Chapter One
Kinematics of Particles

1-1 Introduction to Dynamics

The term dynamic may be defined simply as time-varying
Dynamics includes:

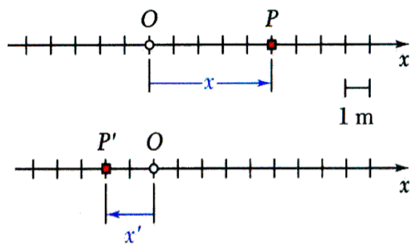
- Kinematics: study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time without reference to the cause of motion.
- Kinetics: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

The motion of particles include:

- Rectilinear motion: position, velocity, and acceleration of a particle as it moves along a straight line.
- Curvilinear motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

1-2 Rectilinear Motion of Particals: Position, Velocity & Acceleration

Particle moving along a straight line is said to be in rectilinear motion. At any given instant t, the particle will occupy a certain position on the straight line. To define the position P of the particle, one can choose a fixed origin O on the straight line and a positive direction along the line. the distance x can be measured from O to P and record it with a plus or minus sign, according to

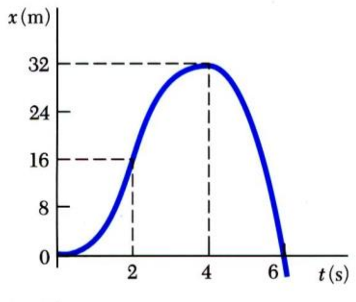


whether P is reached from O by moving along the line in the positive or the negative direction. The distance x, with the appropriate sign, completely defines the position of the particle; it is called the position coordinate of the particle considered.

The motion of a particle is known if the position coordinate for particle is known for every value of time t. Motion of the particle may be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

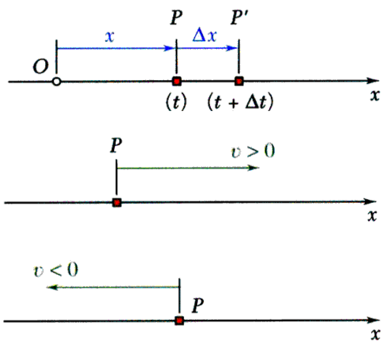
or in the form of a graph x vs. t.



Consider particle which occupies position *P* at time *t* and *P'* at *t+Dt*,

$$Average\ velocity = \frac{\Delta x}{\Delta t}$$

$$Instantaneous\ velocity = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

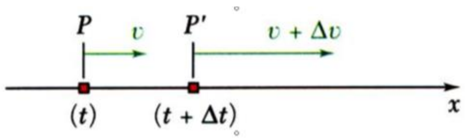


Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*. From the definition of a derivative, e.g.,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

Now consider particle with velocity v at time t and v' at t+Dt;

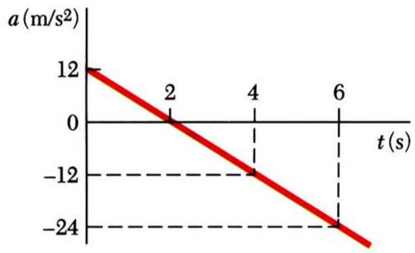


$$Instantaneous\ acceleration = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d^2 x}{dt^2}$$

From the definition of a derivative, e.g.,

$$v = 12t - 3t^2$$

$$a = \frac{dv}{dt} = 12 - 6t$$



1-3 Uniform Rectilinear Motion

For particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant, then:

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

1-4 Uniformly Accelerated Rectilinear Motion

For particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v - v_0 = at$$

$$v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)dt \qquad x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

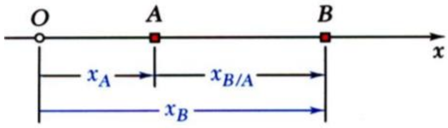
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad \tfrac{1}{2}\left(v^2 - v_0^2\right) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

1-5 Motion of Several Particles: Relative Motion

For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.



$x_{B/A} = x_B - x_A =$ relative position of B with respect to A

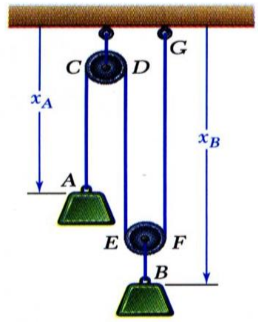
$x_B = x_A + x_{B/A}$

$v_{B/A} = v_B - v_A =$ relative velocity of B with respect to A .... $v_B = v_A + v_{B/A}$

$a_{B/A} = a_B - a_A =$ relative acceleration of B with respect to A .... $a_B = a_A + a_{B/A}$

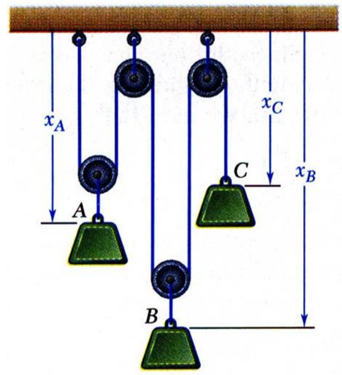
1-6 Motion of Several Particles: Dependent Motion

Position of a particle may depend on position of one or more other particles. Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.



$x_A + 2x_B =$ constant (one degree of freedom)

The positions of three blocks are also dependent as shown:



$2x_A + 2x_B + x_C =$ Positions of three blocks are dependent.

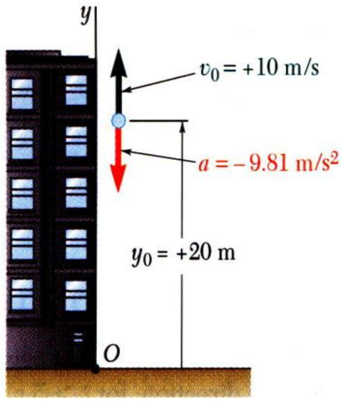
For linearly related positions, similar relations hold between velocities and accelerations.

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

Example1: Ball tossed with 10 m/s vertical velocity from window 20 m above ground. Determine:

1 Velocity and elevation above ground at time t,
2 Highest elevation reached by ball and corresponding time, and
3 Time when ball will hit the ground and corresponding velocity.



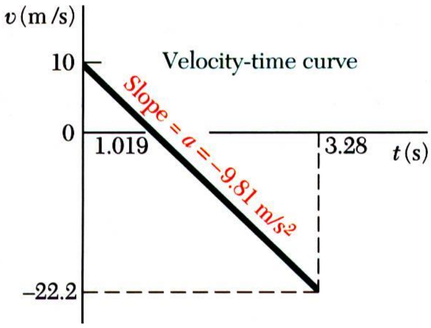
Solution:

- Integrate twice to find v(t) and y(t).

$$\frac{dv}{dt} = a = -9.81\,\mathrm{m/s^2}$$

$$\int_{v_0}^{v(t)} dv = -\int_0^t 9.81\,dt \qquad v(t) - v_0 = -9.81t$$

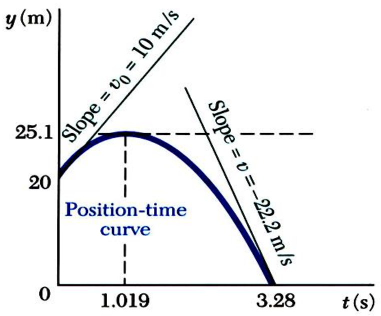
$$v(t) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$



- Solve for t at which velocity equals zero and evaluate corresponding altitude.

$$v(t) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t = 0$$

$$t = 1.019\,\mathrm{s}$$



$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0}^{y(t)} dy = \int_0^t (10 - 9.81t)\,dt$$

$$y(t) - y_0 = 10t - \tfrac{1}{2}9.81t^2$$

$$y(t) = 20\,\mathrm{m} + \left(10\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^{\;2}$$

- Solve for t at which altitude equals zero and evaluate corresponding velocity.

$$y = 20\,\mathrm{m} + \left(10\frac{\mathrm{m}}{\mathrm{s}}\right)(1.019\,\mathrm{s}) - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1.019\,\mathrm{s})^2 \;\ldots\ldots\ldots\; y = 25.1\,\mathrm{m}$$

- Solve for t at which altitude equals zero and evaluate corresponding velocity.

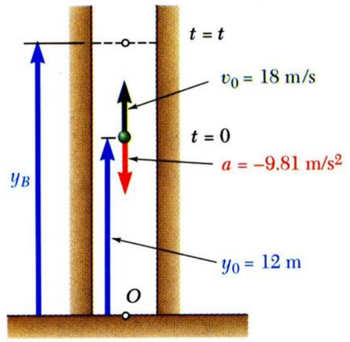
$$y(t) = 20\,\text{m} + \left(10\frac{\text{m}}{\text{s}}\right)t - \left(4.905\frac{\text{m}}{\text{s}^2}\right)t^2 = 0$$

$$t = -1.243\,\text{s}\ \ (\text{meaningles s}) \quad \text{so} \quad t = 3.28\,\text{s}$$

$$v(t) = 10\frac{\text{m}}{\text{s}} - \left(9.81\frac{\text{m}}{\text{s}^2}\right)t \quad \ldots\ldots.. \quad v(3.28\,\text{s}) = 10\frac{\text{m}}{\text{s}} - \left(9.81\frac{\text{m}}{\text{s}^2}\right)(3.28\,\text{s}) = -22.2\frac{\text{m}}{\text{s}}$$

Example 2: Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s. Determine (a) when and where ball hits elevator and (b) relative velocity of ball and elevator at contact.

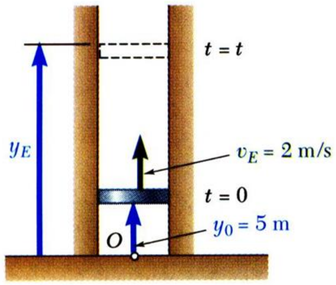
Solution:



1. Substitute initial position and velocity and constant acceleration of ball into general equations for uniformly accelerated rectilinear motion.

$$v_B = v_0 + at = 18\frac{\text{m}}{\text{s}} - \left(9.81\frac{\text{m}}{\text{s}^2}\right)t$$

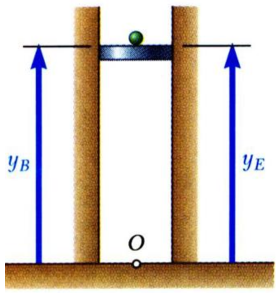
$$y_B = y_0 + v_0 t + \tfrac{1}{2}at^2 = 12\,\text{m} + \left(18\frac{\text{m}}{\text{s}}\right)t - \left(4.905\frac{\text{m}}{\text{s}^2}\right)t^2$$



2. Substitute initial position and constant velocity of elevator into equation for uniform rectilinear motion.

$$v_E = 2\frac{\text{m}}{\text{s}}$$

$$y_E = y_0 + v_E t = 5\,\text{m} + \left(2\frac{\text{m}}{\text{s}}\right)t$$



3. Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.

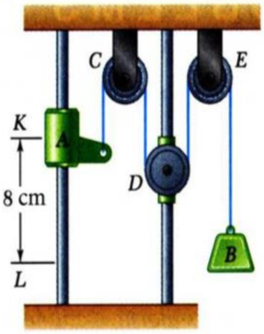
$$y_{B/E} = \left(12 + 18t - 4.905t^2\right) - (5 + 2t) = 0$$

$$t = -0.39\,\text{s}\ \ (\text{meaningles s}) \ \ldots.\ \text{So}\ \ t = 3.65\,\text{s}$$

$$y_E = 5 + 2(3.65) = 12.3\,\text{m}$$

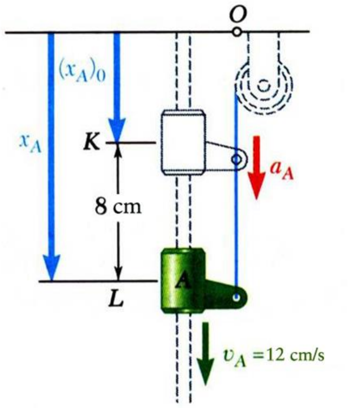
$$v_{B/E} = (18 - 9.81t) - 2 = 16 - 9.81(3.65) = -19.81\frac{\text{m}}{\text{s}}$$

Example 3: Pulley D is attached to a collar which is pulled down at 3 cm/s. At t = 0, collar A starts moving down from K with constant acceleration and zero initial velocity. Knowing that velocity of collar A is 12 cm/s as it passes L, determine the change in elevation, velocity, and acceleration of block B when block A is at L.



Solution:

1. Define origin at upper horizontal surface with positive displacement downward.
2. Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.



$$v_A^2 = (v_A)_0^2 + 2a_A\left[x_A - (x_A)_0\right]$$

$$\left(12\frac{\text{cm}}{\text{s}}\right)^2 = 2a_A(8\,\text{cm}) \qquad a_A = 9\frac{\text{cm}}{\text{s}^2}$$

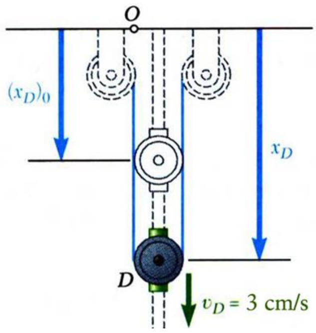
$$v_A = (v_A)_0 + a_A t$$

$$12\frac{\text{cm}}{\text{s}} = 9\frac{\text{cm}}{\text{s}^2}t \qquad t = 1.333\,\text{s}$$

3. Pulley *D* has uniform rectilinear motion. Calculate change of position at time *t*.

$$x_D = (x_D)_0 + v_D t$$

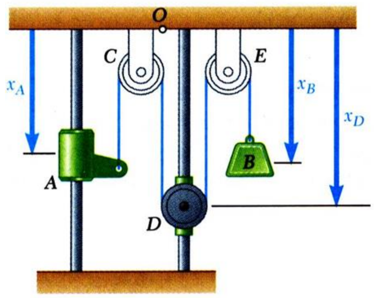
$$x_D - (x_D)_0 = \left(3\frac{\text{cm}}{\text{s}}\right)(1.333\,\text{s}) = 4\,\text{cm}$$



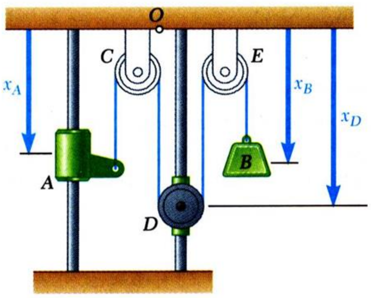
4. Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*. Since Total length of cable remains constant,

$$x_A + 2x_D + x_B = (x_A)_0 + 2(x_D)_0 + (x_B)_0$$
$$\left[x_A - (x_A)_0\right] + 2\left[x_D - (x_D)_0\right] + \left[x_B - (x_B)_0\right] = 0$$
$$(8\,\text{cm}) + 2(4\,\text{cm}) + \left[x_B - (x_B)_0\right] = 0$$

$$x_B - (x_B)_0 = -16\,\text{cm}$$



5. Differentiate motion relation twice to develop equations for velocity and acceleration of block B.



$$x_A + 2x_D + x_B = \text{constant}$$

$$v_A + 2v_D + v_B = 0$$

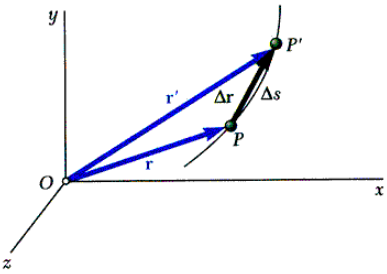
$$\left(12\,\frac{\text{cm}}{\text{s}}\right) + 2\left(3\,\frac{\text{cm}}{\text{s}}\right) + v_B = 0$$

$$v_B = 18\,\frac{\text{cm}}{\text{s}}$$

$$a_A + 2a_D + a_B = 0$$

$$\left(9\,\frac{\text{cm}}{\text{s}^2}\right) + v_B = 0 \qquad \ldots\ldots\ a_B = -9\,\frac{\text{cm}}{\text{s}^2}$$

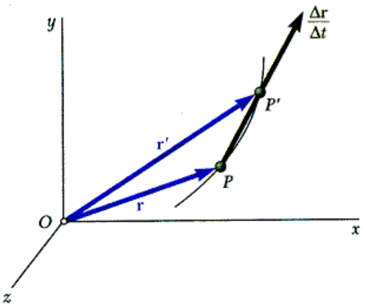
1-7 Curvilinear Motion: Position, Velocity & Acceleration

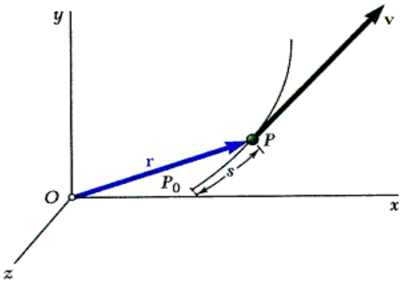


The position vector of a particle at time t is defined by a vector between origin *O* of a fixed reference frame and the position occupied by particle.
Consider a particle which occupies position *P* defined by at time *t* and *P'* defined by at $t + \Delta t$, then:

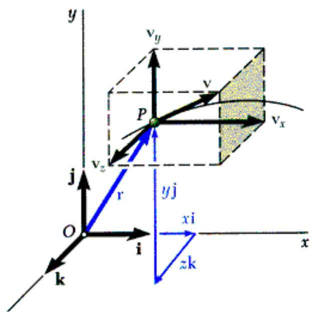
Instantaneous velocity (vector) & Instantaneous speed (scalar)

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \qquad v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$





1-8 Rectangular Components of Velocity & Acceleration



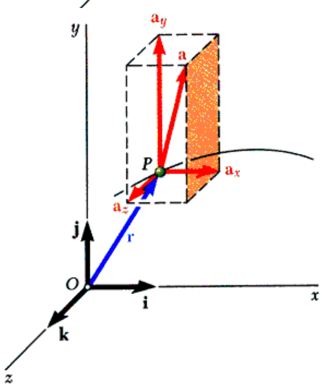
When position vector of particle *P* is given by its rectangular components, $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

Velocity vector,

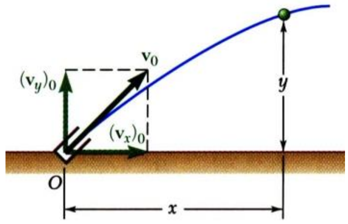
$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k}$$
$$= v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

Acceleration vector,

$$\vec{a} = \frac{d^2x}{dt^2}\vec{i} + \frac{d^2y}{dt^2}\vec{j} + \frac{d^2z}{dt^2}\vec{k} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k}$$
$$= a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$



- Rectangular components particularly effective when component accelerations can be integrated independently, e.g., motion of a projectile,

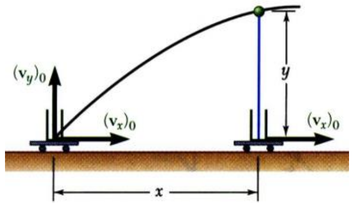


$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

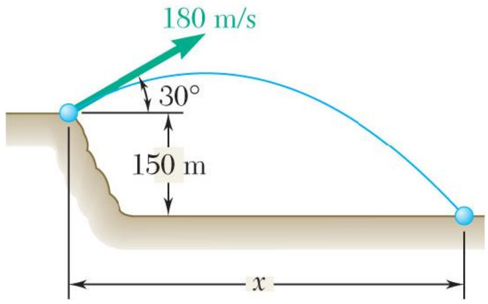
with initial conditions: $x_0 = y_0 = z_0 = 0 \qquad (v_x)_0, (v_y)_0, (v_z)_0 = 0$

Integrating twice yields:
$$\begin{array}{lll} v_x = (v_x)_0 & v_y = (v_y)_0 - gt & v_z = 0 \\ x = (v_x)_0 t & y = (v_y)_0 y - \frac{1}{2} g t^2 & z = 0 \end{array}$$

- Motion in horizontal direction is uniform.
- Motion in vertical direction is uniformly accelerated.
- Motion of projectile could be replaced by two independent rectilinear motions.



Example 4: A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30°with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile.

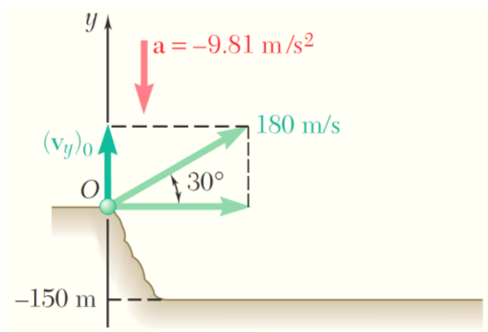


SOLUTION:

- Maximum elevation occurs when $v_y=0$, the Vertical motion is uniformly accelerated, then:

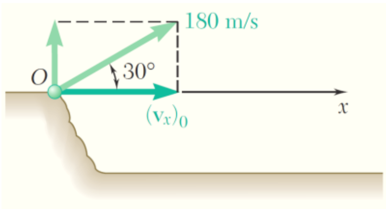
$$(v_y)_0 = (180\ \text{m/s})\ \sin 30° = +90\ \text{m/s}$$

$$\begin{array}{llr} v_y = (v_y)_0 + at & v_y = 90 - 9.81t & (1) \\ y = (v_y)_0 t + \frac{1}{2}at^2 & y = 90t - 4.90t^2 & (2) \\ v_y^2 = (v_y)_0^2 + 2ay & v_y^2 = 8100 - 19.62y & (3) \end{array}$$



Also horizontal motion – uniformly accelerated, then: $(v_x)_0 = (180 \text{ m/s}) \cos 30° = +155.9 \text{ m/s}$

$$x = (v_x)_0 t \qquad x = 155.9t \quad ......(4)$$



$$y = -150 \text{ m}$$

$$-150 = 90t - 4.90t^2 \qquad t^2 - 18.37t - 30.6 = 0 \qquad t = 19.91 \text{ s}$$

Projectile strikes the ground at: Substitute into equation (1)

⇨

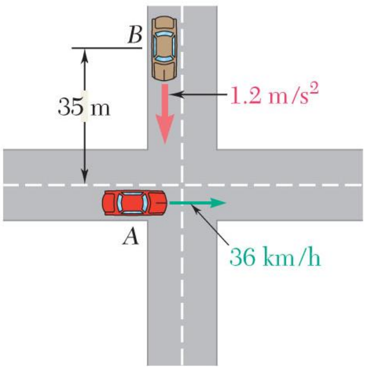
Substitute t into equation (4): $x = 155.9(19.91)$ ⇨ $x = 3100 \text{ m}$

Maximum elevation occurs when $v_y=0$

$$0 = 8100 - 19.62y \qquad y = 413 \text{ m}$$

Maximum elevation above the ground = $150 \text{ m} + 413 \text{ m} = 563 \text{ m}$

Example 4: Automobile *A* is traveling east at the constant speed of 36 km/h. As automobile *A* crosses the intersection shown, automobile *B* starts from rest 35 m north of the intersection and moves south with a constant acceleration of 1.2 m/s^2. Determine the position, velocity, and acceleration of *B* relative to *A* 5 *s* after *A* crosses the intersection.

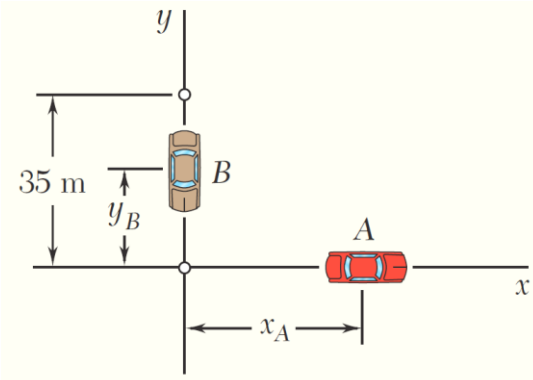


SOLUTION:

Given: v_A=36 km/h, a_A= 0, $(x_A)0$ = 0, $(v_B)0$= 0, a_B= - 1.2 m/s^2, $(y_A)0$ = 35 m

- Determine motion of Automobile A:

$$v_A = \left(36\frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 10 \text{ m/s}$$



We have uniform motion for A so:

$$a_A = 0$$
$$v_A = +10 \text{ m/s}$$
$$x_A = (x_A)_0 + v_A t = 0 + 10t$$

At t = 5 s:

$$a_A = 0$$
$$v_A = +10 \text{ m/s}$$
$$x_A = +(10 \text{ m/s})(5 \text{ s}) = +50 \text{ m}$$

$$\mathbf{a}_A = 0$$
$$\mathbf{v}_A = 10 \text{ m/s} \rightarrow$$
$$\mathbf{r}_A = 50 \text{ m} \rightarrow$$

- Determine motion of Automobile B:

We have uniform acceleration for B so:

$$a_B = -1.2 \text{ m/s}^2$$
$$v_B = (v_B)_0 + at = 0 - 1.2\,t$$
$$y_B = (y_B)_0 + (v_B)_0 t + \tfrac{1}{2}a_B t^2 = 35 + 0 - \tfrac{1}{2}(1.2)t^2$$

At t = 5 s:

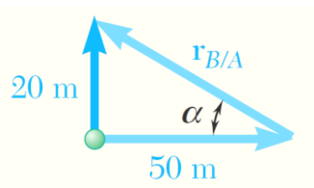
$$a_B = -1.2 \text{ m/s}^2$$
$$v_B = -(1.2 \text{ m/s}^2)(5 \text{ s}) = -6 \text{ m/s}$$
$$y_B = 35 - \tfrac{1}{2}(1.2 \text{ m/s}^2)(5 \text{ s})^2 = +20 \text{ m}$$

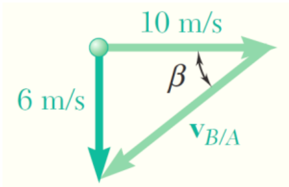
$$\mathbf{a}_B = 1.2 \text{ m/s}^2 \downarrow$$
$$\mathbf{v}_B = 6 \text{ m/s} \downarrow$$
$$\mathbf{r}_B = 20 \text{ m} \uparrow$$

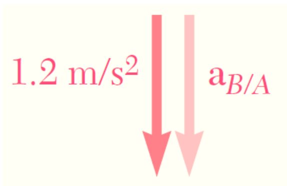
Since :

$$\mathbf{a}_A = 0 \qquad \mathbf{a}_B = 1.2 \text{ m/s}^2 \downarrow$$
$$\mathbf{v}_A = 10 \text{ m/s} \rightarrow \qquad \mathbf{v}_B = 6 \text{ m/s} \downarrow$$
$$\mathbf{r}_A = 50 \text{ m} \rightarrow \qquad \mathbf{r}_B = 20 \text{ m} \uparrow$$

Then the problems can be solve geometrically, and apply the arctangent relationship:







$$r_{B/A} = 53.9 \text{ m} \qquad \alpha = 21.8°$$
$$\mathbf{r}_{B/A} = 53.9 \text{ m} ⦨ 21.8°$$

$$v_{B/A} = 11.66 \text{ m/s} \qquad \beta = 31.0°$$
$$\mathbf{v}_{B/A} = 11.66 \text{ m/s} ⦩ 31.0°$$

$$\mathbf{a}_{B/A} = 1.2 \text{ m/s}^2 \downarrow$$

Or one can solve the problems using vectors to obtain equivalent results:

$$\mathbf{r_B} = \mathbf{r_A} + \mathbf{r_{B/A}} \qquad \mathbf{v_B} = \mathbf{v_A} + \mathbf{v_{B/A}} \qquad \mathbf{a_B} = \mathbf{a_A} + \mathbf{a_{B/A}}$$

$$20\mathbf{j} = 50\mathbf{i} + \mathbf{r_{B/A}} \qquad -6\mathbf{j} = 10\mathbf{i} + \mathbf{v_{B/A}} \qquad -1.2\mathbf{j} = 0\mathbf{i} + \mathbf{a_{B/A}}$$

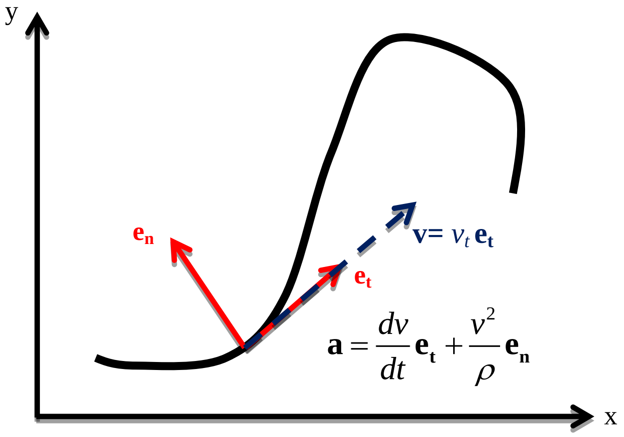
$$\mathbf{r_{B/A}} = 20\mathbf{j} - 50\mathbf{i}\ \ \text{(m)} \qquad \mathbf{v_{B/A}} = -6\mathbf{j} - 10\mathbf{i}\ \ \text{(m/s)} \qquad \mathbf{a_{B/A}} = -1.2\mathbf{j}\ \ (\text{m/s}^2)$$

$$v_{B/A} = 11.66 \text{ m/s}$$

Physically, a rider in car A would "see" car B traveling south and west.

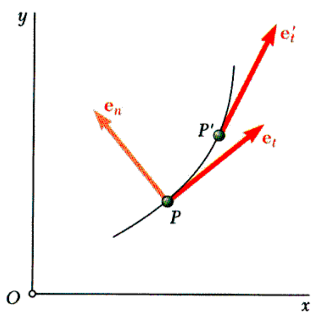
1-9 Tangential and Normal Components

If we have an idea of the path of a vehicle, it is often convenient to analyze the motion using tangential and normal components (sometimes called path coordinates).



- The tangential direction ($\mathbf{e_t}$) is tangent to the path of the particle. This velocity vector of a particle is in this direction
- The normal direction ($\mathbf{e_n}$) is perpendicular to **et** and points towards the inside of the curve.
- The acceleration can have components in both the $\mathbf{e_n}$ and $\mathbf{e_t}$ directions

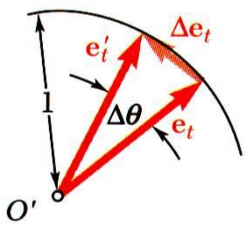
To derive the acceleration vector in tangential and normal components, define the motion of a particle as shown in the figure. $\vec{e}_t$ and $\vec{e}'$ are tangential unit vectors for the particle path at *P* and *P'*. When drawn with respect to the same origin, $\Delta\vec{e}_t = \vec{e}_t' - \vec{e}_t$ and $\Delta\theta$ is the angle between them.



$$\Delta e_t = 2\sin(\Delta\theta/2)$$

$$\lim_{\Delta\theta\to 0}\frac{\Delta\vec{e}_t}{\Delta\theta} = \lim_{\Delta\theta\to 0}\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}\vec{e}_n = \vec{e}_n$$

$$\vec{e}_n = \frac{d\vec{e}_t}{d\theta}$$



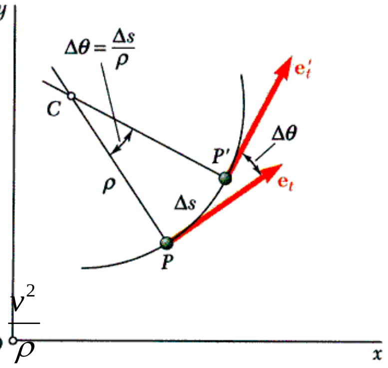
$$\vec{v} = v\vec{e}_t$$

With the velocity vector expressed as

, the particle acceleration may be written as:

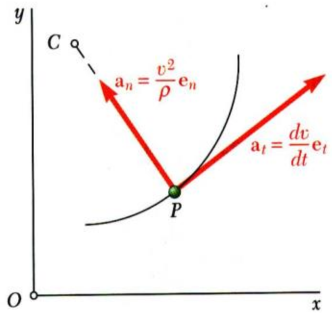
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$$

But, $\frac{d\vec{e}_t}{d\theta} = \vec{e}_n \qquad \rho\, d\theta = ds \qquad \frac{ds}{dt} = v$

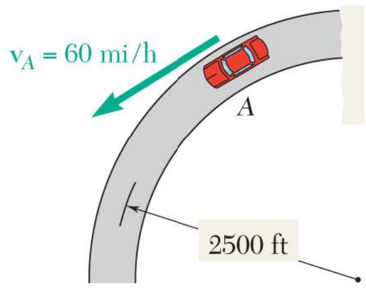


After substituting, $\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n \qquad a_t = \frac{dv}{dt} \qquad a_n = \frac{v^2}{\rho}$

- The tangential component of acceleration reflects change of speed and the normal component reflects change of direction.
- The tangential component may be positive or negative. Normal component always points toward center of path curvature.

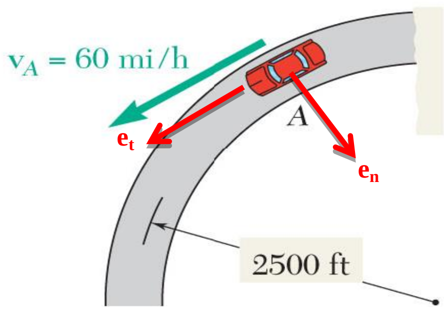


Example 5: A motorist is traveling on a curved section of highway of radius 2500 ft at the speed of 60 mi/h. The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. Knowing that after 8 s the speed has been reduced to 45 mi/h, determine the acceleration of the automobile immediately after the brakes have been applied.



SOLUTION: Define your coordinate system

Then Determine velocity and acceleration in the tangential direction



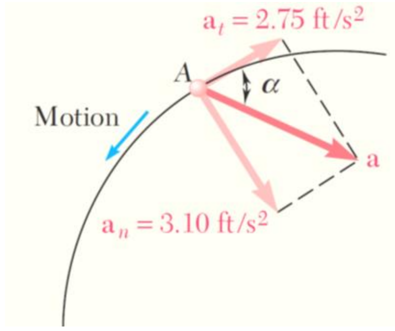
$$60\text{ mi/h} = \left(60\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 88\text{ ft/s}$$

$$45\text{ mi/h} = 66\text{ ft/s}$$

The deceleration constant, therefore;

$$a_t = \text{average } a_t = \frac{\Delta v}{\Delta t} = \frac{66\text{ ft/s} - 88\text{ ft/s}}{8\text{ s}} = -2.75\text{ ft/s}^2$$

Immediately after the brakes are applied, the speed is still 88 ft/s



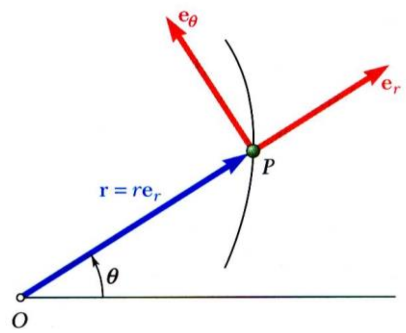
$$a_n = \frac{v^2}{\rho} = \frac{(88\text{ ft/s})^2}{2500\text{ ft}} = 3.10\text{ ft/s}^2$$

$$a = \sqrt{a_n^2 + a_t^2} = \sqrt{2.75^2 + 3.10^2}$$

$$\tan\alpha = \frac{a_n}{a_t} = \frac{3.10\text{ ft/s}^2}{2.75\text{ ft/s}^2}$$

$\mathbf{a} = 4.14\text{ ft/s}^2$ and $\alpha = 48.4°$

1-10 Radial and Transverse Components



The position of a particle *P* is expressed as a distance *r* from the origin *O* to *P*– this defines the radial direction $\mathbf{e_r}$. The transverse direction $\mathbf{e_\theta}$ is perpendicular to $\mathbf{e_r}$: $\vec{r} = r\vec{e}_r$

The particle velocity vector is: $\vec{v} = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$

The particle acceleration vector is: $\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$

One can derive the velocity and acceleration relationships by recognizing that the unit vectors change direction. The particle velocity vector is:

$$\vec{v} = \frac{d}{dt}\left(r\vec{e}_r\right) = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt} = \frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$$

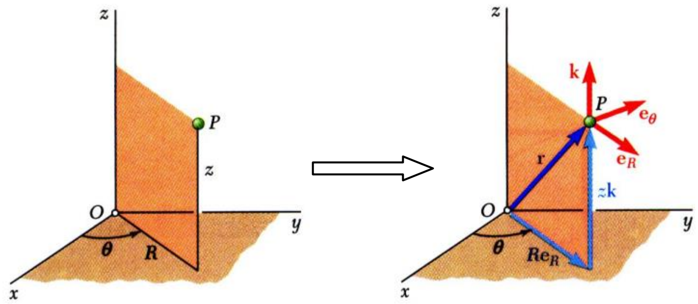
$$\vec{r} = r\vec{e}_r \ldots\ldots\ldots \frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} = \vec{e}_\theta\frac{d\theta}{dt} \ldots\ldots\ldots\ldots \frac{d\vec{e}_\theta}{dt} = \frac{d\vec{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\vec{e}_r\frac{d\theta}{dt}$$

Similarly, the particle acceleration vector is:

$$\vec{a} = \frac{d}{dt}\left(\frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta\right) = \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\vec{e}_r}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta + r\frac{d\theta}{dt}\frac{d\vec{e}_\theta}{dt}$$
$$= \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

When particle position is given in cylindrical coordinates, it is convenient to express the velocity and acceleration vectors using the unit vectors

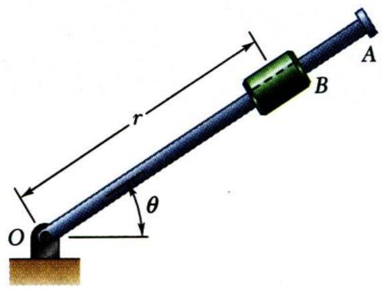


- Position vector: $\vec{r} = R\vec{e}_R + z\vec{k}$
- Velocity vector:

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\vec{e}_R + R\dot{\theta}\vec{e}_\theta + \dot{z}\vec{k}$$

- Acceleration vector: $\vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{R} - R\dot{\theta}^2\right)\vec{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\vec{e}_\theta + \ddot{z}\vec{k}$

Example 6: Rotation of the arm about O is defined by θ = 0.15t^2 where θ is in radians and t in seconds. Collar B slides along the arm such that r = 0.9 - 0.12t^2 where r is in meters. After the arm has rotated through 30^o, determine *(a)* the total velocity of the collar, *(b)* the total acceleration of the collar, and *(c)* the relative acceleration of the collar with respect to the arm.



SOLUTION:

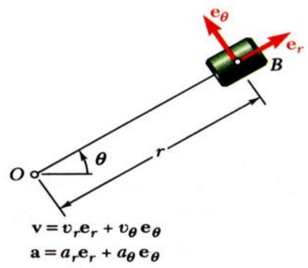
- Evaluate time t for θ = 30^o: $\theta = 0.15t^2 = 30° = 0.524\,\text{rad}$ $t = 1.869\,\text{s}$
- Evaluate radial and angular positions, and first and second derivatives at time t.

$r = 0.9 - 0.12t^2 = 0.481\,\text{m}$ $\dot{r} = -0.24t = -0.449\,\text{m/s}$

.$\ddot{r} = -0.24\,\text{m/s}^2$

$\theta = 0.15t^2 = 0.524\,\text{rad}$ $\dot{\theta} = 0.30t = 0.561\,\text{rad/s}$

$\ddot{\theta} = 0.30\,\text{rad/s}^2$

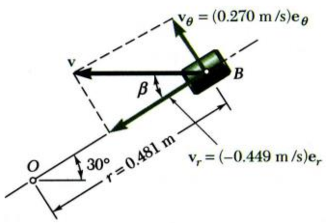


- Calculate velocity and acceleration:

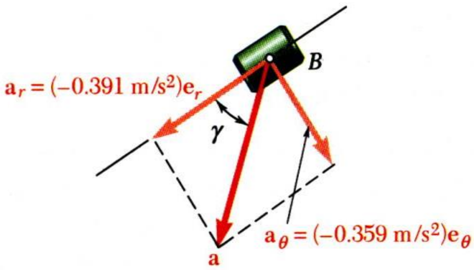
$v_r = \dot{r} = -0.449\,\text{m/s}$

$v_\theta = r\dot{\theta} = (0.481\,\text{m})(0.561\,\text{rad/s}) = 0.270\,\text{m/s}$

$v = \sqrt{v_r^2 + v_\theta^2} = 0.524\,\text{m/s}$ $\beta = \tan^{-1}\frac{v_\theta}{v_r} = 31.0°$



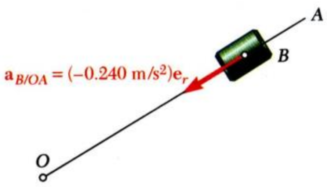
$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.240\,\text{m/s}^2 - (0.481\,\text{m})(0.561\,\text{rad/s})^2$$
$$= -0.391\,\text{m/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.481\,\text{m})(0.3\,\text{rad/s}^2) + 2(-0.449\,\text{m/s})(0.561\,\text{rad/s})$$
$$= -0.359\,\text{m/s}^2$$
$$a = \sqrt{a_r^2 + a_\theta^2} \qquad \gamma = \tan^{-1}\frac{a_\theta}{a_r}$$



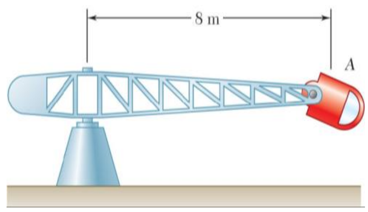
$$a = 0.531\,\text{m/s} \qquad \gamma = 42.6°$$

- Evaluate acceleration with respect to arm. Motion of collar with respect to arm is rectilinear and defined by coordinate *r*.

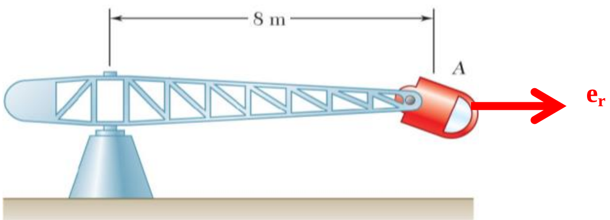
$$a_{B/OA} = \ddot{r} = -0.240\,\text{m/s}^2$$



Example 7: The angular acceleration of the centrifuge arm varies according to $\ddot{\theta} = 0.05\theta$ (rad/s²) where θ is measured in radians. If the centrifuge starts from rest, determine the acceleration magnitude after the gondola has traveled two full rotations.



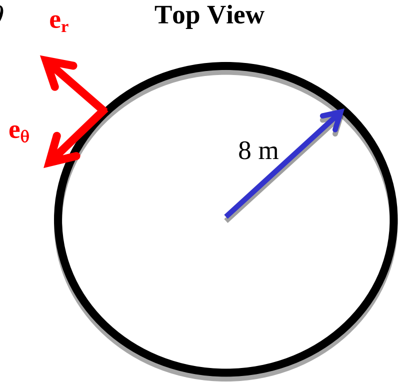
SOLUTION:



- Define your coordinate system
- Determine the angular velocity

$$\ddot{\theta} = 0.05\theta \ \ (\text{rad/s}^2)$$

Acceleration is a function of position, so use: $\ddot{\theta}d\theta = \dot{\theta}d\dot{\theta}$



- Evaluate the integral: $\int_0^{(2)(2\pi)} 0.05\theta d\theta = \int_0^{\dot{\theta}} \dot{\theta} d\dot{\theta}$

$$\left.\frac{0.05\theta^2}{2}\right|_0^{2(2\pi)} = \left.\frac{\dot{\theta}^2}{2}\right|_0^{\dot{\theta}} \qquad \dot{\theta}^2 = 0.05[2(2\pi)]^2$$

- Determine the angular velocity: $\dot{\theta}^2 = 0.05[2(2\pi)]^2$

- Determine the angular acceleration: $\ddot{\theta} = 0.05\theta = 0.05(2)(2\pi) = 0.6283 \text{ rad/s}^2$
- Find the radial and transverse accelerations:

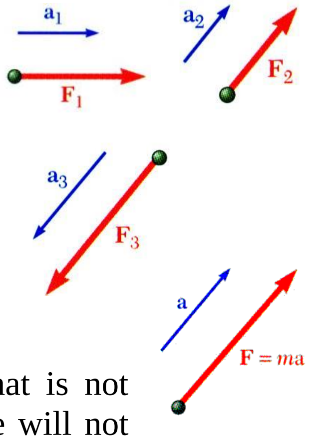
$$\begin{aligned}\vec{a} &= \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta \\ &= \left(0 - (8)(2.8099)^2\right)\vec{e}_r + \left((8)(0.6283) + 0\right)\vec{e}_\theta \\ &= -63.166\,\vec{e}_r + 5.0265\,\vec{e}_\theta \quad (\text{m/s}^2)\end{aligned}$$

$$a_{mag} = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-63.166)^2 + \left[5.0265\right]^2}$$

$$a_{mag} = 63.365 \text{ m/s}^2$$

Chapter TWO
Kinetics of Particles: Force and Acceleration

2-1 Newton's Second Law of Motion



If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of resultant and in the direction of the resultant.

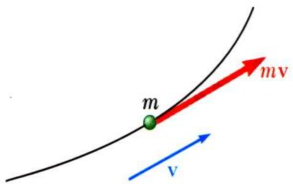
$$\vec{F} = m\vec{a}$$

If particle is subjected to several forces: $\sum \vec{F} = m\vec{a}$

We must use a Newtonian frame of reference, i.e., one that is not accelerating or rotating. If no force acts on particle, particle will not accelerate, i.e., it will remain stationary or continue on a straight line at constant velocity.

2-2 Linear Momentum of a Particle

The principle of conservation of linear momentum is:



$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

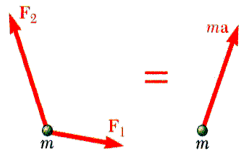
$$= \frac{d}{dt}(m\vec{v}) = \frac{d}{dt}(\vec{L})$$

Where: $\vec{L} = m\vec{v}$ = Linear momentum

Sum of forces = rate of change of linear momentum $\sum \vec{F} = \dot{\vec{L}}$

If $\sum \vec{F} = 0$ then linear momentum is <u>constant</u>

2-3 Equations of Motion

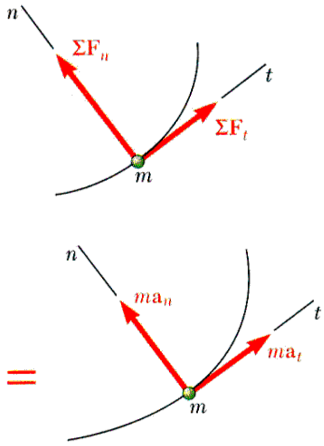


- Newton's second law $\sum \vec{F} = m\vec{a}$
- Convenient to resolve into components:

$$\sum\left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right) = m\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

$$\sum F_x = m\ddot{x} \quad \sum F_y = m\ddot{y} \quad \sum F_z = m\ddot{z}$$



- For tangential and normal components:

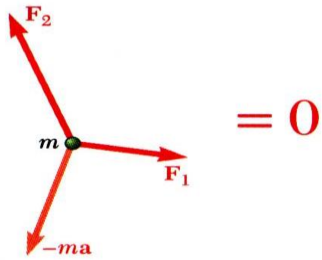
$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$

$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$

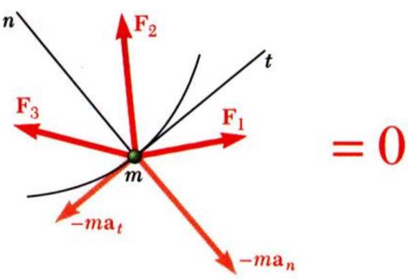
2-4 Dynamic Equilibrium

Alternate expression of Newton's law:: $\sum \vec{F} - m\vec{a} = 0$

Where: $-m\vec{a}$ = inertia vector



If we include inertia vector, the system of forces acting on particle is equivalent to zero. The particle is said to be in dynamic equilibrium.
Inertia vectors are often called *inertia forces* as they measure the resistance that particles offer to changes in motion.



2-5 Equation of Motion for a System of Particles

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\sum F_i = \sum m_i a_i$$

- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the unbalanced force on a particle causes it to accelerate.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

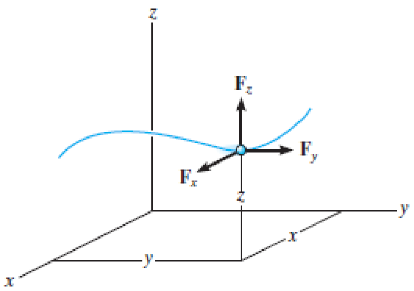
2-6 Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial *x*, *y*, *z* frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their **i**, **j**, **k** components. Applying the equation of motion, we have

$$\Sigma \mathbf{F} = m\mathbf{a}; \qquad \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$

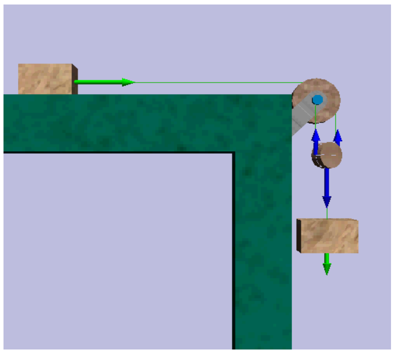
For this equation to be satisfied, the respective i, j, k components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations:

$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$
$$\Sigma F_z = ma_z$$

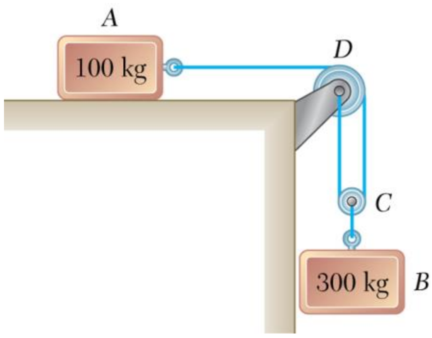


In particular, if the particle is constrained to move only in the x–y plane, then the first two of these equations are used to specify the motion.

Example 1: The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.



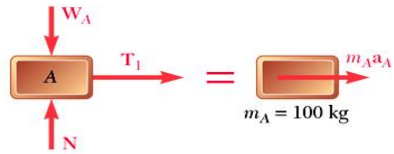
SOLUTION:



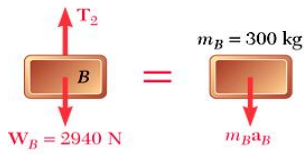
Kinematic relationship: If A moves x_A to the right, B moves down 0.5 x_A:

$$x_B = \tfrac{1}{2}x_A \qquad a_B = \tfrac{1}{2}a_A$$

Draw free body diagrams & apply Newton's law:



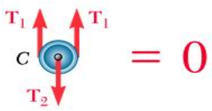
$$\sum F_x = m_A a_A \Longrightarrow T_1 = (100)a_A$$



$$\sum F_y = m_B a_B \Longrightarrow m_B g - T_2 = m_B a_B$$

$$300\times 9.81 - T_2 = (300)a_B$$

$$T_2 = 2940\text{-}(300)a_B$$



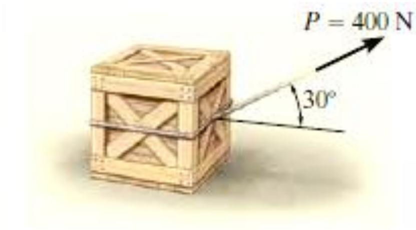
$$\sum F_y = m_C a_C \Longrightarrow T_2 - 2T_1 = 0$$

$$2940\text{-}(300)a_B - 2T_1 = 0 \qquad 2940\text{-}(300)a_B - 200a_A = 0$$

$$2940\text{-}(300)a_B - 2\times 200a_B = 0$$

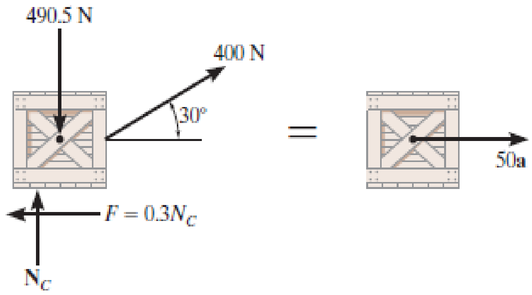
$$a_B = 4.2\,m/s^2 \qquad a_A = 8.4\,m/s^2 \qquad T_1 = 840\,N \qquad T_2 = 1680\,N$$

Example 2: The 50-kg crate shown in Fig. rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.



SOLUTION:

$W = mg = 50$ kg $(9.81$ m/s$^2) = 490.5$ N.



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 400\cos 30° - 0.3N_C = 50a$$
$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400\sin 30° = 0$$

$$N_C = 290.5 \text{ N}$$
$$a = 5.185 \text{ m/s}^2$$
$$v = v_0 + a_c t = 0 + 5.185(3)$$
$$= 15.6 \text{ m/s} \rightarrow$$

Example 3: A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s. Determine the maximum height to which it will travel if (*a*) atmospheric resistance is neglected; and (*b*) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

SOLUTION:

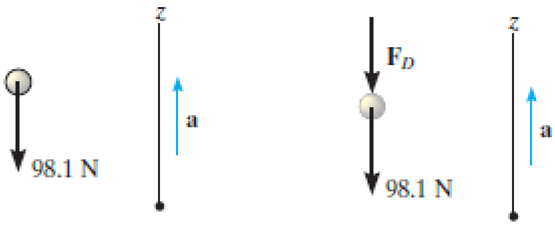
$W = mg = 10(9.81) = 98.1$ N

$$+\uparrow \Sigma F_z = ma_z; \qquad -98.1 = 10a, \qquad a = -9.81 \text{ m/s}^2$$

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$(+\uparrow)$
$$v^2 = v_0^2 + 2a_c(z - z_0)$$
$$0 = (50)^2 + 2(-9.81)(h - 0)$$
$$h = 127 \text{ m} \qquad \textit{Ans.}$$

$F_D = (0.01v^2)$



$+\uparrow \Sigma F_z = ma_z; \quad -0.01v^2 - 98.1 = 10a, \quad a = -(0.001v^2 + 9.81)$

Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$(+\uparrow)\ a\,dz = v\,dv; \qquad -(0.001v^2 + 9.81)\,dz = v\,dv$

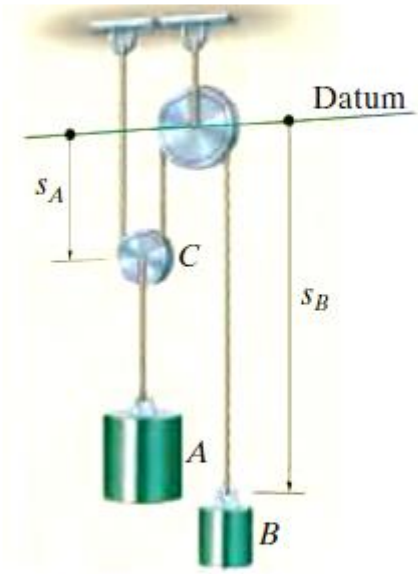
Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at $z = h$, $v = 0$, we have

$$\int_0^h dz = -\int_{50\text{ m/s}}^{0} \frac{v\,dv}{0.001v^2 + 9.81} = -500\ln(v^2 + 9810)\Big|_{50\text{ m/s}}^{0}$$

$$h = 114 \text{ m} \qquad \textit{Ans.}$$

NOTE: The answer indicates a lower elevation than that obtained in part (*a*) due to atmospheric resistance or drag.

Example 4: The 100-kg block *A* shown in Fig. is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block *B* in 2 s.

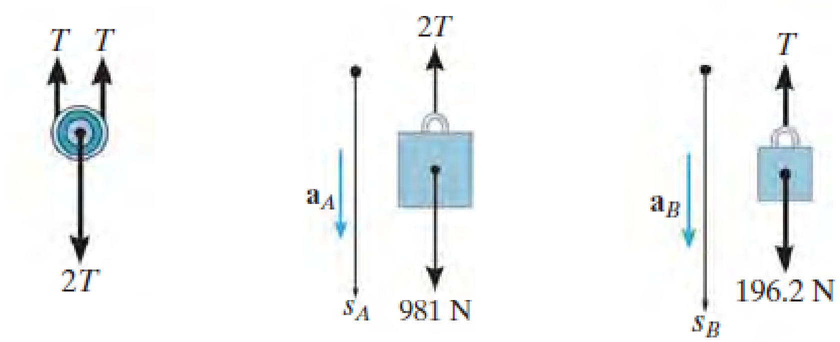


SOLUTION:

Notice from free body diagram that for *A* to remain stationary:
$T = 1/2 * 9.81 * 100 = 490.5$ N,

whereas for *B* to remain static:

$T = 9.81 * 20 = 196.2$ N.



Equations of Motion. Block *A*,

$+\downarrow \Sigma F_y = ma_y;$ $\quad 981 - 2T = 100a_A$

Block *B*,

$+\downarrow \Sigma F_y = ma_y;$ $\quad 196.2 - T = 20a_B$

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B$$

$$T = 327.0 \text{ N}$$
$$a_A = 3.27 \text{ m/s}^2$$
$$a_B = -6.54 \text{ m/s}^2$$

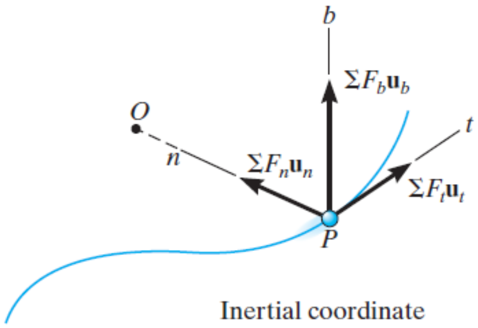
Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

$(+\downarrow)$
$$v = v_0 + a_B t$$
$$= 0 + (-6.54)(2)$$
$$= -13.1 \text{ m/s} \qquad \textit{Ans.}$$

The negative sign indicates that block B is moving upward.

2-7 Equations of Motion: Normal and Tangential Coordinates

When a particle moves along a curved path which is known, the equation of motion for the particle may be written in the tangential, normal, and binomial directions. Note that there is no motion of the particle in the binomial direction, since the particle is constrained to move along the path. We have



Inertial coordinate system

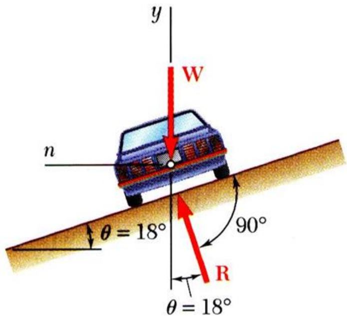
$$\Sigma \mathbf{F} = m\mathbf{a}$$
$$\Sigma F_t \mathbf{u}_t + \Sigma F_n \mathbf{u}_n + \Sigma F_b \mathbf{u}_b = m\mathbf{a}_t$$

This equation is satisfied provided

$$\Sigma F_t = ma_t$$
$$\Sigma F_n = ma_n$$

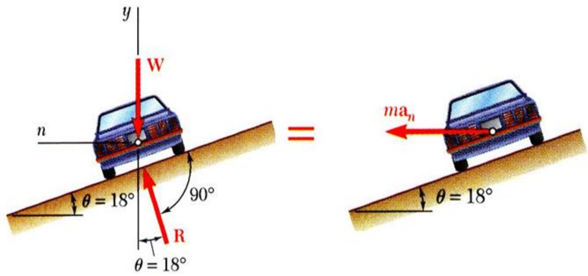
Recall that $a_t (= dv/dt)$ represents the time rate of change in the magnitude of velocity. So if $\Sigma \mathbf{F}_t$ acts in the direction of motion, the particle's speed will increase, whereas if it acts in the opposite direction, the particle will slow down. Likewise, $a_n (= v^2/\rho)$ represents the time rate of change in the velocity's direction. It is caused by $\Sigma \mathbf{F}_n$, which *always* acts in the positive n direction, i.e., toward the path's center of curvature. For this reason it is often referred to as the *centripetal force*.

Example 5: Determine the rated speed of a highway curve of radius r = 400 ft banked through an angle θ = 18^o. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted at its wheels.



SOLUTION:

Resolve the equation of motion for the car into vertical and normal components:



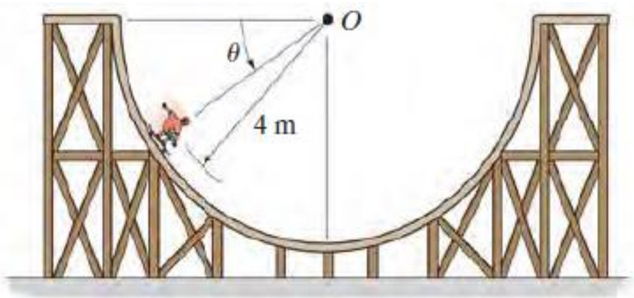
$$\sum F_y = 0 : R\cos\theta - W = 0 \ldots\ldots\ldots\ldots R = \frac{W}{\cos\theta}$$

$$R\sin\theta = \frac{W}{g}a_n \ldots\ldots\ldots \frac{W}{\cos\theta}\sin\theta = \frac{W}{g}\frac{v^2}{\rho}$$

Solve for the vehicle speed:

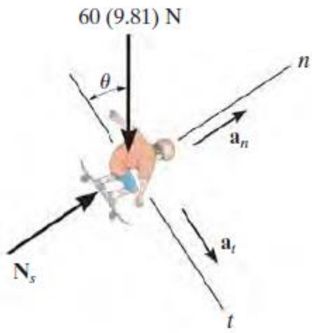
$$v^2 = g\rho\tan\theta = \left(32.2\,\text{ft/s}^2\right)\left(400\,\text{ft}\right)\tan 18° = 64.7\,\text{ft/s} = 44.1\,\text{mi/h}$$

Example 6: The 60-kg skateboarder in Fig. coasts down the circular track. If he starts from rest when θ = 0^o, determine the magnitude of the normal reaction the track exerts on him when θ= 60^o. Neglect his size for the calculation.



SOLUTION:

Equations of Motion.



$$+\nearrow \Sigma F_n = ma_n;\quad N_s - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{ m}}\right)$$

$$+\searrow \Sigma F_t = ma_t;\quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\,a_t$$

$$a_t = 9.81\cos\theta$$

Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60^o$. Using the geometric relation $s = \theta\, r$, where $ds = r\, d\theta = (4\text{ m})\, d\theta$, and the initial condition $v = 0$ at $\theta = 0^o$, we have,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60^\circ} 9.81\cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24\sin\theta\Big|_0^{60^\circ}$$

$$\frac{v^2}{2} - 0 = 39.24(\sin 60^\circ - 0)$$

$$v^2 = 67.97\ \text{m}^2/\text{s}^2$$

Substituting this result and $\theta = 60^o$ into Eq. of N_s, yields

$$N_s = 1529.23\text{ N} = 1.53\text{ kN}$$

2-8 Equations of Motion: Cylindrical Coordinates

When all the forces acting on a particle are resolved into cylindrical components, i.e., along the unit-vector directions $\mathbf{u}_r$, $\mathbf{u}_\theta$, $\mathbf{u}_z$, the equation of motion can be expressed as

$$\Sigma\mathbf{F} = m\mathbf{a}$$

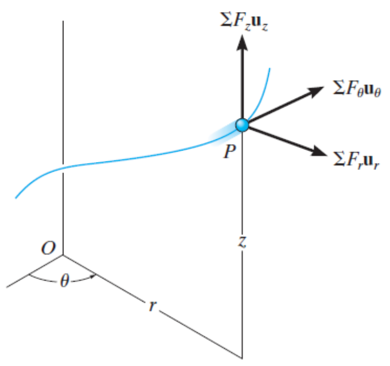
$$\Sigma F_r\mathbf{u}_r + \Sigma F_\theta\mathbf{u}_\theta + \Sigma F_z\mathbf{u}_z = ma_r\mathbf{u}_r + ma_\theta\mathbf{u}_\theta + ma_z\mathbf{u}_z$$

To satisfy this equation, we require

$$\Sigma F_r = ma_r$$

$$\Sigma F_\theta = ma_\theta$$

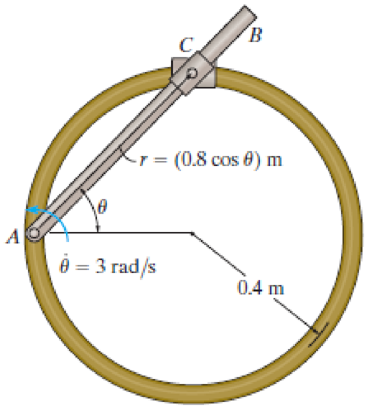
$$\Sigma F_z = ma_z$$



Inertial coordinate system

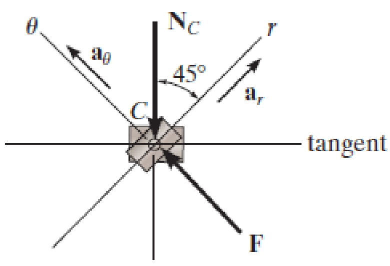
Example 7: The smooth 0.5-kg double-collar in Fig. can freely slide on arm AB and the circular guide rod.

If the arm rotates with a constant angular velocity of $\dot{\theta}$ = 3 rad/s, determine the force the arm exerts on the ollar at the instant θ= 45°. Motion is in the horizontal plane.



SOLUTION:

Free-Body Diagram. The normal reaction $\boldsymbol{N}_C$ of the circular guide rod and the force $\boldsymbol{F}$ of arm AB act on the collar in the plane of motion. Note that $\boldsymbol{F}$ acts perpendicular to the axis of arm AB, that is, in the direction of the u axis, while $\boldsymbol{N}_C$ acts perpendicular to the tangent of the circular path at θ= 45°. The four unknowns are N_C, F, a_r, a_θ.



Equations of Motion.

$$+\nearrow \Sigma F_r = ma_r: \qquad -N_C \cos 45° = (0.5 \text{ kg})\, a_r \qquad (1)$$

$$+\nwarrow \Sigma F_\theta = ma_\theta: \qquad F - N_C \sin 45° = (0.5 \text{ kg})\, a_\theta \qquad (2)$$

Kinematics. Using the chain rule, the first and second time derivatives of r when θ= 45°, $\dot{\theta}$ = 3 rad/s, θ = 0, are

$$r = 0.8 \cos\theta = 0.8 \cos 45° = 0.5657 \text{ m}$$

$$\dot{r} = -0.8 \sin\theta\, \dot{\theta} = -0.8 \sin 45°(3) = -1.6971 \text{ m/s}$$

$$\ddot{r} = -0.8\left[\sin\theta\, \ddot{\theta} + \cos\theta\, \dot{\theta}^2\right]$$

$$= -0.8[\sin 45°(0) + \cos 45°(3^2)] = -5.091 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -5.091 \text{ m/s}^2 - (0.5657 \text{ m})(3 \text{ rad/s})^2 = -10.18 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.5657 \text{ m})(0) + 2(-1.6971 \text{ m/s})(3 \text{ rad/s})$$

$$= -10.18 \text{ m/s}^2$$

Substituting these results into Eqs. (1) and (2) and solving, we get

$$N_C = 7.20 \text{ N}$$

$$F = 0 \qquad \textit{Ans.}$$

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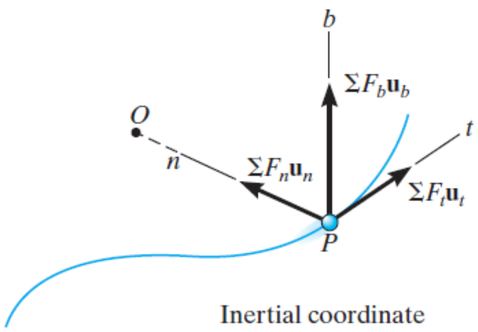

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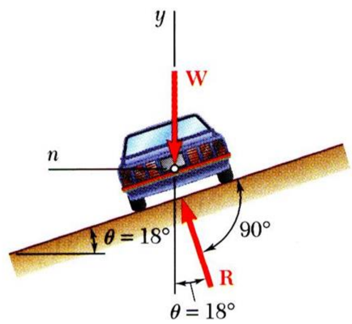
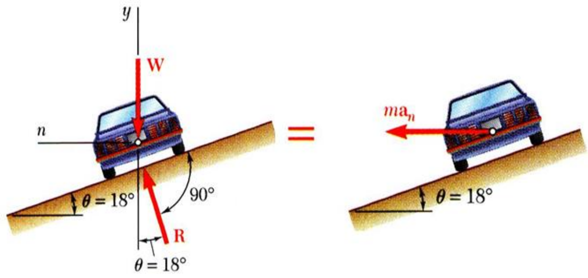


SOLUTION:



Resolve the equation of motion for the car into vertical and normal components:

$$\sum F_y = 0 : R\cos\theta - W = 0 \text{..........} R = \frac{W}{\cos\theta}$$

$$R\sin\theta = \frac{W}{g}a_n \text{.......} \frac{W}{\cos\theta}\sin\theta = \frac{W}{g}\frac{v^2}{\rho}$$

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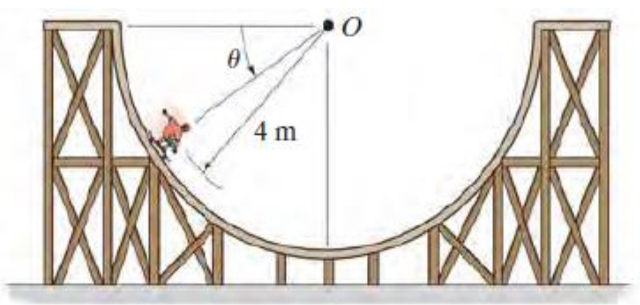
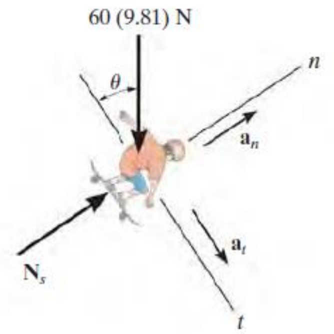


SOLUTION:

Equations of Motion.



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$$a_t = 9.81\cos\theta$$

Since a_t is expressed in terms of θ, the equation *v dv* = a_t *ds* must be used to determine the speed of the skateboarder when *θ* = *60°*. Using the geometric relation *s* = θ *r*, where *ds* = *r dθ* = (4 m) *dθ*, and the initial condition *v* = 0 at *θ* = *0°*, we have,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60°} 9.81\cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24\sin\theta\Big|_0^{60°}$$

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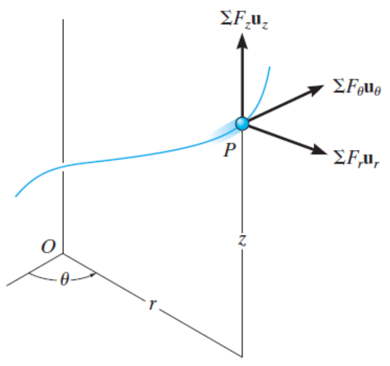

Inertial coordinate system

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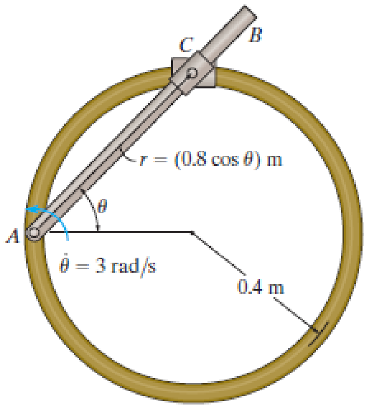
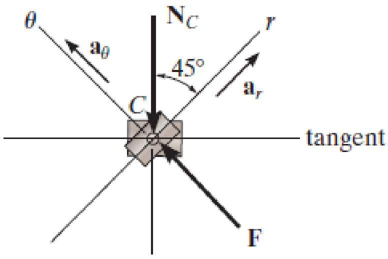
If the arm rotates with a constant angular velocity of $\dot{\theta}$ = 3 rad/s, determine the force the arm exerts on the ollar at the instant θ= 45^o. Motion is in the horizontal plane.



SOLUTION:

Free-Body Diagram. The normal reaction N_C of the circular guide rod and the force F of arm AB act on the collar in the plane of motion. Note that F acts perpendicular to the axis of arm AB, that is, in the direction of the u axis, while N_C acts perpendicular to the tangent of the circular path at θ= 45°. The four unknowns are N_C, F, a_r, a_θ.



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Kinematics. Using the chain rule, the first and second time derivatives of r when θ= 45°, $\dot{\theta}$ = 3 rad/s, $\ddot{\theta}$ = 0, are

$$r = 0.8 \cos\theta = 0.8 \cos 45° = 0.5657 \text{ m}$$

$$\dot{r} = -0.8 \sin\theta\, \dot{\theta} = -0.8 \sin 45°(3) = -1.6971 \text{ m/s}$$

$$\ddot{r} = -0.8\left[\sin\theta\, \ddot{\theta} + \cos\theta\, \dot{\theta}^2\right]$$

$$= -0.8[\sin 45°(0) + \cos 45°(3^2)] = -5.091 \text{ m/s}^2$$

We have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -5.091 \text{ m/s}^2 - (0.5657 \text{ m})(3 \text{ rad/s})^2 = -10.18 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.5657 \text{ m})(0) + 2(-1.6971 \text{ m/s})(3 \text{ rad/s})$$

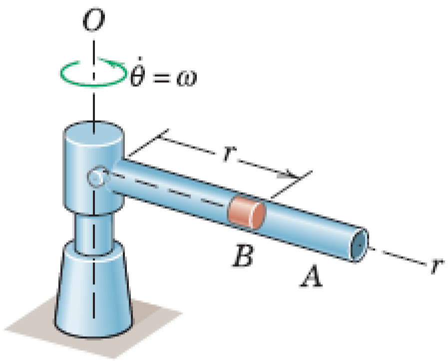
$$= -10.18 \text{ m/s}^2$$

Substituting these results into Eqs. (1) and (2) and solving, we get

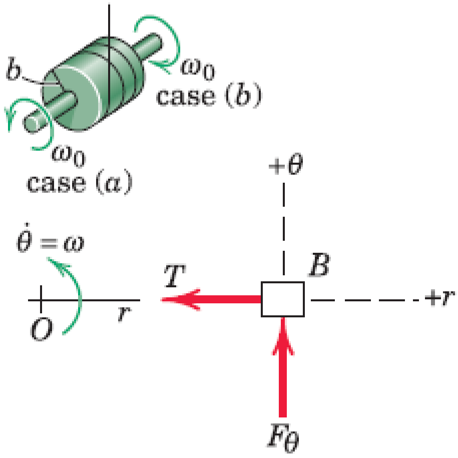
$$N_C = 7.20 \text{ N}$$

$$F = 0 \qquad \textit{Ans.}$$

Example 8: Tube A rotates about the vertical O-axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug B of mass m whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius b. Determine the tension T in the cord and the horizontal component F_θ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (a) and second in the direction for case (b). Neglect friction.



SOLUTION:



$[\Sigma F_r = ma_r]$ $\qquad -T = m(\ddot{r} - r\dot{\theta}^2)$

$[\Sigma F_\theta = ma_\theta]$ $\qquad F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

Case (a). With $\dot{r} = +b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \qquad F_\theta = 2mb\omega_0\omega$$

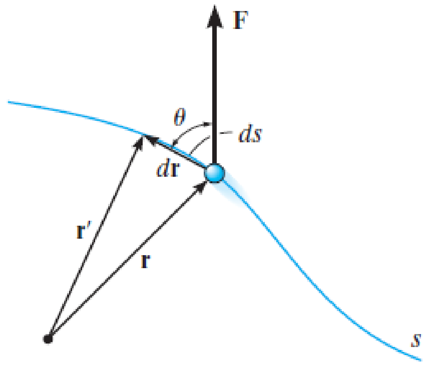
Case (b). With $\dot{r} = -b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr\omega^2 \qquad F_\theta = -2mb\omega_0\omega$$

CHAPTER TWO
KINETICS OF PARTICLES: WORK AND ENERGY

2-9 The Work of a Force

A force *F* will do work on a particle only when the particle undergoes a displacement in the direction of the force. For example, if the force *F* in Fig. causes the particle to move along the path s from position r to a new position *r'*, the displacement is then *dr* = *r'* - *r*. The magnitude of *dr* is *ds*, the length of the differential segment along the path. If the angle between the tails of *dr* and *F* is θ, then the work done by *F* is a scalar quantity, defined by



$$dU = F\, ds \cos\theta$$

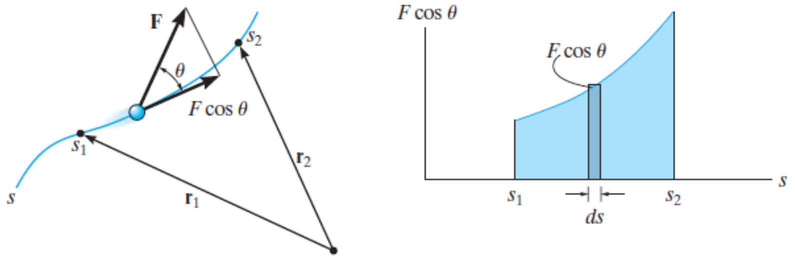
By definition of the dot product this equation can also be written as

$$dU = F \cdot dr$$

2-9-1 Work of a Variable Force

If the particle acted upon by the force F undergoes a finite displacement along its path from r_1 to r_2 or s_1 to s_2 , the work of force F is determined by integration. Provided *F* and θ can be expressed as a function of position, then

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos\theta \, ds$$



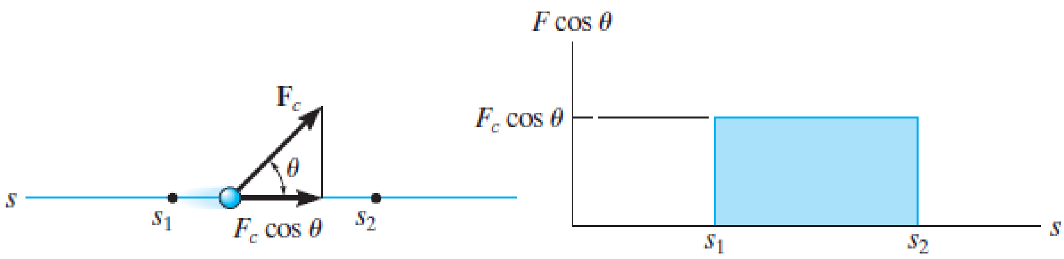
2-9-2 Work of a Constant Force Moving Along a Straight Line

If the force $\boldsymbol{F_c}$ has a constant magnitude and acts at a constant angle θ from its straight-line path, then the component of $\boldsymbol{F_c}$ in the direction of displacement is always $\boldsymbol{F_c}\cos\theta$. The work done by $\boldsymbol{F_c}$ when the particle is displaced from s_1 to s_2 is determined from, in which case

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

$$U_{1-2} = F_c \cos\theta (s_2 - s_1)$$

Here the work of $\boldsymbol{F_c}$ represents the *area of the rectangle* as in Figure below:



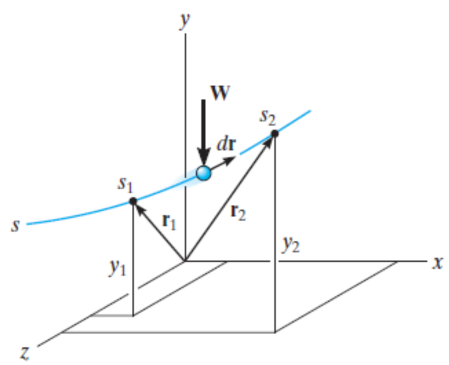
2-9-3 Work of a Weight

Consider a particle of weight $\boldsymbol{W}$, which moves up along the path s shown in Fig. from position s_1 to position s_2 .

$$U_{1-2} = \int \mathbf{F}\cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j})\cdot(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W\,dy = -W(y_2 - y_1)$$

$$U_{1-2} = -W\,\Delta y$$

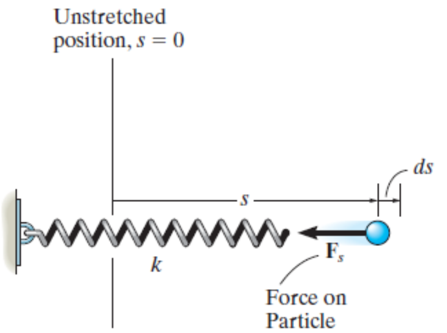


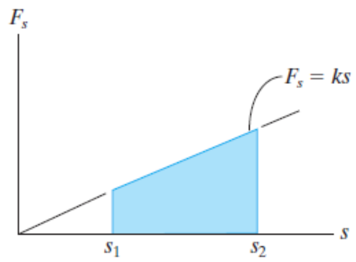
2-9-4 Work of a Spring Force

If an elastic spring is elongated a distance *ds*, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks\ ds$. The work is *negative* since $\mathbf{F}_s$ acts in the opposite sense to *ds*. If the particle displaces from s_1 to s_2, the work of $\mathbf{F}_s$ is then

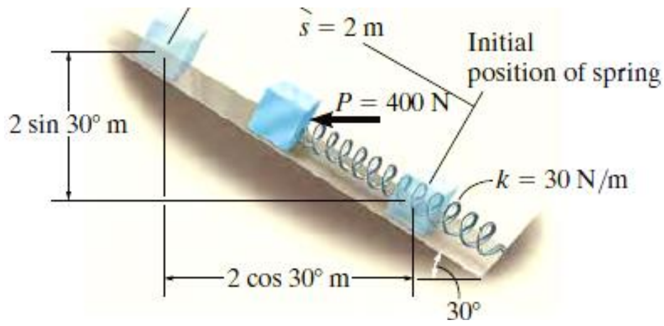
$$U_{1-2} = \int_{s_1}^{s_2} F_s\, ds = \int_{s_1}^{s_2} -ks\, ds$$

$$U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$



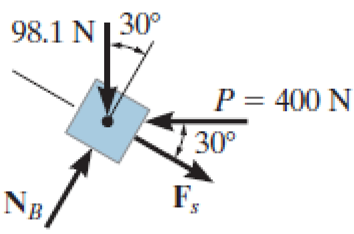


Example 9: The 10-kg block shown in Fig. rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force *P* = 400 N pushes the block up the plane *s* = 2 m.



SOLUTION:

$$U_P = 400\text{ N}\,(2\text{ m}\cos 30°) = 692.8\text{ J}$$



Spring Force F_s. In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5 \text{ m} + 2 \text{ m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\mathbf{F}_s$ is thus

$$U_s = -\left[\tfrac{1}{2}(30 \text{ N/m})(2.5 \text{ m})^2 - \tfrac{1}{2}(30 \text{ N/m})(0.5 \text{ m})^2\right] = -90 \text{ J}$$

Weight W. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1 \text{ N})\,(2 \text{ m} \sin 30°) = -98.1 \text{ J}$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

$$U_T = 692.8 \text{ J} - 90 \text{ J} - 98.1 \text{ J} = 505 \text{ J} \qquad \textit{Ans.}$$

2-10 The Principle of Work and Energy

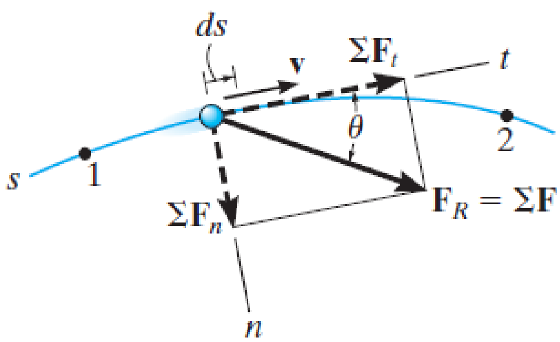
Consider the particle in Fig. which is located on the path defined relative to an inertial coordinate system. If the particle has a mass *m* and is subjected to a system of external forces represented by the resultant $\mathbf{F}_R = \sum \mathbf{F}$, then the equation of motion for the particle in the tangential direction is $\sum F_t = ma_t$. Applying the kinematic equation $a_t = v\,dv>ds$ and integrating both sides, assuming initially that the particle has a position $s = s_1$ and a speed $v = v_1$, and later at $s = s_2$, $v = v_2$, we have

$$\sum \int_{s_1}^{s_2} F_t\, ds = \int_{v_1}^{v_2} mv\, dv$$

$$\sum \int_{s_1}^{s_2} F_t\, ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$\sum U_{1-2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$T_1 + \sum U_{1-2} = T_2$$



Where:

$$\boldsymbol{T = kinetic\ energy\ \frac{1}{2}mv^2}$$

Principle of Work and Energy.

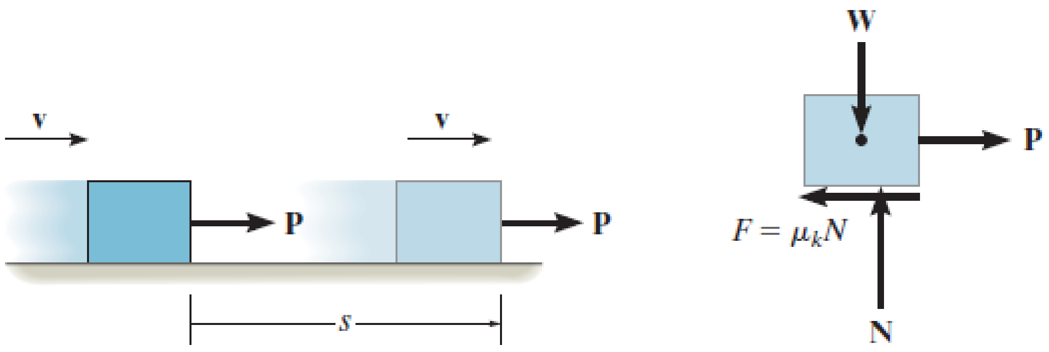
- Apply the principle of work and energy, $T_1 + \Sigma U_{1-2} = T_2$.
- The kinetic energy at the initial and final points is *always positive*, since it involves the speed squared $\left(T = \frac{1}{2}mv^2\right)$.
- A force does work when it moves through a displacement in the direction of the force.
- Work is *positive* when the force component is in the *same sense of direction* as its displacement, otherwise it is negative.
- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.
- The work of a weight is the product of the weight magnitude and the vertical displacement, $U_W = \pm Wy$. It is positive when the weight moves downwards.
- The work of a spring is of the form $U_s = \frac{1}{2}ks^2$, where k is the spring stiffness and s is the stretch or compression of the spring.

If we apply the principle of work and energy to this and each of the other particles in the system, then since work and energy are scalar quantities, the equations can be summed algebraically, which gives

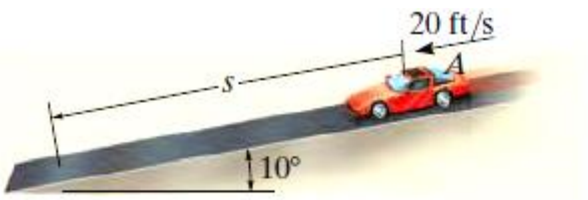
$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

Problems involve cases where a body slides over the surface of another body in the presence of friction considers as special class of problems which requires a careful application. Consider, for example, a block which is translating a distance *s* over a rough surface as shown in Fig. If the applied force ***P*** just balances the *resultant* frictional force $\mu_k N$.

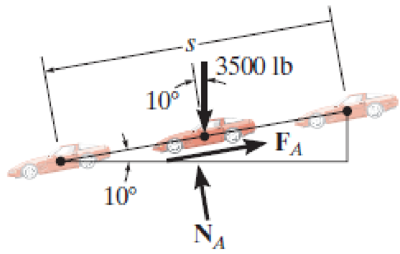
$$\frac{1}{2}mv^2 + Ps - \mu_k Ns = \frac{1}{2}mv^2$$



Example 10: The 3500-lb automobile shown in Fig. travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far *s* the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.



SOLUTION:



Applying the equation of equilibrium normal to the road, we have

$$+\nwarrow \Sigma F_n = 0; \qquad N_A - 3500 \cos 10^\circ \text{ lb} = 0 \qquad N_A = 3446.8 \text{ lb}$$

Thus,

$$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10^\circ) - (1723.4 \text{ lb})s = 0$$

Solving for *s* yields

$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$

NOTE: If this problem is solved by using the equation of motion, *two steps* are involved. First, from the free-body diagram, Fig. , the equation of motion is applied along the incline. This yields

$$+\swarrow \Sigma F_s = m a_s; \qquad 3500 \sin 10^\circ \text{ lb} - 1723.4 \text{ lb} = \frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2} a$$

$$a = -10.3 \text{ ft/s}^2$$

Then, since *a* is constant, we have

$$(+\swarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0);$$

$$(0)^2 = (20 \text{ ft/s})^2 + 2(-10.3 \text{ ft/s}^2)(s - 0)$$

$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$

2-11 Power and Efficiency

The term "power" provides a useful basis for choosing the type of motor or machine which is required to do a certain amount of work in a given time. For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner. The *power* generated by a machine or engine that performs an amount of work *dU* within the time interval *dt* is therefore

$$P = \frac{dU}{dt}$$

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

The basic units of power used in the SI and FPS systems are the watt (W) and horsepower (hp), respectively. These units are defined as

1 W = 1 J/s = 1 N . m/s

1 hp = 550 ft . lb/s

For conversion between the two systems of units, 1 hp = 746 W.

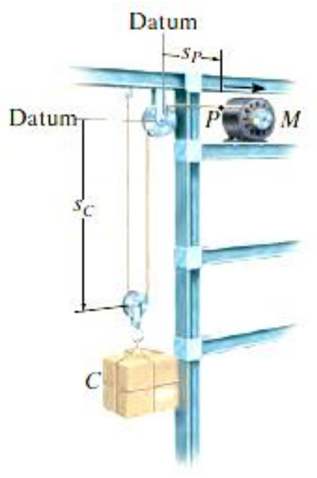
The *mechanical efficiency* of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\varepsilon = \frac{\text{power output}}{\text{power input}}$$

If energy supplied to the machine occurs during the *same time interval* at which it is drawn, then the efficiency may also be expressed in terms of the ratio. Since machines consist of a series of moving parts, frictional forces will always be developed within the machine, and as a result, extra energy or power is needed to overcome these forces. Consequently, power output will be less than power input and so *the efficiency of a machine is always less than 1*. The procedure for analysis is as follow:

- First determine the external force **F** acting on the body which causes the motion. This force is usually developed by a machine or engine placed either within or external to the body.
- If the body is accelerating, it may be necessary to draw its free-body diagram and apply the equation of motion ($\Sigma \mathbf{F} = m\mathbf{a}$) to determine **F**.
- Once **F** and the velocity **v** of the particle where **F** is applied have been found, the power is determined by multiplying the force magnitude with the component of velocity acting in the direction of **F**, (i.e., $P = \mathbf{F} \cdot \mathbf{v} = Fv\cos\theta$).
- In some problems the power may be found by calculating the work done by **F** per unit of time ($P_{avg} = \Delta U/\Delta t$,).

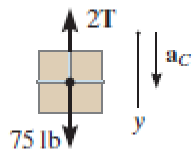
Example 11: The motor of the hoist shown in Fig. lifts the 75-lb crate *C* so that the acceleration of point *P* is 4 ft/s^2. Determine the power that must be supplied to the motor at the instant *P* has a velocity of 2 ft/s. Neglect the mass of the pulley and cable and take e = 0.85.



SOLUTION:

$$+\downarrow \quad \Sigma F_y = ma_y; \qquad -2T + 75\text{ lb} = \frac{75\text{ lb}}{32.2\text{ ft/s}^2}a_c \qquad (1)$$

$$2a_C = -a_P \qquad (2)$$



Since $a_P = +4\text{ ft/s}^2$, then $a_C = -(4\text{ ft/s}^2)/2 = -2\text{ ft/s}^2$. What does the negative sign indicate? Substituting this result into Eq. 1 and *retaining* the negative sign since the acceleration in *both* Eq. 1 and Eq. 2 was considered positive downward, we have

$$-2T + 75\text{ lb} = \left(\frac{75\text{ lb}}{32.2\text{ ft/s}^2}\right)(-2\text{ ft/s}^2)$$

$$T = 39.83\text{ lb}$$

The power output, measured in units of horsepower, required to draw the cable in at a rate of 2 ft/s is therefore

$$P = \mathbf{T}\cdot\mathbf{v} = (39.83\text{ lb})(2\text{ ft/s})[1\text{ hp}/(550\text{ ft}\cdot\text{lb/s})]$$
$$= 0.1448\text{ hp}$$

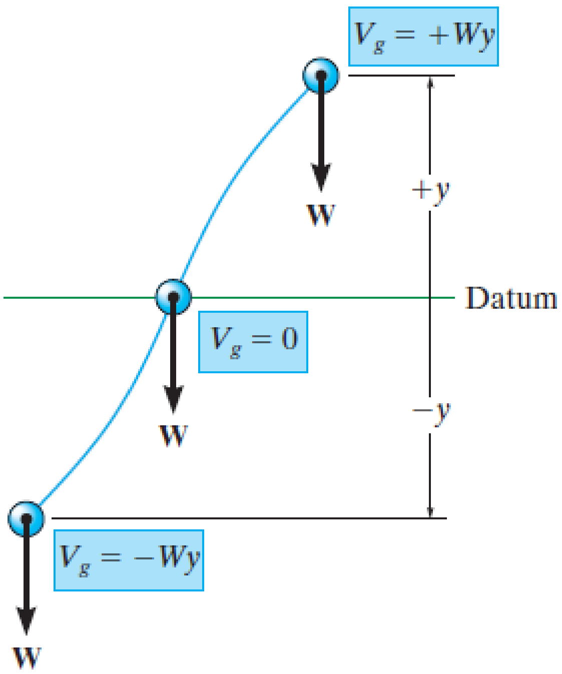
This *power output* requires that the motor provide a *power input* of

$$\text{power input} = \frac{1}{\varepsilon}(\text{power output})$$
$$= \frac{1}{0.85}(0.1448\text{ hp}) = 0.170\text{ hp} \qquad \textit{Ans.}$$

NOTE: Since the velocity of the crate is constantly changing, the power requirement is *instantaneous*.

2-12 Conservative Forces and Potential Energy

If the work of a force is *independent of the path* and depends only on the force's initial and final positions on the path, then we can classify this force as a *conservative force*. Examples of conservative forces are the weight of a particle and the force developed by a spring. The work done by the weight depends *only* on the *vertical displacement* of the weight, and the work done by a spring force depends *only* on the spring's *elongation* or *compression*.



Energy is defined as the capacity for doing work. For example, if a particle is originally at rest, then the principle of work and energy states that $\sum U_{1\text{-}2} = T_2$. In other words, the kinetic energy is equal to the work that must be done on the particle to bring it from a state of rest to a speed v. Thus, the *kinetic energy* is a measure of the particle's *capacity to do work*, which is associated with the *motion* of the particle. When energy comes from the *position* of the particle, measured from a fixed datum or reference plane, it is called potential energy. Thus, *potential energy* is a measure of the amount of work a conservative force will do when it moves from a given position to the datum. In mechanics, the potential energy created by gravity (weight) and an elastic spring is important.

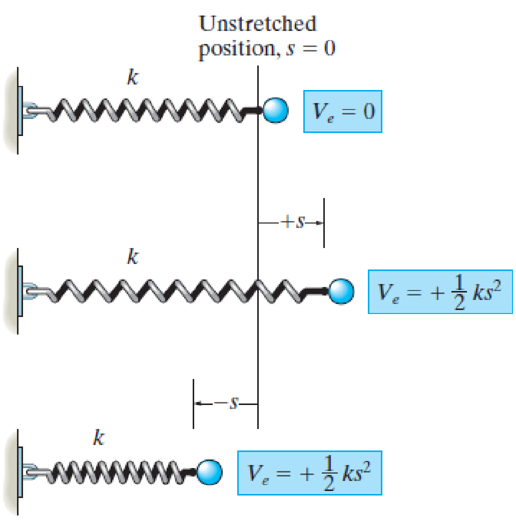
In general, if y is *positive upward*, the gravitational potential energy of the particle of weight W is

$$V_g = Wy$$

When an elastic spring is elongated or compressed a distance s from its unstretched position, elastic potential energy V_e can be stored in the spring. This energy is

$$V_e = +\tfrac{1}{2}ks^2$$

Here V_e is *always positive* since, in the deformed position, the force of the spring has the *capacity* or "potential" for always doing positive work on the particle when the spring is returned to its unstretched position.



Elastic potential energy

In the general case, if a particle is subjected to both gravitational and elastic forces, the particle's potential energy can be expressed as a *potential function*, which is the algebraic sum

$$V = V_g + V_e$$

The work done by a conservative force in moving the particle from one point to another point is measured by the *difference* of this function, i.e.,

$$U_{1-2} = V_1 - V_2$$

For example, the potential function for a particle of weight W suspended from a spring can be expressed in terms of its position, s, measured from a datum located at the unstretched length of the spring, We have

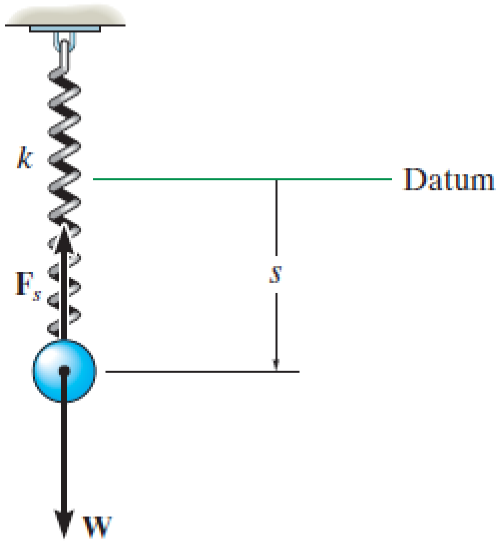
$$V = V_g + V_e$$

$$= -Ws + \tfrac{1}{2}ks^2$$

If the particle moves from $s1$ to a lower position s_2 , it can be seen that the work of **W** and $\mathbf{F}_s$ is

$$U_{1-2} = V_1 - V_2 = \left(-Ws_1 + \tfrac{1}{2}ks_1^2\right) - \left(-Ws_2 + \tfrac{1}{2}ks_2^2\right)$$

$$= W(s_2 - s_1) - \left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$



2-13 Conservation of Energy

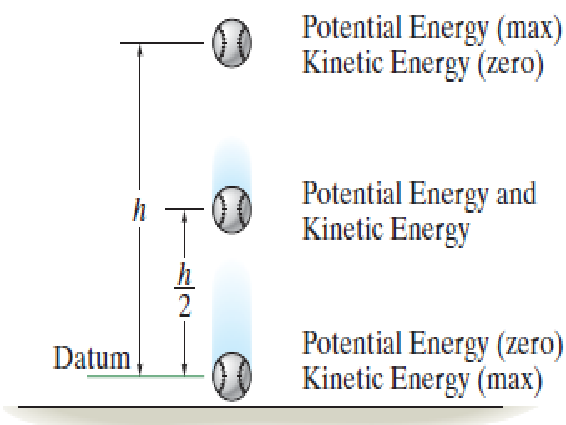
When a particle is acted upon by a system of *both* conservative and nonconservative forces, the portion of the work done by the *conservative forces* can be written in terms of the difference in their potential energies , i.e., $(\sum U_{1\text{-}2})_{cons.} = V_1 - V_2$. As a result, the principle of work and energy can be written as

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{noncons.}} = T_2 + V_2$$

Here $(\sum U_{1\text{-}2})_{noncons.}$ represents the work of the nonconservative forces acting on the particle. If *only conservative forces* do work then we have

$$T_1 + V_1 = T_2 + V_2$$

This equation is referred to as the *conservation of mechanical energy* or simply the *conservation of energy*. It states that during the motion the sum of the particle's kinetic and potential energies remains *constant*. For this to occur, kinetic energy must be transformed into potential energy, and vice versa. For example, if a ball of weight **W** is dropped from a height *h* above the ground (datum), the potential energy of the ball is maximum before it is dropped, at which time its kinetic energy is zero. The total mechanical energy of the ball in its initial position is thus



$$E = T_1 + V_1 = 0 + Wh = Wh$$

When the ball has fallen a distance *h*>2, its speed can be determined by using

$$v^2 = v_0^2 + 2a_c(y - y_0)$$

which yields

$$v = \sqrt{2g(h/2)} = \sqrt{gh}.$$

The energy of the ball at the mid-height position is therefore

$$E = T_2 + V_2 = \frac{1}{2}\frac{W}{g}(\sqrt{gh})^2 + W\left(\frac{h}{2}\right) = Wh$$

Just before the ball strikes the ground, its potential energy is zero and its speed is

$$v = \sqrt{2gh}$$

Here, again, the total energy of the ball is

$$E = T_3 + V_3 = \frac{1}{2}\frac{W}{g}(\sqrt{2gh})^2 + 0 = Wh$$

Note that when the ball comes in contact with the ground, it deforms somewhat, and provided the ground is hard enough, the ball will rebound off the surface, reaching a new height *h*', which will be *less* than the height *h* from which it was first released. Neglecting air friction, the difference in height accounts for an energy loss,

$$El = W(h - h')$$

Which occurs during the collision. Portions of this loss produce noise, localized deformation of the ball and ground, and heat.

If a system of particles is *subjected only to conservative forces*, then an equation can be written for the particles. Applying the ideas of the preceding discussion, ($\sum T_1 + \sum U_{1\text{-}2} = \sum T_2$) becomes

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

Here, the sum of the system's initial kinetic and potential energies is equal to the sum of the system's final kinetic and potential energies. In other words, $\sum T + \sum V$ = const. The conservation of energy equation can be used to solve problems involving *velocity*, *displacement*, and *conservative force systems*. It is generally *easier to apply* than the principle of work and energy because this equation requires specifying the particle's kinetic and potential energies at only *two points* along the path, rather than determining the work when the particle moves through a *displacement*. For application it is suggested that the following procedure be used.

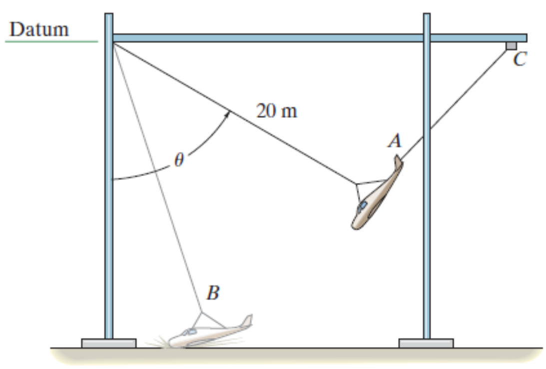
Potential Energy.

- Draw two diagrams showing the particle located at its initial and final points along the path.
- If the particle is subjected to a vertical displacement, establish the fixed horizontal datum from which to measure the particle's gravitational potential energy *Vg* .
- Data pertaining to the elevation *y* of the particle from the datum and the stretch or compression *s* of any connecting springs can be determined from the geometry associated with the two diagrams.
- Recall *Vg* = *Wy*, where *y* is positive upward from the datum and negative downward from the datum; also for a spring, *Ve* = 12 *ks*2, which is *always positive*.

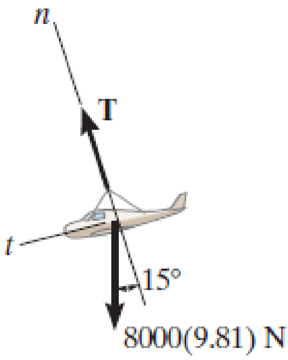
Conservation of Energy.

- Apply the equation *T*1 + *V*1 = *T*2 + *V*2 .
- When determining the kinetic energy, *T* = 12 *mv*2, remember that the particle's speed v must be measured from an inertial reference frame.

Example 12: The gantry structure in the photo is used to test the response of an airplane during a crash. As shown in Fig. the plane, having a mass of 8 Mg, is hoisted back until $\theta = 60^o$, and then the pull-back cable *AC* is released when the plane is at rest. Determine the speed of the plane just before it crashes into the ground, $\theta = 15^o$. Also, what is the maximum tension developed in the supporting cable during the motion? Neglect the size of the airplane and the effect of lift caused by the wings during the motion.



SOLUTION:



$$T_A + V_A = T_B + V_B$$

$$0 - 8000 \text{ kg } (9.81 \text{ m/s}^2)(20 \cos 60° \text{ m}) =$$

$$\tfrac{1}{2}(8000 \text{ kg})v_B^2 - 8000 \text{ kg } (9.81 \text{ m/s}^2)(20 \cos 15° \text{ m})$$

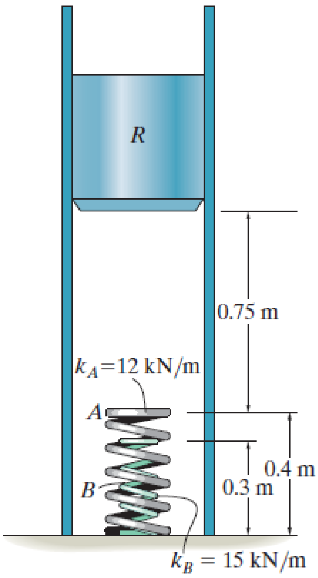
$$v_B = 13.52 \text{ m/s} = 13.5 \text{ m/s} \qquad \textit{Ans.}$$

$+\nwarrow \quad \Sigma F_n = ma_n;$

$$T - (8000(9.81) \text{ N}) \cos 15° = (8000 \text{ kg})\frac{(13.52 \text{ m/s})^2}{20 \text{ m}}$$

$$T = 149 \text{ kN} \qquad \textit{Ans.}$$

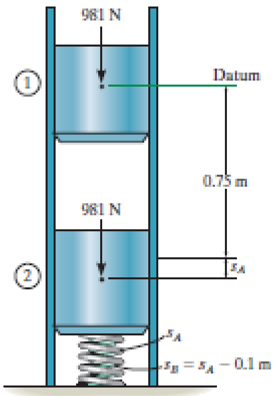
Example 13: The ram *R* shown in Fig. has a mass of 100 kg and is released from rest 0.75 m from the top of a spring, *A*, that has a stiffness k_A = 12 kN/m. If a second spring *B*, having a stiffness k_B = 15 kN/m, is "nested" in *A*, determine the maximum displacement of *A* needed to stop the downward motion of the ram. The unstretched length of each spring is indicated in the figure. Neglect the mass of the springs.



SOLUTION:

Potential Energy. We will *assume* that the ram compresses *both* springs at the instant it comes to rest. The datum is located through the center of gravity of the ram at its initial position, Fig. When the kinetic energy is reduced to zero (v_2 = 0), *A* is compressed a distance s_A and s_B compresses $s_B = s_A$ - 0.1 m.

Conservation of Energy.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \left\{\tfrac{1}{2}k_A s_A^2 + \tfrac{1}{2}k_B(s_A - 0.1)^2 - Wh\right\}$$

$$0 + 0 = 0 + \left\{\tfrac{1}{2}(12\,000 \text{ N/m})s_A^2 + \tfrac{1}{2}(15\,000 \text{ N/m})(s_A - 0.1 \text{ m})^2 - 981 \text{ N } (0.75 \text{ m} + s_A)\right\}$$

Rearranging the terms,

$$13\,500 s_A^2 - 2481 s_A - 660.75 = 0$$

Using the quadratic formula and solving for the positive root, we have

$$s_A = 0.331 \text{ m} \qquad \textit{Ans.}$$

Since $s_B = 0.331 \text{ m} - 0.1 \text{ m} = 0.231 \text{ m}$, which is positive, the assumption that *both* springs are compressed by the ram is correct.

NOTE: The second root, $s_A = -0.148$ m, does not represent the physical situation. Since positive s is measured downward, the negative sign indicates that spring A would have to be "extended" by an amount of 0.148 m to stop the ram.

CHAPTER TWO
KINETICS OF PARTICLES: IMPULSE AND MOMENTUM

2-14 Principle of Linear Impulse and Momentum

Using kinematics, the equation of motion for a particle of mass m can be written as

$$\Sigma \mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

Where **a** and **v** are both measured from an inertial frame of reference. Rearranging the terms and integrating between the limits $\mathbf{v} = \mathbf{v}_1$ at $t = t_1$ and $\mathbf{v} = \mathbf{v}_2$ at $t = t_2$, we have

$$\Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m \int_{\mathbf{v}_1}^{\mathbf{v}_2} d\mathbf{v} \qquad \Sigma \int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v}_2 - m\mathbf{v}_1$$ ………(1)

This equation is referred to as the *principle of linear impulse and momentum*.
Each of the two vectors of the form $\mathbf{L} = m\mathbf{v}$ in Eq. 1 is referred to as the particle's linear momentum. The integral $\mathbf{I} = \int \mathbf{F}\, dt$ in Eq. 1is referred to as the *linear impulse*.
For problem solving, Eq. 1will be rewritten in the form

$$m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$$

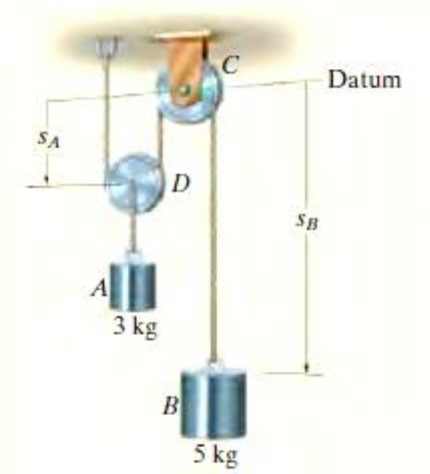
equation states that the initial linear momentum of the system plus the impulses of all the *external forces* acting on the system from t_1 to t_2 is equal to the system's final linear momentum.

$$\Sigma m_i(\mathbf{v}_i)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}_i\, dt = \Sigma m_i(\mathbf{v}_i)_2$$ ……..(2)

When the sum of the *external impulses* acting on a system of particles is *zero*, Eq. 2 reduces to a simplified form, namely,

$$\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2$$

This equation is referred to as the *conservation of linear momentum*.



Example 14: Blocks *A* and *B* shown in Fig. have a mass of 3 kg and 5 kg, respectively. If the system is released from rest, determine the velocity of block *B* in 6 s. Neglect the mass of the pulleys and cord.

SOLUTION:

Since the weight of each block is constant, the cord tensions will also be constant. Furthermore, since the mass of pulley *D* is neglected, the cord tension $T_A = 2T_B$. Note that the blocks are both assumed to be moving downward in the positive coordinate directions, s_A and s_B .

Principle of Impulse and Momentum.

Block *A*:

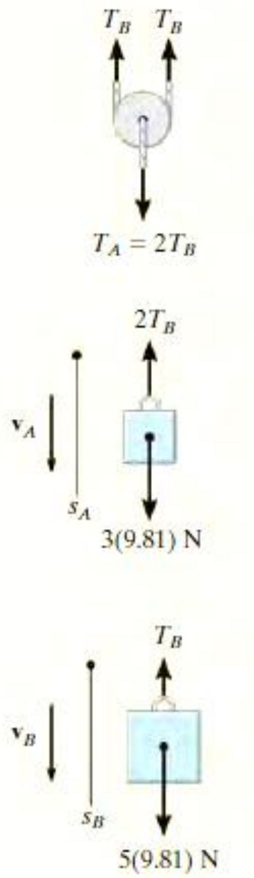
$(+\downarrow)$
$$m(v_A)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_A)_2$$

$$0 - 2T_B(6\text{ s}) + 3(9.81)\text{ N}(6\text{ s}) = (3\text{ kg})(v_A)_2 \qquad (1)$$

Block *B*:

$(+\downarrow)$
$$m(v_B)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_B)_2$$

$$0 + 5(9.81)\text{ N}(6\text{ s}) - T_B(6\text{ s}) = (5\text{ kg})(v_B)_2 \qquad (2)$$



$$2s_A + s_B = l$$

Taking the time derivative yields

$$2v_A = -v_B \qquad (3)$$

As indicated by the negative sign, when *B* moves downward *A* moves upward. Substituting this result into Eq. 1 and solving Eqs. 1 and 2 yields

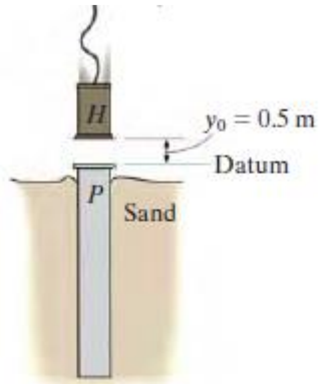
$$(v_B)_2 = 35.8\text{ m/s}\downarrow \qquad \textit{Ans.}$$

$$T_B = 19.2\text{ N}$$

Example 15: An 800-kg rigid pile is driven into the ground using a 300-kg hammer. The hammer falls from rest at a height y_0 = 0.5 m and strikes the top of the pile. Determine the impulse which the pile exerts on the hammer if the pile is surrounded entirely by loose sand so that after striking, the hammer does *not* rebound off the pile.

SOLUTION

Conservation of Energy. The velocity at which the hammer strikes the pile can be determined using the conservation of energy
equation applied to the hammer. With the datum at the top of the pile, Fig. 15–10*a*, we have



$$T_0 + V_0 = T_1 + V_1$$

$$\frac{1}{2}m_H(v_H)_0^2 + W_H y_0 = \frac{1}{2}m_H(v_H)_1^2 + W_H y_1$$

$$0 + 300(9.81)\text{ N}(0.5\text{ m}) = \frac{1}{2}(300\text{ kg})(v_H)_1^2 + 0$$

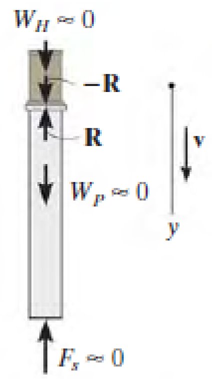
$$(v_H)_1 = 3.132\text{ m/s}$$

Conservation of Momentum. Since the hammer does not rebound off the pile just after collision, then $(v_H)_2 = (v_P)_2 = v_2$.

$$(+\downarrow) \qquad m_H(v_H)_1 + m_P(v_P)_1 = m_H v_2 + m_P v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) + 0 = (300\text{ kg})v_2 + (800\text{ kg})v_2$$

$$v_2 = 0.8542\text{ m/s}$$

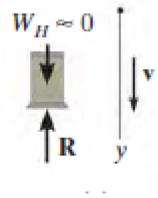


Principle of Impulse and Momentum. The impulse which the pile imparts to the hammer can now be determined since v_2 is known. From the free-body diagram for the hammer, Fig. *c*, we have

$$(+\downarrow) \qquad m_H(v_H)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m_H v_2$$

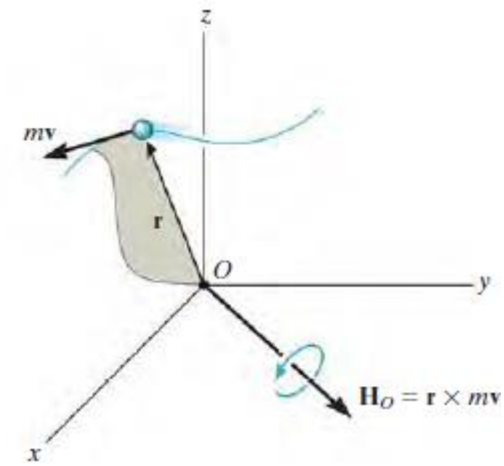
$$(300\text{ kg})(3.132\text{ m/s}) - \int R\,dt = (300\text{ kg})(0.8542\text{ m/s})$$

$$\int R\,dt = 683\text{ N}\cdot\text{s} \qquad \textit{Ans.}$$



NOTE: The equal but opposite impulse acts on the pile. Try finding this impulse by applying the principle of impulse and momentum to the pile.

2-15 Angular Momentum



The *angular momentum* of a particle about point O is defined as the "moment" of the particle's linear momentum about O. Since this concept is analogous to finding the moment of a force about a point, the angular momentum, $\mathbf{H}_O$, is sometimes referred to as the *moment of momentum*.

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} \quad \text{........(1)}$$

$$\Sigma \mathbf{M}_O = \dot{\mathbf{H}}_O \quad \text{........(2)}$$

The equation 2 states that *the resultant moment about point O of all the forces acting on the particle is equal to the time rate of change of the particle's angular momentum about point O.*

If Eq. 2 is rewritten in the form $\sum \mathbf{M}_O\, dt = d\,\mathbf{H}_O$ and integrated, assuming that at time $t = t_1$, $\mathbf{H}_O = (\mathbf{H}_O)_1$ and at time $t = t_2$, $\mathbf{H}_O = (\mathbf{H}_O)_2$, we have

$$\Sigma \int_{t_1}^{t_2} \mathbf{M}_O\, dt = (\mathbf{H}_O)_2 - (\mathbf{H}_O)_1 \qquad (\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O\, dt = (\mathbf{H}_O)_2 \quad \text{.......(3)}$$

$$\text{angular impulse} = \int_{t_1}^{t_2} \mathbf{M}_O\, dt = \int_{t_1}^{t_2} (\mathbf{r} \times \mathbf{F})\, dt$$

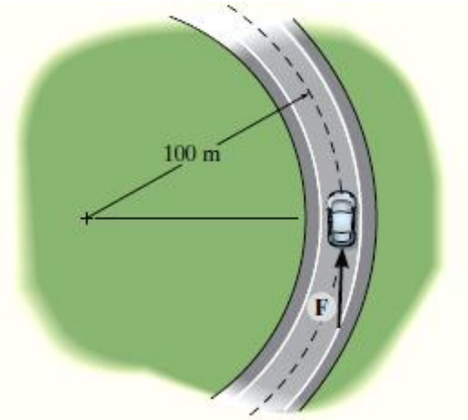
When the angular impulses acting on a particle are all zero during the time t_1 to t_2, Eq. 3 reduces to the following simplified form:

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

This equation is known as the *conservation of angular momentum*. we can also write the conservation of angular momentum for a system of particles as

$$\Sigma(\mathbf{H}_O)_1 = \Sigma(\mathbf{H}_O)_2$$

Example 16: The 1.5-Mg car travels along the circular road as shown in Fig. If the traction force of the wheels on the road is $F = (150t^2)$ N, where t is in seconds, determine the speed of the car when $t = 5$ s. The car initially travels with a speed of 5 m/s. Neglect the size of the car.



SOLUTION:

Principle of Angular Impulse and Momentum.

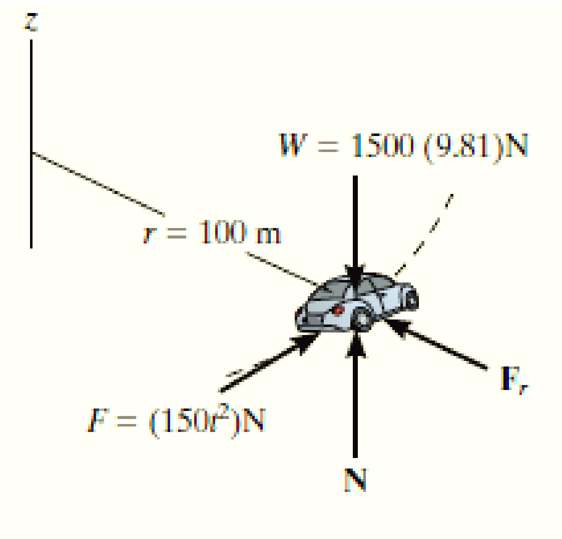
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$r\, m_c (v_c)_1 + \int_{t_1}^{t_2} r\, F\, dt = r\, m_c (v_c)_2$$

$$(100 \text{ m})(1500 \text{ kg})(5 \text{ m/s}) + \int_0^{5\text{ s}} (100 \text{ m})[(150t^2) \text{ N}]\, dt = (100 \text{ m})(1500 \text{ kg})(v_c)_2$$

$$750(10^3) + 5000t^3 \Big|_0^{5\text{ s}} = 150(10^3)(v_c)_2$$

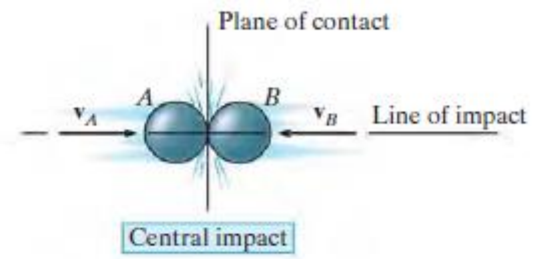
$$(v_c)_2 = 9.17 \text{ m/s} \qquad \textit{Ans.}$$



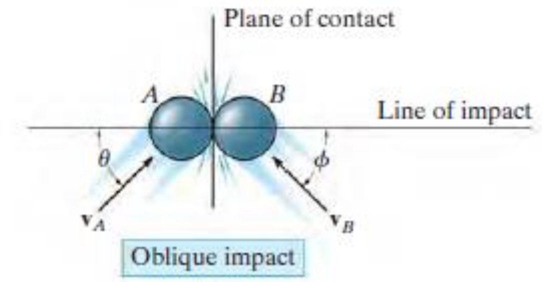
2-15 Impact

Impact occurs when two bodies collide with each other during a very *short* period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.

In general, there are two types of impact. *Central impact* occurs when the direction of motion of he mass centers of the two colliding particles is along a line passing through the mass centers of the particles. This line is called the *line of impact*, which is perpendicular to the plane of contact, Fig. *a*. When the motion of one or both of the particles make an angle with the line of impact, Fig. *b*, the impact is said to be *oblique impact*.



(a)



(b)

2-15-1 Central Impact

In most cases the *final velocities* of two smooth particles are to be determined *just after* they are subjected to direct central impact. Provided the coefficient of restitution, the mass of each particle, and each particle's initial velocity *just before* impact are known, the solution to this problem can be obtained using the following two equations:

- The conservation of momentum applies to the system of particles,

$$\sum mv_1 = \sum mv_2$$

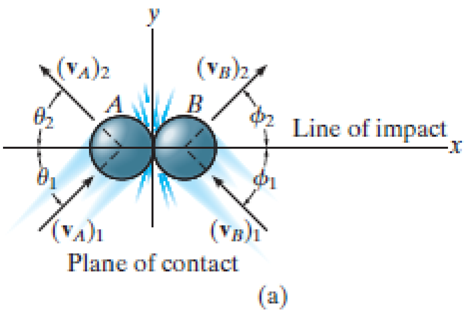
- The coefficient of restitution *e*,

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

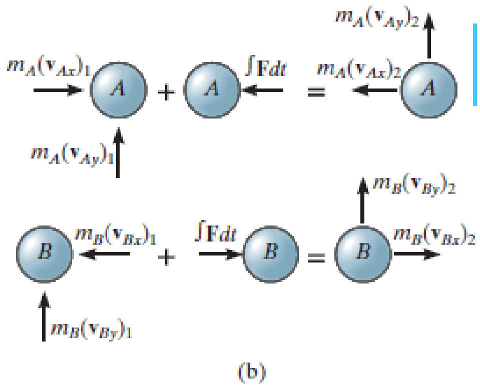
e is equal to the ratio of the relative velocity of the particles' separation *just after impact*, $(v_B)_2$ - $(v_A)_2$, to the relative velocity of the particles' approach *just before impact*, $(v_B)_1$ - $(v_A)_1$. By measuring these relative velocities experimentally, it has been found that *e* varies appreciably with impact velocity as well as with the size and shape of the colliding bodies. For these reasons the coefficient of restitution is reliable only when used with data which closely approximate the conditions which were known to exist when measurements of it were made. In general *e* has a value between zero and one, and one should be aware of the physical meaning of these two limits. If the solution yields a negative magnitude, the velocity acts in the opposite sense.

- If the collision between the two particles is *perfectly elastic*, the deformation impulse ($\int \mathbf{P}\, dt$) is equal and opposite to the restitution impulse ($\int \mathbf{R}\, dt$). Although in reality this can never be achieved, $e = 1$ for an elastic collision.
- The impact is said to be *inelastic or plastic* when $e = 0$. In this case there is no restitution impulse ($\int \mathbf{R}\, dt = \mathbf{0}$), so that after collision both particles couple or stick *together* and move with a common velocity.

2-15-2 Oblique Impact



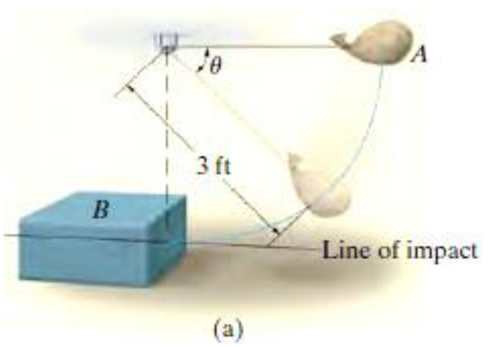
When oblique impact occurs between two smooth particles, the particles move away from each other with velocities having unknown directions as well as unknown magnitudes. Provided the initial velocities are known, then four unknowns are present in the problem. As shown in Fig. *a*, these unknowns may be represented either as $(v_A)_2$, $(v_B)_2$, θ_2, and ϕ_2 , or as the *x* and *y* components of the final velocities.



If the *y* axis is established within the plane of contact and the *x* axis along the line of impact, the impulsive forces of deformation and restitution act *only in the x direction*, Fig. *b*. By resolving the velocity or momentum vectors into components along the *x* and *y* axes, Fig. *b*, it is then possible to write four independent scalar equations in order to determine $(v_{Ax})_2$, $(v_{Ay})_2$, $(v_{Bx})_2$, and $(v_{By})_2$.

- Momentum of the system is conserved *along the line of impact*, *x* axis, so that $\sum mv_1 = \sum mv_2$.
- The coefficient of restitution, $e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$, relates the relative-velocity *components* of the particles *along the line of impact* (*x* axis).
- If these two equations are solved simultaneously, we obtain $(v_{Ax})_2$ and $(v_{Bx})_2$.
- Momentum of particle *A* is conserved along the *y* axis, perpendicular to the line of impact, since no impulse acts on particle *A* in this direction. As a result $m_A(v_{Ay})_1 = m_A(v_{Ay})_2$ or $(v_{Ay})_1 = (v_{Ay})_2$
- Momentum of particle *B* is conserved along the *y* axis, perpendicular to the line of impact, since no impulse acts on particle *B* in this direction. Consequently $(v_{By})_1 = (v_{By})_2$.

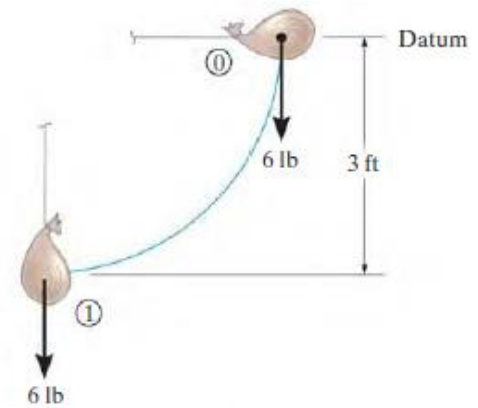
Example 17: The bag *A*, having a weight of 6 lb, is released from rest at the position $\theta = 0^o$, as shown in Fig. *a*. After falling to $\theta = 90^o$, it strikes an 18-lb box *B*. If the oefficient of restitution between the bag and box is $e = 0.5$, determine the velocities of the bag and box just after impact. What is the loss of energy during collision?



(a)

SOLUTION

This problem involves central impact. Why? Before analyzing the mechanics of the impact, however, it is first necessary to obtain the velocity of the bag *just before* it strikes the box.



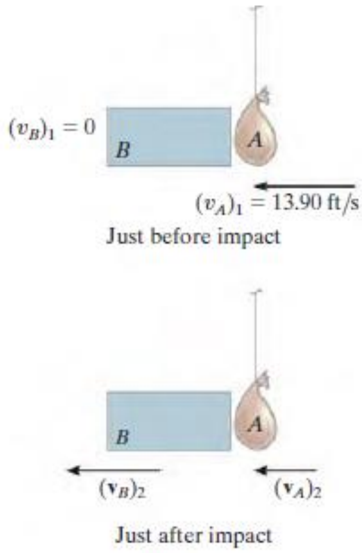
(b)

Conservation of Energy. With the datum at $\theta = 0^o$, Fig. b, we have

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 0 = \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_1^2 - 6\text{ lb}(3\text{ ft});(v_A)_1 = 13.90\text{ ft/s}$$

Conservation of Momentum. After impact we will assume *A* and *B* travel to the left. Applying the conservation of momentum to the system, Fig. *c*, we have



(c)

$$(\overset{+}{\leftarrow}) \qquad m_B(v_B)_1 + m_A(v_A)_1 = m_B(v_B)_2 + m_A(v_A)_2$$

$$0 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.90\text{ ft/s}) = \left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_B)_2 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_2$$

$$(v_A)_2 = 13.90 - 3(v_B)_2 \qquad (1)$$

Coefficient of Restitution. Realizing that for separation to occur after collision $(v_B)_2 > (v_A)_2$, Fig. *c*, we have

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.5 = \frac{(v_B)_2 - (v_A)_2}{13.90\text{ ft/s} - 0}$$

$$(v_A)_2 = (v_B)_2 - 6.950 \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously yields

$(v_A)_2 = -1.74\text{ ft/s} = 1.74\text{ ft/s} \rightarrow$ and $(v_B)_2 = 5.21\text{ ft/s} \leftarrow$ *Ans.*

Loss of Energy. Applying the principle of work and energy to the bag and box just before and just after collision, we have

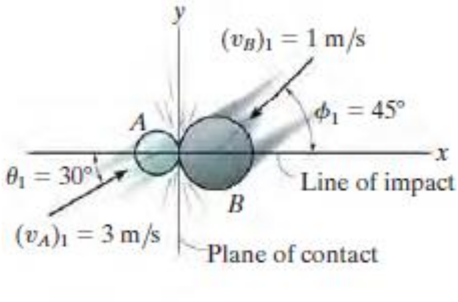
$$\Sigma U_{1-2} = T_2 - T_1;$$

$$\Sigma U_{1-2} = \left[\frac{1}{2}\left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(5.21\text{ ft/s})^2 + \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(1.74\text{ ft/s})^2\right] - \left[\frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.9\text{ ft/s})^2\right]$$

$$\Sigma U_{1-2} = -10.1\text{ ft}\cdot\text{lb} \qquad \textit{Ans.}$$

NOTE: The energy loss occurs due to inelastic deformation during the collision.

Example 18: Two smooth disks *A* and *B*, having a mass of 1 kg and 2 kg, respectively, collide with the velocities shown in Fig. *a*. If the coefficient of restitution for the disks is *e* = 0.75, determine the *x* and *y* components of the final velocity of each disk just after collision.



(a)

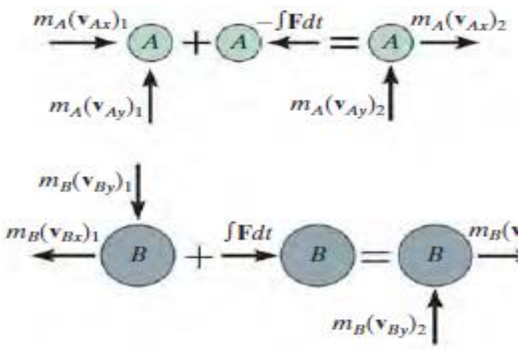
SOLUTION

This problem involves *oblique impact*. Why? In order to solve it, we have established the *x* and *y* axes along the line of impact and the plane of contact, respectively, Fig. *a*. Resolving each of the initial velocities into *x* and *y* components, we have

$$(v_{Ax})_1 = 3\cos 30^\circ = 2.598\text{ m/s} \qquad (v_{Ay})_1 = 3\sin 30^\circ = 1.50\text{ m/s}$$

$$(v_{Bx})_1 = -1\cos 45^\circ = -0.7071\text{ m/s} \qquad (v_{By})_1 = -1\sin 45^\circ = -0.7071\text{ m/s}$$

The four unknown velocity components after collision are *assumed to act in the positive directions*, Fig. *b*. Since the impact occurs in the *x* direction (line of impact), the conservation of momentum for *both* disks can be applied in this direction.



(b)

Conservation of "x" Momentum. In reference to the momentum diagrams, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$1\text{ kg}(2.598\text{ m/s}) + 2\text{ kg}(-0.707\text{ m/s}) = 1\text{ kg}(v_{Ax})_2 + 2\text{ kg}(v_{Bx})_2$$

$$(v_{Ax})_2 + 2(v_{Bx})_2 = 1.184 \qquad (1)$$

Coefficient of Restitution (x).

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \; 0.75 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{2.598 \text{ m/s} - (-0.7071 \text{ m/s})}$$

$$(v_{Bx})_2 - (v_{Ax})_2 = 2.482 \qquad (2)$$

Solving Eqs. 1 and 2 for $(v_{Ax})_2$ and $(v_{Bx})_2$ yields

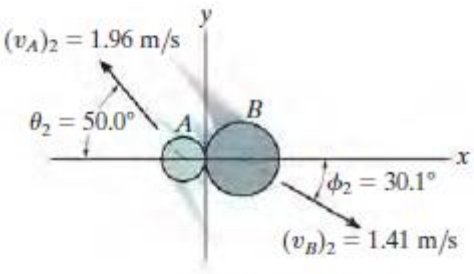
$(v_{Ax})_2 = -1.26 \text{ m/s} = 1.26 \text{ m/s} \leftarrow \quad (v_{Bx})_2 = 1.22 \text{ m/s} \rightarrow$ *Ans.*

Conservation of "*y*" Momentum. The momentum of *each disk* is *conserved* in the *y* direction (plane of contact), since the disks are smooth and therefore *no* external impulse acts in this direction. From Fig. *b*,

$(+\uparrow) \; m_A(v_{Ay})_1 = m_A(v_{Ay})_2; \quad (v_{Ay})_2 = 1.50 \text{ m/s} \uparrow$ *Ans.*

$(+\uparrow) \; m_B(v_{By})_1 = m_B(v_{By})_2; \quad (v_{By})_2 = -0.707 \text{ m/s} = 0.707 \text{ m/s} \downarrow$ *Ans.*

NOTE: Show that when the velocity components are summed vectorially, one obtains the results shown in Fig. *c*.



(c)

Chapter Three
Kinematics of Rigid Bodies

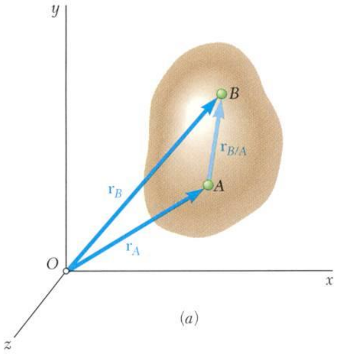
3-1 Introduction to Dynamics

Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.

Classification of rigid body motions:

- translation:
  - rectilinear translation
  - curvilinear translation
- rotation about a fixed axis
- general plane motion
- motion about a fixed point
- general motion

3-2 Translation



(a)

- Consider rigid body in translation:
  - direction of any straight line inside the body is constant,
  - all particles forming the body move in parallel lines.

- For any two particles in the body:

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

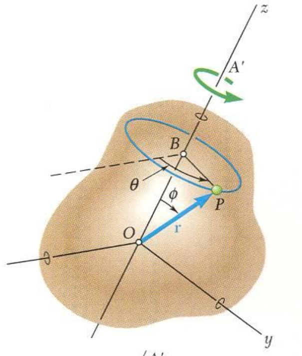
- Differentiating with respect to time: $\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A \ldots\ldots \vec{v}_B = \vec{v}_A$

  Then all particles have the same velocity.
- Differentiating with respect to time again:

  $\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A \ldots\ldots\ldots\ldots \vec{a}_B = \vec{a}_A$

And all particles have the same acceleration.

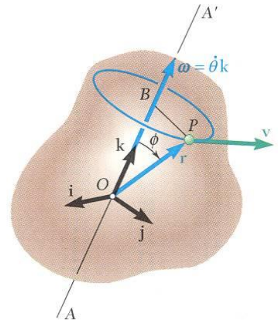
3-3 Rotation About a Fixed Axis: Velocity and Acceleration



Consider rotation of rigid body about a fixed axis *AA'*. Velocity vector of the particle *P* is tangent to the path with magnitude:

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$



The same result is obtained from:

$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity$$

Differentiating to determine the acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega}\times\vec{r}) = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt} = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}$$

$$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration = \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

Acceleration of *P* is combination of two vectors: $\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{\omega}\times\vec{r}$

$\vec{\alpha}\times\vec{r}$ = tangential acceleration component

$\vec{\omega}\times\vec{\omega}\times\vec{r}$ = radial acceleration component

Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

$$\omega = \frac{d\theta}{dt} \quad \text{or} \quad dt = \frac{d\theta}{\omega} \ldots\ldots \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega\frac{d\omega}{d\theta}$$

- *Uniform Rotation, a* = 0: $\theta = \theta_0 + \omega t$
- *Uniformly Accelerated Rotation, a = constant:*

  $\omega = \omega_0 + \alpha t$

  $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

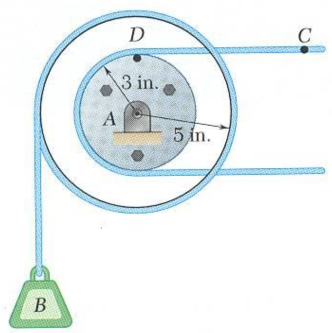
  $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

3-4 Comparison Between Rotational and Linear Equations

The kinematics equations for rotational and translation motion:

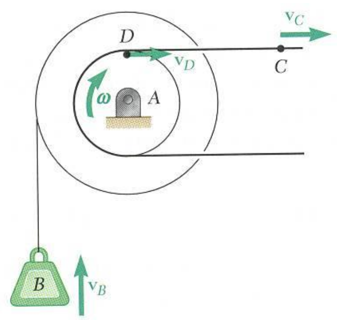
Rigid Body Under Constant Angular Acceleration	Particle Under Constant Acceleration
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Example1: Cable C has a constant acceleration of 9 in/s2 and an initial velocity of 12 in/s, both directed to the right. Determine (a) the number of revolutions of the pulley in 2 s, (b) the velocity and change in position of the load B after 2 s, and (c) the acceleration of the point D on the rim of the inner pulley at t = 0.



SOLUTION:

The tangential velocity and acceleration of *D* are equal to the velocity and acceleration of *C*.

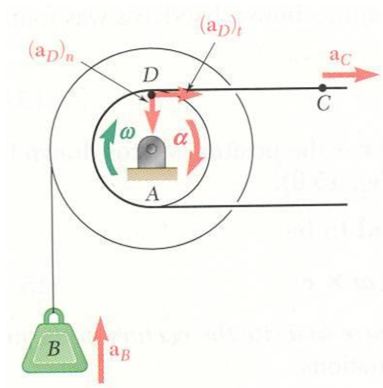


$$(\vec{v}_D)_0 = (\vec{v}_C)_0 = 12\,\text{in./s} \rightarrow \qquad (\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(v_D)_0 = r\omega_0 \qquad (a_D)_t = r\alpha$$

$$\omega_0 = \frac{(v_D)_0}{r} = \frac{12}{3} = 4\,\text{rad/s} \qquad \alpha = \frac{(a_D)_t}{r} = \frac{9}{3} = 3\,\text{rad/s}^2$$

Apply the relations for uniformly accelerated rotation to determine velocity and angular position of pulley after 2 s.



$$\omega = \omega_0 + \alpha t = 4\,\text{rad/s} + (3\,\text{rad/s}^2)(2\,\text{s}) = 10\,\text{rad/s}$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4\,\text{rad/s})(2\,\text{s}) + \tfrac{1}{2}(3\,\text{rad/s}^2)(2\,\text{s})^2 = 14\,\text{rad}$$

$$N = (14\,\text{rad})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) = \text{number of revs} \qquad N = 2.23\,\text{rev}$$

$$v_B = r\omega = (5\,\text{in.})(10\,\text{rad/s}) \ldots\ldots\ldots \vec{v}_B = 50\,\text{in./s} \uparrow$$

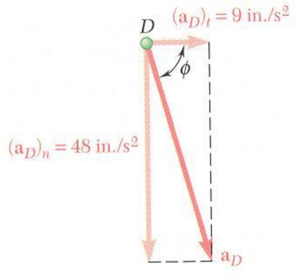
$$\Delta y_B = r\theta = (5\,\text{in.})(14\,\text{rad}) \ldots\ldots\ldots\ldots \Delta y_B = 70\,\text{in.}$$

Evaluate the initial tangential and normal acceleration components of D.

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(a_D)_n = r_D\omega_0^2 = (3\,\text{in.})(4\,\text{rad/s})^2 = 48\,\text{in/s}^{\;2} \downarrow$$

Magnitude and direction of the total acceleration:

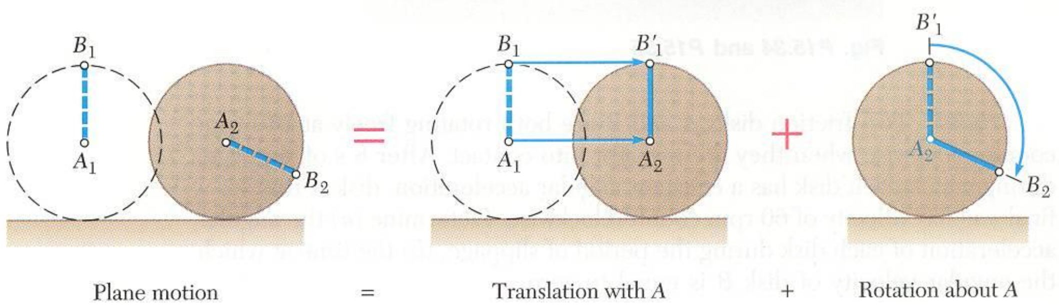


$$a_D = \sqrt{(a_D)_t^2 + (a_D)_n^2} = \sqrt{9^2 + 48^2} = 48.8\,\text{in./s}^2$$

$$\tan\phi = \frac{(a_D)_n}{(a_D)_t} = \frac{48}{9} \quad \ldots\ldots\ldots\; \phi = 79.4°$$

3-5 General Plane Motion

General plane motion is neither a translation nor a rotation. General plane motion can be considered as the *sum* of a translation and rotation.

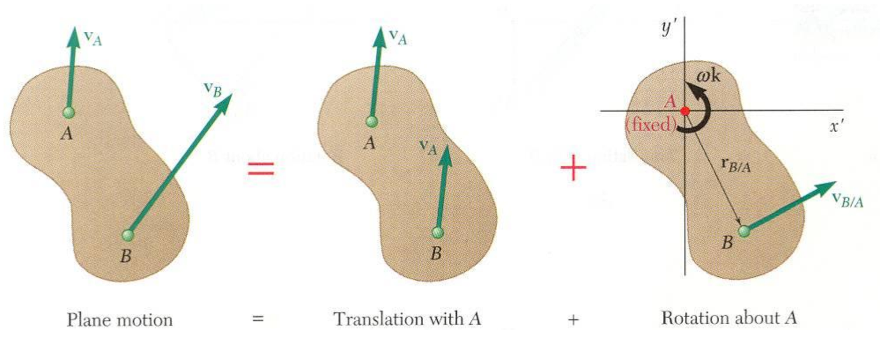


Displacement of particles A and B to A_2 and B_2 can be divided into two parts:

- translation to A_2 and
- rotation of about A_2 to B_2

3-6 Absolute and Relative Velocity in Plane Motion

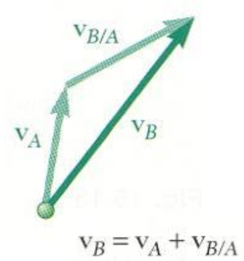
Any plane motion can be replaced by a translation of an arbitrary reference point *A* and a simultaneous rotation about *A*.



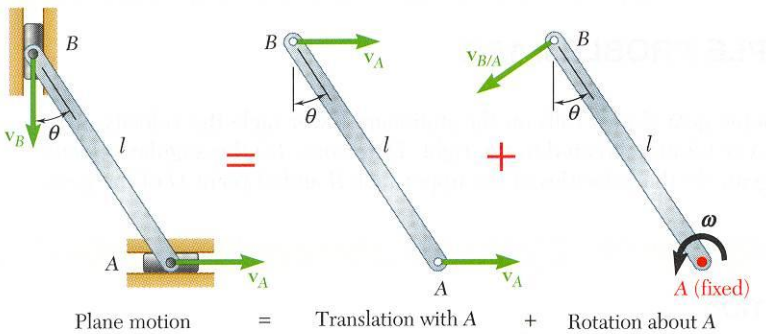
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

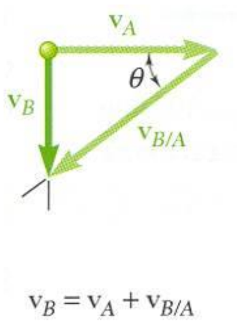
$$\vec{v}_{B/A} = \omega\vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega$$

$$\vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$



Assuming that the velocity v_A of end A is known, wish to determine the velocity v_B of end B and the angular velocity ω in terms of v_A, l, and θ.



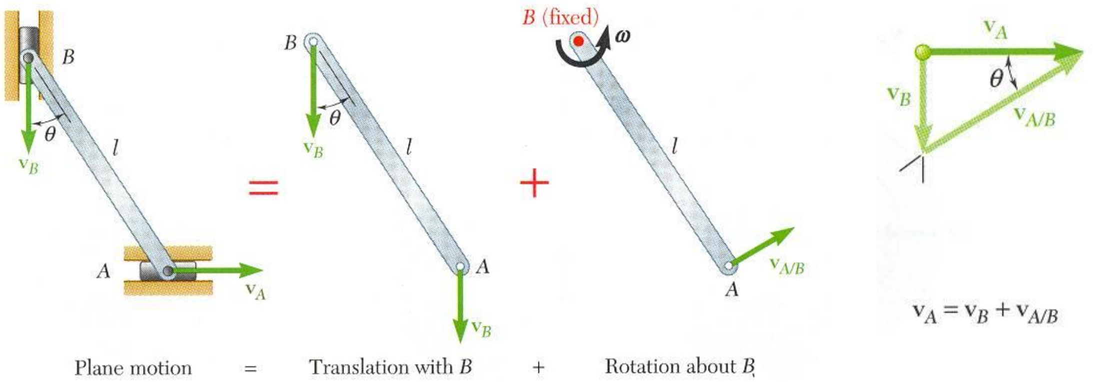


The direction of v_B and $v_{B/A}$ are known. Complete the velocity diagram.

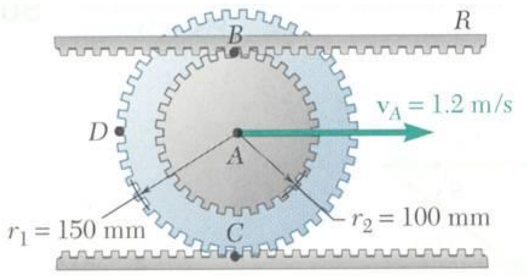
$$\frac{v_B}{v_A} = \tan\theta \qquad \frac{v_A}{v_{B/A}} = \frac{v_A}{l\omega} = \cos\theta$$

$$v_B = v_A \tan\theta \qquad \omega = \frac{v_A}{l\cos\theta}$$

Selecting point *B* as the reference point and solving for the velocity v_A of end *A* and the angular velocity ω leads to an equivalent velocity triangle.



$v_{A/B}$ has the same magnitude but opposite sense of $v_{B/A}$. The sense of the relative velocity is dependent on the choice of reference point. Angular velocity ω of the rod in its rotation about *B* is the same as its rotation about *A*. Angular velocity is not dependent on the choice of reference point.



Example 2: The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s. Determine *(a)* the angular velocity of the gear, and *(b)* the velocities of the upper rack *R* and point *D* of the gear.

SOLUTION:

The displacement of the gear center in one revolution is equal to the outer circumference. For $x_A > 0$ (moves to right), $\theta < 0$ (rotates clockwise).

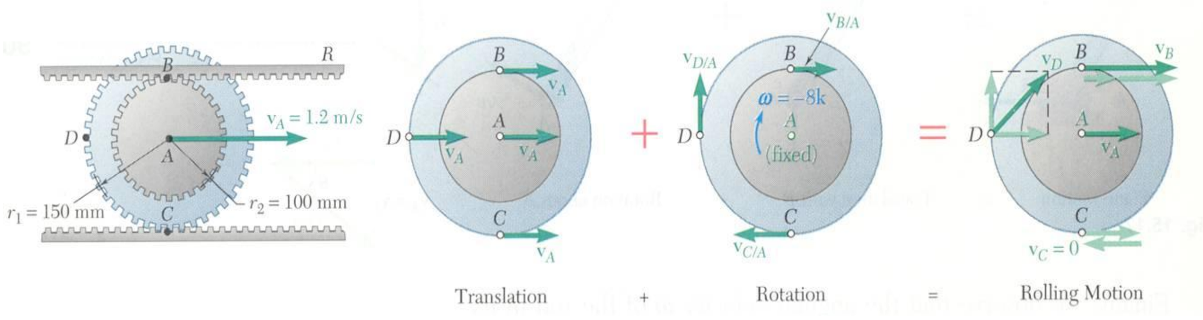
$$\frac{x_A}{2\pi r} = -\frac{\theta}{2\pi} \qquad x_A = -r_1\theta$$

Differentiate to relate the translational and angular velocities.

$$v_A = -r_1\omega$$

$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\text{m/s}}{0.150\,\text{m}} \qquad \vec{\omega} = \omega\vec{k} = -(8\,\text{rad/s})\vec{k}$$

For any point *P* on the gear:



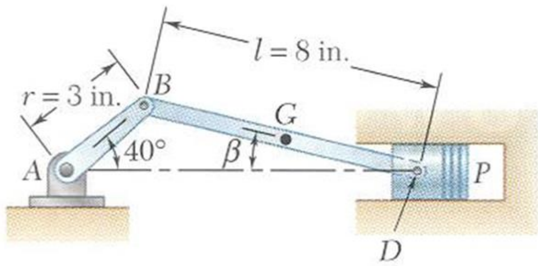
Velocity of the upper rack is equal to velocity of point B:

$$\vec{v}_R = \vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (0.10\,\mathrm{m})\vec{j}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (0.8\,\mathrm{m/s})\vec{i} = (2\,\mathrm{m/s})\vec{i}$$

Velocity of the point *D*:

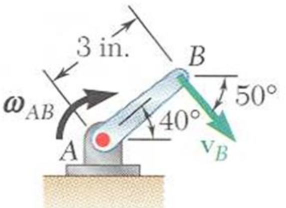
$$\vec{v}_D = \vec{v}_A + \omega\vec{k} \times \vec{r}_{D/A}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (-0.150\,\mathrm{m})\vec{i}$$
$$= (1.2\,\mathrm{m/s})\vec{i} + (1.2\,\mathrm{m/s})\vec{j} v_D = 1.697\,\mathrm{m/s}$$

Example 3: The crank *AB* has a constant clockwise angular velocity of 2000 rpm. For the crank position indicated, determine *(a)* the angular velocity of the connecting rod *BD*, and *(b)* the velocity of the piston *P*.



SOLUTION:

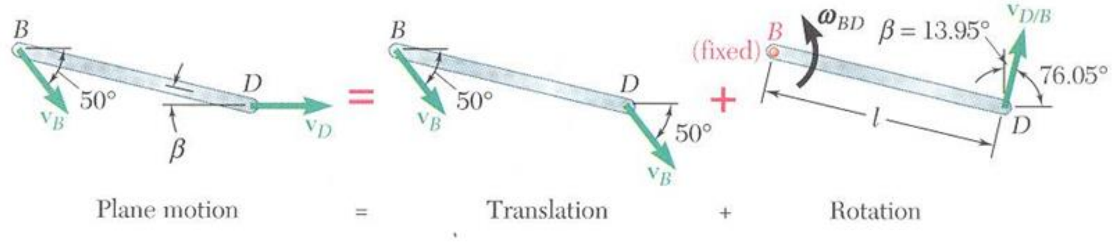
The velocity $\vec{v}_B$ is obtained from the crank rotation data:



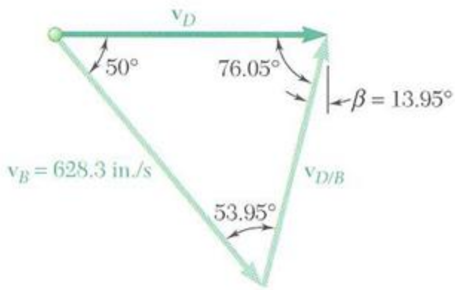
$$\omega_{AB} = \left(2000\,\frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{\mathrm{min}}{60\,\mathrm{s}}\right)\left(\frac{2\pi\,\mathrm{rad}}{\mathrm{rev}}\right) = 209.4\,\mathrm{rad/s}$$
$$v_B = (AB)\omega_{AB} = (3\,\mathrm{in.})(209.4\,\mathrm{rad/s})$$

The direction of the absolute velocity $\vec{v}_D$ is horizontal. The direction of the relative velocity $\vec{v}_{D/B}$ is perpendicular to *BD*. Compute the angle between the horizontal and the connecting rod from the law of sines.

$$\frac{\sin 40°}{8\,\mathrm{in.}} = \frac{\sin\beta}{3\,\mathrm{in.}} \qquad \beta = 13.95°$$



Determine the velocity magnitudes from the vector triangle:



$$\frac{v_D}{\sin 53.95°} = \frac{v_{D/B}}{\sin 50°} = \frac{628.3\,\text{in./s}}{\sin 76.05°}$$

$$v_D = 523.4\,\text{in./s} = 43.6\,\text{ft/s} \qquad \ldots\ldots\ldots v_P = v_D = 43.6\,\text{ft/s}$$
$$v_{D/B} = 495.9\,\text{in./s}$$

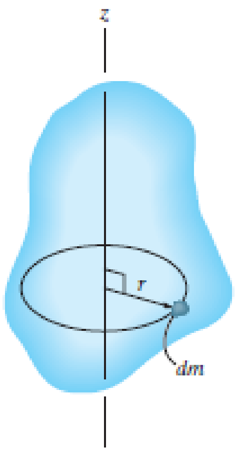
$$v_{D/B} = l\omega_{BD} \ldots\ldots\ldots \omega_{BD} = \frac{v_{D/B}}{l} = \frac{495.9\,\text{in./s}}{8\,\text{in.}} = 62.0\,\text{rad/s}$$
$$\vec{\omega}_{BD} = (62.0\,\text{rad/s})\vec{k}$$

CHAPTER FOUR
PLANE MOTION OF RIGID BODIES:
FORCES AND ACCELERATIONS

4-1 Introduction

In this chapter we will be concerned with the *kinetics* of rigid bodies, i.e., relations between the forces acting on a rigid body, the shape and mass of the body, and the motion produced.

4-2 The Mass Moment of Inertia



Since a body has a definite size and shape, an applied nonconcurrent force system can cause the body to both translate and rotate. The translational aspects of the motion were studied in Chapter 13 and are governed by the equation **F** = *m***a**. It will be shown in the next section that the rotational aspects, caused by a moment **M**, are governed by an equation of the form **M** = *I*A. The symbol *I* in this equation is termed the mass moment of inertia. By comparison, the *moment of inertia* is a measure of the resistance of a body to *angular acceleration* (**M** = *I*A) in the same way that *mass* is a measure of the body's resistance to *acceleration* (**F** = *m***a**). We define the *moment of inertia* as the integral of the "second moment" about an axis of all the elements of mass *dm* which compose the body. For example, the body's moment of inertia about the *z* axis in Fig. is

$$I = \int_m r^2\, dm$$

If the body consists of material having a variable density, r = r (*x*,*y*,*z*), the elemental mass *dm* of the body can be expressed in terms of its density and volume as *dm* = r *dV*. Substituting *dm* into Eq. above, the body's moment of inertia is then computed using *volume elements* for integration; i.e.,

$$I = \int_V r^2 \rho\, dV$$

In the special case of r being a *constant*, this term may be factored out of the integral, and the integration is then purely a function of geometry,

$$I = \rho \int_V r^2\, dV$$

If the moment of inertia of the body about an axis passing through the body's mass center is known, then the moment of inertia about any other *parallel axis* can be determined by using the *parallel-axis theorem*. the moment of inertia about the z axis can be written as

$$I = I_G + md^2$$

where

I_G = moment of inertia about the z-axis passing through the mass center G
m = mass of the body
d = perpendicular distance between the parallel z and z- axes

Occasionally, the moment of inertia of a body about a specified axis is reported in handbooks using the *radius of gyration*, k. This is a geometrical property which has units of length. When it and the body's mass m are known, the body's moment of inertia is determined from the equation

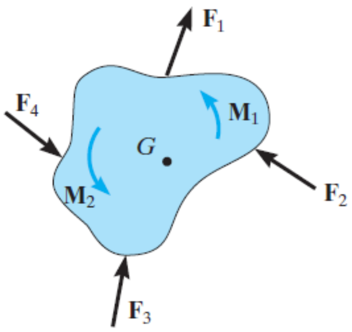
$$I = mk^2 \quad \text{or} \quad k = \sqrt{\frac{I}{m}}$$

4-3 Planar Kinetic Equations of Motion

In the following analysis we will limit our study of planar kinetics to rigid bodies which, along with their loadings, are considered to be symmetrical with respect to a fixed reference plane.

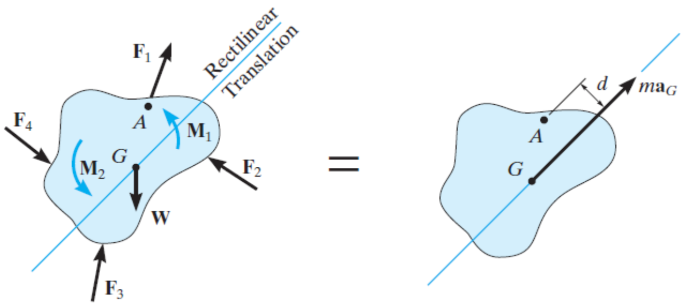
4-3-1 Equations of Motion: Translation

When the rigid body in Fig. *a* undergoes a *translation*, all the particles of the body have the *same acceleration*. Furthermore, A = **0**, in which case the rotational equation of motion applied at point G reduces to a simplified form, namely, $\sum M_G = 0$. Application of this and the force equations of motion will now be discussed for each of the two types of translation. When a body is subjected to *rectilinear translation*, all the particles of the body (slab) travel along parallel straightline paths. The free-body and kinetic diagrams are shown in Fig. *b*. Since I_G A = **0**, only $m\mathbf{a}_G$ is shown on the kinetic diagram. Hence, the equations of motion which apply in this case become



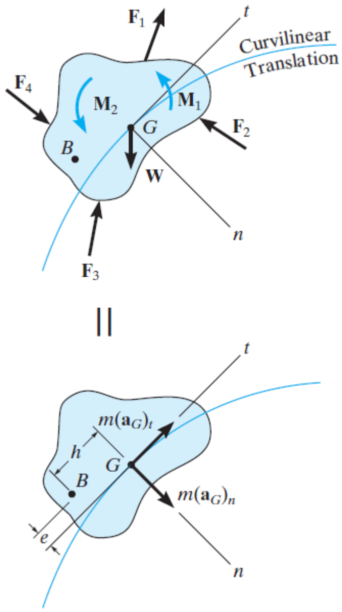
(a)

$$\Sigma F_x = m(a_G)_x$$
$$\Sigma F_y = m(a_G)_y$$
$$\Sigma M_G = 0$$



(b)

When a rigid body is subjected to *curvilinear translation*, all the particles of the body have the same accelerations as they travel along *curved*. For analysis, it is often convenient to use an inertial coordinate system having an origin which coincides with the body's mass center at the instant considered, and axes which are oriented in the normal and tangential directions to the path of motion, Fig. *c*. The three scalar equations of motion are then



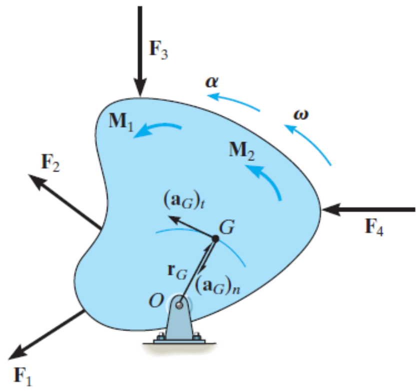
(c)

$$\Sigma F_n = m(a_G)_n$$
$$\Sigma F_t = m(a_G)_t$$
$$\Sigma M_G = 0$$

4-3-2 Equations of Motion: Rotation about a Fixed Axis

Consider the rigid body (or slab) shown in Fig. *a*, which is constrained to rotate in the vertical plane about a fixed axis perpendicular to the page and passing through the pin at *O*. The angular velocity and angular acceleration are caused by the external force and couple moment system acting on the body. Because the body's center of mass *G* moves around a *circular path*, the acceleration of this point is best represented by its tangential and normal components. The *tangential component of acceleration* has a *magnitude* of $(a_G)_t$ = ar_G and must act in a *direction* which is *consistent* with the body's angular acceleration A. The *magnitude* of the *normal component of acceleration* is $(a_G)_n$ = v^2rG. This component is *always directed* from point *G* to *O*, regardless of the rotational sense of V. The free-body and kinetic diagrams for the body are shown in Fig. *b*. The two components $m(\mathbf{a}_G)_t$ and $m(\mathbf{a}_G)_n$, shown on the kinetic diagram, are associated with the tangential and normal components of acceleration of the body's mass center. The *IG* A vector acts in the same *direction* as A and has a *magnitude* of *IG*a, where *IG* is the body's moment of inertia calculated about an axis which is perpendicular to the page and passes through *G*. The equations of motion which apply to the body can be written in the form

$$\Sigma F_n = m(a_G)_n = m\omega^2 r_G$$
$$\Sigma F_t = m(a_G)_t = m\alpha r_G$$
$$\Sigma M_G = I_G\alpha$$



(a)

 Asst. Prof. Dr. Sheelan M. Hama

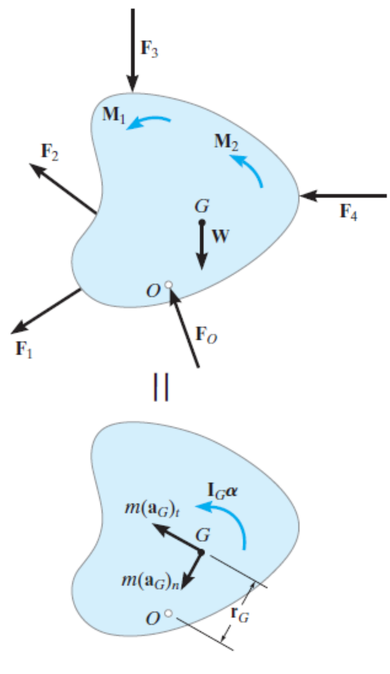
Often it is convenient to sum moments about the pin at O in order to eliminate the *unknown* force $\mathbf{F}_O$. From the kinetic diagram, Fig. *b*, this requires

$$\zeta + \Sigma M_O = \Sigma(\mathcal{M}_k)_O; \qquad \Sigma M_O = r_G m(a_G)_t + I_G\alpha$$

We can write the three equations of motion for the body as

$$\Sigma F_n = m(a_G)_n = m\omega^2 r_G$$
$$\Sigma F_t = m(a_G)_t = m\alpha r_G$$
$$\Sigma M_O = I_O\alpha$$

When using these equations, remember that "$I_O\alpha$" accounts for the "moment" of *both* $m(\mathbf{a}_G)_t$ *and* $I_G\alpha$ about point O, Fig. *b*. In other words, $\sum M_O = \sum(M_k)O = I_O\alpha$,

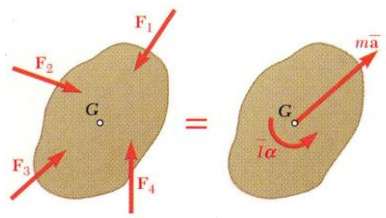


(b)

4-5 Plane Motion of a Rigid Body: D'Alembert's Principle

Motion of a rigid body in plane motion is completely defined by the resultant and moment resultant about G of the external forces

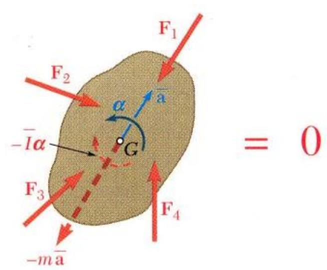
$$\sum F_x = m\bar{a}_x \quad \sum F_y = m\bar{a}_y \quad \sum M_G = \bar{I}\alpha$$



d'Alembert's Principle: The external forces acting on a rigid body are equivalent to the effective forces of the various particles forming the body. The most general motion of a rigid body that is symmetrical with respect to the reference plane can be replaced by the sum of a translation and a centroidal rotation. The fundamental relation between the forces acting on a rigid body in plane motion and the acceleration of its mass center and the angular acceleration of the body is illustrated in a free-body-diagram equation.

The techniques for solving problems of static equilibrium may be applied to solve problems of plane motion by utilizing

- d'Alembert's principle, or
- principle of dynamic equilibrium



These techniques may also be applied to problems involving plane motion of connected rigid bodies by drawing a free-body-diagram equation for each body and solving the corresponding equations of motion simultaneously.

4-6 Uniform Free Body Diagrams and Kinetic Diagrams

The free body diagram is the same as you have done in statics , we will add the kinetic diagram in our dynamic analysis.

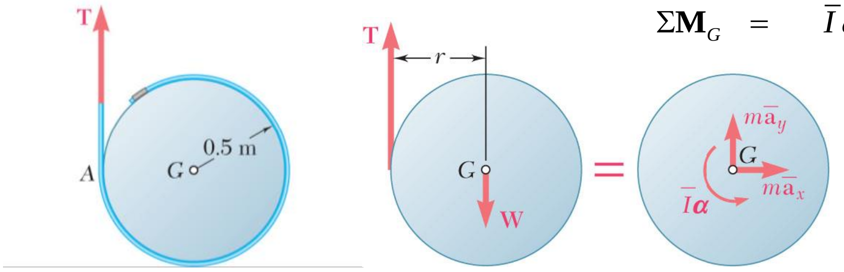
1. Isolate the body of interest (free body)
2. Draw your axis system (Cartesian, polar, path)
3. Add in applied forces (e.g., weight)
4. Replace supports with forces (e.g., tension force)
5. Draw appropriate dimensions (angles and distances)

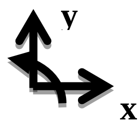
Put the inertial terms for the body of interest on the kinetic diagram.

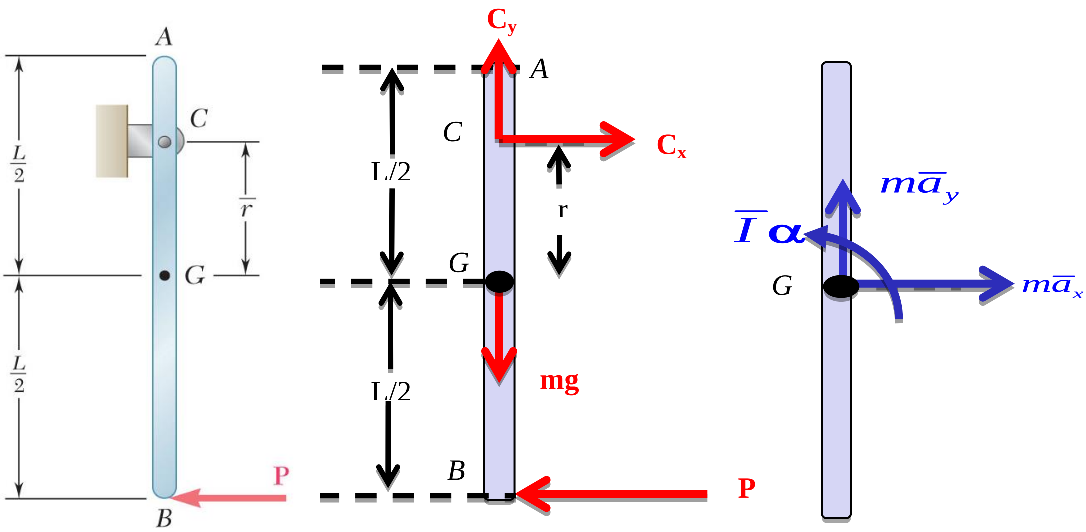
1. Isolate the body of interest (free body)
2. Draw in the mass times acceleration of the particle; if unknown, do this in the positive direction according to your chosen axes. For rigid bodies, also include the rotational term, $I_G\alpha$.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

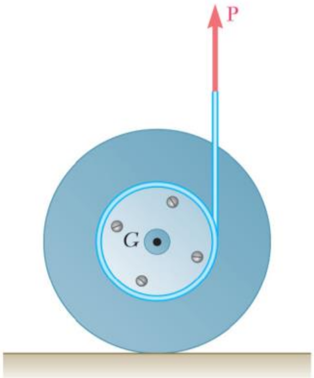
$$\Sigma \mathbf{M}_G = \bar{I}\alpha$$



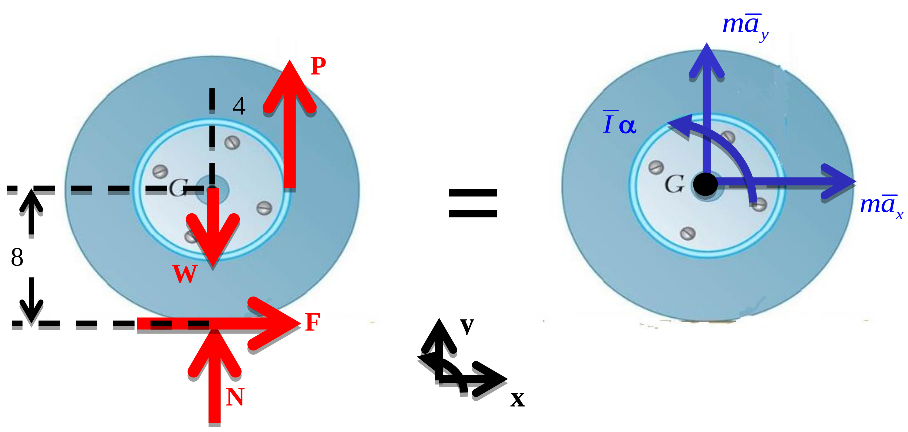




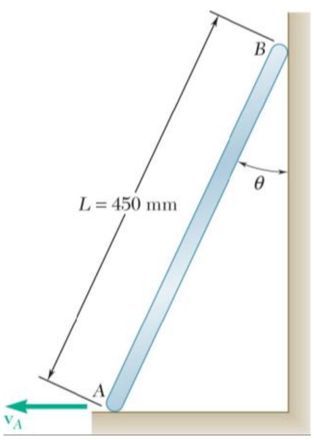
Example1: A drum of 4 inch radius is attached to a disk of 8 inch radius. The combined drum and disk had a combined mass of 10 lbs. A cord is attached as shown, and a force of magnitude P=5 lbs is applied. The coefficients of static and kinetic friction between the wheel and ground are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively. Draw the FBD and KD for the wheel.

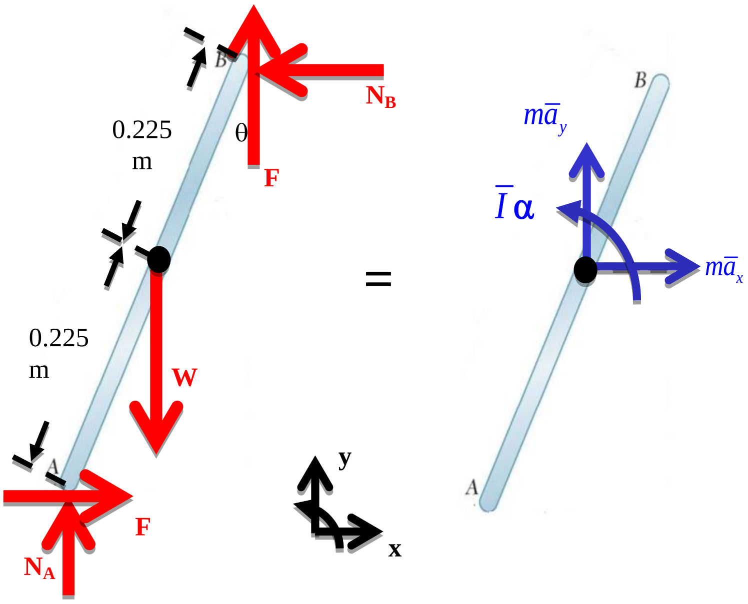


SOLUTION:

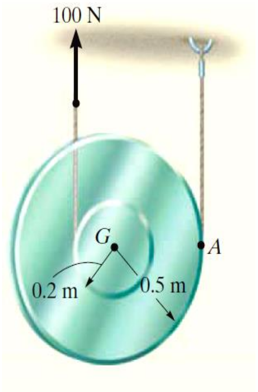


Example 2: The ladder AB slides down the wall as shown. The wall and floor are both rough. Draw the FBD and KD for the ladder.





Example 3: Determine the angular acceleration of the spool in Fig. *a*. The spool has a mass of 8 kg and a radius of gyration of k_G = 0.35 m. The cords of negligible mass are wrapped around its inner hub and outer rim.



(a)

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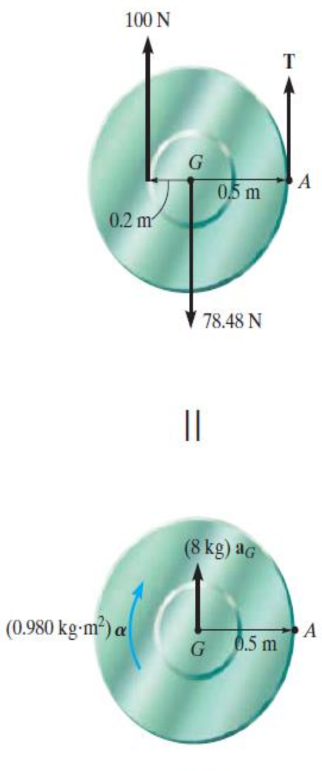
SOLUTION:

$$I_G = mk_G^2 = 8\text{ kg}(0.35\text{ m})^2 = 0.980\text{ kg}\cdot\text{m}^2$$

Equations of Motion.

$$+\uparrow\Sigma F_y = m(a_G)_y; \qquad T + 100\text{ N} - 78.48\text{ N} = (8\text{ kg})a_G \qquad (1)$$

$$\circlearrowleft +\Sigma M_G = I_G\alpha; \quad 100\text{ N}(0.2\text{ m}) - T(0.5\text{ m}) = (0.980\text{ kg}\cdot\text{m}^2)\alpha \qquad (2)$$



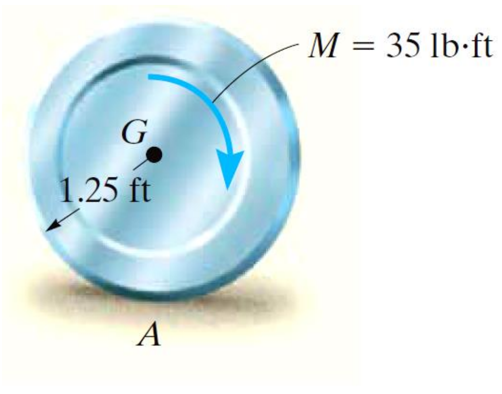
(b)

$$(\circlearrowleft +)\ a_G = \alpha r; \qquad a_G = \alpha\,(0.5\text{ m})$$

Solving Eqs. 1 to 3, we have

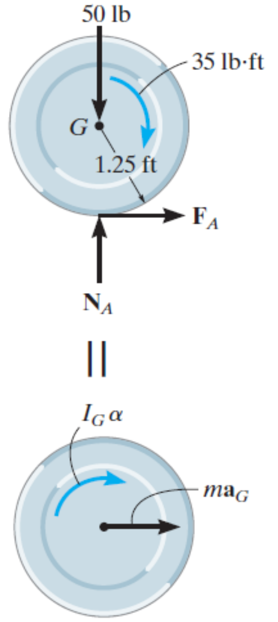
$$\alpha = 10.3\text{ rad/s}^2$$
$$a_G = 5.16\text{ m/s}^2$$
$$T = 19.8\text{ N}$$

Example 4: The 50-lb wheel shown in Fig. has a radius of gyration k_G = 0.70 ft. If a 35 lb.ft couple moment is applied to the wheel, determine the acceleration of its mass center *G*. The coefficients of static and kinetic friction between the wheel and the plane at *A* are μ_s = 0.3 and μ_k = 0.25, respectively.



(a)

SOLUTION:



(b)

$$I_G = mk_G^2 = \frac{50\text{ lb}}{32.2\text{ ft/s}^2}(0.70\text{ ft})^2 = 0.7609\text{ slug}\cdot\text{ft}^2$$

The unknowns are N_A, F_A, a_G, and α.

Equations of Motion.

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\quad F_A = \left(\frac{50\text{ lb}}{32.2\text{ ft/s}^2}\right)a_G$ (1)

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad N_A - 50\text{ lb} = 0$ (2)

$\circlearrowright + \Sigma M_G = I_G\alpha;$ $\quad 35\text{ lb}\cdot\text{ft} - 1.25\text{ ft}(F_A) = (0.7609\text{ slug}\cdot\text{ft}^2)\alpha$ (3)

A fourth equation is needed for a complete solution.

Kinematics (No Slipping). If this assumption is made, then

$(\circlearrowright +)$ $\quad a_G = (1.25\text{ ft})\alpha$ (4)

Solving Eqs. 1 to 4,

$$N_A = 50.0\text{ lb} \qquad F_A = 21.3\text{ lb}$$
$$\alpha = 11.0\text{ rad/s}^2 \qquad a_G = 13.7\text{ ft/s}^2$$

This solution requires that no slipping occurs, i.e., $F_A \le \mu_s N_A$. However, since $21.3\text{ lb} > 0.3(50\text{ lb}) = 15\text{ lb}$, the wheel slips as it rolls.

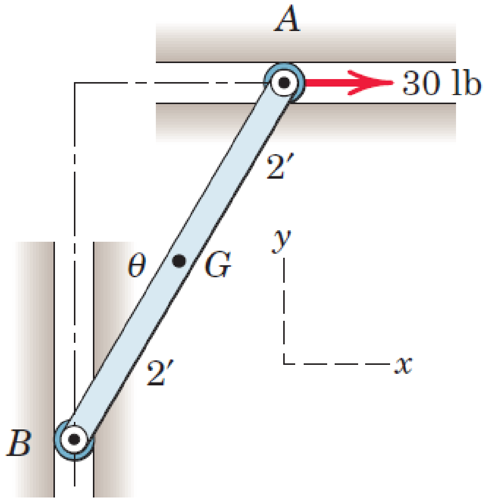
(Slipping). Equation 4 is not valid, and so $F_A = \mu_k N_A$, or

$$F_A = 0.25N_A \qquad (5)$$

Solving Eqs. 1 to 3 and 5 yields

$$N_A = 50.0\text{ lb} \qquad F_A = 12.5\text{ lb}$$
$$\alpha = 25.5\text{ rad/s}^2$$
$$a_G = 8.05\text{ ft/s}^2 \rightarrow \qquad \textit{Ans.}$$

Example 5: The slender bar *AB* weighs 60 lb and moves in the vertical plane, with its ends constrained to follow the smooth horizontal and vertical guides. If the 30-lb force is applied at *A* with the bar initially at rest in the position for which $\theta = 30^o$, calculate the resulting angular acceleration of the bar and the forces on the small end rollers at *A* and *B*.



SOLUTION:

$$\bar{a}_x = \bar{a}\cos 30° = 2\alpha\cos 30° = 1.732\alpha \text{ ft/sec}^2$$

$$\bar{a}_y = \bar{a}\sin 30° = 2\alpha\sin 30° = 1.0\alpha \text{ ft/sec}^2$$

$[\Sigma M_G = \bar{I}\alpha]$

$$30(2\cos 30°) - A(2\sin 30°) + B(2\cos 30°) = \frac{1}{12}\frac{60}{32.2}(4^2)\alpha$$

$[\Sigma F_x = m\bar{a}_x]$ $\quad 30 - B = \frac{60}{32.2}(1.732\alpha)$

$[\Sigma F_y = m\bar{a}_y]$ $\quad A - 60 = \frac{60}{32.2}(1.0\alpha)$

Solving the three equations simultaneously gives us the results

$$A = 68.2 \text{ lb} \qquad B = 15.74 \text{ lb} \qquad \alpha = 4.42 \text{ rad/sec}^2$$