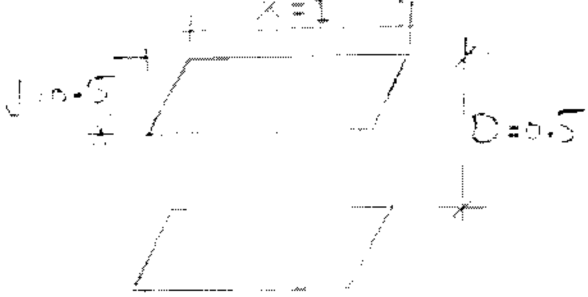
[illegible] 0.241

$F_{11} = 1 -$ [illegible] $= 1 - 0.425 -$ [illegible]

5- Two parallel black plates (0.5 x 1) m are spaced (0.5 m) one plate is maintained at (1000°C) & the other at (500°C) What is the net radiant heat exchange between the two plates?

Solution.-

from shape factors chart



$\frac{y}{D} = \frac{0.5}{0.5} = 1$

$\frac{x}{D} = \frac{1}{0.5} = 2 \Rightarrow F_{12} = 0.285$

$q = A_1 F_{12} (E_{b1} - E_{b2}) = A_1 F_{12}\, \sigma (T_1^4 - T_2^4)$

$= (5.67 \times 10^{-8})(0.5)(0.285)(1273^4 - 773^4) = 15.33\ KW$

Heat Transfer I

"Heat Exchange Between Non-Black Bodies"

&

"Radiation Shields"

Lecture No. (12)

Heat Exchange Between Nonblack Bodies

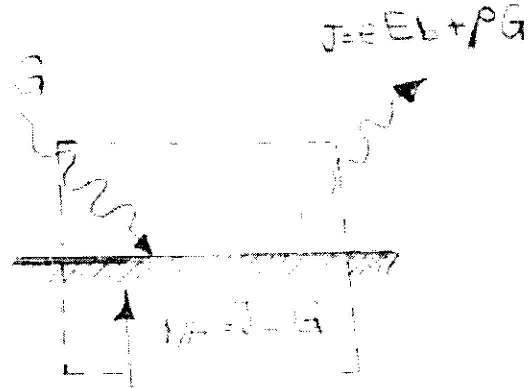
We [illegible] that [illegible] uniform [illegible] properties are constant [illegible] the surface.

[illegible]

G. [illegible]

J. [illegible]

[illegible]



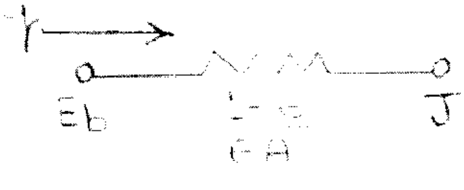
$$J = \epsilon E_b + \rho G$$

where

ϵ [illegible]

E_b [illegible] blackbody emissivity

[illegible]



[illegible]

$$\rho = 1 - \alpha = 1 - \epsilon$$ [illegible]

$$J = \epsilon E_b + (1-\epsilon)G \Rightarrow \frac{q}{A} = J - G = \epsilon E_b + (1-\epsilon)G - G$$

$$q = \frac{\epsilon A}{1-\epsilon}(E_b - J) \quad \text{or} \quad q = \frac{E_b - J}{\frac{1-\epsilon}{\epsilon A}}$$

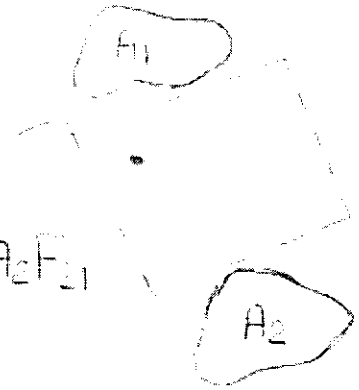
[illegible] the exchange [illegible] surfaces A_1 [illegible] A_2 [illegible] of that [illegible] radiation [illegible] surface 1, the [illegible] unit [illegible] 2 [illegible]

$J_1 A_1 F_{12}$

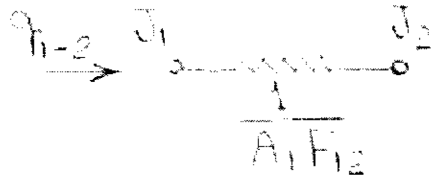
[illegible] of that [illegible] surface 2, [illegible] that [illegible] 1 [illegible]

$J_2 A_2 F_{21}$

$q_{1-2} = J_1A_1F_{12} - J_2A_2F_{21}$



The [illegible] exchange between [illegible]



$$q_{1-2} = J_1A_1F_{12} - J_2A_2F_{21}$$

[illegible] $A_1F_{12} = A_2F_{21}$

[illegible]

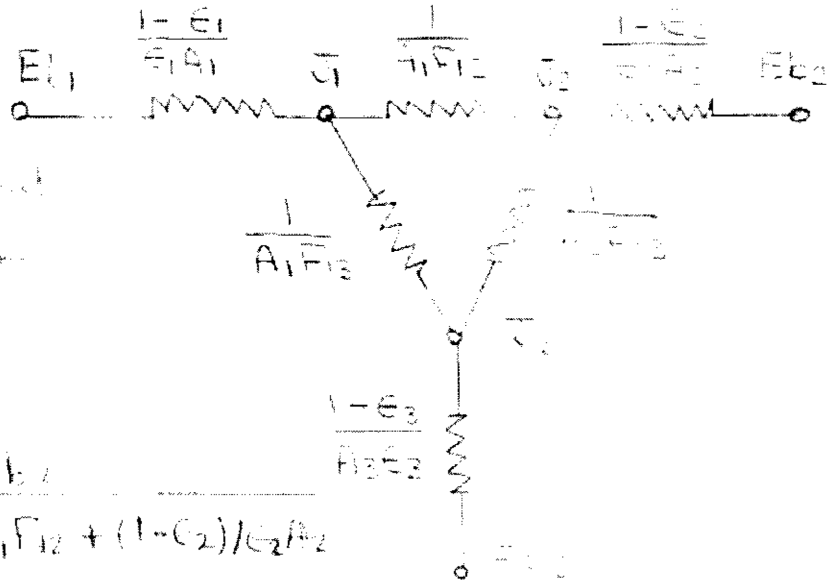
$$q_{1-2} = (J_1 - J_2)A_1F_{12} = (J_1 - J_2)A_2F_{21}$$

$$\text{or} \quad q_{1-2} = \frac{J_1 - J_2}{1/A_1F_{12}}$$

To [illegible] construct a network for a particular [illegible] heat transfer problem we need only connect a ([illegible] resistance) $(1-\epsilon)/\ldots$ [illegible] each [illegible] (space [illegible]) $(1/A_mF_{mn})$ between [illegible] radiosity potentials.

[illegible] network for two surfaces which [illegible] each other [illegible]

[illegible]



[illegible]

$$q_{1-2} = \frac{E_{b1} - E_{b2}}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1F_{12} + (1-\epsilon_2)/\epsilon_2 A_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1F_{12} + (1-\epsilon_2)/\epsilon_2 A_2}$$

[illegible]

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1F_{12}}$$

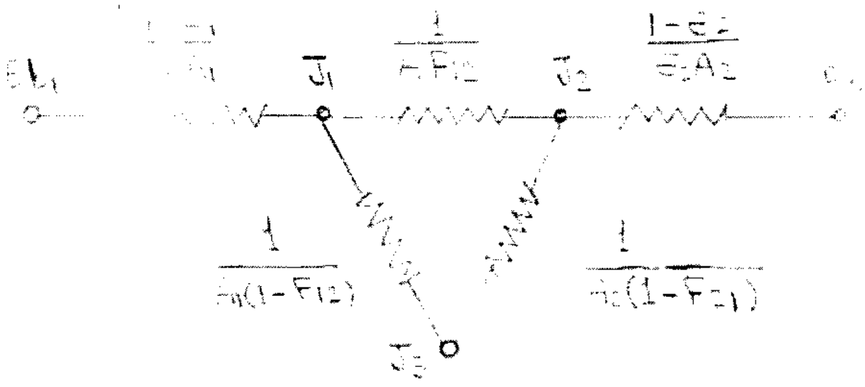
[illegible]

$$q_{1-3} = \frac{J_1 - J_3}{1/A_1F_{13}}$$

In[illegible]

We [illegible]

[illegible]



N[illegible]

$$F_{13} = 1 - F_{12} \quad , \quad F_{23} = 1 - F_{21}$$

[illegible]

S[illegible]

Holman

Ex 8.3 // It is desired to transmit energy from one spaceship to another. A 1.5 m square plate is available on each ship to accomplish this. The ships are guided so that the plates are parallel and 30 cm apart. One plate is maintained at 800°C and the other at 280°C. The emissivities are 0.5 and 0.8 respectively. Find (a) the net heat transfer between the spaceships in watts and (b) the total heat lost by the hot plate in watts. Assume that outer space is a blackbody at 0 K.

Sol:

$A_1 = A_2 = 1.5^2 = 2.25\ m^2 \qquad T_1 = 1073\ K,\ T_2 = 553\ K$

$\epsilon_1 = 0.5,\ \epsilon_2 = 0.8 \qquad T_3 = 0\ K$

from chart $\quad Y/D = X/D = 1.5/0.3 = 5 \Rightarrow F_{12} = 0.7$

$F_{11} = F_{22} = F_{33} = 0 \qquad A_1 = A_2 \Rightarrow F_{12} = F_{21} = 0.7$

$F_{12} + F_{11} + F_{13} = 1 \Rightarrow F_{13} = 0.3$

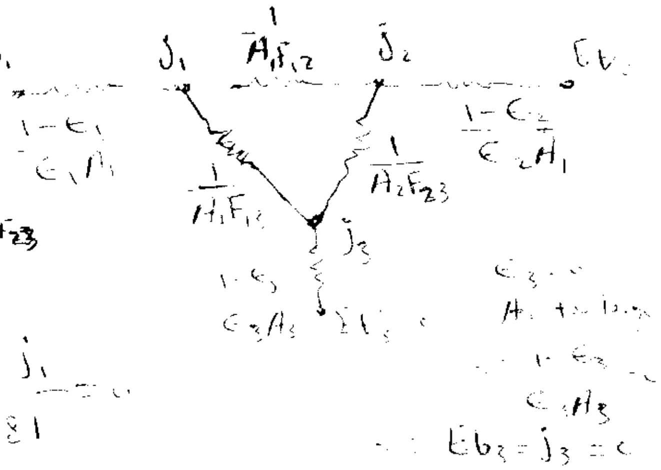
$F_{21} + F_{22} + F_{23} = 1 \Rightarrow F_{23} = 0.3$

$E_{b_1} = \sigma T_1^4 = 75146\ W/m^2,\quad E_{b_2} = \sigma T_2^4 = 5302\ W/m^2$

$T_3 = 0 \Rightarrow E_{b_3} = 0$

$\frac{1-\epsilon_1}{\epsilon_1 A_1} = 0.444,\quad \frac{1}{A_1F_{12}} = 0.6349$

$\frac{1-\epsilon_2}{\epsilon_2 A_2} = 0.1111,\quad \frac{1}{A_1F_{13}} = 1.481 = \frac{1}{A_2F_{23}}$



node J_1

$$\frac{75146 - J_1}{0.444} + \frac{J_2 - J_1}{0.6349} + \frac{0 - J_1}{1.481} = 0$$

node J_2

$$\frac{J_1 - J_2}{0.6349} + \frac{0 - J_2}{1.481} + \frac{5302 - J_2}{0.1111} = 0$$

$\Rightarrow J_1 = 41070\ W/m^2,\quad J_2 = 9992\ W/m^2$

$$q_1 = \frac{75146 - 41070}{0.444} = 76671\ W$$

$$q_2 = \frac{5302 - 9992}{0.1111} = -42210\ W$$

$$q_{1-2} = \frac{[illegible]\,(T_1^4 - T_2^4)}{\dfrac{A_1 + A_2 - 2A_1F_{12}}{[illegible]\,F_{12})^2} + \left(\dfrac{1}{\epsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)}$$

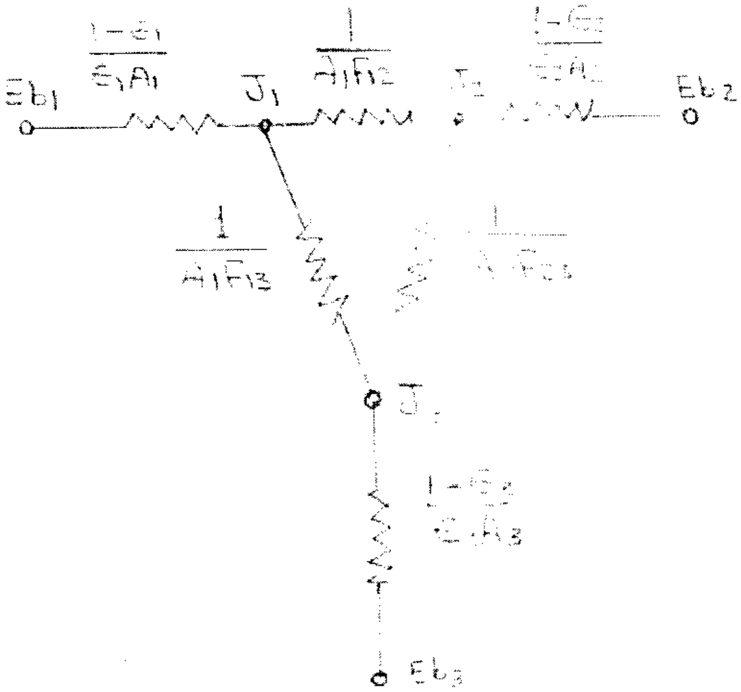
where [illegible] reciprocity relation $A_1F_{12} = A_2F_{21}$

Ex [illegible] 5

1 – [illegible] cylinder [illegible] plate [illegible] at [illegible] 2000 C [illegible] of the other [illegible] emiss- ivities [illegible] 0.2 & 0.5 respectively [illegible] plates are [illegible] large room [illegible] of which [illegible] 27 C. [illegible] exchange heat with each other [illegible] with the [illegible] the plate surfaces facing each other [illegible] be [illegible] find the net heat transfer to [illegible] each plate [illegible] the room [illegible]

[illegible]

[illegible]



T_1 [illegible] K

T_2 [illegible] K

T_3 [illegible] 300 K

A_1 [illegible]

$\epsilon_1 = 0.2$ [illegible] 0.5

[illegible]

$= 0.285$ from chart [illegible]

[illegible] 0.415 [illegible] 0.715

[illegible]

$= 2$

[illegible] 0.285 [illegible]

[illegible] 0.715 [illegible]

[illegible] 0.415 [illegible]

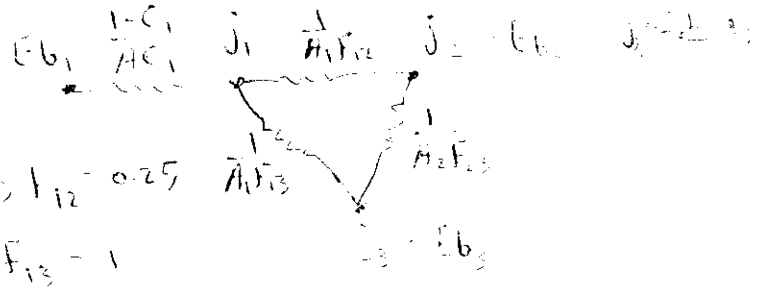
[illegible] $= 0$

[illegible]

[illegible] 2.747 [illegible]

EX 8-51: Two parallel planes 90 by 60 cm are separated by a distance of 60 cm. One plane is maintained at a temperature of 800 K and has an emissivity of 0.6. The other plane is insulated. The planes are placed in a large room that is maintained at 290 K. Calculate the temperature of insulated plane and the energy lost by the heated plane.

$A_1 = A_2 = 0.9 \times 0.6 = 0.54\ m^2$



From chart $Y/D = 90/60 = 1.5$

$X/D = 60/60 = 1 \Rightarrow F_{12} = 0.25$

$F_{11} = F_{22} = F_{33} = 0$ $\quad F_{12} + F_{11} + F_{13} = 1$

$A_1 = A_2 \Rightarrow F_{12} = F_{21} = 0.25$ $\quad F_{21} + F_{22} + F_{23} = 1 \Rightarrow F_{13} = F_{23} = 0.75$

$\Rightarrow \frac{1-\epsilon_1}{A_1\epsilon_1} = 1.235 \quad \frac{1}{A_1F_{13}} = \frac{1}{A_2F_{23}} = 2.469 \quad , \quad \frac{1}{A_1F_{12}} = 7.407$

$E_{b1} = \sigma T_1^4 = 23224 \quad , \quad J_3 = E_{b3} = 401$

$$q = \frac{23224 - 401}{1.235 + 1.9752} = 7109.5 = \frac{23224 - J_1}{1.235}$$

$\Rightarrow J_1 = 14444\ W/m^2$

$$\frac{14444 - J_2}{7.407} = \frac{J_2 - 401}{2.469} \Rightarrow J_2 = 3912 = E_{b2} = \sigma T_2^4$$

$\Rightarrow T_2 = 512.5\ K = 239.5°C$

$R_s = \frac{1}{A_1F_{12}} + \frac{1}{A_2F_{23}} = 7.407 + 2.469$

$R_s = 9.876$

$$\frac{1}{R_T} = \frac{1}{(1/A_1F_{13})} + \frac{1}{R_s} = \frac{1}{2.469} + \frac{1}{9.876}$$

$\Rightarrow R_T = 1.9752$

[illegible]

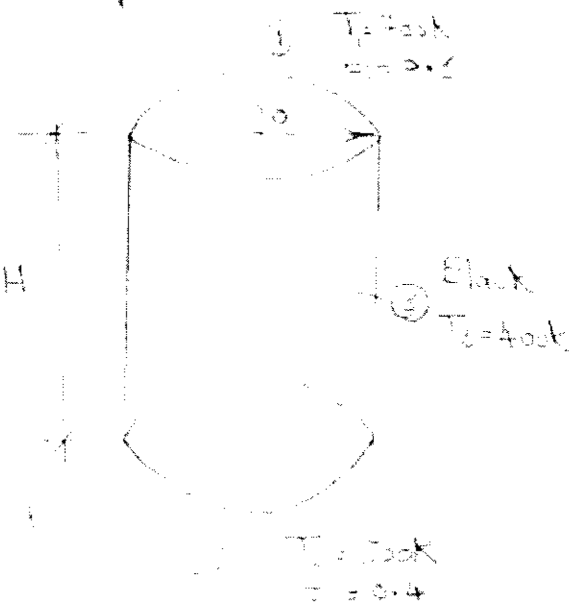
$$\frac{2\,[illegible] - [illegible]\,54}{2}$$

[illegible]

[illegible] has [illegible] $\epsilon_2 = 0.4$ [illegible] temperature $T_1 = 700$ [illegible] is maintained at a temperature of $T_2 = 400\,K$. Determine the net [illegible] radiation heat transfer [illegible] surface [illegible]

Can [illegible] maintained at specified temperature [illegible]

[illegible]



[illegible]

$A_1 = A_2 = [illegible] = \pi (1m)^2 = [illegible] m^2$

$A_3 = \pi D H = \pi (1) \times (1) = [illegible]$

[illegible] $\bar{F}_{12}$ [illegible]

[illegible] $\bar{F}_{12} = 0.38$ [illegible]

$F_{11} + F_{12} + F_{13} = 1$ [illegible] $F_{11} = 0$

$F_{13} = 1 - 0.38 = 0.62$ [illegible]

[illegible]

[illegible] $F_{21} = F_{12} = 0.38$ [illegible]

$F_{23} = F_{13} = 0.62$ [illegible]

[illegible] F_{31} [illegible]

[illegible] $A_1 F_{13} = A_3 F_{31} \Rightarrow F_{31} = F_{13}(A_1/A_3) = 0.62 \left(\frac{3.14}{6.28}\right)$

$F_{31} = 0.31 = F_{32}$ [illegible]

[illegible] $= \sigma T_1^4 = J_1 + \frac{1-\epsilon_1}{\epsilon_1} (F_{12} [illegible] + F_{13} [illegible])$

[illegible] $(i=2)$: $\sigma T_2^4 = J_2 + \frac{1-\epsilon_2}{\epsilon_2} (F_{21} [illegible] + F_{23} [illegible])$

[illegible] $(i=3)$: $\sigma T_3^4 = J_3 + 0$ [illegible]

[illegible]

[illegible] $= J_1 + \frac{1-}{}$ [illegible] $(J_1 - J_2)$ + [illegible] $(J_1 - J_3))$

[illegible] $= J_2 + \frac{1-}{}$ [illegible] $(J_2 - J_1)$ + [illegible] $(J_2 - J_3))$

[illegible] $= J_3$

[illegible]

$J_1 =$ [illegible] $J_2 =$ [illegible] W/m^2 [illegible] $J_3 =$ [illegible] W/m^2

[illegible] $\sum_{j=1}^{N} F_{ij} (J_i - J_j)$

$Q_1 = A_1$ [illegible] $J_1 - J_2) + F_{13} (J_1 - J_3))$

= [illegible] (11?? − ????) + [illegible]) = ?75? W

$Q_2 = A_2$ [illegible] $- J_1) + F$ [illegible] $- J_3))$

= [illegible] −441 [illegible] + [illegible] (4?? − [illegible] = −1??.W

$Q_3 = A_3$ [illegible] $J_3 - J_1) + F_{32} (J_3 - J_2))$

= [illegible] (14?2 − ???) + [illegible] (14?? − [illegible] = −???.6W

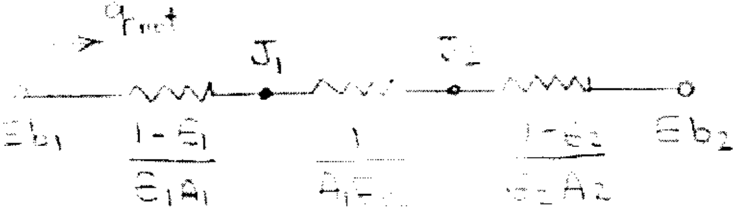
Infinite Parallel Surfaces :-

* [illegible] A_1 & A_2 [illegible]

[illegible]

[illegible] $A_1 = A_2$ [illegible] $F_{12} = 1$ [illegible]

$\frac{1}{A}$ [illegible] $\frac{(\ldots - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$

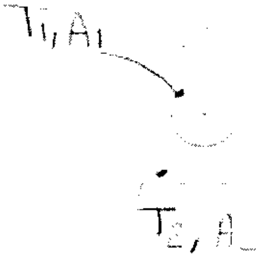


[illegible]

* [illegible]

[illegible]

[illegible] A_1 [illegible]



[illegible] $\frac{A_1}{A_2}$ [illegible]

[illegible]

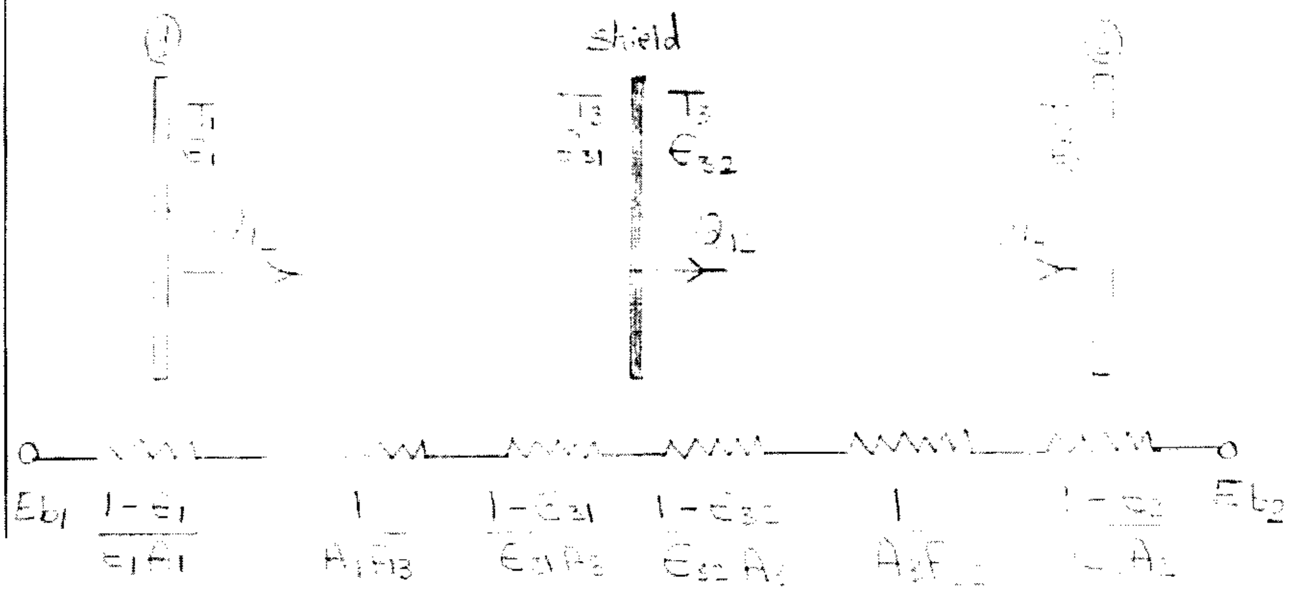
[illegible]

Radiation Shields :-

[illegible]

$$Q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

[illegible]



$$\frac{1}{A} = \frac{1}{A}\bigg|_{1-3} = \frac{1}{A}\bigg|_{3-2}$$

$$\frac{1}{A} = \frac{\sigma (T_1^4 - T_3^4)}{1/\epsilon_1 + 1/\epsilon_3 - 1} = \frac{\sigma (T_3^4 - T_2^4)}{1/\epsilon_3 + 1/\epsilon_2 - 1}$$

[illegible]

$$q_{1-2} = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1F_{13}} + \frac{1-\epsilon_3}{A_3\epsilon_3} + \frac{1-\epsilon_3}{A_3\epsilon_3} + \frac{1}{A_3F_{32}} + \frac{1-\epsilon_2}{A_2\epsilon_2}}$$

[illegible] $F_{13} = F_{32} = 1$ [illegible] $A_1 = A_3 = A_2$ [illegible] parallel plates [illegible]

$$q_{12} = \frac{A\sigma (T_1^4 - T_2^4)}{(1/\epsilon_1 + 1/\epsilon_2 - 1) + (1/\epsilon_3 + 1/\epsilon_3 - 1)}$$

[illegible]

[illegible]

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1} = [illegible]$$

$= [illegible]$

[illegible]

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{(1/\epsilon_1 + 1/\epsilon_2 - 1) + (1/\epsilon_3 + 1/\epsilon_2 - 1)}$$

$$\frac{Q}{A} = [illegible]$$

[illegible] %

2 — [illegible] plates at temperature [illegible] radiation shield [illegible]

[illegible]

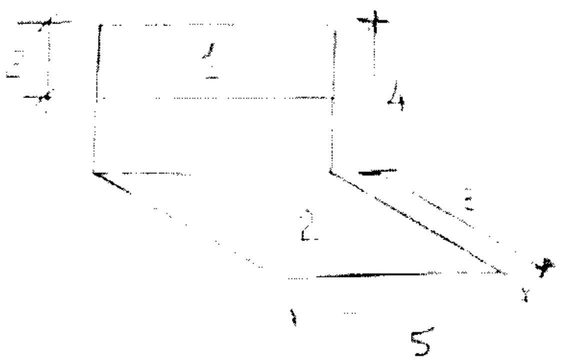
$$\frac{[illegible]}{[illegible]} = \frac{1/\epsilon_1 + 1/\epsilon_2 - 1}{(1/\epsilon_1 + 1/\epsilon_2 - 1) + (1/\epsilon_3 + [illegible] - 1)}$$

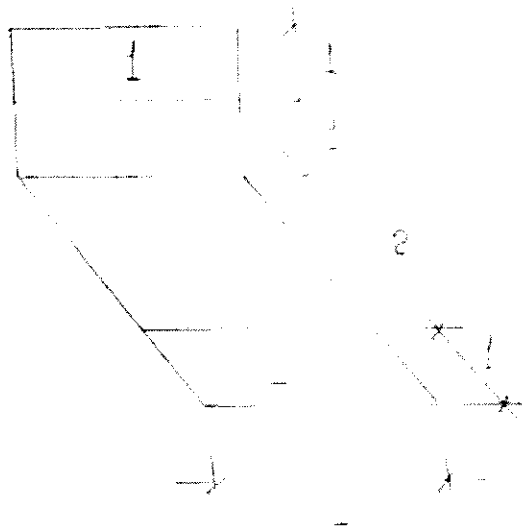
$$\frac{1}{10} = \frac{[illegible]}{[illegible]}$$

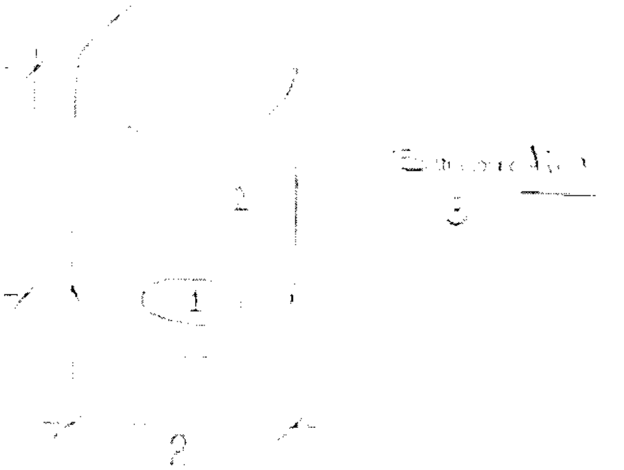
[illegible]

He…

1- … F …







2- … black … K …