REINFORCED CONCRETE DESIGN-I

Course Code

3319

First Semester, 2020 / 2021

Asst.Prof.Dr. Akram S. Mahmoud

Course Description :

This course introduced Material properties, Flexural theories, Un-cracked section, Working stress method, Ultimate strength, Design and analysis of Singly Rectangular, doubly, T-section, irregular section beams, Shear analysis and design, Continuous beams, One way slab, Short columns, Long columns, Bond, anchorage, development length, Cracked and deflection.

Course Learning Outcomes:

By the end of successful completion of this course, the student will be able to:

1. How to analyze the RC section using working stress method
2. How to design reinforced concrete beams for flexure and shear according ultimate strength method.
3. How to design reinforced concrete one-way and two-way slabs.
4. Achieve the serviceability requirements of RC members
5. Use ACI 318-19 code specifications in various design problems.

Course Topics/Contents

Week	Topic	Comments*
1.	Introduction and revision, materials and properties of concrete and reinforcing bars. ACI safety code provisions.	
2.		
3.	Analysis and design of singly reinforced concrete beams.	
4.		
5.	Analysis and Design of doubly reinforced concrete beams	
6.		
7.	Analysis and design of T and L reinforced concrete beams	
8.	Analysis and design of beams for shear and diagonal tension	
9.		Midterm exam
10.	Analysis and Design of continuous beam for flexure using ACI coefficients method	
11.	Analysis and design of Reinforced Concrete solid one-way slabs.	
12.	Analysis and design of Reinforced Concrete solid Two-way slabs.	
13.		
14.	Design for bond, anchorage and development length	
15.		
16.	**Final Exam**	

References

1. Building Code Requirements for Structural Concrete, ACI 318-19, American Concrete Institute, Farmington Hills, MI, 2019.
2. Arthur H. Nilson, David Darwin, Charles W. Dolan, Design of Concrete Structures, McGraw-Hill,14th ed., 2020
3. Structural Concrete- Theory and Design", Hasson, M. N. and Al-Manseer. 5th Edition, John Wiley $ Sons, Inc. 2012.
4. الدكتور جمال عبد الواحد فرحان الظاهر- تصاميم المنشآت الخرسانية المسلحة وفقاً الكود (ACI 318M-14)
5. Design of Reinforced Concrete, 10th Edition, By Jack C. McCormac and Russell H. Brown, Wiley, ISBN: 978-1-118-87910-8.

CHAPTER 1

REINFORCED CONCRETE STRUCTURES

1.1 INTRODUCTION

Many structures are built of reinforced concrete: bridges, viaducts, buildings, retaining walls, tunnels, tanks, conduits, and others.

Reinforced concrete is a logical union of two materials: plain concrete, which possesses high compressive strength but little tensile strength, and steel bars embedded in the concrete, which can provide the needed strength in tension.

First practical use of reinforced concrete was known in the mid-1800s. In the first decade of the 20th century, progress in reinforced concrete was rapid. Since the mid-1950s, reinforced concrete design practice has made the transition from that based on elastic methods to one based on strength. Understanding of reinforced concrete behavior is still far from complete; building codes and specifications that give design procedures are continually changing to reflect latest knowledge.

1.2 REINFORCED CONCRETE MEMBERS

Every structure is proportioned as to both architecture and engineering to serve a particular function. Form and function go hand in hand, and the beat structural system is the one that fulfills most of the needs of the user while being serviceable, attractive, and economically cost efficient. Although most structures are designed for a life span of 50 years, the durability performance record indicates that properly proportioned concrete structures have generally had longer useful lives.

Reinforced concrete structures consist of a series of "members" (components) that interact to support the loads placed on the structures.

The components can be broadly classified into:

1. Floor Slabs

Floor slabs are the main horizontal elements that transmit the moving live loads as well as the stationary dead loads to the vertical framing supports of a structure. They can be:

- Slabs on beams,
- Waffle slabs,
- Slabs without beams (Flat Plates) resting directly on columns,
- Composite slabs on joists.

They can be proportioned such that they act in one direction (one-way slabs) or proportioned so that they act in two perpendicular directions (two-way slabs and flat plates).

2. Beams

Beams are the structural elements that transmit the tributary loads from floor slabs to vertical supporting columns. They are normally cast monolithically with the slabs and are structurally reinforced on one face, the lower tension side, or both the top and bottom faces. As they are cast monolithically with the slab, they form a T-beam section for interior beams or an L beam at the building exterior.

The plan dimensions of a slab panel determine whether the floor slab behaves essentially as a one-way or two-way slab.

3. Columns

The vertical elements support the structural floor system. They are compression members subjected in most cases to both bending and axial load and are of major importance in the safety considerations of any structure. If a structural system is also composed of horizontal compression members, such members would be considered as beam-columns.

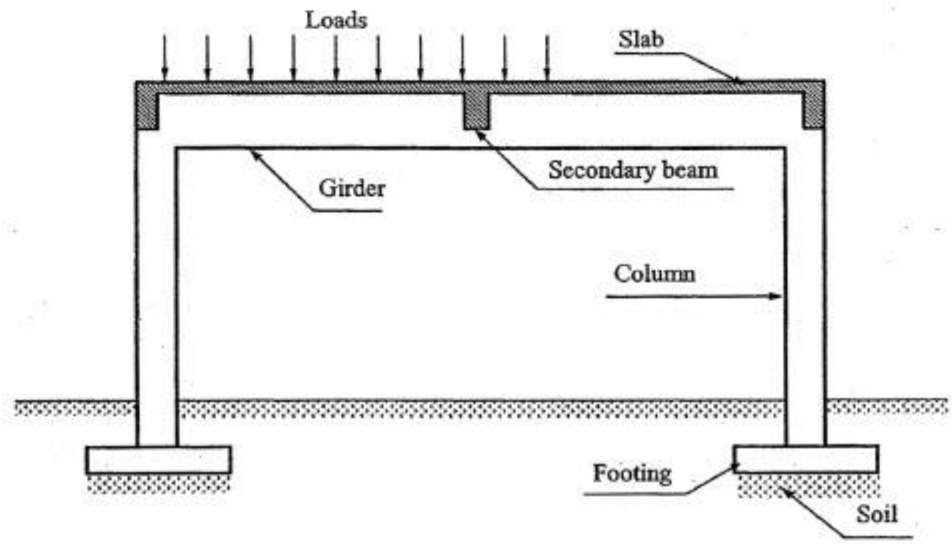
4. Walls

Walls are the vertical enclosures for building frames. They are not usually or necessarily made of concrete but of any material that esthetically fulfills the form and functional needs of the structural system. Additionally, structural concrete walls are often necessary as foundation walls, stairwell walls, and shear walls that resist horizontal wind loads and earthquake-induced loads.

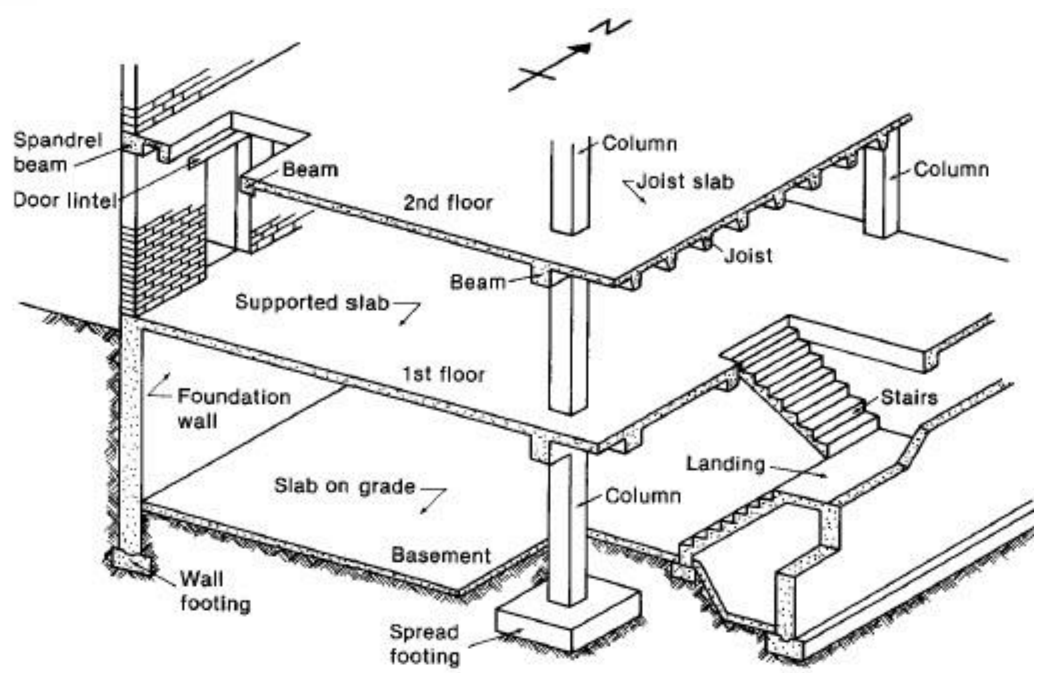
5. Foundations

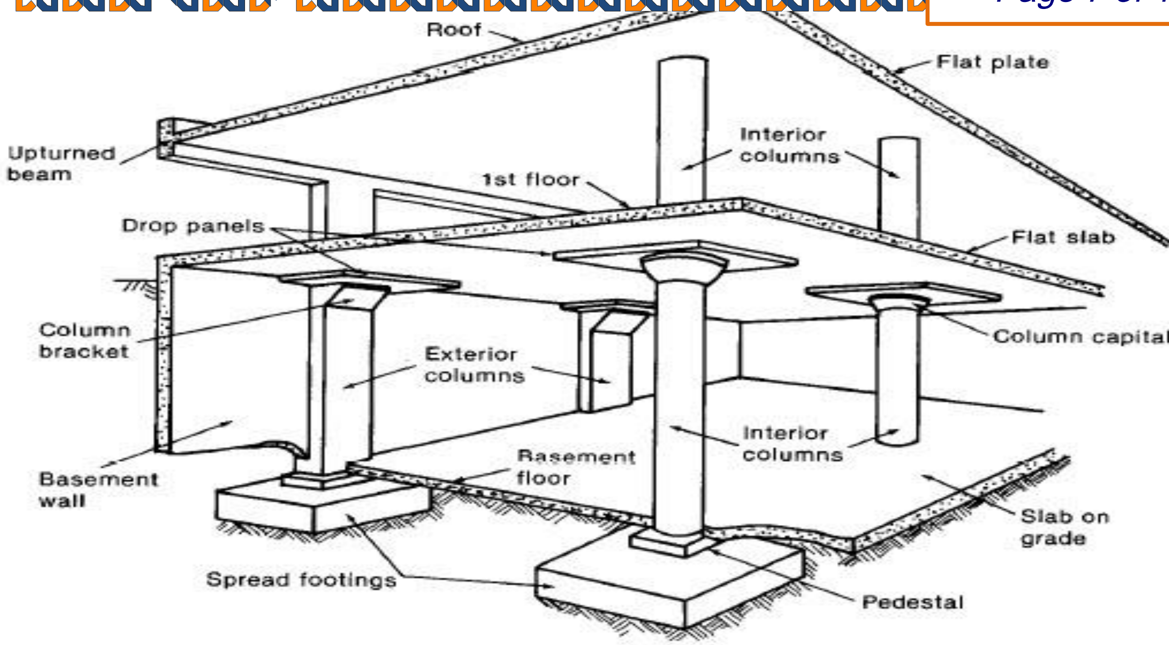
Foundations are the structural concrete elements that transmit the weight of the superstructure to the supporting soil. They could be in many forms:

- Isolated footing - the simplest one. It can be viewed as an inverted slab transmitting a distributed load from the soil to the column.
- Combined footings supporting more than one column.
- Mat foundations, and rafts which are basically inverted slab and beam construction.
- Strip footing or wall footing supporting walls.
- Piles driven to rock.



Typical reinforced concrete structural framing system

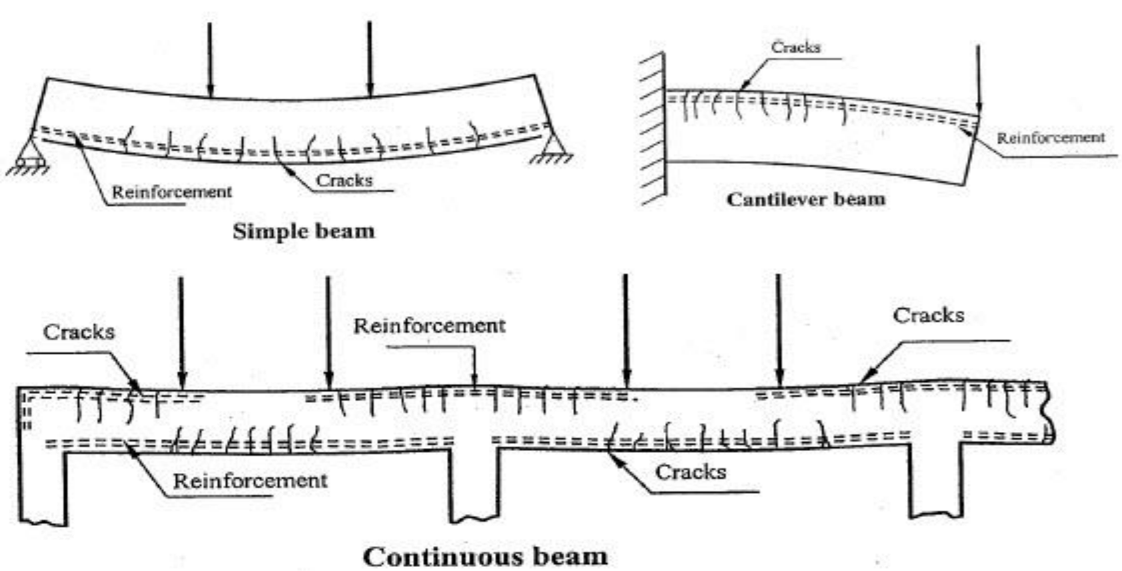




Reinforced concrete building elements

1.3 REINFORCED CONCRETE BEHAVIOR

The addition of steel reinforcement that bonds strongly to concrete produces a relatively ductile material capable of transmitting tension and suitable for any structural elements, e.g., slabs, beam, columns. Reinforcement should be placed in the locations of anticipated tensile stresses and cracking areas. For example, the main reinforcement in a simple beam is placed at the bottom fibers where the tensile stresses develop. However, for a cantilever, the main reinforcement is at the top of the beam at the location of the maximum negative moment. Finally for a continuous beam, a part of the main reinforcement should be placed near the bottom fibers where the positive moments exist and the other part is placed at the top fibers where the negative moments exist.



MATERIALS AND PROPERTIES

2.1 CONCRETE

Plain concrete is made by mixing cement, fine aggregate, coarse aggregate, water, and frequently admixtures.

Structural concrete can be classified into:

- Lightweight concrete with a unit weight from about $1350\ to\ 1850\ kg/m^3$ produced from aggregates of expanded shale, clay, slate, and slag.

Other lightweight materials such as pumice, scoria, perlite, vermiculite, and diatomite are used to produce insulating lightweight concretes ranging in density from about $250\ to\ 1450\ kg/m^3$.

- Normal-weight concrete with a unit weight from about $1800\ to\ 2400\ kg/m^3$produced from the most commonly used aggregates—sand, gravel, crushed stone.
- Heavyweight concrete with a unit weight from about $3200\ to\ 5600\ kg/m^3$produced from such materials such as barite, limonite, magnetite, ilmenite, hematite, iron, and steel punching or shot. It is used for shielding against radiations in nuclear reactor containers and other structures.

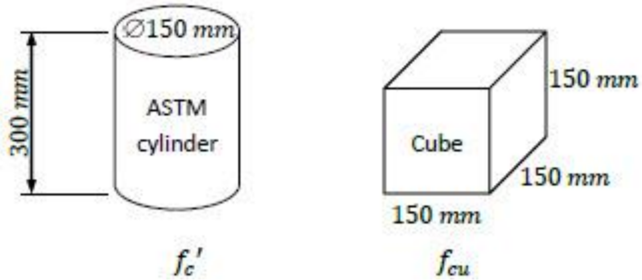
2.2 COMPRESSIVE STRENGTH

The strength of concrete is controlled by the proportioning of cement, coarse and fine aggregates, water, and various admixtures. The most important variable is (W/C) ratio.

Concrete strength (f_c') – uniaxial compressive strength measured by a compression test of a standard test cylinder (diameter by high) on the 28th day–ASTM C31, C39.

In many countries, the standard test unit is the cube (200 X 200X200 mm).

The concrete strength depends on the size and shape of the test specimen and the manner of testing. For this reason the cylinder (Ø150mm by 300mm high) strength is 80% of the 150mm cube strength and 83% of the 200mm cube strength.



$$f_c' \approx 0.80\, f_{cu}$$

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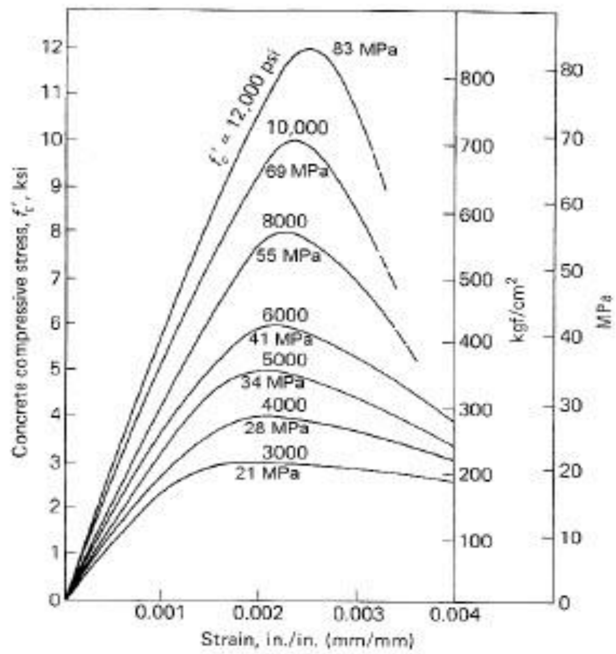
Stress-strain relationship:

Typical curves for specimens (150 X 300mm cylinders) loaded in compression at 28 days.

Lower-strength concrete has greater deformability (ductility) than higher-strength concrete (length of the portion of the curve after the maximum stress is reached at a strain between 0.002 and 0.0025).
Ultimate strain at crushing of concrete varies from 0.003 to as high as 0.008.

- In usual reinforced concrete design $f_c'$of (*24 to 35 MPa*) are used for nonprestressed structures.
- $f_c'$of (*35 to 42 MPa*) are used for prestressed structures.
- $f_c'$of (*42 to 97 MPa*) are used particularly in columns of tall buildings.

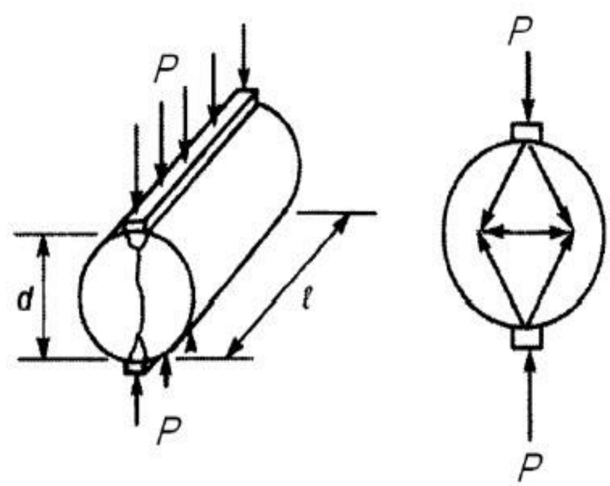


2.3 TENSILE STRENGTH

Concrete tensile strength is about 10 to 15% of its compressive strength.
The strength of concrete in tension is an important property that greatly affects that extent and size of cracking in structures.
Tensile strength is usually determined by using:

- Split-cylinder test (ASTM C496). A standard 150 X 300mm compression test cylinder is placed on its side and loaded in compression along a diameter. The splitting tensile strength f_{ct} is computed as

$$f_{ct} = \frac{2P}{\pi l d}$$

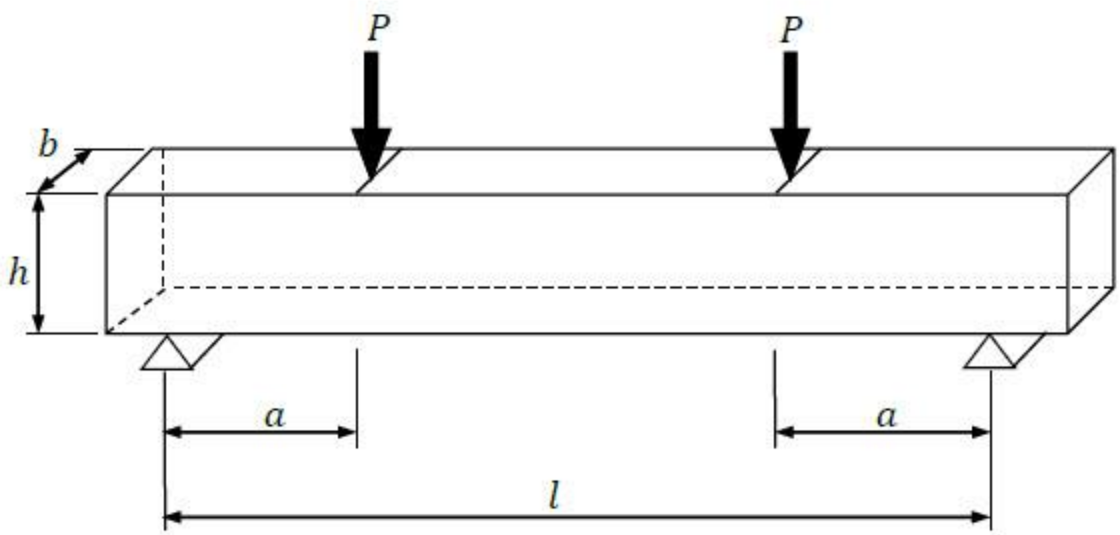


- Tensile strength in flexure (modulus of rupture) (ASTM C78 or C293). A plain concrete beam *150 X 150 mm X 750 mm long* is loaded in flexure at the third points of 600-mm span until it fails due to cracking on the tension face. Modulus of rupture f_r is computed as

$$f_r = \frac{M}{I}c = \frac{6M}{bh^2}$$

It is accepted (ACI 9.5.2.3) that an average value for f_r may be taken as

$f_r = 0.62\lambda\sqrt{f_c'}$; f_c' $in\ MPa$
Where $\lambda = 1\ for\ normalweight\ concrete$



- Direct axial tension test. It is difficult to measure accurately and not in use today.

2.4 MODULUS OF ELASTICITY

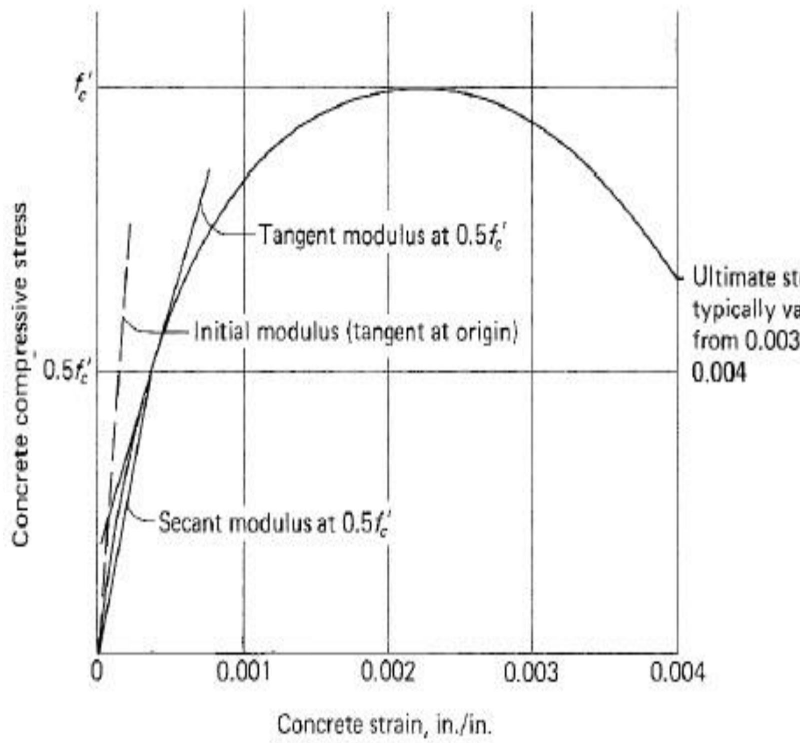
The modulus of elasticity of concrete varies, unlike that of steel, with strength.

A typical stress-strain curve for concrete in compression is shown. The initial modulus (tangent at origin), the tangent modulus (at $0.5f_c'$), and the secant modulus are noted. Usually the secant modulus at from 25 to 50% of the compressive strength f_c' is considered to be the modulus of elasticity. The empirical formula given by ACI-8.5.1

$$E_c = 0.043 w_c^{1.5}\sqrt{f_c'}$$

For normalweight concrete, E_c shall be permitted to be taken as

$$E_c = 4700\sqrt{f_c'}$$



Where:

$$1440 \le w_c \le 2560\ kg/m^3 \quad and\ f_c'\ in\ MPa.\ .$$

2.5 CREEP AND SHRINKAGE

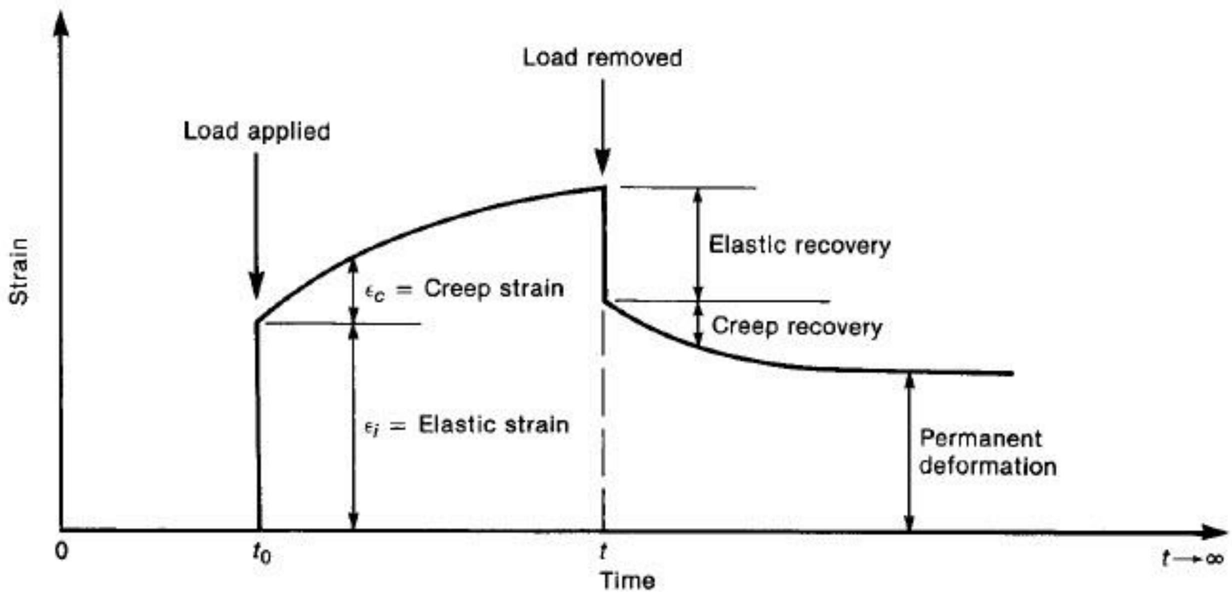
Creep and shrinkage are time-dependent deformations that, along with cracking, provide the greatest concern for the designer because of the inaccuracies and unknowns that surround them. Concrete is elastic only under loads of short duration; and, because of additional deformation with time, the effective behavior is that of an inelastic material. Deflection after a long period of time is therefore difficult to predict, but its control is needed to assure serviceability during the life of the structure.

Creep (or plastic flow) is the property of concrete (and other materials) by which it continues to deform with time under sustained loads at unit stresses within the accepted elastic range (say, below $0.5f_c'$). This inelastic deformation increases at a decreasing rate during the time of loading, and its total magnitude may be several times as large as the short-time elastic deformation.

Frequently creep is associated with shrinkage, since both are occurring simultaneously and often provide the same net effect: increased deformation with time.

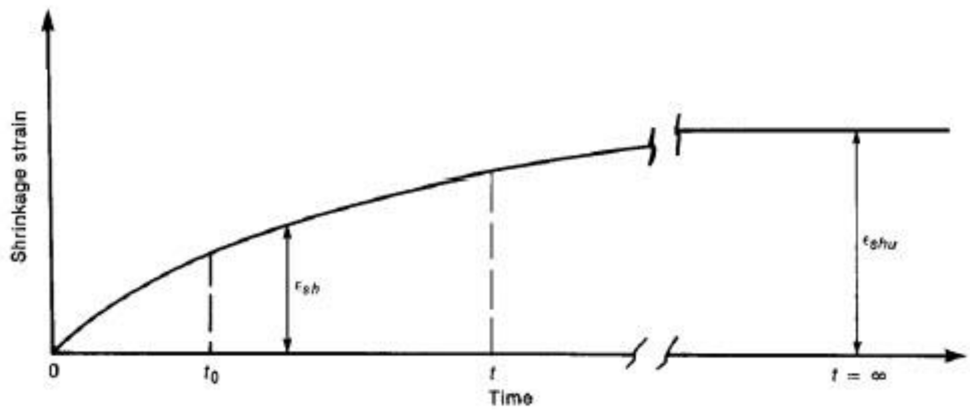
The internal mechanism of creep, or "plastic flow" as it is sometimes called, may be due to any one or a combination of the following: (1) crystalline flow in the aggregate and hardened cement paste; (2) plastic flow of the cement paste surrounding the aggregate; (3) closing of internal voids; and (4) the flow of water out of the cement gel due to external load and drying.

Factors affecting the magnitude of creep are (1) the constituents—such as the composition and fineness of the cement, the admixtures, and the size, grading, and mineral content of the aggregates: (2) proportions such as water content and water-cement ratio; (3) curing temperature and humidity; (4) relative humidity during period of use; (5) age at loading; (6) duration of loading; (7) magnitude of stress; (8) surface-volume ratio of the member; and (9) slump.



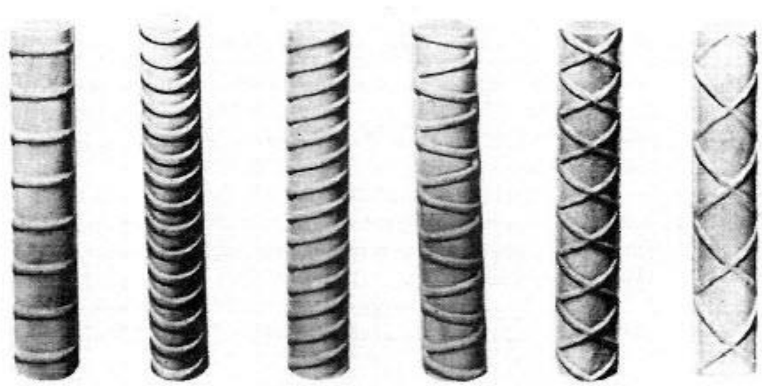
Creep of concrete will often cause an increase in the long-term deflection of members. Unlike concrete, steel is not susceptible to creep. For this reason, steel reinforcement is often provided in the compression zone of beams to reduce their long-term deflection.

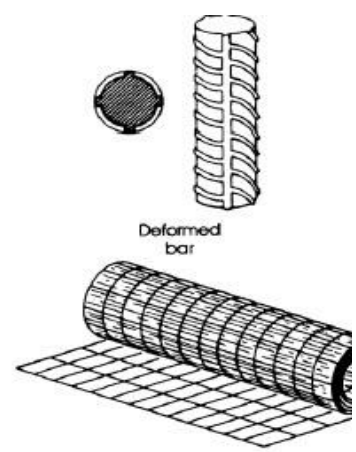
Shrinkage, broadly defined, is the volume change during hardening and curing of the concrete. It is unrelated to load application. The main cause of shrinkage is the loss of water as the concrete dries and hardens. It is possible for concrete cured continuously under water to increase in volume; however, the usual concern is with a decrease in volume. In general, the same factors have been found to influence shrinkage strain as those that influence creep—primarily those factors related to moisture loss.

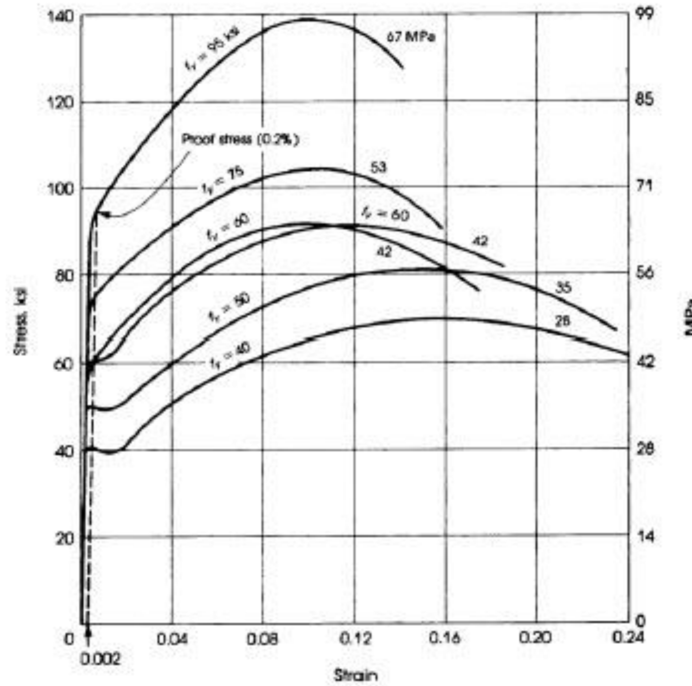


2.6 STEEL REINFORCEMENT

The useful strength of ordinary reinforcing steels in tension as well as compression, the yield strength is about 15 times the compressive strength of common structural concrete and well over 100 times its tensile strength.







Steel reinforcement may consist of:

- Bars (deformed bars, as in picture below) – for usual construction.
- Welded wire fabric – is used in thins slabs, thin shells.
- Wires – are used for prestressed concrete.

The "Grade" of steel is the minimum specified yield stress (point) expressed in:

- *MPa* for SI reinforcing bar Grades 300, 350, 420, and 520.
- *Ksi* for Inch-Pound reinforcing bar Grades 40, 50, 60, and 75.

The introduction of carbon and alloying additives in steel increases its strength but reduces its ductility. The proportion of carbon used in structural steels varies between 0.2% and 0.3%. The steel modulus of elasticity (E_s) is constant for all types of steel. The ACI Code has adopted a value of $E_s = 2 \times 10^5\ MPa\ (2 \times 10^6\ psi)$.

Summary of minimum ASTM strength requirements

Product	ASTM Specification	Designation	Minimum Yield Strength, psi (MPa)	Minimum Tensile Strength, psi (MPa)
Reinforcing bars	A615	Grade 40 Grade 60 Grade 75	40,000 (280) 60,000 (420) 75,000 (520)	60,000 (420) 90,000 (620) 100,000 (690)
	A706	Grade 60	60,000 (420) [78,000 (540) maximum]	80,000 (550)[a]
	A996	Grade 40 Grade 50 Grade 60	40,000 (280) 50,000 (350) 60,000 (420)	60,000 (420) 80,000 (550) 90,000 (620)
	A1035	Grade 100	100,000 (690)	150,000 (1030)
Deformed bar mats	A184	Same as reinforcing bars		
Zinc-coated bars	A767	Same as reinforcing bars		
Epoxy-coated bars	A775, A934	Same as reinforcing bars		
Stainless-steel bars[b]	A955	Same as reinforcing bars		
Wire Plain	A82		70,000 (480)	80,000 (550)
Deformed	A496		75,000 (515)	85,000 (585)
Welded wire reinforcement Plain W1.2 and larger Smaller than W1.2	A185		65,000 (450) 56,000 (385)	75,000 (515) 70,000 (485)
Deformed	A497		70,000 (480)	80,000 (550)
Prestressing tendons Seven-wire strand	A416	Grade 250 (stress-relieved)	212,500 (1465)	250,000 (1725)
		Grade 250 (low-relaxation)	225,000 (1555)	250,000 (1725)
		Grade 270 (stress-relieved)	229,500 (1580)	270,000 (1860)
		Grade 270 (low-relaxation)	243,000 (1675)	270,000 (1860)
Wire	A421	Stress-relieved	199,750 (1375) to 212,500 (1465)[c]	235,000 (1620) to 250,000 (1725)[c]
		Low-relaxation	211,500 (1455) to 225,000 (1550)[c]	235,000 (1620) to 250,000 (1725)[c]
Bars	A722	Type I (plain) Type II (deformed)	127,500 (800) 120,000 (825)	150,000 (1035) 150,000 (1035)
Compacted strand[b]	A779	Type 245 Type 260 Type 270	241,900 (1480) 228,800 (1575) 234,900 (1620)	247,000 (1700) 263,000 (1810) 270,000 (1860)

[a] But not less than 1.25 times the actual yield strength.
[b] Not listed in ACI 318.
[c] Minimum strength depends on wire size.

Cross sectional Areas of standard steel bars for reinforced concrete structures

Diameter, mm	Area of bars for Number of bars , cm²									Mass, Kg/ m
	1	2	3	4	5	6	7	8	9	
6	0.283	0.565	0.848	1.131	1.414	1.696	1.979	2.262	2.545	0.222
8	0.503	1.005	1.508	2.011	2.513	3.016	3.519	4.021	4.524	0.395
10	0.785	1.571	2.356	3.142	3.927	4.712	5.498	6.283	7.069	0.617
12	1.131	2.262	3.393	4.524	5.655	6.786	7.917	9.048	10.179	0.888
14	1.539	3.079	4.618	6.158	7.697	9.236	10.776	12.315	13.854	1.208
16	2.011	4.021	6.032	8.042	10.053	12.064	14.074	16.085	18.096	1.578
18	2.545	5.089	7.634	10.179	12.723	15.268	17.813	20.358	22.902	1.998
20	3.142	6.283	9.425	12.566	15.708	18.850	21.991	25.133	28.274	2.466
22	3.801	7.603	11.404	15.205	19.007	22.808	26.609	30.411	34.212	2.984
25	4.909	9.817	14.726	19.635	24.544	29.452	34.361	39.270	44.179	3.854
28	6.158	12.315	18.473	24.630	30.788	36.945	43.103	49.260	55.418	4.834
32	8.042	16.085	24.127	32.170	40.212	48.255	56.297	64.340	72.382	6.314
36	10.179	20.358	30.536	40.715	50.894	61.073	71.251	81.430	91.609	7.991
40	12.566	25.133	37.699	50.265	62.832	75.398	87.965	100.531	113.097	9.865
45	15.904	31.809	47.713	63.617	79.522	95.426	111.330	127.235	143.139	12.486

BEAM DESIGN METHODS AND REQUIREMENTS

Design and Analysis Philosophies

Design involves the determination of the type of structural system to be used, the cross sectional dimensions, and the required reinforcement. The designed structure should be able to resist all forces expected to act during the life span of the structure safely and without excessive deformation or cracking.
Analysis involves the determination of the capacity of a section of known dimensions, material properties and steel reinforcement, if any to external forces and moments.

Design Methods (Philosophies)

Two methods of design have long prevalent.

- **Working Stress Method** focuses on conditions at service loads.
- **Strength Design Method** focusing on conditions at loads greater than the service loads when failure may be imminent.

The Strength Design Method is deemed conceptually more realistic to establish structural safety.

1 The Working-Stress Design Method

This method is based on the condition that the stresses caused by service loads without load factors are not to exceed the allowable stresses which are taken as a fraction of the ultimate stresses of the materials, fc' for concrete and f_y for steel.

.2 The Ultimate – Strength Design Method

At the present time, the ultimate-strength design method is the method adopted by most prestigious design codes.
In this method, elements are designed so that the internal forces produced by factored loads do not exceed the corresponding reduced strength capacities.

Reduced strength provided ≥ [factored loads]

The factored loads are obtained by multiplying the working loads (service loads) by factors usually greater than unity.

Safety Provisions (the strength requirement)

Safety is required to insure that the structure can sustain all expected loads during its construction stage and its life span with an appropriate factor of safety.

There are three main reasons why some sorts of safety factor are necessary in structural design.

• **Variability in resistance**. *Variability of fc' and fy, *assumptions are made during design and *differences between the as-built dimensions and those found in structural drawings.

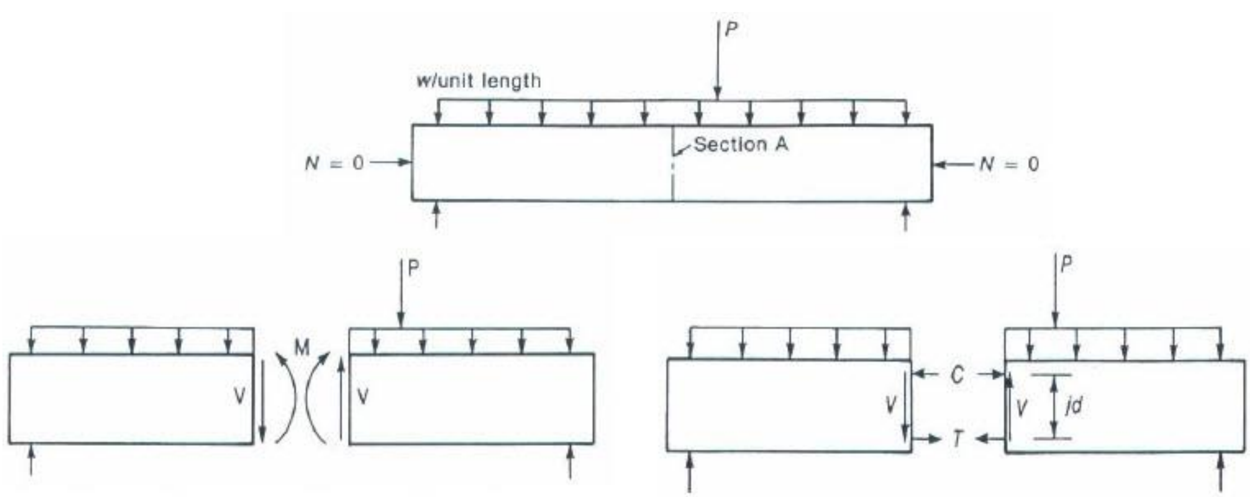
• **Variability in loading**. Real loads may differ from assumed design loads, or distributed differently.

• **Consequences of failure**. *Potential loss of life, *cost of clearing the debris and replacement of the structure and its contents and *cost to society.

Analysis of beams in bending at service loads

1Introduction

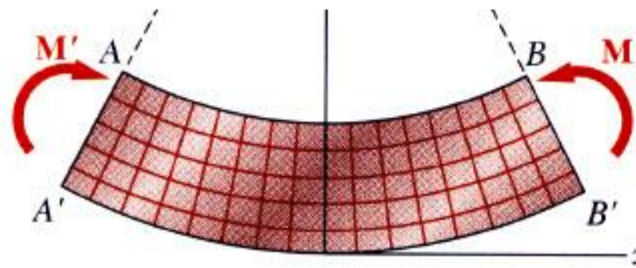
A beam is a structural member used to support the internal moments and shears and in some cases torsion.



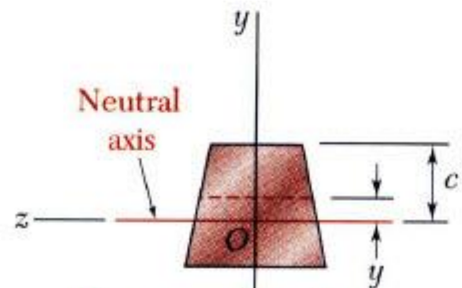
$$C = T \rightarrow \quad M = C\,(jd) = T\,(jd)$$

.2 Basic Assumptions in Beam Theory

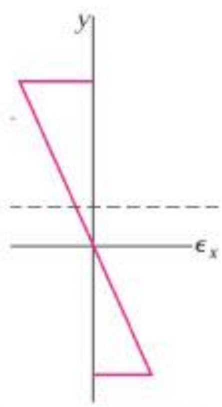
1. Plane sections remain plane after bending. This means that in an initially straight beam, strain varies linearly over the depth of the section after bending.



Beam after bending

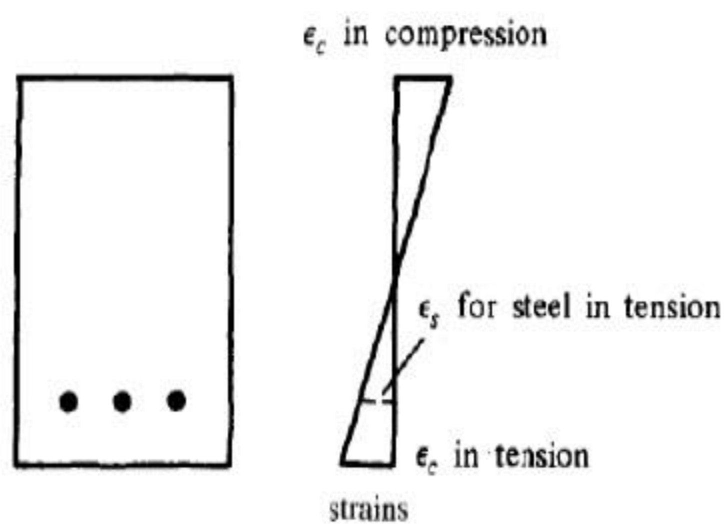


Its cross section



Strain distribution

2. The strain in the reinforcement is equal to the strain in the concrete at the same level, i.e. $\varepsilon_s = \varepsilon_c$ at same level.
3. Concrete is assumed to fail in compression, when $\varepsilon_c = 0.003$.
4. Tensile strength of concrete is neglected in flexural strength.
5. Perfect bond is assumed between concrete and steel.



WORKING STRESS METHOD

In the working stress method, a structural element is so designed that the stresses resulting from the action of service loads (also called working loads) and computed by the mechanics of elastic members do not exceed some predesigned allowable values.

Service load is the load, such as dead, live, snow, wind, and earthquake, which is assumed actually to occur when the structure is in service.
The working stress method may be expressed by the following:

$$f \le f_{allow}$$

Where:

f: an elastic stress, such as by using the flexure formula $f = \frac{MC}{I}$ for a beam, computed under service load.

f_{allow} a limiting or allowable stress prescribed by a building code as a percentage of the compressive strength f_c' for concrete, or of the yield stress for the steel reinforcing bars f_y.

In this section, it is assumed that a small transverse load is placed on a concrete beam with tensile reinforcing and that the load is gradually increased in magnitude until the beam fails. As this takes place, the beam will go through three distinct stages before collapse occurs. These are: (1) the uncracked concrete stage, (2) the concrete cracked–elastic stresses stage, and (3) the ultimate-strength stage. A relatively long beam is considered for this discussion so that shear will not have a large effect on its behavior.

Stages of flexural behavior

1 Uncracked Concrete Stage

At small loads when the tensile stresses are less than the modulus of rupture (the bending tensile stress at which the concrete begins to crack), the entire cross section of the beam resists bending, with compression on one side and tension on the other as shown in Figure below. According to ACI 318-11 code the modulus of rupture is:

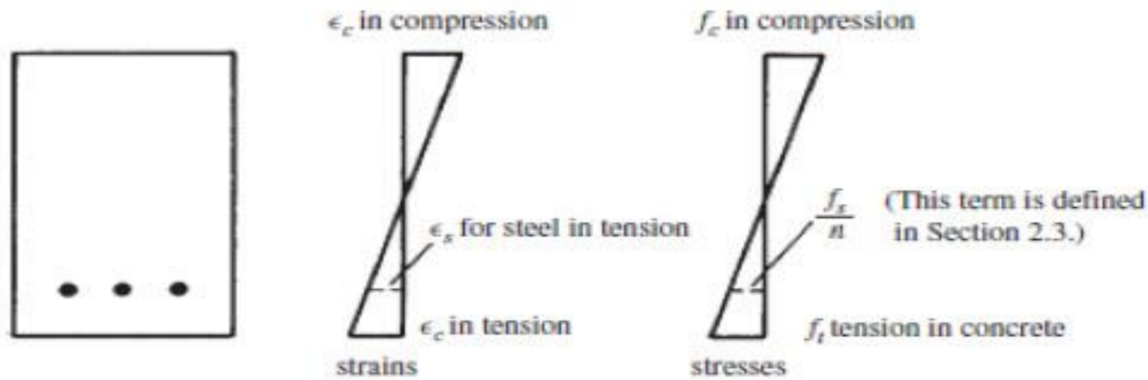
$f_r = 0.62\lambda\sqrt{f_{c'}}$

Where λ =

1 for normal concrete.

0.85 For sand light weight concrete.

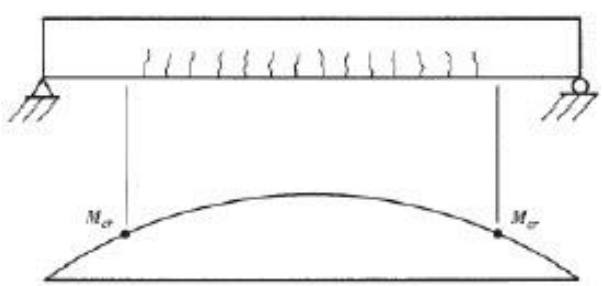
0.75 For all light weight concrete.



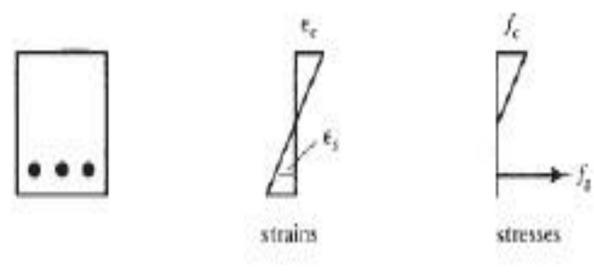
2 Concrete Cracked–Elastic Stresses Stage

As the load is increased after the modulus of rupture of the concrete is exceeded, cracks begin to develop in the bottom of the beam. The moment at which these cracks begin to form—that is, when the tensile stress in the bottom of the beam equals the modulus of rupture—is referred to as the cracking moment, **M_{cr}**. As the load is further increased, these cracks quickly spread up to the vicinity of the neutral axis, and then the neutral axis begins to move upward. The cracks occur at those places along the beam where the actual moment is greater than the cracking moment, as shown in Figure (a).

Now that the bottom has cracked, another stage is present because the concrete in the cracked zone obviously cannot resist tensile stresses—the steel must do it. This stage will continue as long as the compression stress in the top fibers is less than about one-half of the concrete's compression strength, f_c', and as long as the steel stress is less than its yield stress. The stresses and strains for this range are shown in Figure (b). In this stage, the compressive stresses vary linearly with the distance from the neutral axis or as a straight line.



(a)



(b)

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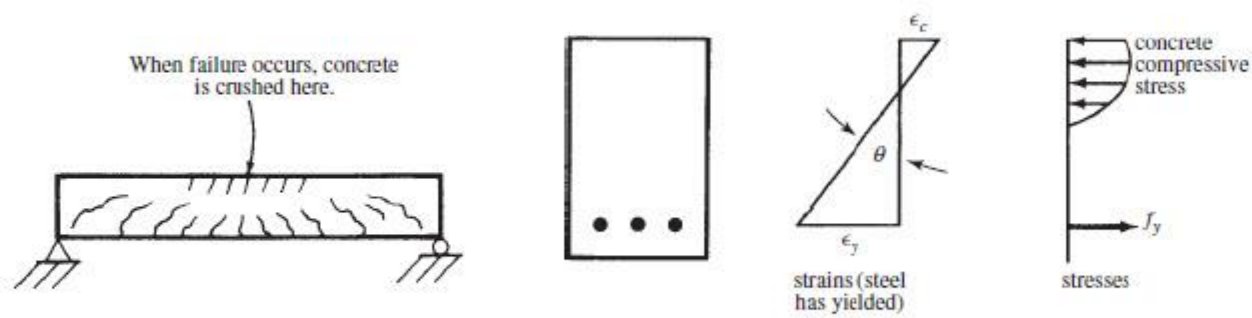
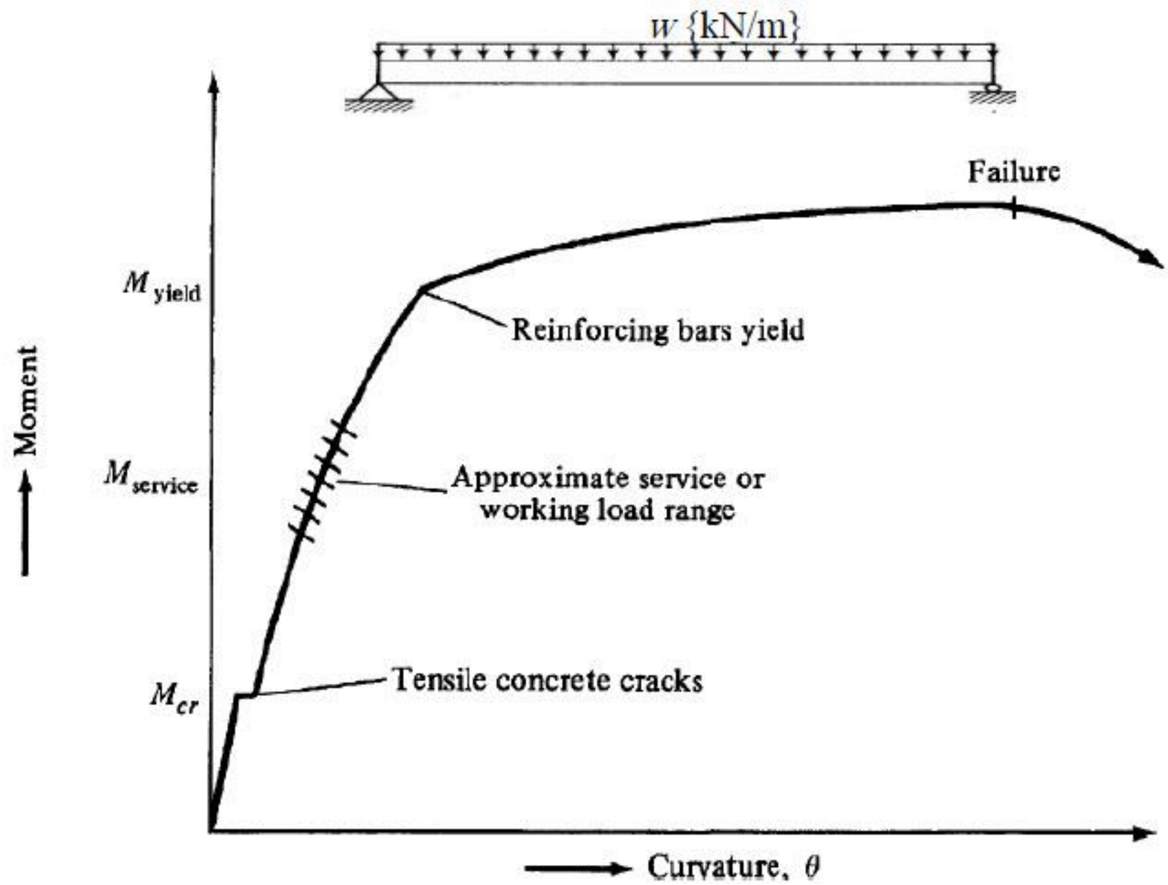
The straight-line stress–strain variation normally occurs in reinforced concrete beams under normal service-load conditions because at those loads, the stresses are generally less than $0.5f_c'$. To compute the concrete and steel stresses in this range, the transformed-area method is used. The service or working loads are the loads that are assumed to actually occur when a structure is in use or service. Under these loads, moments develop that are considerably larger than the cracking moments. Obviously, the tensile side of the beam will be cracked.

3 Beam Failure—Ultimate-Strength Stage

As the load is increased further so that the compressive stresses are greater than $0.5f_c'$, the tensile cracks move farther upward, as does the neutral axis, and the concrete compression stresses begin to change appreciably from a straight line. For this initial discussion, it is assumed that the reinforcing bars have yielded. The stress variation is much like that shown in Figure below.



To further illustrate the three stages of beam behavior that have just been described, a moment–curvature diagram is shown in Figure below. For this diagram, θ is defined as the angle change of the beam section over a certain length and is computed by the following expression in which ε is the strain in a beam fiber at some distance, y, from the neutral axis of the beam:

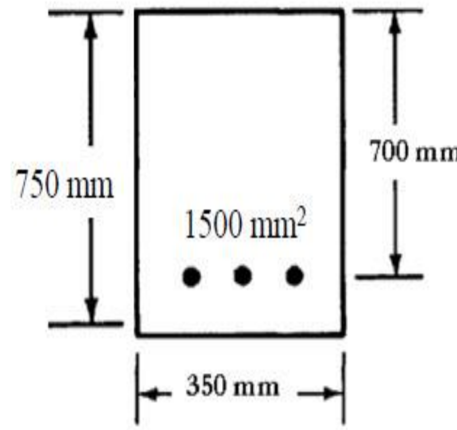


Stages of flexural behavior

The first stage of the diagram is for small moments less than the cracking moment, $\mathbf{M_{cr}}$, where the entire beam cross section is available to resist bending. In this range, the strains are small, and the diagram is nearly vertical and very close to a straight line. When the moment is increased beyond the cracking moment, the slope of the curve will decrease a little because the beam is not quite as stiff as it was in the initial stage before the concrete cracked. The diagram will follow almost a straight line from $\mathbf{M_{cr}}$ to the point where the reinforcing is stressed to its yield point. Until the steel yields, a fairly large additional load is required to appreciably increase the beam's deflection. After the steel yields, the beam has very little additional moment capacity, and only a small additional load is required to substantially increase rotations as well as deflections. The slope of the diagram is now very flat.

<u>Example 1:</u>

Calculate the cracking moment for the section shown below if the value of $f_c' = 30\ MPa$

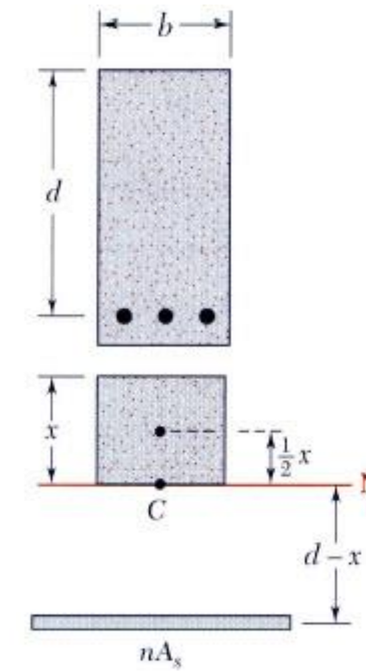


$$I_g = \frac{1}{12}bh^3 = \frac{350 \times 750^3}{12} = 1.2305 \times 10^{10} mm^4$$

$$f_r = 0.62\sqrt{f_c'} = 0.62\sqrt{30} = 3.4MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.4 \times 1.2305 \times 10^{10}}{750/2} \times 10^{-6} = 111.43kN.m$$

NOTE



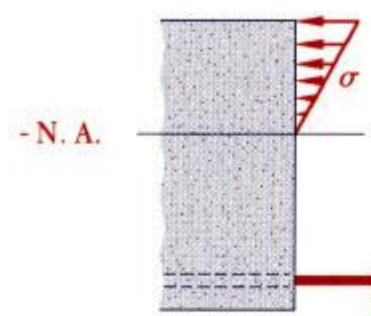
- After cracking, the steel bars carry the entire tensile load below the neutral surface. The upper part of the concrete beam carries the compressive load.
- In the transformed section, the cross sectional area of the steel, A_s, is replaced by the equivalent area nA_s.
  Where:

n = the modular ratio = E_s / E_c

- To determine the location of the neutral axis,

$$bx\frac{x}{2} - nA_s(d - x) = 0$$

$$\frac{1}{2}bx^2 + nA_s - nA_s d = 0$$



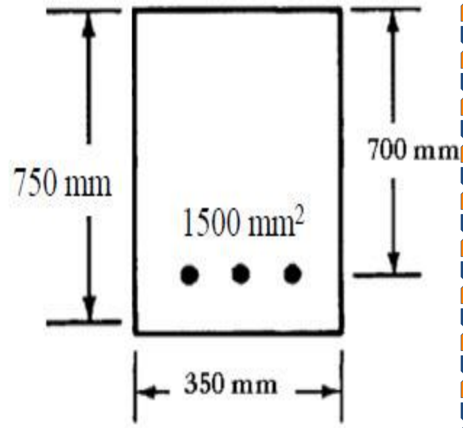
- The height of the concrete compression block is x.
- The normal stress in the concrete and steel.

$$\sigma_x = \frac{My}{I};$$

$$\sigma_x = \sigma_c = f_c \ \ and \ \sigma_s = nf_c$$

Example 2:

Calculate the bending stresses for the section shown, M= 180 kN.m, $f_c' = 30\ MPa$.



solution

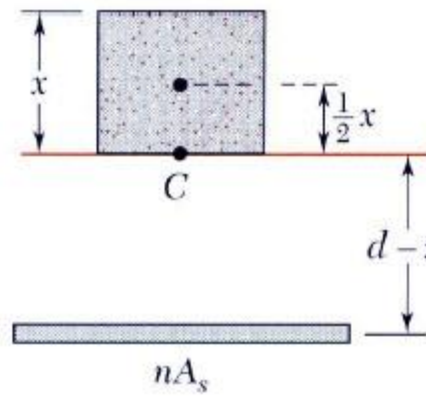
Note:

M > M_{cr} = 111 kN.m from previous example. Thus, section is cracked.

$$E_c = 4700\sqrt{f_c'} = 4700\sqrt{30} = 25743MPa$$

$$n = \frac{E_s}{E_c} = \frac{200000}{25743} = 7.77$$

$$\frac{350x^2}{2} + 8(1500) - (700 - x) = 0 \quad \rightarrow x = 187.45mm$$



$$I_{cr} = \frac{bx^3}{3} + nA_s(d-x)^2 = \frac{350 \times 187.45^3}{3} + 8(1500) \times (700 - 187.45)^2$$

$$I_{cr} = 3.92 \times 10^9 mm^4$$

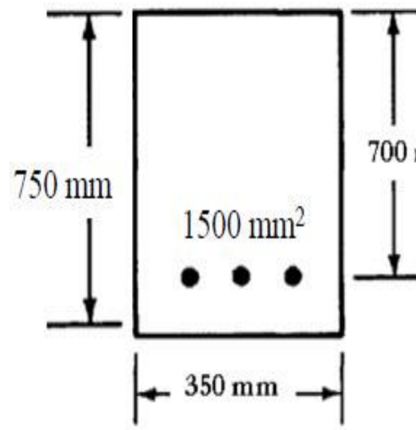
$$f_c = \frac{My}{I_{cr}} = \frac{180 \times 10^6 \times 187.45}{3.92 \times 10^9} = 8.6\ MPa$$

$$f_c = 8.6\ MPa < 0.45f'_c = 0.45(30) = 13.5MPa \quad ok$$

$$f_s = n\frac{My}{I_{cr}} = 8\frac{180 \times 10^6(700 - 187.45)}{3.92 \times 10^9} = 188.3\ MPa$$

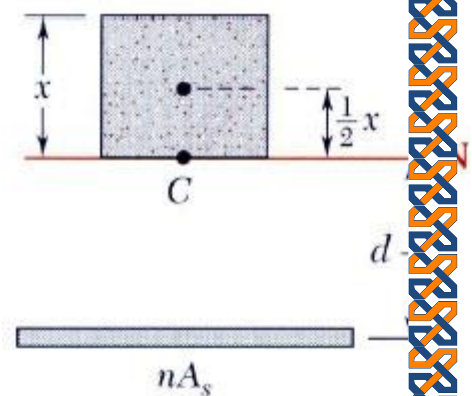
Example 3:

Calculate the allowable moment for the section shown, $f_s(allowable)$=180 MPa, $f_c(allowable)$=12 MPa, $f_c' = 30MPa$.



Solution

$$M_s = \frac{f_s I_{cr}}{ny} = \frac{180\times3.92\times10^9}{8(700-187.45)\times10^6} = 172kN.m$$

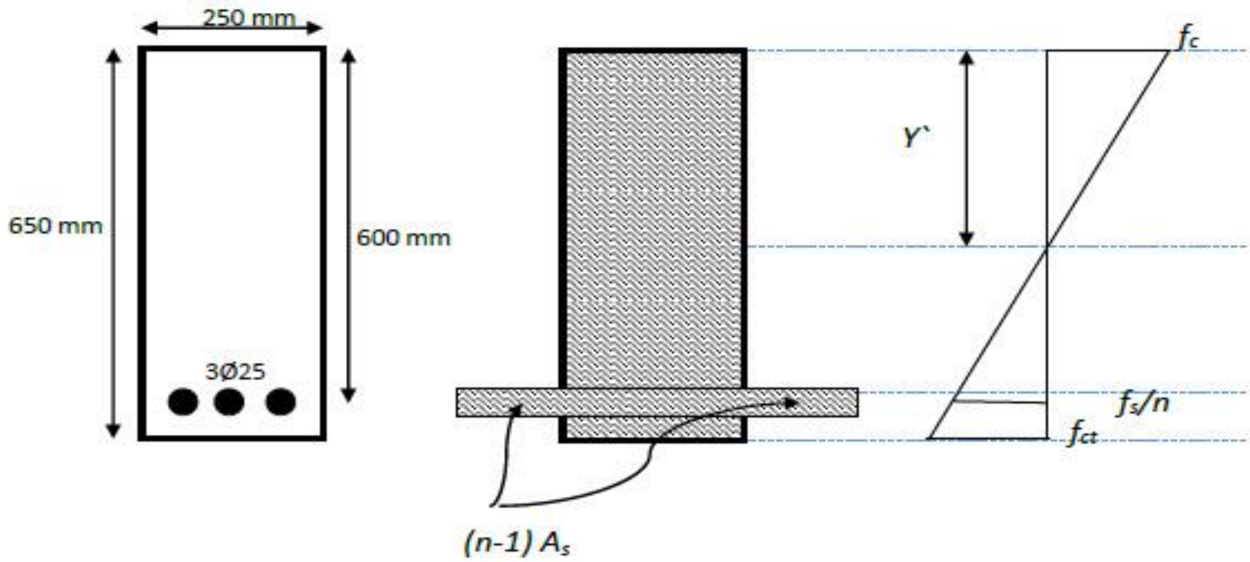


$$M_c = \frac{f_c I_{cr}}{y} = \frac{12\times3.92\times10^9}{187.45\times10^6} = 250.95kN.m$$

$$M_{allowable} = 172kN.m$$

Example 5:

For reinforced concrete beam shown in figure determine the stresses caused by a bending moment M=61 kN.m. Take ***f `c=28 MPa and f_y=420 MPa.***



Solution:-

$n = Es/Ec = 200000/4700\sqrt{fc'} = 8$

$As = 3(510) = 1530mm^2$

$Y` = (650\times250\times6502 + (8-1)\times1530\times600)/(650\times250 + (8-1)\times1530)$

$= 342\ mm.$

$$I = \frac{250\times650^3}{12} + 250\times650\times(342-325)^2 + (8-1)\times1530\times(600-342)^2$$
$$= 6.48\times10^9 mm^4$$

$$f_{ct} = \frac{M(650 - Y`)}{I} = \frac{61\times10^6\times(650-342)}{6.48\times10^9} = 2.9\ MPa$$

$$f_r = 0.62\sqrt{f_c'} = 3.28\ MPa > f_{ct} = 2.9\ MPa \rightarrow the\ section\ is\ uncracked.$$

$$f_c = \frac{MY'}{I} = \frac{61\times10^6\times342}{6.48\times10^9} = 3.22\ MPa = 11.5\% f_c'$$

$$f_s = n\frac{f_c}{Y'}(600 - Y') = 8\times\frac{3.22}{342}\times(600-342) = 19.43\ MPa = 4.6\% f_y.$$

Elastic Stresses—Concrete Cracked

When the bending moment is sufficiently large to cause the tensile stress in the extreme fibers to be greater than the modulus of rupture, it is assumed that all of the concrete on the tensile side of the beam is cracked and must be neglected in the flexure calculations.

The cracking moment of a beam is normally quite small compared to the service load moment. Thus, when the service loads are applied, the bottom of the beam cracks. The cracking of the beam does not necessarily mean that the beam is going to fail. The reinforcing bars on the tensile side begin to pick up the tension caused by the applied moment.

According to ACI-code $\mathbf{M_{cr}}$ can be calculated as shown

$$M_{cr} = \frac{f_r I_g}{y_t}$$

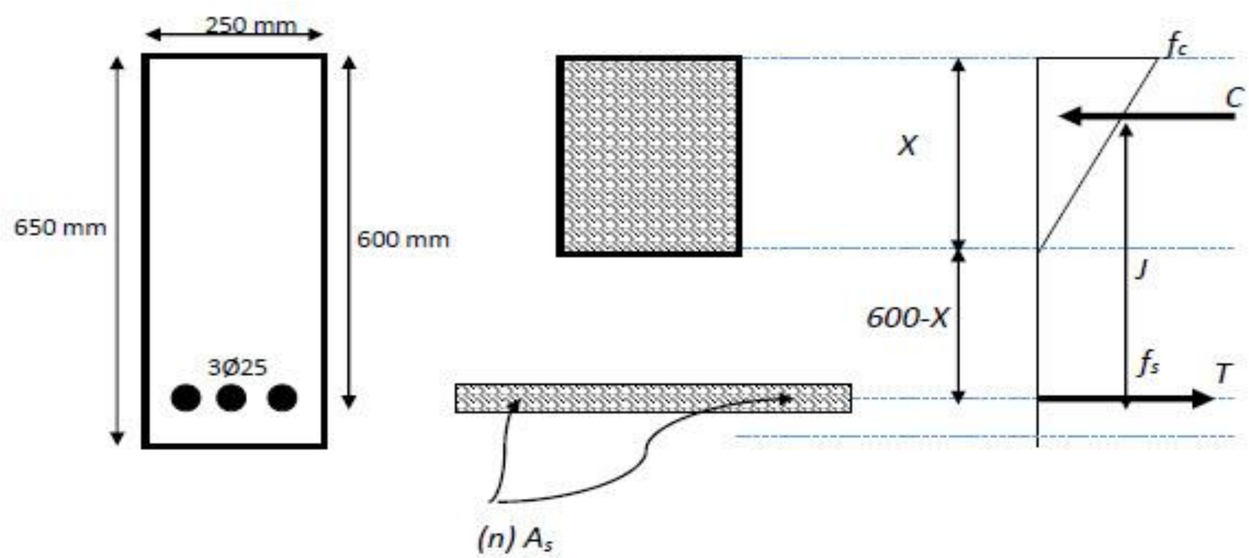
Where:

I_g= moment of inertia of cross section of beam.

y_t = is the distance between the centroid of beam to the extreme fiber on tension side.

Example 6:

For reinforced concrete beam shown in figure determine the stresses caused by a bending moment M=122 kN.m. Take f_c'**=28 MPa and** $\mathbf{f_y}$**=420 MPa.**



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Solution:-

$$n = \frac{E_s}{E_c} = \frac{200000}{4700\sqrt{f_c'}} = 8$$

$$A_s = 3(510) = 1530mm^2$$

$$f_r = 0.62\sqrt{f_c'} = 3.28\ MPa$$

$$I_g = \frac{250 \times 650^3}{12} = 5.72 \times 10^9 mm^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.28 \times 5.72 \times 10^9}{650/2} \times 10^{-6} = 57.73kN.m < 122kN.m$$

The section is cracked.

Moment of compression area= Moment of tension area

$$250 \times \frac{x^2}{2} = 8 \times 1530(600 - x) \rightarrow solve\ to\ get\ x = 198.32mm$$

$$I_{cr} = \frac{250 \times 198.32^3}{3} + 8 \times 1530(600 - 198.23)^2 = 2.62 \times 10^9 mm^4$$

$$f_c = \frac{Mx}{I_{cr}} = \frac{122 \times 10^6 \times 198.32}{2.62 \times 10^9} = 9.23\ MPa$$

$$f_s = n\frac{M(600 - x)}{I_{cr}} = 8\frac{22 \times 10^6(600 - 198.32)}{2.62 \times 10^9} = 149.6\ MPa$$

From above, we can note that the existence of cracks leads to reduce the moment of inertia by about 54%.

Working Stresses Method : Design of RC Beams

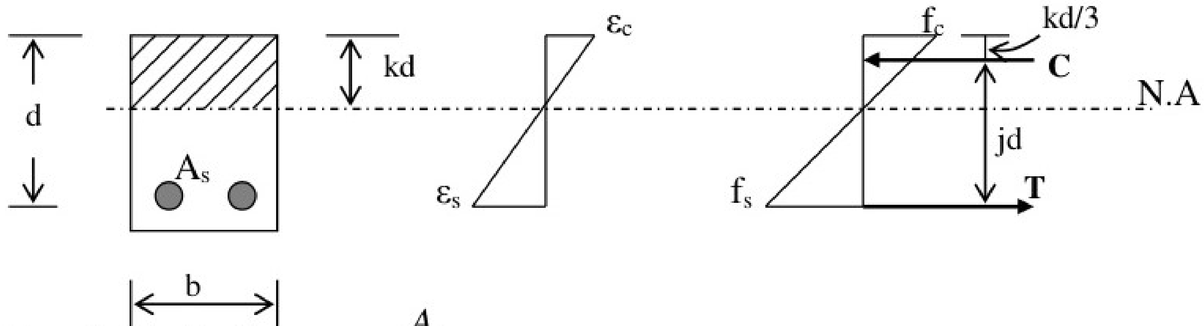
Internal forces in a singly reinforced concrete beam are represented by the following:

Compression force in concrete = $C = \frac{1}{2} f_c \times b \times kd$

Tension force in steel = $T = A_s \times f_s$

From equilibrium : $\sum f = 0 \quad \Rightarrow \; C = T$

$$\frac{1}{2} f_c \times b \times kd = A_s f_s$$



If tensile steel ratio = $\rho = \frac{A_s}{b \times d}$

$$\frac{1}{2} f_c \times b \times kd = \rho \times b \times d \times f_s$$

$$\therefore \rho = \frac{1}{2} \frac{f_c}{f_s} k \qquad \text{..... (1)}$$

$$\sum M = 0$$

$$M = C \times jd = \frac{1}{2} f_c \times b \times kd(jd) = \frac{1}{2} f_c (kj) bd^2$$

OR;

$$M = T \times jd = A_s f_s jd$$

Determination of " k " value:

1- From general expression:

By taking moment of areas about the neutral axis,

$$\frac{b(kd)^2}{2} - nA_s(d - kd) = 0$$

$$\rho = \frac{A_s}{bd} \quad \text{OR} \quad A_s = \rho bd$$

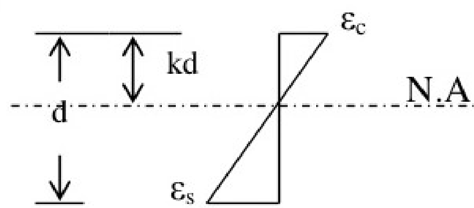
$$\frac{b(kd)^2}{2} - n\rho bd(d - kd) = 0 \qquad \times \frac{2}{bd^2}$$

$$k^2 + 2n\rho k - 2n\rho = 0$$

$$k = \sqrt{n^2\rho^2 + 2n\rho} - n\rho \qquad ..... (2)$$

Equation (2) is important to calculate the strength capacity of a given section (analysis).

2- From the geometry of the strain diagram,



$$\frac{\varepsilon_c}{\varepsilon_s} = \frac{kd}{d - kd}$$

$$\frac{\varepsilon_c}{\varepsilon_s} = \frac{k}{1-k} \qquad ..... (3)$$

$$\varepsilon_c = \frac{f_c}{E_c} \quad \text{and} \quad \varepsilon_s = \frac{f_s}{E_s} \quad \text{and} \quad n = \frac{E_s}{E_c}$$

And let $r = \frac{f_s}{f_c}$

$\therefore$ equation (3) yields,

$$\frac{f_c}{f_s} \times \frac{E_s}{E_c} = \frac{k}{1-k} \quad \Rightarrow \quad \frac{1}{r} \times n = \frac{k}{1-k} \quad \Rightarrow \quad \frac{n}{r} - k\frac{n}{r} = k$$

$$k\frac{n}{r} + k = \frac{n}{r} \quad \Rightarrow \quad k\left(\frac{n}{r} + 1\right) = \frac{n}{r} \quad \Rightarrow \quad k = \frac{\frac{n}{r}}{\left(\frac{n}{r} + 1\right)}$$

$$k = \frac{n}{n + r} \qquad ..... (4)$$

Equation (4) is important in the design of reinforced concrete section.

Determination of " j " value:

$$jd = 1 - \frac{kd}{3} \quad \Rightarrow \quad j = 1 - \frac{k}{3}$$

Balanced steel ratio (ρ_e)***:***

It is the stage at steel at which steel and concrete reaches to their allowable stresses (f_{ca} and f_{sa}) at same time and under same applied loads.

$$\frac{1}{2} f_c \times b \times kd = A_s f_s$$

$$\left[\frac{1}{2} f_c \times b \times kd = \rho \times b \times d \times f_s\right] \div bd \quad \Rightarrow \quad \frac{1}{2} f_c k = \rho f_s$$

$$\rho_e = \frac{1}{2}\frac{f_c}{f_s}k \quad \Rightarrow \quad \rho_e = \frac{k}{2r}$$

From equation (4), $k = \dfrac{n}{n+r}$

$$\therefore \rho_e = \frac{n}{2r(n+r)}$$ where, at balanced state $r = \dfrac{f_{sa}}{f_{ca}}$

Under reinforced and over reinforced sections:

Under reinforced section: Occurs when steel reaches its allowable stress at lower load than for concrete (steel reaches its allowable stress before concrete).

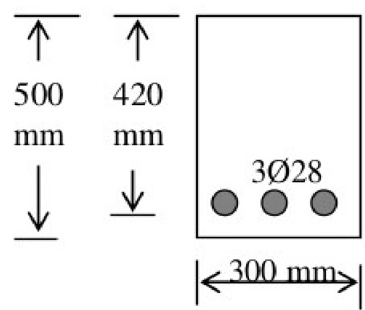
If $\rho = \dfrac{A_s}{bd} < \rho_e \quad \Rightarrow \quad M_{allowable} = T \cdot jd$

Over reinforced section: Occurs when Concrete reaches its allowable stress at lower load than for steel (concrete reaches its allowable stress before steel).

If $\rho = \dfrac{A_s}{bd} > \rho_e \quad \Rightarrow \quad M_{allowable} = C \cdot jd$

Ex.5)

Calculate the maximum flexural stresses for the beam section shown below, if the applied moment is 95 kN.m , and n = 9.



Sol.)

$$\rho = \frac{A_s}{bd} = \frac{1847}{300 \times 420} = 0.0147$$

$$\rho n = 0.0147 \times 9 = 0.1323$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$k = \sqrt{2(0.1323) + (0.1323)^2} - 0.1323 = 0.399$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.399}{3} = 0.867$$

$$M = \frac{1}{2} f_c \times b \times kd(jd) = \frac{1}{2} f_c (kj) bd^2$$

$$f_c = \frac{2M}{bd^2 kj} = \frac{2(95 \times 10^6)}{300(420)^2 0.399 \times 0.867} = 10.37 MPa$$

$$M = A_s f_s jd$$

$$f_s = \frac{M}{A_s jd} = \frac{95 \times 10^6}{1847 \times 0.867 \times 420} = 141.3 MPa$$

Spacing limits of reinforcements: [ACI 318-05 sec. 7.6]

1- The minimum clear spacing between parallel pars in a layer shall be d_b but not less than 25mm.
2- If two layers of bars are used, the clear vertical spacing between layers shall not be less than 25mm.

Minimum concrete cover of beams: [ACI 318-05 sec. 7.7.1]

1- Concrete not exposed to weather or earth 40 mm.
2- Concrete exposed to weather or earth:
No.19 through No.57 bars 50 mm
No.16 and smaller 40 mm

Economical section:

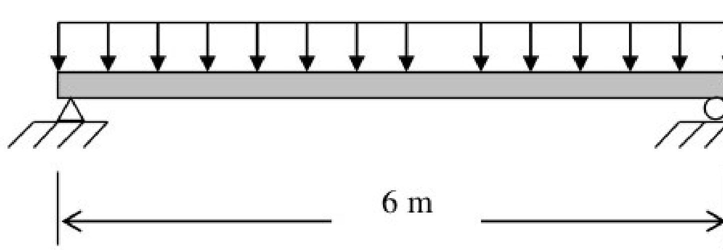
If the section dimensions are not restricted by architectural requirements, best economical section dimensions are :

$$\frac{2}{3} > \frac{b}{d} > \frac{1}{2} \quad \text{or} \quad \boxed{\frac{3}{2}b < d < 2b}$$

Ex.6)

Design the simply supported beam shown below to resist a dead load of 35 kN/m and alive load of 15 kN/m. Use $f'_c = 21MPa$ and $f_y = 300MPa$.

Sol.)



1- $M = \frac{wl^2}{8} = \frac{50(6)^2}{8} = 225kN.m$

2- Use $f_s = f_{sa} = 140MPa$

And $f_c = f_{ca} = 0.45 f'_c = 9.45MPa$

$E_s = 200000MPa$

$E_c = 4700\sqrt{f'_c} = 4700\sqrt{21} = 21538\,MPa$

$r = \frac{f_{sa}}{f_{ca}} = \frac{140}{9.45} = 14.8$, $n = \frac{E_s}{E_c} = \frac{200000}{21538} = 9.3$ say 9.0

$k = \frac{n}{n+r} = \frac{9}{9+14.8} = 0.378$

$j = 1 - \frac{k}{3} = 1 - \frac{0.378}{3} = 0.874$

3- Find concrete section dimensions b & d:

$M = \frac{1}{2} f_c (kj) bd^2$

$$225 \times 10^6 = \frac{1}{2}(9.45)(0.378 \times 0.874)bd^2$$

$$\therefore bd^2 = 144137662 mm^3$$

Now assume ' b ' in mm and calculate the corresponding ' d ':

b	d	
250 mm	759 mm	d > 2b $\therefore$ try greater ' b ' value
300 mm	693 mm	d > 2b $\therefore$ try greater ' b ' value
350 mm	641 mm	3/2 b < d < 2b $\therefore$ OK.

h = 641 + 40 (cover) + 12 (Stirrups) + 28/2 (bar diameter /2)
h = 707mm $\therefore$ use h = 710 mm
4- Calculate steel reinforcement amount:

$$M = A_s f_s jd$$

$$225 \times 10^6 = A_s(140)0.874 \times 644$$

$$\therefore A_s = 2855 mm^2$$

Try using $\phi 28mm$ bars

$$A_b = \frac{\pi(28)^2}{4} = 616 mm^2$$

Required Number of bars = $\frac{2855}{616} = 4.6$ bars

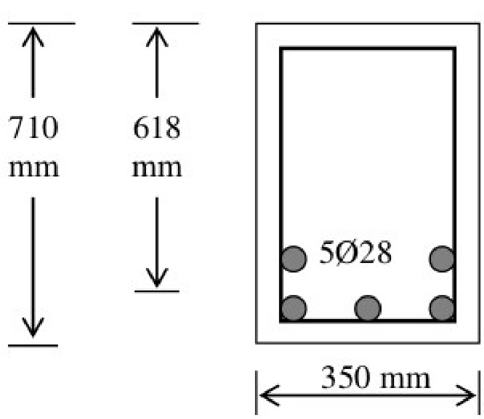
$\therefore$ Use $5\phi 28 \Rightarrow A_{s)provided} = 5 \times 616 = 3080 mm^2 > A_{s)Required} = 2855 mm^2$ $\therefore$ OK.

Check bar spacing $= \frac{350 - 5(28) - 2(40) - 2(12)}{4} = 26.5mm < d_b = 28mm$

$\therefore$ Not Good $\Rightarrow$ Arrange bars in two layers: 3 +2

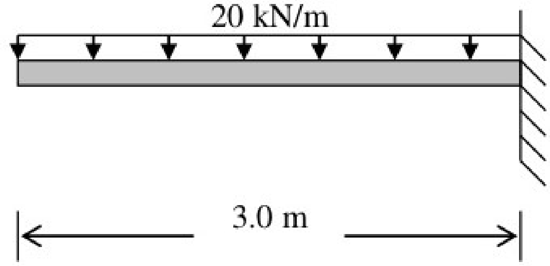
Check bar spacing $= \frac{350 - 3(28) - 2(40) - 2(12)}{2} = 81mm > d_b = 28mm$ $\therefore$ OK.

d =710 - 40 - 12 – 28(Lower layer)+25/2(bar spacing)= 618 mm



Ex.7)

Design the cantilever beam shown below, if $f_c' = 28MPa$, $f_y = 420MPa$ and $n = 8$



Sol.)

1- $M = \frac{wl^2}{2} = \frac{20(3)^2}{2} = 90kN.m$

2- Use $f_s = f_{sa} = 170MPa$

And $f_c = f_{ca} = 0.45 f_c' = 12.6MPa$

$E_s = 200000MPa$

$E_c = 4700\sqrt{f_c'} = 4700\sqrt{28} = 24870\ MPa$

$r = \frac{f_{sa}}{f_{ca}} = \frac{170}{12.6} = 13.5$, $n = \frac{E_s}{E_c} = \frac{200000}{24870} = 8.0$

$k = \frac{n}{n+r} = \frac{8}{8+13.5} = 0.372$

$j = 1 - \frac{k}{3} = 1 - \frac{0.372}{3} = 0.876$

3- Find concrete section dimensions b & d:

$M = \frac{1}{2} f_c (kj) bd^2$

$90 \times 10^6 = \frac{1}{2}(12.6)(0.372 \times 0.876) bd^2 \quad \Rightarrow bd^2 = 43838422mm^3$

Now assume ' b ' in mm and calculate the corresponding ' d ':

b	d	
200 mm	468 mm	d > 2b ∴ try greater ' b ' value
250 mm	419 mm	3/2 b < d < 2b ∴ OK.
300 mm	382 mm	d <3/2b ∴ Not good

h = 419 + 40 (cover) + 10 (Stirrups) + 25/2 (bar diameter /2)

h = 482mm ∴ use h = 500 mm

d = 500 – 40 -10 -25/2 = 438mm

4- Calculate steel reinforcement amount:

$M = A_s f_s jd$

$90 \times 10^6 = A_s (170) 0.876 \times 438$

$\therefore A_s = 1380 mm^2$

Try using $2\phi 25mm + 2\phi 16mm$ bars

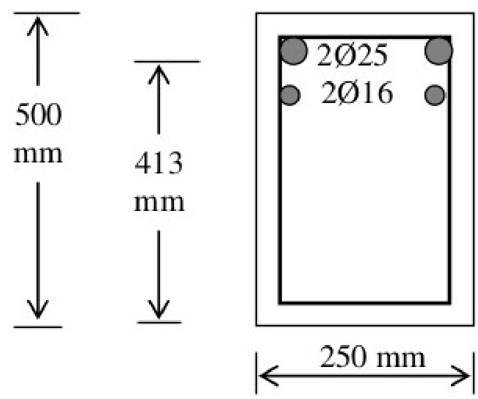
$A_{s)provided} = 2 \times 491 + 2 \times 201 = 1384 mm^2 > A_{s)required} = 1380 mm^2$

Check bar spacing $= \dfrac{250 - 2(25) - 2(16) - 2(40) - 2(10)}{3} = 22.6mm < 25mm$

Not Good $\Rightarrow$ Arrange bars in two layers: $2\phi 25mm + 2\phi 16mm$

Check bar spacing $= \dfrac{250 - 2(25) - 2(40) - 2(10)}{1} = 100mm > 25mm$ OK.

d =500- 40 - 10 - 25(Lower layer)-25/2(bar spacing) = 413 mm



Doubly reinforced rectangular beams:

When beam cross section is restricted for architectural requirements or others and when the external applied moment > internal moment resistance, then compression reinforcement should be used:

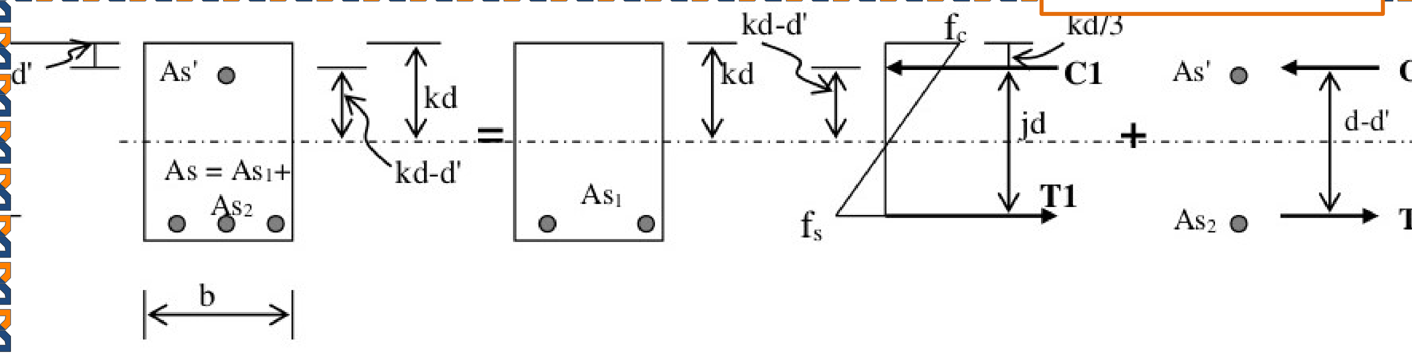
Compression reinforcement is used:

- For limited dimensions.
- $M_{external} > M_{internal}$

In the analysis and design of doubly reinforced concrete beam, the external moment is divided into two parts:

M_1 : resulted from ideally reinforced section with tension reinforcement As_1

M_2 : resulted from the remaining area of tension steel or the area of compression steel (As_2 Or $A's$).



M = **M_1** + **M_2**

$$M_1 = C_1 jd = \frac{1}{2} f_c (kj) bd^2$$

$$k = \frac{n}{n+r} \quad \text{and} \quad j = 1 - \frac{k}{3}$$

$$M_1 = T_1 \cdot jd = (A_{s1} \cdot f_s) jd$$

$$\therefore A_{s1} = \frac{M_1}{f_s \cdot jd}$$

$$M_2 = M - M_1$$

$$M_2 = A_{s2} \cdot f_s (d - d')$$

Additional area of tension steel.

$$\therefore A_{s2} = \frac{M_2}{f_s \cdot (d - d')}$$

Total area of tension steel reinforcement:

$$A_s = A_{s1} + A_{s2}$$

From triangles of stress diagram, the stress in concrete at compression steel level f_{c1} is:

$$\frac{f_{c1}}{kd - d'} = \frac{f_c}{kd}$$

Where : f_c is the allowable compression stress in concrete.

$$f_{c1} = \left(\frac{kd - d'}{kd} \right) f_c$$

Stress in compression steel is:

$$f'_s = n \cdot f_{c1}$$

But the ACI code requires using $2n$ to transform compression steel to concrete in stress calculations. Thus;

$$f'_s = 2n \cdot f_{c1} = 2n \left(\frac{kd - d'}{kd} \right) f_c \le f_{sa}$$

Where : f_{sa} is the allowable stress in steel.

$$M_2 = C_2(d-d') = f_s' \cdot A_s'(d-d') = 2n\left(\frac{kd-d'}{kd}\right)f_c \cdot A_s'(d-d')$$

$$A_s' = \frac{M_2 \cdot kd}{2n(kd-d')(d-d')f_c} \quad \text{OR} \quad A_s' = \frac{M_2}{f_s'(d-d')} \quad \text{If} \quad 2nf_{c1} < f_{sa}$$

And ;

$$A_s' = \frac{M_2}{(f_{sa} - f_{c1})jd} \quad \text{If} \quad 2nf_{c1} > f_{sa}$$

Ex.8)

Find the required amount of reinforcement of a rectangular beam section of b=320 mm, d=400 mm, if the beam is subjected to a positive bending moment of 120 kN.m. Use $f_c' = 21MPa$, $f_y = 300MPa$

Sol.)

$$f_{sa} = 140MPa$$

$$f_{ca} = 0.45f_c' = 0.45(21) = 9.45MPa$$

$$n\frac{E_s}{E_c} = \frac{200000}{4700\sqrt{21}} = 9.3 \quad \text{say } n = 9$$

$$r = \frac{f_{sa}}{f_{ca}} = \frac{140}{9.45} = 14.8$$

$$k = \frac{n}{n+r} = \frac{9}{9+14.8} = 0.378 \quad \text{and} \quad j = 1 - \frac{k}{3} = 1 - \frac{0.378}{3} = 0.874$$

$$\rho_e = \frac{1}{2}\frac{f_c}{f_s}k = \frac{9.45}{2 \times 140} \times 0.378 = 0.0127$$

$$M_1 = \frac{1}{2}f_c(kj)bd^2 = \frac{9.45(0.378 \times 0.874)320(400)^2}{2} = 79.9 \times 10^6 N.mm = 79.9kN.m$$

$$A_{s1} = \frac{M_1}{f_s \cdot jd} = \frac{79.9 \times 10^6}{140(0.874 \times 400)} = 1632mm^2 \quad \text{OR.}$$

$$A_{s1} = \rho_e \cdot b \cdot d = 0.0127 \times 320 \times 400 = 1626mm^2$$

$$M_2 = M - M_1 = 120 - 79.9 = 40.1kN.m$$

$$\therefore A_{s2} = \frac{M_2}{f_s \cdot (d-d')} = \frac{40.1 \times 10^6}{140(400-70)} = 868mm^2$$

$$A_s = A_{s1} + A_{s2} = 1632 + 868 = 2500mm^2$$

$$f_s' = 2n\left(\frac{kd-d'}{kd}\right)f_c = 2 \times 9\left(\frac{0.378 \times 400 - 70}{0.378 \times 400}\right)9.45 = 91.35MPa \le f_{sa} = 140MPa$$

$\therefore OK$ the use of $2n$ is accepted and;

$$A_s' = \frac{M_2}{f_s'(d-d')} = \frac{40.1 \times 10^6}{91.35(400-70)} = 1330mm^2$$

STRENGTH DESIGN AND ANALYSIS METHOD according to ACI Code

1 INTODUCTION

In the strength design method (formerly called ultimate strength method), the service loads are increased by factors to obtain the load at which failure is considered to be "imminent". This load is called the factored load or factored service load. The structure or structural element is then proportioned such that the strength is reached when the factored load is acting. The computation of this strength takes into account the nonlinear stress-strain behavior of concrete.

The strength design method may be expressed by the following:

Strength provided ≥ [strength required to carry factored loads]

Where the "strength provided" (such as moment strength) is computed in accordance with the provisions of a building code, and the "strength required" is that obtained by performing a structural analysis using factored loads.

2 SAFETY PROVISIONS

Structures and structural members must always be designed to carry some reserve load above what is expected under normal use. Such reserve capacity is provided to account for a variety of factors, which may be grouped in two general categories:

- Factors relating to overload.
- Factors relating to understrength (that is, less strength than computed by acceptable calculating procedures).

Overloads may arise from changing the use for which the structure was designed, from underestimation of the effects of loads by oversimplification in calculation procedures, and from effects of construction sequence and methods. Understrength may result from adverse variations in material strength, workmanship, dimensions, control, and degree of supervision, even though individually these items are within required tolerances.

In the strength design method, the member is designed to resist factored loads, which are obtained by multiplying the service loads by load factors. Different factors are used for different loadings. Because dead loads can be estimated quite accurately, their load factors are smaller than those of live loads, which have a high degree of uncertainty. Several load combinations must be considered in the design to compute the maximum and minimum

design forces. Reduction factors are used for some combinations of loads to reflect the low probability of their simultaneous occurrences. The ACI Code presents specific values of load factors to be used in the design of concrete structures.
In addition to load factors, the ACI Code specifies another factor to allow an additional reserve in the capacity of the structural member. The nominal strength is generally calculated using accepted analytical procedure based on statistics and equilibrium; however, in order to account for the degree of accuracy within which the nominal strength can be calculated, and for adverse variations in materials and dimensions, a strength reduction factor, , should be used in the strength design method.

To summarize the above discussion, the ACI Code has separated the safety provision into an overload or load factor and to an under capacity (or strength reduction) factor, Ø. A safe design is achieved when the structure's strength, obtained by multiplying the nominal strength by the reduction factor, Ø, exceeds or equals the strength needed to withstand the factored loadings (service loads times their load factors).
The requirement for strength design may be expressed:

$$Design\ Strenght\ \geq Factored\ Load\ (i.e., requird\ Strenght)$$

$$\emptyset P_n \geq P_u$$

$$\emptyset M_n \geq M_u$$

$$\emptyset V_n \geq V_u$$

Where $P_n, M_n\ and\ V_n$ are "nominal" strengths in axial compression, bending moment, and shear, respectively, using the subscript n.

$P_u, M_u\ and\ V_u$ are the factored load effects in axial compression, bending moment, and shear, respectively, using the subscript u.
Given a load factor of 1.2 for dead load and a load factor of 1.6 for live load, the overall safety factor for a structure loaded be a dead load, D, and a live load, L, is:

$$Factor\ of\ safety = \frac{1.2D + 1.6L}{D + L}\left(\frac{1}{\emptyset}\right)$$

3 LOAD FACTORS AND STRENGTH REDUCTION FACTORS

Overload Factors *U*

The factors ***U*** for overload as given by ACI-9.2 are:

$$U = 1.4(D + F)$$

$$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r or\ S\ or\ R)$$

$$U = 1.2D + 1.6(\ L_r or\ S\ or\ R) + (1.0L\ or\ 0.8W)$$

$$U = 1.2D + 1.6W + 1.0L + 0.5(L_r or\ S\ or\ R)$$

$$U = 1.2D + 1.0E + 1.0L + 0.2S$$

$$U = 0.9D + 1.6W + 1.6H$$

$$U = 0.9D + 1.0E + 1.6H$$

Where:

$D =$ dead load; $\mathrm{L} =$ live load ; $L_r =$ roof live load; $\mathrm{S} =$ snow load;

$\mathrm{F} =$ load due to weights and pressures of fluids;

$\mathrm{R} =$ rain load; $\mathrm{W} =$ wind load;

$\mathrm{E} =$ earthquake load; $-$defined densities and controllable maximum heights;

$\mathrm{H} =$ load due to weight and pressure of soil, water in soil or other materials;

$\mathrm{T} =$ the cumulative effect of temperature, creep, shrinkage and differential settlement.

Strength Reduction Factors ∅

The factors ∅ for understrength are called strength reduction factors according to ACI-9.3 and are as follows:

Strength Condition $\emptyset$ ***Factors***

1-Flexure (with or without axial force)

- Tension-controlled sections …………………………….. 0.9
- Compression-controlled sections
  - Spirally reinforced ………………..……………….. 0.75
  - Others ………………………………..……………. 0.65

2-Shear and torsion ……………………………….……..…….... 0.75

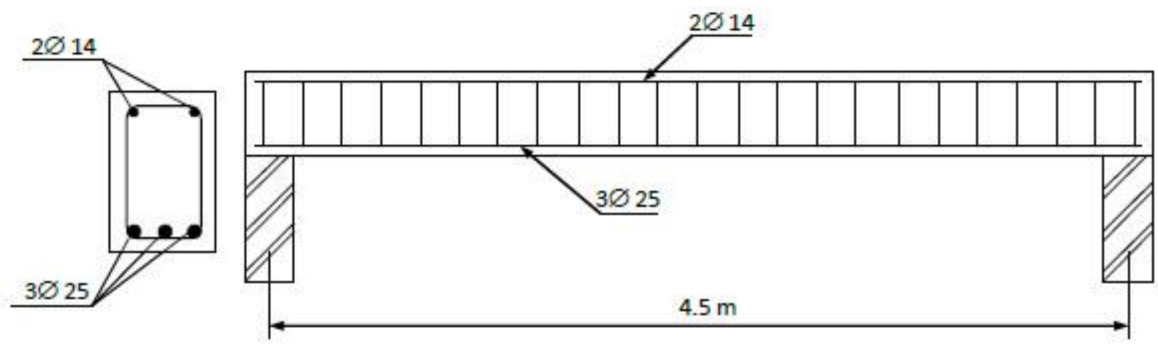
3-Bearing on concrete …………………………….…..………... 0.65

4-Post-tensioned anchorage zones …………………………….. 0.85

5-Struts, ties, nodal zones, and bearing areas in strut-and-tie models….0.75

Example:

A simple beam is loaded with a dead load of $40kN/m$ and a live load of$30kN/m$. Check the strength requirement according to ACI code if the nominal bending moment $M_n = 275kN.m$.



<u>Solution:</u>

$$M_n = 275\ KN.m \qquad and \qquad \phi = 0.9$$

$$w_u = 1.2D + 1.6L = 1.2 \cdot 40 + 1.6 \cdot 30 = 96\ \ KN/m$$

$$M_u = M_{max} = \frac{w_u l^2}{8} = \frac{96 \cdot 4.5^2}{8} = 243\ KN \cdot m$$

$$\phi M_n \geq M_u$$

$0.9 \cdot 275 = 247.5\ KN \cdot m > 243\ KN \cdot m$ OK Strength requirement is satisfied

$$\text{Factor of Safety} = \frac{1.2D + 1.6\,L}{D + L}\left(\frac{1}{\phi}\right) = \frac{96}{40 + 30}\left(\frac{1}{0.9}\right) = 1.52$$

4 FLEXURE IN BEAMS

Reinforced concrete beams are nonhomogeneous in that they are made of two entirely different materials. The methods used in the analysis of reinforced concrete beams are therefore different from those used in the design or investigation of beams composed entirely of steel, wood, or any other structural material.

Two different types of problems arise in the study of reinforced concrete:

1. Analysis. Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.
2. Design. Given a factored design moment, normally designated as M_u select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.

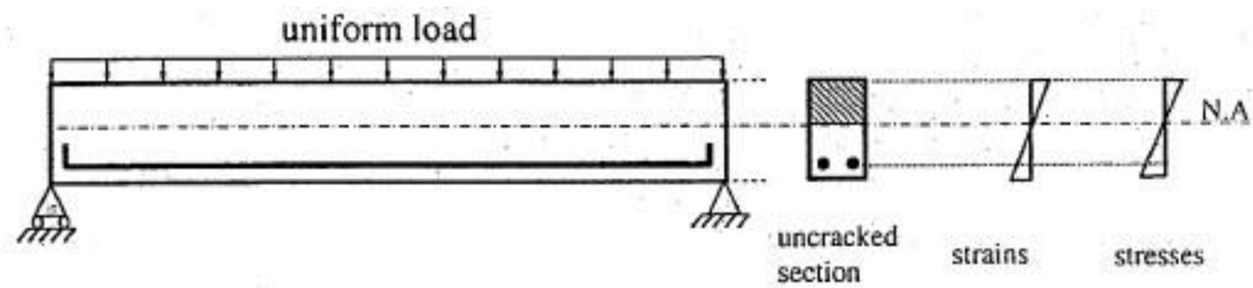
The Strength Design Method requires the conditions of static equilibrium and strain compatibility across the depth of the section to be satisfied.

The following are the assumptions for Strength Design Method:

1. Strains in reinforcement and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
2. Concrete sections are considered to have reached their flexural capacities when they develop 0.003 strain in the extreme compression fiber.
3. Stress in reinforcement varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
4. Tensile strength of concrete is neglected.
5. Compressive stress distribution of concrete can be represented by the corresponding stress-strain relationship of concrete. This stress distribution may be simplified by a rectangular stress distribution as described later.

5 REINFORCED CONCRETE BEAM BEHAVIOR

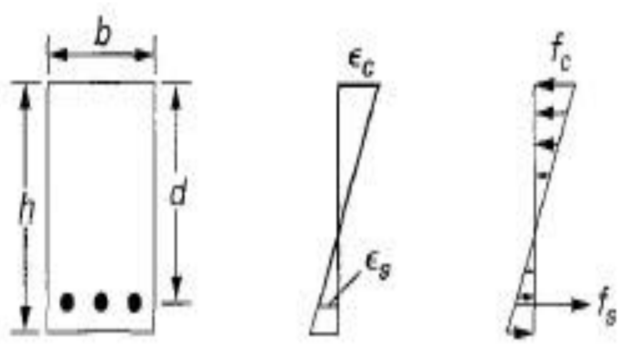
Consider a simply supported and reinforced concrete beam with uniformly distributed load on top. Under such loading and support conditions, flexure-induced stresses will cause compression at the top and tension at the bottom of the beam. Concrete, which is strong in compression, but weak in tension, resists the force in the compression zone, while steel reinforcing bars are placed in the bottom of the beam to resist the tension force. As the applied load is gradually increased from zero to failure of the beam (ultimate condition), the

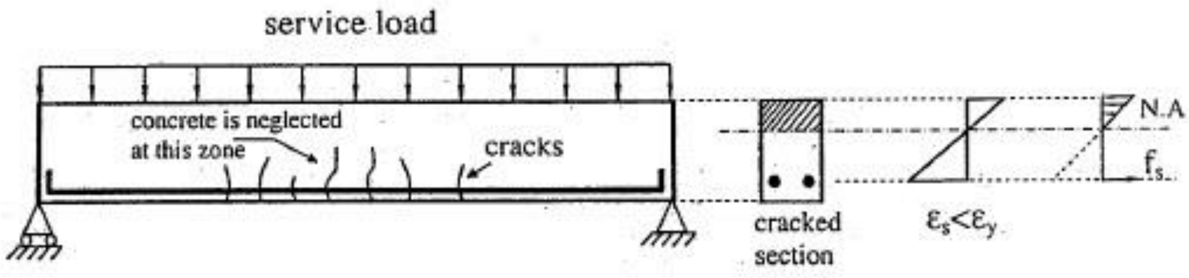


Stage I : before cracking

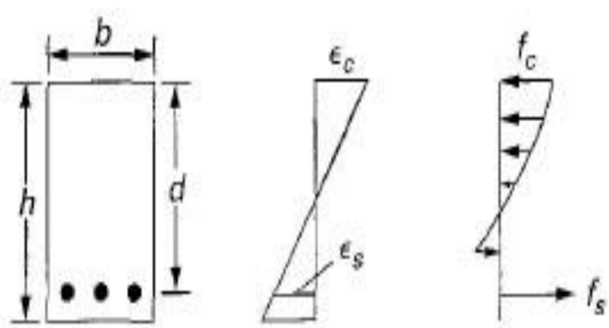
beam may be expected to behave in the following manner:

Stage I: when the applied load is low, the stress distribution is essentially linear over the depth of the section. The tensile stresses in the concrete are low enough so that the entire cross-section remains uncracked and the stress distribution is as shown in below. In the compression zone, the concrete stresses are low enough (less than about $0.5f_c'$) so that their distribution is approximately linear.

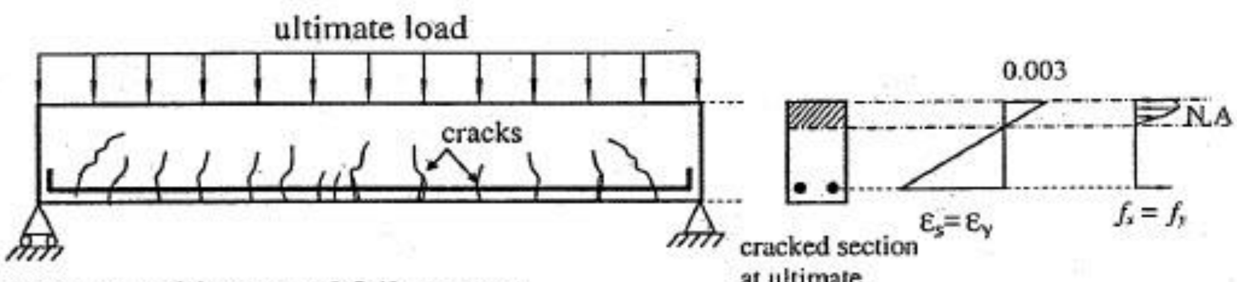




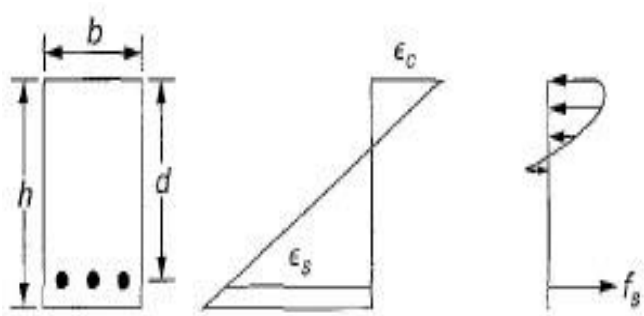
Stage II : cracking stage, before yield, working load



Stage II: On increasing the applied load, the tensile stresses at the bottom of the beam become high enough to exceed the tensile strength at which the concrete cracks. After cracking, the tensile force is resisted mainly by the steel reinforcement. Immediately below the neutral axis, a small portion of the beam remains uncracked. These tensile stresses in the concrete offer, however, only a small contribution to the flexural strength. The concrete stress distribution in the compression zone becomes nonlinear.



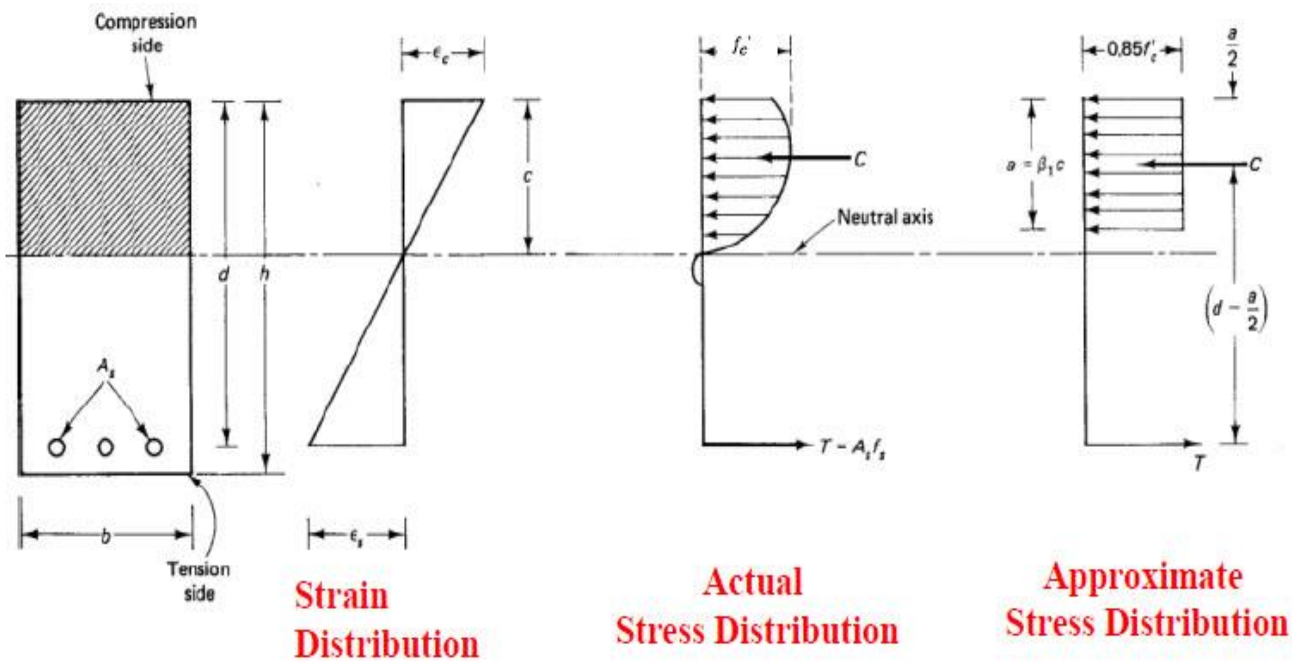
Stage III : ultimate and failure stage



Stage III: at nominal (so,-called ultimate) strength, the neutral axis has moved farther up-ward as flexural cracks penetrate more and more toward the compression face. The steel reinforcement has yielded and the concrete stress distribution in the compression zone becomes more nonlinear. Below the neutral axis, the concrete is cracked except for a very small zone.

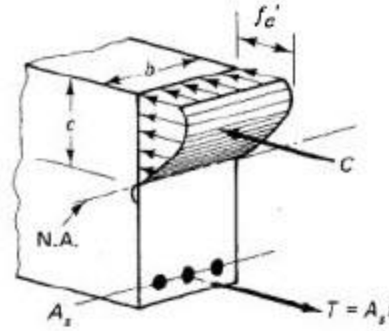
At the ultimate stage, two types of failure can be noticed. If the beam is reinforced with a small amount of steel, ductile failure will occur. In this type of failure, the steel yields and the concrete crushes after experiencing large deflections and lots of cracks. On the other hand, if the beam is reinforced with a large amount of steel, brittle failure will occur. The failure in this case is sudden and occurs due to the crushing of concrete in the compression zone without yielding of the steel and under relatively small deflections and cracks. This is not a preferred mode of failure because it does not give enough warning before final collapse.

6 THE EQUIVALENT RECTANGULAR COMPRESSIVE STRESS DISTRIBUTION (COMPRESSIVE STRESS BLOCK).



The actual distribution of the compressive stress in a section has the form of rising parabola. It is time consuming to evaluate the volume of compressive stress block. An equivalent rectangular stress block can be used without loss of accuracy.

The flexural strength M_u , using the equivalent rectangular, is obtained as follows:



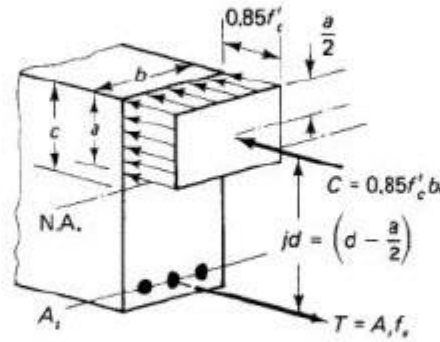
$$C = 0.85f_c'ab \qquad\qquad T = A_sf_y$$

$$\sum F_x = 0 \qquad gives: \ C = T$$

$$A_sf_y = 0.85f_c'ab \quad or \quad a = \frac{A_sf_y}{0.85f_c'b}$$

$$M_n = T\left(d - \frac{a}{2}\right) = C\left(d - \frac{a}{2}\right)$$

$$M_n = A_sf_y\left(d - \frac{a}{2}\right) \qquad or \qquad M_n = 0.85f_c'ab\left(d - \frac{a}{2}\right)$$



Notation:

a= depth of rectangular compressive stress block,

b= width of the beam at the compression side,

c= depth of the neutral axis measured from the extreme compression fibers,

d= effective depth of the beam, measured from the extreme compression fibers to the centroid of the steel area,

h= total depth of the beam,

ε_c= strain in extreme compression fibers,

ε_s= strain at tension steel,

f_c'= compressive strength of concrete,

f_y=yield stress of steel,

A_s= area of the tension steel,

C= resultant compression force in concrete,

T= resultant tension force in steel,

M_n= nominal moment strength of the section.

Example:

Determine the nominal moment strength of the beam section. Take $f_c' = 20\ MPa\ and\ f_y = 400\ MPa$.

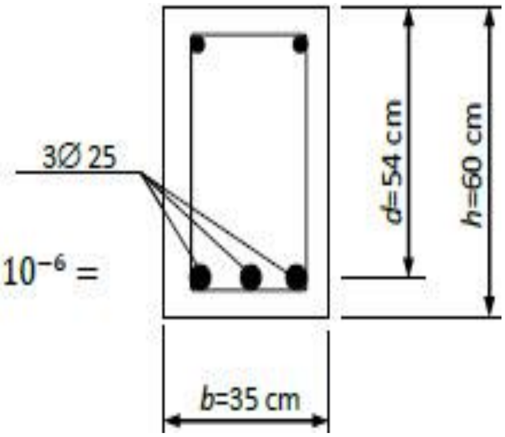
Solution:

$A_s(3\varnothing 25) = 14.72\ cm^2$

$$a = \frac{A_s f_y}{0.85\, f_c' b} = \frac{14.72 \cdot 100 \cdot 400}{0.85 \cdot 20 \cdot 350} = 98.96\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 14.72 \cdot 100 \cdot 400 \left(540 - \frac{98.96}{2}\right) \cdot 10^{-6} =$$

$$= 288.82\ KN \cdot m$$



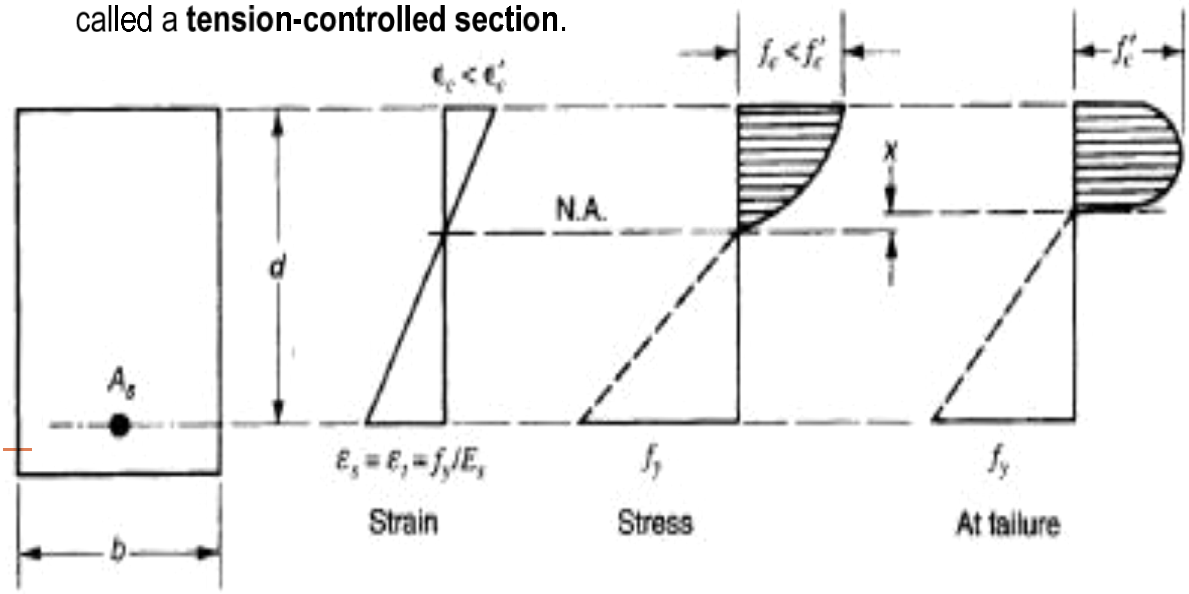
Asst.Prof.Dr. Akram S. Mahmoud

7 TYPES OF FAILURE AND STRAIN LIMITS

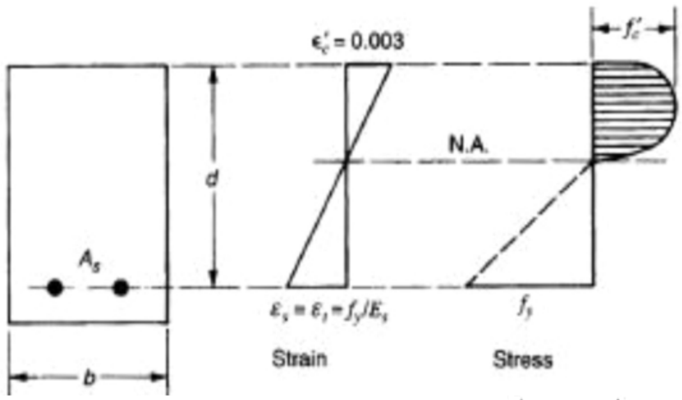
Types of failure

Three types of flexural failure of a structural member can be expected depending on the percentage of steel used in the section.

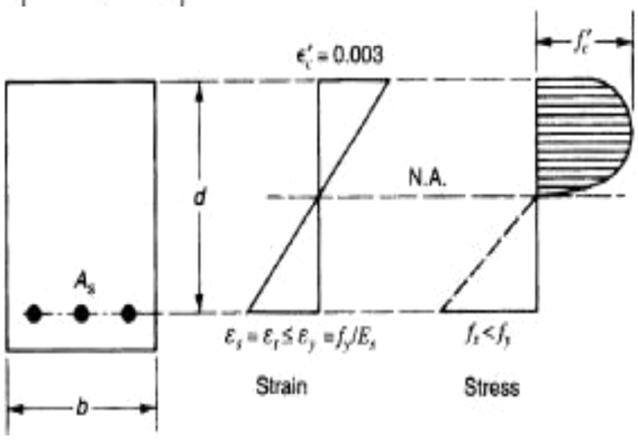
1. Steel may reach its yield strength before the concrete reaches its maximum strength, In this case, the failure is due to the yielding of steel reaching a high strain equal to or greater than $\varepsilon_t \geq \varepsilon_{ty} + 0.003$. The section contains a relatively small amount of steel and is called a **tension-controlled section**.



2. Steel may reach its yield strength at the same time as concrete reaches its ultimate strength. The section is called a **balanced section**.



3. Concrete may fail before the yield of steel, due to the presence of a high percentage of steel in the section. In this case, the concrete strength and its maximum strain of 0.003 are reached, but the steel stress is less than the yield strength, that is f_s, is less than f_y. The strain in the steel is equal to or less than ε_{ty} =0.002. This section is called a **compression-controlled section**.



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21.2—Strength reduction factors for structural concrete members and connections

ACI 318-19 Provisions

21.2.1 Strength reduction factors ϕ shall be in accordance with Table 21.2.1,

Table 21.2.1—Strength reduction factors ϕ

Action or structural element		ϕ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

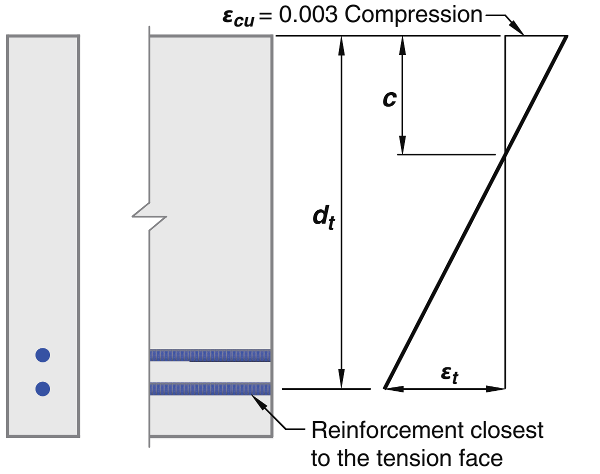

Fig. R21.2.2a—Strain distribution and net tensile strain in a nonprestressed member.

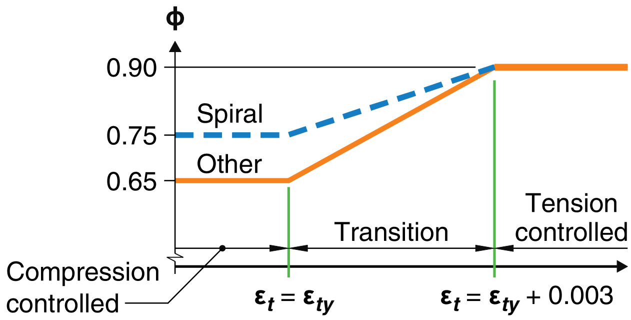

Fig. R21.2.2b—Variation of ϕ with net tensile strain in extreme tension reinforcement, ε_t.

Table 21.2.2—Strength reduction factor ϕ for moment, axial force, or combined moment and axial force

Net tensile stain ε_t	Classification	ϕ			
		Type of transverse reinforcement			
		Spirals conforming to 25.7.3		Other	
$\varepsilon_t \le \varepsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003$	Transition[1]	$0.75 + 0.15\dfrac{(\varepsilon_t - \varepsilon_{ty})}{(0.003)}$	(c)	$0.65 + 0.25\dfrac{(\varepsilon_t - \varepsilon_{ty})}{(0.003)}$	(d)
$\varepsilon_t \ge \varepsilon_{ty} + 0.003$	Tension-controlled	0.90	(e)	0.90	(f)

[1]For sections classified as transition, it shall be permitted to use ϕ corresponding to compression-controlled sections.