*University of Anbar*
*Engineering College*
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# CHAPTER ONE

# INTRODUCTION

**LECTURE**
**DR. AHMED H. ABDULKAREEM**
**2019 - 2020**

## 1.1 Geotechnical Engineering

In the general sense of engineering, ***soil*** is defined as the uncemented aggregate of mineral grains and decayed organic matter (solid particles) along with the liquid and gas that occupy the empty spaces between the solid particles. Soil is used as a construction material in various civil engineering projects, and it supports structural foundations. Thus, civil engineers must study the properties of soil, such as its origin, grain-size distribution, ability to drain water, compressibility, shear strength, load bearing capacity, and so on.

***Soil mechanics*** is the branch of science that deals with the study of the physical properties of soil and the behavior of soil masses subjected to various types of forces.

***Rock mechanics*** is a branch of science that deals with the study of the properties of rocks. It includes the effect of the network of fissures and pores on the nonlinear stress strain behavior of rocks as strength anisotropy. Rock mechanics (as we know now) slowly grew out of soil mechanics. So, collectively, soil mechanics and rock mechanics are generally referred to as ***geotechnical engineering***.

## 1.2 FOUNDATIONS: THEIR IMPORTANCE AND PURPOSE

All engineered construction resting on the earth must be carried by some kind of interfacing element called *a* ***foundation (substructure)*** as in Fig.1 .1 . The foundation is the part of an engineered system that transmits to, and into, the underlying soil or rock the loads supported by the foundation and its self-weight. The resulting soil stresses—except at the ground surface—are in addition to those presently existing in the earth mass from its self-weight and geological history.

The term ***superstructure*** is commonly used to describe the engineered part of the system bringing load to the foundation, or substructure as in Fig. 1.1. The term ***superstructure*** has particular significance for buildings and bridges; however, foundations also may carry only machinery, support industrial equipment (pipes, towers, tanks), act as sign bases, and the like. For these reasons it is better to describe a foundation as that part of the engineered system that interfaces the load-carrying components to the ground.

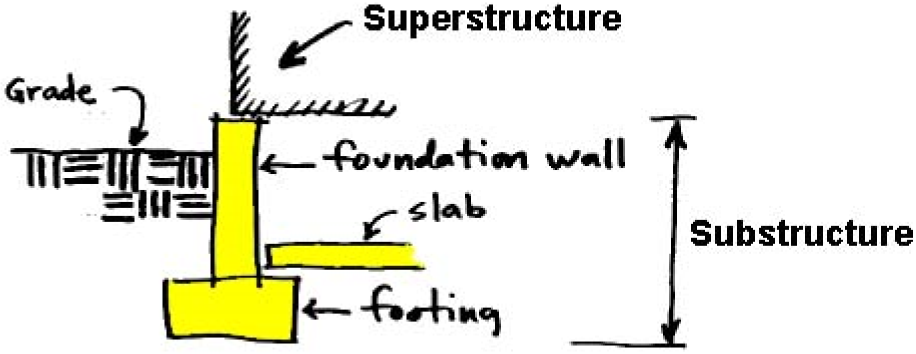


**Fig. 1.1 The foundation (substructure) and superstructure**

## 1.3 Foundation Engineering

Foundation engineering is the application and practice of the fundamental principles of soil mechanics and rock mechanics (i.e., geotechnical engineering) in the design of foundations of various structures. These foundations include those of columns and walls of buildings, bridge abutments, embankments, and others. It also involves the analysis and design of earth-retaining structures such as retaining walls, sheet-pile walls, and braced cuts. This notes is prepared, in general, to elaborate upon the foundation engineering aspects of these structures.

## 1.4 General Format of the Notes

This Notes is divided into four major parts.

- **Part I—Exploration of Soil (Chapters 2)**
- **Part II—Foundation Analysis (Chapters 3 through 6).**

Foundation analysis, in general, can be divided into two categories: shallow foundations and deep foundations.

Spread footings and mat (or raft) foundations are referred to as shallow foundations. A *spread footing* is simply an enlargement of a load-bearing wall or column that makes it possible to spread the load of the structure over a larger area of the soil. In soil with low load-bearing capacity, the size of the spread footings is impracticably large. In that case, it is more economical to construct the entire structure over a concrete pad. This is called a *mat foundation*. Piles and drilled shafts are deep foundations. They are structural members used for heavier structures when the depth requirement for supporting the load is large. They transmit the load of the superstructure to the lower layers of the soil.

- **Part III—Lateral Earth Pressure and Earth-Retaining Structures (Foundation Engineering II ch1 through ch6).**

This part includes discussion of the general principles of lateral earth pressure on vertical or near-vertical walls based on wall movement and analyses of retaining walls, sheet pile walls, braced cuts and slope stability.

The types foundations shown in Fig.1.2, 1.3 and Table 1.1.

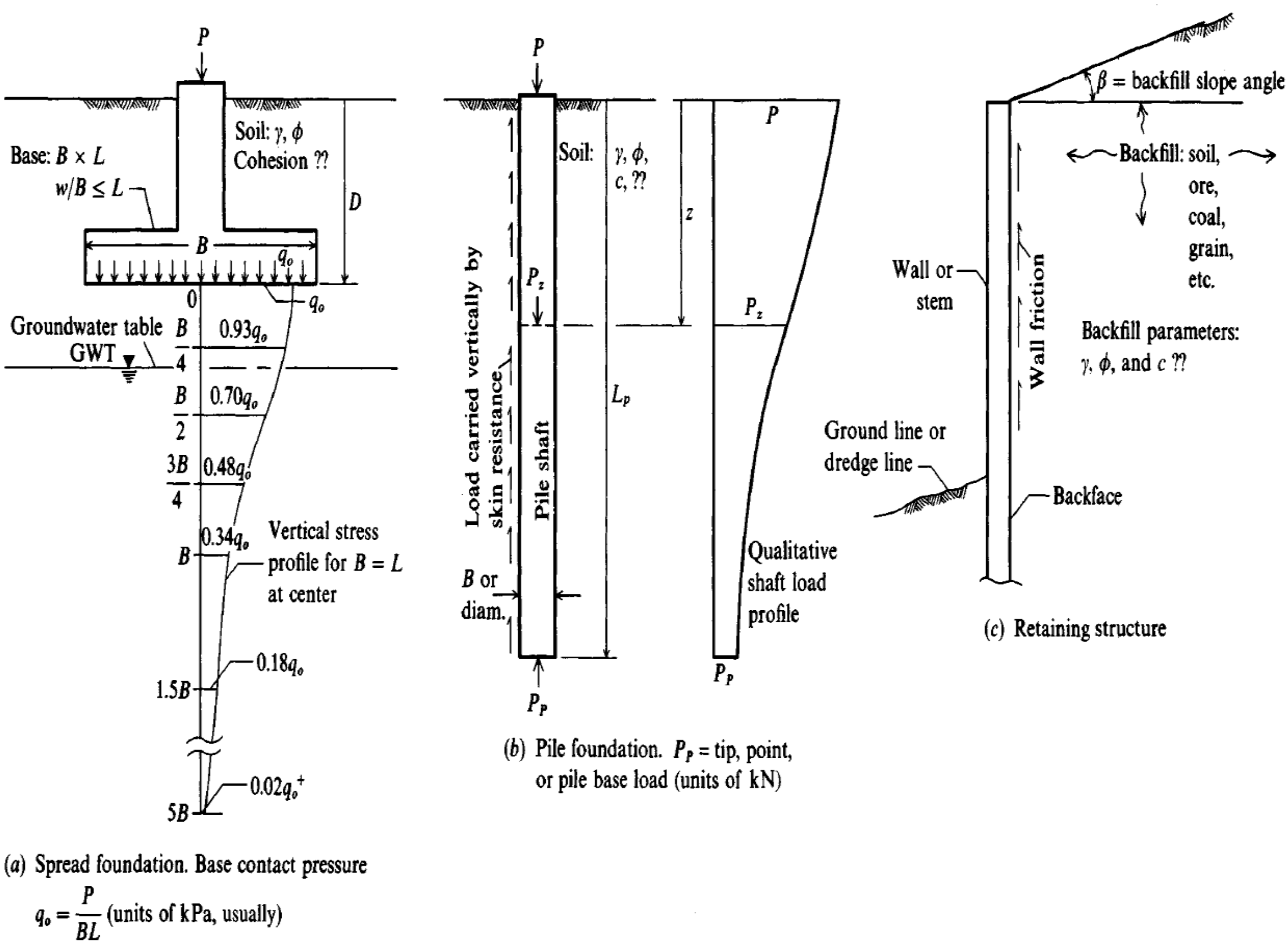


**Fig. 1.2 Types of foundation engineering**.

**TABLE 1-1**
**Foundation types and typical usage**

| Foundation type | Use | Applicable soil conditions |
|---|---|---|
| | **Shallow foundations (generally $D/B \leq 1$)** | |
| Spread footings, wall footings | Individual columns, walls | Any conditions where bearing capacity is adequate for applied load. May use on a single stratum; firm layer over soft layer or soft layer over firm layer. Check settlements from any source. |
| Combined footings | Two to four columns on footing and/or space is limited | Same as for spread footings above. |
| Mat foundations | Several rows of parallel columns; heavy column loads; use to reduce differential settlements | Soil bearing capacity is generally less than for spread footings, and over half the plan area would be covered by spread footings. Check settlements from any source. |
| | **Deep foundations (generally $L_p/B \geq 4^+$)** | |
| Floating pile | In groups of $2^+$ supporting a cap that interfaces with column(s) | Surface and near-surface soils have low bearing capacity and competent soil is at great depth. Sufficient skin resistance can be developed by soil-to-pile perimeter to carry anticipated loads. |
| Bearing pile | Same as for floating pile | Surface and near-surface soils not relied on for skin resistance; competent soil for point load is at a practical depth (8–20 m). |
| Drilled piers or caissons | Same as for piles; use fewer; For large column loads | Same as for piles. May be floating or point-bearing (or combination). Depends on depth to competent bearing stratum. |
| | **Retaining structures** | |
| Retaining walls, bridge abutments | Permanent material retention | Any type of soil but a specified zone (Chaps. 11, 12) in backfill is usually of controlled fill. |
| Sheeting structures (sheet pile, wood sheeting, etc.) | Temporary or permanent for excavations, marine cofferdams for river work | Retain any soil or water. Backfill for waterfront and cofferdam systems is usually granular for greater drainage. |

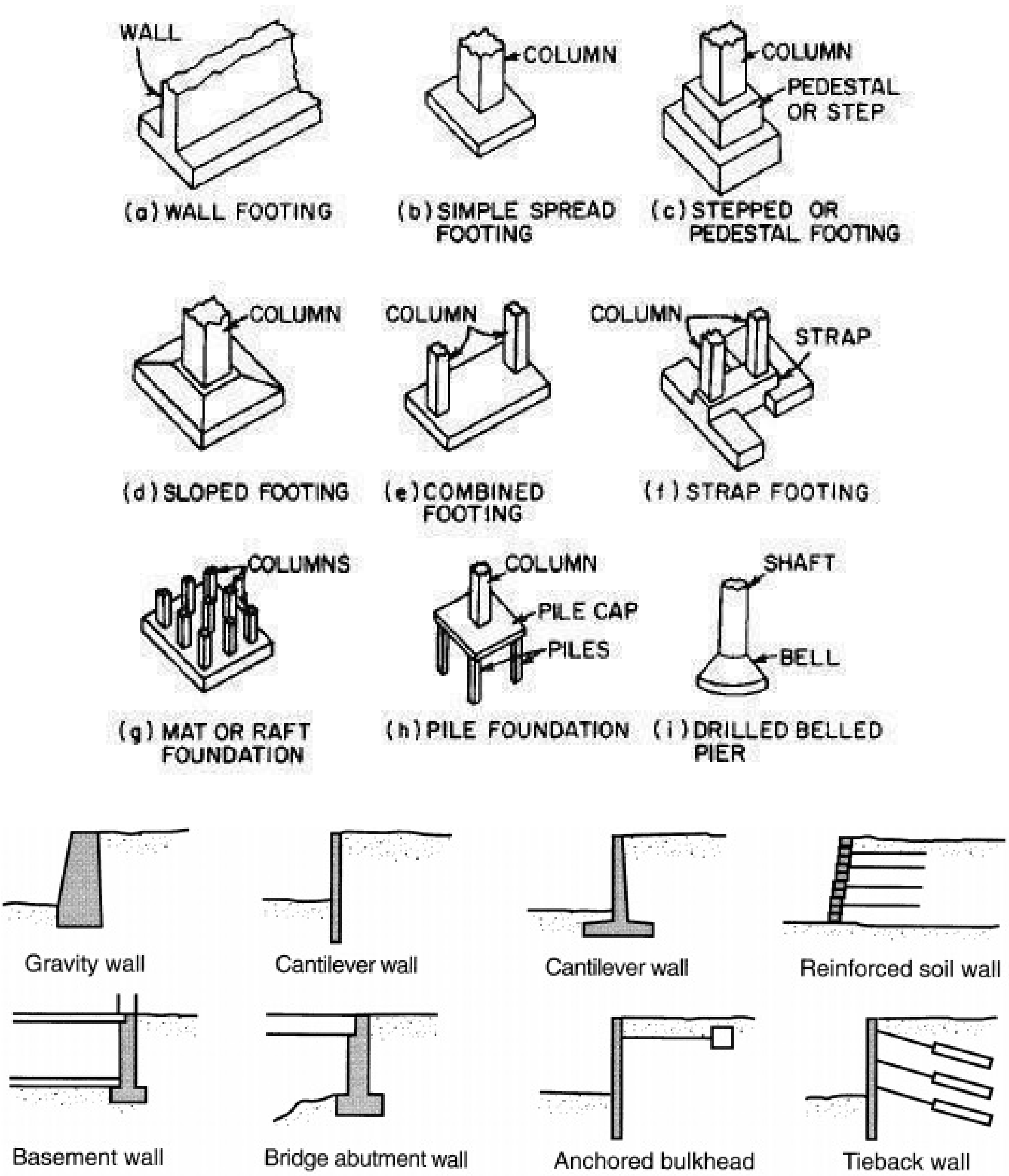


Fig. 1.3 Various types of foundations

## 1.5 Design Methods

The *allowable stress design* (ASD) has been used for over a century in foundation design and is also used in this edition of the notes. The ASD is a deterministic design method which is based on the concept of applying a factor of safety (FS) to an ultimate load $Q_u$ (which is an ultimate limit state). Thus, the allowable load $Q_{all}$ can be expressed as

$$Q_{all} = Q_u / FS \tag{1.1}$$

According to ASD,

$$Q_{design} \le Q_{all} \tag{1.2}$$

where $Q_{design}$ is the design (working) load.

Over the last several years, *reliability based design methods* are slowly being incorporated into civil engineering design. This is also called the *load and resistance factor* design method (LRFD). It is also known as the ultimate strength design (USD). The LRFD was initially brought into practice by the American Concrete Institute (ACI) in the 1960s.

Several codes in North America now provide parameters for LRFD.

- American Association of State Highway and Transportation Officials (AASHTO) (1994, 1998)
- American Petroleum Institute (API) (1993)
- American Concrete Institute (ACI) (2002)

According to LRFD, the factored nominal load Qu is calculated as

$$Q_u = (LF)_1 Q_{u(1)} + (LF\mathrm{d})_2 Q_{u(2)} + \ . \ . \ . \tag{1.3}$$

where

$Q_u$ = factored nominal load

$(LF)_i$ ($i$ = 1, 2, . . .) is the load factor for nominal load $Q_{u(i)}$ ($i$= 1, 2, . . .)

Most of the load factors are greater than one. As an example, according to AASHTO (1998), the load factors are

| Load | LF |
|---|---|
| Dead load | 1.25 t0 1.95 |
| Live Load | 1.35 to 1.75 |
| Wind Load | 1.4 |
| Seismic | 1.0 |

The basic design inequality then can be given as

$$Q_u \leq \phi Q_n \tag{1.4}$$

where

$Q_n$ = nominal load capacity

$\phi$ = resistance factor ( <1)

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# CHAPTER THREE

# SHALLOW FOUNDATIONS
## ULTIMATE BEARING CAPACITY

**LECTURE**
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**2019 - 2020**

## 3.1. Introduction

To perform satisfactorily, shallow foundations must have two main characteristics:

**1.** They have to be safe against overall shear failure in the soil that supports them.

**2.** They cannot undergo excessive displacement, or settlement. (The term ***excessive*** is relative, because the degree of settlement allowed for a structure depends on several considerations.)

The load per unit area of the foundation at which shear failure in soil occurs is called the ***ultimate bearing capacity***, which is the subject of this chapter. In this chapter, we will discuss the following:

- Fundamental concepts in the development of the theoretical relationship for ultimate bearing capacity of shallow foundations subjected to centric vertical loading
- Effect of the location of water table and soil compressibility on ultimate bearing capacity
- Bearing capacity of shallow foundations subjected to vertical eccentric loading and eccentrically inclined loading.

## 4.2 General Concept

## 4.2.1 Types of Shear Failure

**Shear Failure:** Also called "Bearing capacity failure" and it's occur when the shear stresses in the soil exceed the shear strength of the soil.

### There are three types of shear failure in the soil:

**1. General Shear Failure (Fig. 3.1)**

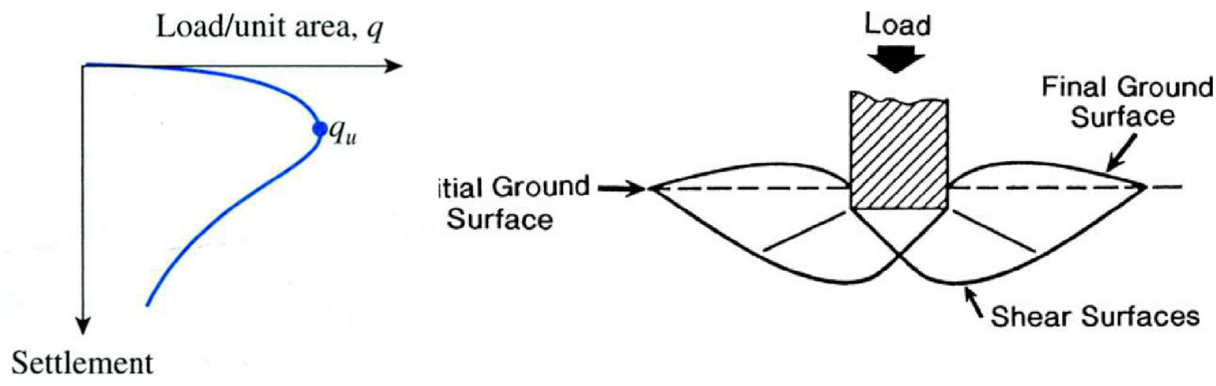



*Fig. 3.1 General shear failure.*

**The following are some characteristics of general shear failure:**

- Occurs over dense sand or stiff cohesive soil.
- Involves total rupture of the underlying soil.
- There is a continuous shear failure of the soil from below the footing to the ground surface (solid lines on the figure above).
- When the (load / unit area) plotted versus settlement of the footing, there is a distinct load at which the foundation fails ($Q_u$)
- The value of ($Q_u$) divided by the area of the footing is considered to be the ultimate bearing capacity of the footing($q_u$).
- For general shear failure, the ultimate bearing capacity has been defined as the bearing stress that causes **a sudden** catastrophic failure of the foundation.
- As shown in the above **Fig. 3.1**, a general shear failure ruptures occur and pushed up the soil on both sides of the footing (In laboratory).
- However, for actual failures on the field, the soil is often pushed up on **only one side** of the footing with **subsequent tilting** of the structure as shown in **Fig. 3.2** below:

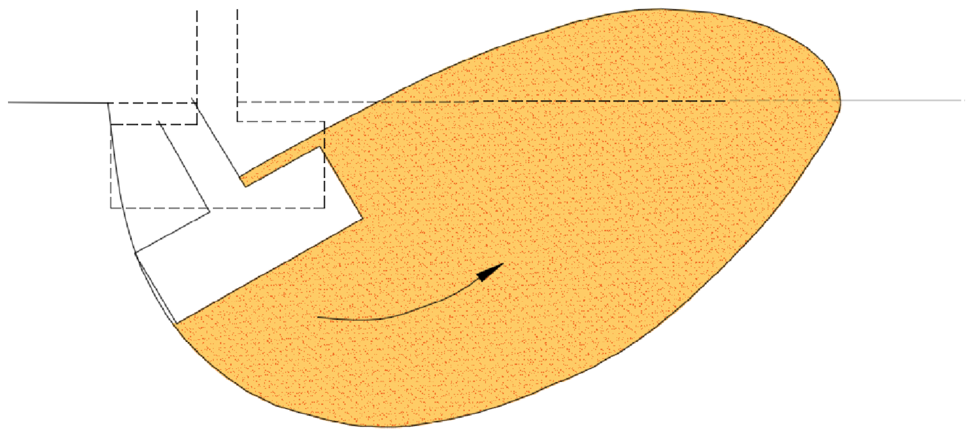


*Fig. 3.2 Actual general shear failure.*

**2. Local Shear Failure (Fig. 3.3):**

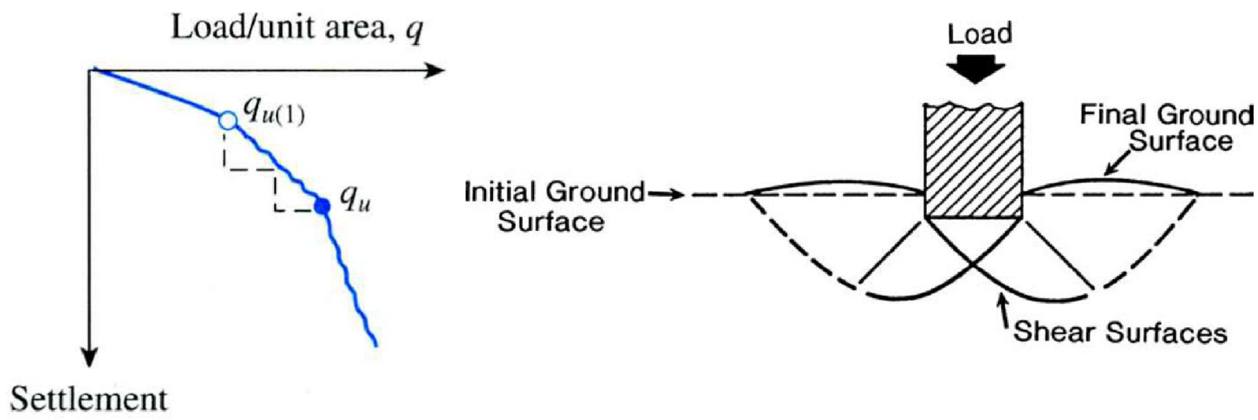



*Fig. 3.3 Local shear failure.*

**The following are some characteristics of local shear failure:**

- Occurs over sand or clayey soil of medium compaction.
- Involves rupture of the soil only immediately below the footing.
- There is soil bulging on both sides of the footing, but the bulging is not as significant as in general shear. That's because the underlying soil compacted less than the soil in general shear.
- The failure surface of the soil will **gradually** (not sudden) extend outward from the foundation (not the ground surface) as shown by **solid lines** in **Fig. 3.3.**

- So, local shear failure can be considered as a transitional phase between general shear and punching shear.
- Because of the transitional nature of local shear failure, the ultimate bearing capacity could be defined as the firs failure load ($q_{u(1)}$) which occur at the point which have the first measure nonlinearity in the load/unit area-settlement curve (open circle), or at the point where the settlement starts rabidly increase ($q_u$) (closed circle).
- This value of ($q_u$) is the required (load/unit area) to extends the failure surface to the ground surface (dashed lines as in **Fig. 3.3**).
- In this type of failure, the value of ($q_u$) it's not the peak value so, this failure called (Local Shear Failure).
- The actual local shear failure in field is proceed as shown in **Fig. 3.4**.

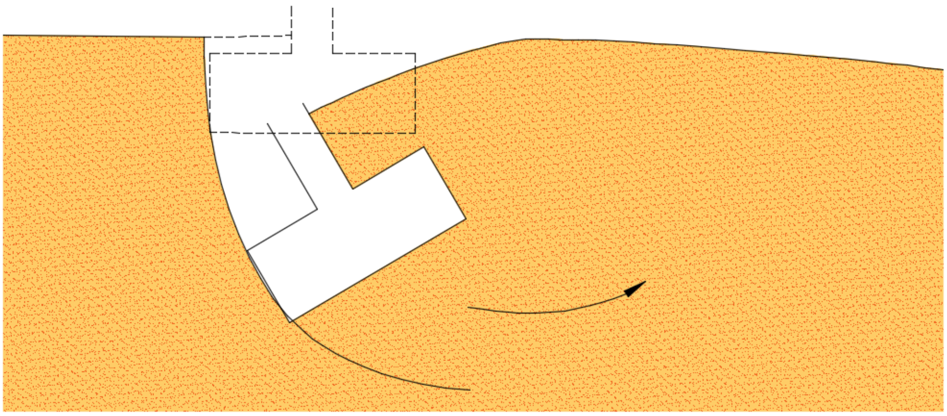


*Fig. 3.4 Actual Local shear failure.*

**3. Punching Shear Failure ( Fig. 3.5):**

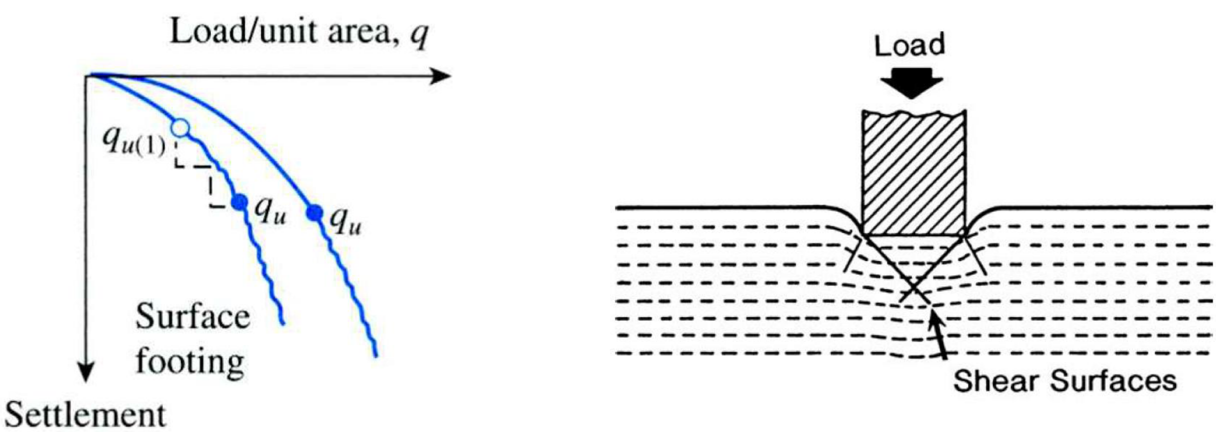



*Fig. 3.5 Punching shear failure.*

**The following are some characteristics of punching shear failure:**

- Occurs over fairly loose soil.
- Punching shear failure does not develop the distinct shear surfaces associated with a general shear failure.
- The soil outside the loaded area remains relatively uninvolved and there is a minimal movement of soil on both sides of the footing.
- The process of deformation of the footing involves compression of the soil directly below the footing as well as the vertical shearing of soil around the footing perimeter.
- As shown in **Fig. 3.5** above, the (q)-settlement curve does not have a dramatic break, and the bearing capacity is often defined as the first measure nonlinearity in the (q)-settlement curve($q_{u(1)}$).
- Beyond the ultimate failure (load/unit area) ($q_{u(1)}$), the (load/unit area)-settlement curve will be steep and practically linear.
- The actual punching shear failure in field is proceed as shown in **Fig.3.6**:

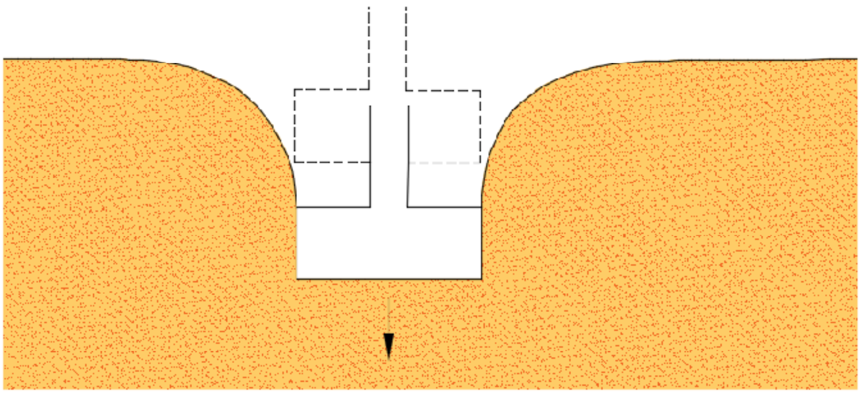


***Fig. 3.6 Actual Punching shear failure.***

**Note:**

**Ultimate Bearing Capacity (qu)**:

It's the **minimum load per unit area** of the foundation that causes shear failure in the underlying soil.

Or, it's the **maximum load per unit area** of the foundation can be resisted by the underlying soil without occurs of shear failure (if this load is exceeded, the shear failure will occur in the underlying soil).

**Allowable Bearing Capacity (qall)**

It's the load per unit area of the foundation can be resisted by the underlying soil without any unsafe movement occurs (shear failure) and if this load is exceeded, the shear failure **will not** occur in the underlying soil **till** reaching the ultimate load.

## 3.3 Terzaghi's Bearing Capacity Theory

Terzaghi (1943) was the first to present a comprehensive theory for evaluation of the ultimate bearing capacity of rough shallow foundation. **This theory is based on the following assumptions:**

1. The foundation is considered to be shallow if ($D_f \leq B$).
2. The foundation is considered to be strip or continuous if (Width to length ratio approaches zero), and the derivation of the equation is to a strip footing.
3. The effect of soil above the bottom of the foundation may be assumed to be replaced by an equivalent surcharge ($q=\gamma\times D_f$). So, the shearing resistance of this soil along the failure surfaces is neglected (Lines ab and cd in the below **Fig. 3.7**)
4. The failure surface of the soil is similar to **general shear failure** (i.e. equation is derived for general shear failure) as shown in **Fig. 3.7**.

**Note:**

1. In recent studies, investigators have suggested that, foundations are considered to be shallow if [ $D_f \leq (3\rightarrow4)B$], otherwise, the foundation is deep.
2. Always the value of (q) is the **effective stress** at the bottom of the foundation.

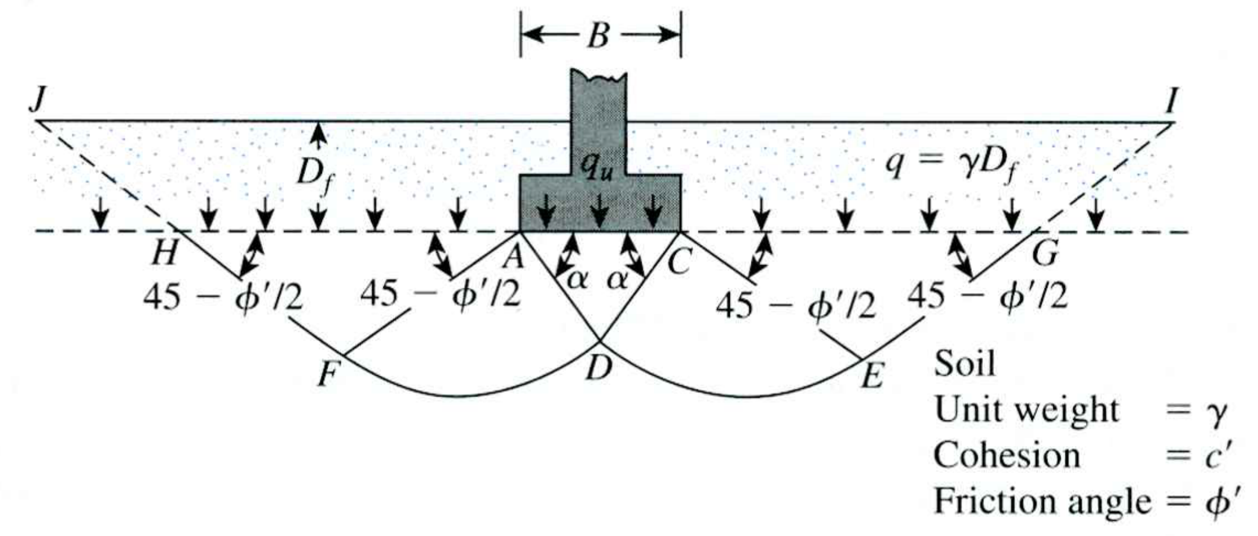



***Fig. 3.7 Bearing capacity failure in soil under a rough rigid continouus foundation***

The failure zone under the foundation can be separated into three parts (see Fig. 3.7):

**1.** The *triangular zone ACD* immediately under the foundation

**2.** The *radial shear zones ADF* and *CDE,* with the curves *DE* and *DF* being arcs of a logarithmic spiral

**3.** Two triangular *Rankine passive zones AFH* and *CEG*

The angles *CAD* and *ACD* are assumed to be equal to the soil friction angle $\phi$'. Note that, with the replacement of the soil above the bottom of the foundation by an equivalent surcharge *q,* the shear resistance of the soil along the failure surfaces *GI* and *HJ* was neglected.

## Terzaghi's Bearing Capacity Equations

As mentioned previously, the equation was derived for a **strip footing** and **general shear failure,** this equation is:

$q_u=cN_c+qN_q+0.5B\gamma N_\gamma$ (for continuous or strip footing) (3.1)

Where

$q_u$=Ultimate bearing capacity of the soil (kN/m$^2$)

c=Cohesion of soil (kN/m$^2$)

q=**Efeective** stress at the bottom of the foundation= $\gamma D_f$ (kN/m$^2$)

$N_c$, $N_q$, $N_\gamma$=Bearing capacity factors (nondimensional)and are functions **only** of the soil friction angle,$\phi'$

The bearing capacity factors $N_c$, $N_q$, and $N_\gamma$ are defined by

$$N_c = \cot\phi'\left[\frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(\frac{\pi}{4}+\frac{\phi'}{2}\right)}-1\right] = \cot\phi'(N_q-1) \tag{3.2}$$

$$N_q = \frac{e^{2(3\pi/4-\phi'/2)\tan\phi'}}{2\cos^2\left(45+\frac{\phi'}{2}\right)} \tag{3.3}$$

and

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'}-1\right)\tan\phi' \tag{3.4}$$

$K_{p\gamma}$= passive pressure coefficient

The variations of the bearing capacity factors shown in **Table 3.1 for general shear failure**.

The above equation (for strip footing) was modified to be useful for both **square** and **circular footings** as following:

$q_u=1.3cN_c+qN_q+0.4B\gamma N_\gamma$ B (square foundation) (3.5)

B=The dimension of each side of the foundation .

$q_u=1.3cN_c+qN_q+0.3B\gamma N_\gamma$ B (circular foundation) (3.6)

B=The diameter of the foundation .

**Note:**

These two equations are also for **general shear failure,** and all factors in the two equations are the same as explained for strip footing.

**Table 3.1 Terzaghi's bearing capacity factors**

| $\phi'$ | $N_c$ | $N_q$ | $N_\gamma$[a] | $\phi'$ | $N_c$ | $N_q$ | $N_\gamma$[a] |
|---|---|---|---|---|---|---|---|
| 0 | 5.70 | 1.00 | 0.00 | 26 | 27.09 | 14.21 | 9.84 |
| 1 | 6.00 | 1.10 | 0.01 | 27 | 29.24 | 15.90 | 11.60 |
| 2 | 6.30 | 1.22 | 0.04 | 28 | 31.61 | 17.81 | 13.70 |
| 3 | 6.62 | 1.35 | 0.06 | 29 | 34.24 | 19.98 | 16.18 |
| 4 | 6.97 | 1.49 | 0.10 | 30 | 37.16 | 22.46 | 19.13 |
| 5 | 7.34 | 1.64 | 0.14 | 31 | 40.41 | 25.28 | 22.65 |
| 6 | 7.73 | 1.81 | 0.20 | 32 | 44.04 | 28.52 | 26.87 |
| 7 | 8.15 | 2.00 | 0.27 | 33 | 48.09 | 32.23 | 31.94 |
| 8 | 8.60 | 2.21 | 0.35 | 34 | 52.64 | 36.50 | 38.04 |
| 9 | 9.09 | 2.44 | 0.44 | 35 | 57.75 | 41.44 | 45.41 |
| 10 | 9.61 | 2.69 | 0.56 | 36 | 63.53 | 47.16 | 54.36 |
| 11 | 10.16 | 2.98 | 0.69 | 37 | 70.01 | 53.80 | 65.27 |
| 12 | 10.76 | 3.29 | 0.85 | 38 | 77.50 | 61.55 | 78.61 |
| 13 | 11.41 | 3.63 | 1.04 | 39 | 85.97 | 70.61 | 95.03 |
| 14 | 12.11 | 4.02 | 1.26 | 40 | 95.66 | 81.27 | 115.31 |
| 15 | 12.86 | 4.45 | 1.52 | 41 | 106.81 | 93.85 | 140.51 |
| 16 | 13.68 | 4.92 | 1.82 | 42 | 119.67 | 108.75 | 171.99 |
| 17 | 14.60 | 5.45 | 2.18 | 43 | 134.58 | 126.50 | 211.56 |
| 18 | 15.12 | 6.04 | 2.59 | 44 | 151.95 | 147.74 | 261.60 |
| 19 | 16.56 | 6.70 | 3.07 | 45 | 172.28 | 173.28 | 325.34 |
| 20 | 17.69 | 7.44 | 3.64 | 46 | 196.22 | 204.19 | 407.11 |
| 21 | 18.92 | 8.26 | 4.31 | 47 | 224.55 | 241.80 | 512.84 |
| 22 | 20.27 | 9.19 | 5.09 | 48 | 258.28 | 287.85 | 650.67 |
| 23 | 21.75 | 10.23 | 6.00 | 49 | 298.71 | 344.63 | 831.99 |
| 24 | 23.36 | 11.40 | 7.08 | 50 | 347.50 | 415.14 | 1072.80 |
| 25 | 25.13 | 12.72 | 8.34 | | | | |

[a]From Kumbhojkar (1993)

Now for **local shear failure** the above three equations were modified to be useful for local shear failure as following:

$q_u = 2/3 c' N_c' + qN_q' + 0.5B\gamma N_\gamma'$ (for continuous or strip footing) (3.7)

$q_u = 0.867\, c' N_c' + qN_q' + 0.4B\gamma N_\gamma'$ (for square footing) (3.8)

$q_u = 0.867\, c' N_c' + qN_q' + 0.3B\gamma N_\gamma'$ (for circular footing) (3.9)

$N_c'$, $N_q'$, $N_\gamma'$=Modified bearing capacity factors can be calculated by using the bearing capacity factor equations ( for $N_c$ , $N_q$, and $N_\gamma$, respectively) by replacing $\phi'$ by $\phi'' = \tan^{-1}(2/3 \tan \phi')$. The variation of $N_c'$, $N_q'$, and $N_\gamma'$ with the soil friction angle $\phi'$ is given in **Table 3.2**.

**Table 3.2 Terzaghi's modified bearing capacity factors $N_c'$, $N_q'$, and $N_\gamma'$**

| $\phi'$ | $N_c'$ | $N_q'$ | $N_\gamma'$ | $\phi'$ | $N_c'$ | $N_q'$ | $N_\gamma'$ |
|---|---|---|---|---|---|---|---|
| 0 | 5.70 | 1.00 | 0.00 | 26 | 15.53 | 6.05 | 2.59 |
| 1 | 5.90 | 1.07 | 0.005 | 27 | 16.30 | 6.54 | 2.88 |
| 2 | 6.10 | 1.14 | 0.02 | 28 | 17.13 | 7.07 | 3.29 |
| 3 | 6.30 | 1.22 | 0.04 | 29 | 18.03 | 7.66 | 3.76 |
| 4 | 6.51 | 1.30 | 0.055 | 30 | 18.99 | 8.31 | 4.39 |
| 5 | 6.74 | 1.39 | 0.074 | 31 | 20.03 | 9.03 | 4.83 |
| 6 | 6.97 | 1.49 | 0.10 | 32 | 21.16 | 9.82 | 5.51 |
| 7 | 7.22 | 1.59 | 0.128 | 33 | 22.39 | 10.69 | 6.32 |
| 8 | 7.47 | 1.70 | 0.16 | 34 | 23.72 | 11.67 | 7.22 |
| 9 | 7.74 | 1.82 | 0.20 | 35 | 25.18 | 12.75 | 8.35 |
| 10 | 8.02 | 1.94 | 0.24 | 36 | 26.77 | 13.97 | 9.41 |
| 11 | 8.32 | 2.08 | 0.30 | 37 | 28.51 | 15.32 | 10.90 |
| 12 | 8.63 | 2.22 | 0.35 | 38 | 30.43 | 16.85 | 12.75 |
| 13 | 8.96 | 2.38 | 0.42 | 39 | 32.53 | 18.56 | 14.71 |
| 14 | 9.31 | 2.55 | 0.48 | 40 | 34.87 | 20.50 | 17.22 |
| 15 | 9.67 | 2.73 | 0.57 | 41 | 37.45 | 22.70 | 19.75 |
| 16 | 10.06 | 2.92 | 0.67 | 42 | 40.33 | 25.21 | 22.50 |
| 17 | 10.47 | 3.13 | 0.76 | 43 | 43.54 | 28.06 | 26.25 |
| 18 | 10.90 | 3.36 | 0.88 | 44 | 47.13 | 31.34 | 30.40 |
| 19 | 11.36 | 3.61 | 1.03 | 45 | 51.17 | 35.11 | 36.00 |
| 20 | 11.85 | 3.88 | 1.12 | 46 | 55.73 | 39.48 | 41.70 |
| 21 | 12.37 | 4.17 | 1.35 | 47 | 60.91 | 44.45 | 49.30 |
| 22 | 12.92 | 4.48 | 1.55 | 48 | 66.80 | 50.46 | 59.25 |
| 23 | 13.51 | 4.82 | 1.74 | 49 | 73.55 | 57.41 | 71.45 |
| 24 | 14.14 | 5.20 | 1.97 | 50 | 81.31 | 65.60 | 85.75 |
| 25 | 14.80 | 5.60 | 2.25 | | | | |

Terzaghi's bearing capacity equations have now been modified to take into account the effects of the foundation shape *(B/L)*, depth of embedment ($D_f$) , and the load inclination. This is given in Section 3.6. Many design engineers, however, still use Terzaghi's equation, which provides **fairly good results** considering the uncertainty of the soil conditions at various sites.

## 3.4 Factor of Safety

Calculating the gross *allowable load-bearing capacity* of shallow foundations requires the application of a factor of safety (FS) to the gross ultimate bearing capacity, or

$$q_{all} = q_u / FS \qquad (3.10)$$

However, some practicing engineers prefer to use a factor of safety such that

Net stress increase on soil = (net ultimate bearing capacity) / FS (3.11)

The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the foundation that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level. If the difference between the unit weight of concrete used in the foundation and the unit weight of soil surrounding is assumed to be negligible, then

$$q_{\text{net}(u)} = q_u - q \tag{3.12}$$

where

$q_{\text{net}(u)}$ = net ultimate bearing capacity

$q = \gamma D_f$

So

$$q_{\text{all(net)}} = (q_u - q) / \text{FS} \tag{3.13}$$

The factor of safety as defined by Eq. (3.13) should be at least 3 in all cases.

## 3.5 Modification of Bearing Capacity Equations

for Water Table Equations (3.1) and (3.5) through (3.6) give the ultimate bearing capacity, based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, some modifications of the bearing capacity equations will be necessary. (See Figure 3.8.)

***Case I.*** If the water table is located so that $0 \le D_1 \le D_f$, the factor $q$ in the bearing capacity equations takes the form

$$q = \text{effective surcharge} = D_1 \gamma + D_2 (\gamma_{sat} - \gamma_w) \tag{3.14}$$

where

$\gamma_{sat}$ = saturated unit weight of soil

$\gamma_w$ = unit weight of water

Also, the value of $\gamma$ in the last term of the equations has to be replaced by $\gamma' = (\gamma_{sat} - \gamma_w)$.

***Case II.*** For a water table located so that $0 \le d \le B$,

$$q = \gamma D_f \tag{3.15}$$

In this case, the factor $\gamma$ in the last term of the bearing capacity equations must be replaced by the factor

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

The preceding modifications are based on the assumption that there is no seepage force in the soil.

***Case III.*** When the water table is located so that $d \ge B$, the water will have no effect on the ultimate bearing capacity.

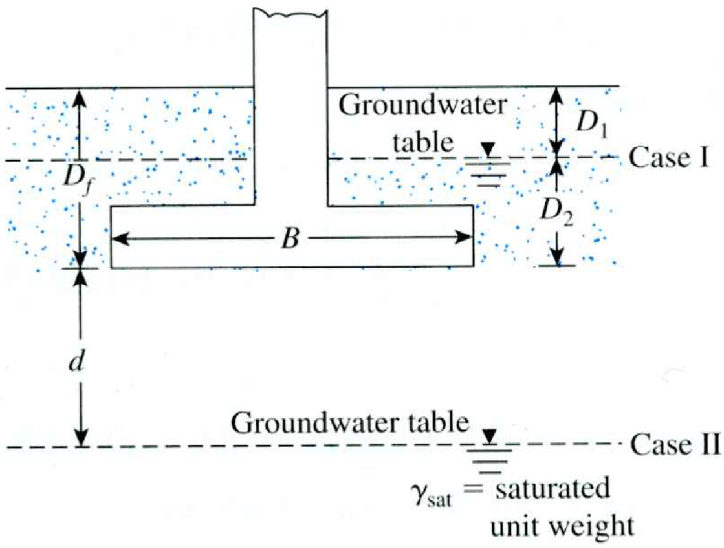



*Figure 3.8* Modification of bearing capacity equations for water table

## 3.6 The General Bearing Capacity Equation (Meyerhof Equation)

Terzagi's equations shortcomings:

- They don't deal with rectangular foundations (0<B/L<1).
- The equations do not take into account the shearing resistance along the failure surface in soil above the bottom of the foundation (as mentioned previously).
- The inclination of the load on the foundation is not considered (if exist).on the foundation may be inclined.

To account for all these shortcomings, **Meyerhof (1963)** suggested the following form of the general bearing capacity equation:

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B\gamma N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i} \qquad (3.15)$$

Where

c=Cohesion of the underlying soil

q=Effective stress at the level of the bottom of the foundation. γ=unit weight of the underlying soil

B=Width of footing (=diameter for a circular foundation).

$N_c$, $N_q$, $N_\gamma$=Bearing capacity factors

$F_{cs}$, $F_{qs}$, $F_{\gamma s}$=Shape factors

$F_{cd}$, $F_{qd}$, $F_{\gamma d}$=Depth factors

$F_{ci}$, $F_{qi}$, $F_{\gamma i}$=Inclination factors

**Notes:**

1. This equation is valid for both general and local shear failure.
2. This equation is similar to original equation for ultimate bearing capacity (Terzaghi's equation) which derived for continuous foundation . The shape, depth, and load inclination factors are empirical factors based on experimental data.

## Bearing Capacity Factors:

The basic nature of the failure surface in soil suggested by Terzaghi now appears to have been borne out by laboratory and field studies of bearing capacity (Vesic, 1973). However, the angle α shown in **Fig. 3.6** is closer to $45+\phi'/2$ than to $\phi'$. If this change is accepted, the values of $N_c$, $N_q$, and $N_\gamma$ for a given soil friction angle will also change from those given in **Table 3.1**. With $\alpha = 45+\phi'/2$, it can be shown that

$$N_q = \tan^2\left(45 + \frac{\phi'}{2}\right)e^{\pi \tan \phi'} \qquad (3.16)$$

and

$$N_c = (N_q - 1)\cot \phi' \qquad (3.17)$$

Equation (3.17) for $N_c$ was originally derived by Prandtl (1921), and Eq. (316) for $N_q$ was presented by Reissner (1924). Caquot and Kerisel (1953) and Vesic (1973) gave the relation for $N_\gamma$ as

$$N_\gamma = 2(N_q + 1)\tan\phi' \tag{3.18}$$

**Table 3.3** shows the variation of the preceding bearing capacity factors with soil friction angles.

*Table 3.3* Bearing Capacity Factors

| $\phi'$ | $N_c$ | $N_q$ | $N_\gamma$ | $\phi'$ | $N_c$ | $N_q$ | $N_\gamma$ |
|---|---|---|---|---|---|---|---|
| 0 | 5.14 | 1.00 | 0.00 | 26 | 22.25 | 11.85 | 12.54 |
| 1 | 5.38 | 1.09 | 0.07 | 27 | 23.94 | 13.20 | 14.47 |
| 2 | 5.63 | 1.20 | 0.15 | 28 | 25.80 | 14.72 | 16.72 |
| 3 | 5.90 | 1.31 | 0.24 | 29 | 27.86 | 16.44 | 19.34 |
| 4 | 6.19 | 1.43 | 0.34 | 30 | 30.14 | 18.40 | 22.40 |
| 5 | 6.49 | 1.57 | 0.45 | 31 | 32.67 | 20.63 | 25.99 |
| 6 | 6.81 | 1.72 | 0.57 | 32 | 35.49 | 23.18 | 30.22 |
| 7 | 7.16 | 1.88 | 0.71 | 33 | 38.64 | 26.09 | 35.19 |
| 8 | 7.53 | 2.06 | 0.86 | 34 | 42.16 | 29.44 | 41.06 |
| 9 | 7.92 | 2.25 | 1.03 | 35 | 46.12 | 33.30 | 48.03 |
| 10 | 8.35 | 2.47 | 1.22 | 36 | 50.59 | 37.75 | 56.31 |
| 11 | 8.80 | 2.71 | 1.44 | 37 | 55.63 | 42.92 | 66.19 |
| 12 | 9.28 | 2.97 | 1.69 | 38 | 61.35 | 48.93 | 78.03 |
| 13 | 9.81 | 3.26 | 1.97 | 39 | 67.87 | 55.96 | 92.25 |
| 14 | 10.37 | 3.59 | 2.29 | 40 | 75.31 | 64.20 | 109.41 |
| 15 | 10.98 | 3.94 | 2.65 | 41 | 83.86 | 73.90 | 130.22 |
| 16 | 11.63 | 4.34 | 3.06 | 42 | 93.71 | 85.38 | 155.55 |
| 17 | 12.34 | 4.77 | 3.53 | 43 | 105.11 | 99.02 | 186.54 |
| 18 | 13.10 | 5.26 | 4.07 | 44 | 118.37 | 115.31 | 224.64 |
| 19 | 13.93 | 5.80 | 4.68 | 45 | 133.88 | 134.88 | 271.76 |
| 20 | 14.83 | 6.40 | 5.39 | 46 | 152.10 | 158.51 | 330.35 |
| 21 | 15.82 | 7.07 | 6.20 | 47 | 173.64 | 187.21 | 403.67 |
| 22 | 16.88 | 7.82 | 7.13 | 48 | 199.26 | 222.31 | 496.01 |
| 23 | 18.05 | 8.66 | 8.20 | 49 | 229.93 | 265.51 | 613.16 |
| 24 | 19.32 | 9.60 | 9.44 | 50 | 266.89 | 319.07 | 762.89 |
| 25 | 20.72 | 10.66 | 10.88 | | | | |

## Shape, Depth, and Inclination Factors

Commonly used shape, depth, and inclination factors are given in Table 3.4.

*Table 3.4* Shape, Depth and Inclination Factors [DeBeer (1970); Hansen (1970); Meyerhof (1963); Meyerhof and Hanna (1981)]

| Factor | Relationship | Reference |
|---|---|---|
| Shape | $F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$<br>$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$<br>$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$ | DeBeer (1970) |
| Depth | $\frac{D_f}{B} \le 1$<br>For $\phi = 0$:<br>$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$<br>$F_{qd} = 1$<br>$F_{\gamma d} = 1$<br>For $\phi' > 0$:<br>$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$<br>$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$<br>$F_{\gamma d} = 1$<br>$\frac{D_f}{B} > 1$<br>For $\phi = 0$:<br>$F_{cd} = 1 + 0.4\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$<br>$F_{qd} = 1$<br>$F_{\gamma d} = 1$<br>For $\phi' > 0$:<br>$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$<br>$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$<br>$F_{\gamma d} = 1$ | Hansen (1970) |
| Inclination | $F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$<br>$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$<br>$\beta$ = inclination of the load on the foundation with respect to the vertical | Meyerhof (1963); Hanna and Meyerhof (1981) |

## 3.7 Eccentrically Loaded Foundation

If the load applied on the foundation is in the center of the foundation without eccentricity, the bearing capacity of the soil will be uniform at any point under the foundation (as shown in **Fig. 3.9**) because there is no any moments on the foundation, and the general equation for stress under the foundation is:

Stress= $Q/A \pm M_x\, y / I_x \pm M_y\, X / I_y$

In this case, the load is in the center of the foundation and there are no moments so,

Stress= Q/A (uniform at any point below the foundation)

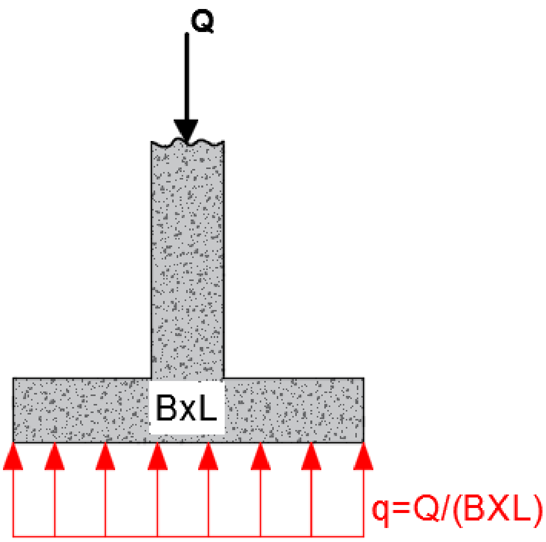



**Fig. 3.9 Non-Eccentricity loaded foundations**

However, in several cases, as with the base of a retaining wall or neighbor footing, the loads does not exist in the center, so foundations are subjected to moments in addition to the vertical load (as shown **Fig. 3.10**). In such cases, the distribution of pressure by the foundation on the soil is not uniform because there is a moment applied on the foundation and the stress under the foundation will be calculated from the general relation:

Stress= Stress= $Q/A \pm M_x\, y / I_x \pm M_y\, X / I_y$ (in case of two way eccentricity)

But, in this section we deal with (one way eccentricity), the equation will be:

Stress= $Q/A \pm M\, c / I$

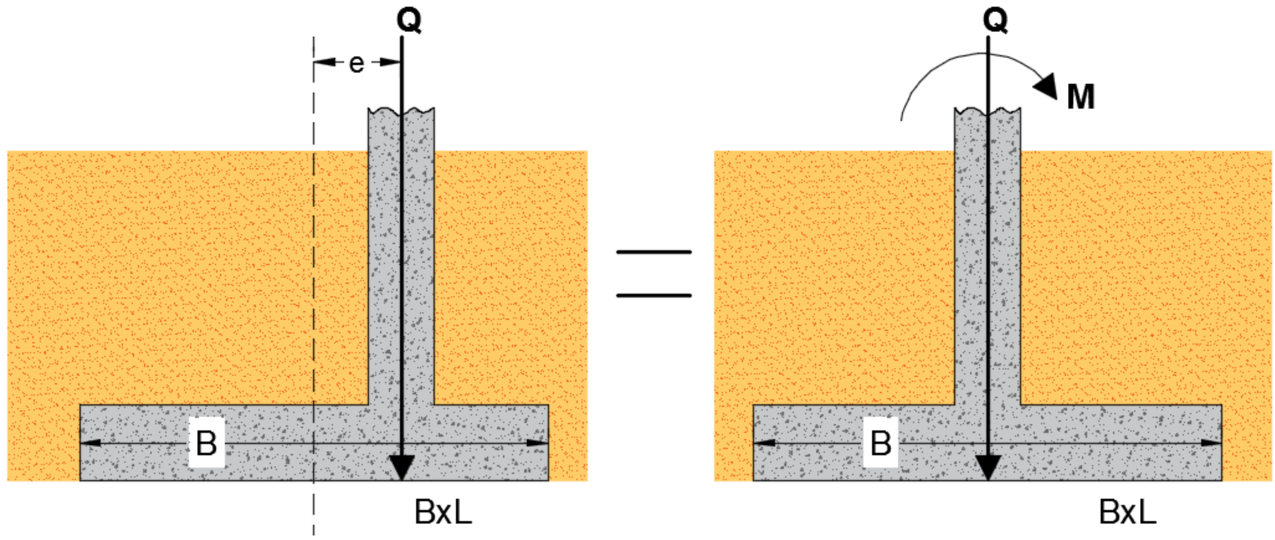
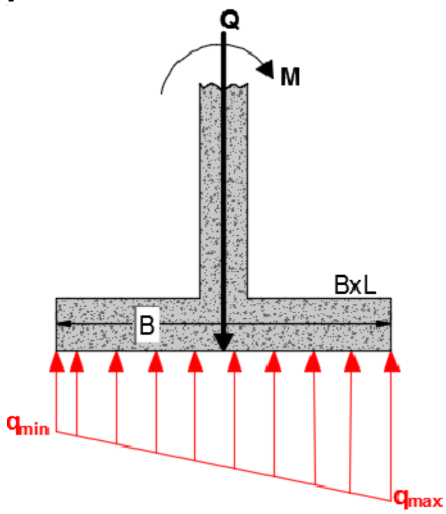



**Fig. 3.10 Eccentricity loaded foundation**

Since the pressure under the foundation is not uniform, there are maximum and minimum pressures (under the two edges of the foundation) and we concerned about calculating these two pressures.

**General equation for calculating maximum and minimum pressure:**




Assume the eccentricity is in direction of (B)

Stress=q= Q/A ± M c / I

A= B × L

M= Q × e

c= B/2 (maximum distance from the center)

I= ($B^3$×L) /12 (I is about the axis that resists the moment)

Substitute in the equation, the equation will be:

q= Q/(B×L) ± (Q×e×B)/(( 2$B^3$×L)/12)→q=Q/(B×L)± (6eQ/($B^2$L)→

$q = \frac{Q}{B \times L}\left(1 \mp \frac{6e}{B}\right)$ **General Equation**

Now, there are three cases for calculating maximum and minimum pressures according to the values of (e and B/6 ) **to maintain minimum pressure always≥**0

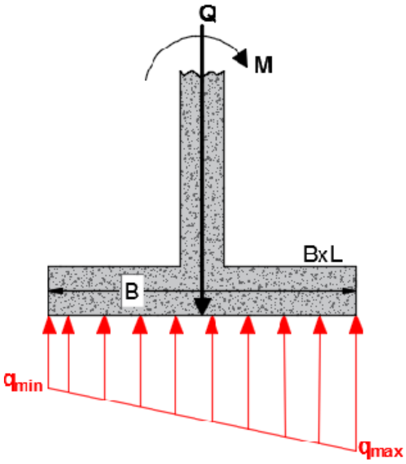
**Case I. (For e<B/6)**:

$$q_{max} = \frac{Q}{B \times L}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{Q}{B \times L}(1 - \frac{6e}{B})$$

Note that when e< B/6 the value of $q_{min}$ will be positive (i.e. compression).

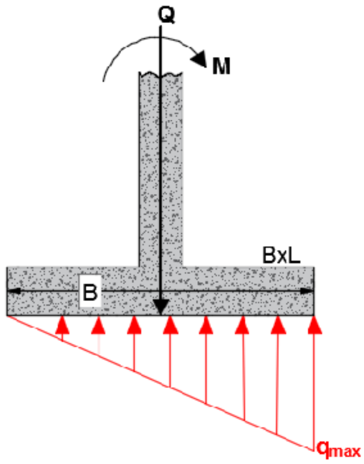
If eccentricity in (L) direction: (For e<L/6):




$$q_{max} = \frac{Q}{B \times L}(1 + \frac{6e}{L})$$

$$q_{min} = \frac{Q}{B \times L}(1 - \frac{6e}{L})$$

**Case II. (For e=B/6)**:




$$q_{max} = \frac{Q}{B \times L}(1 + \frac{6e}{B})$$

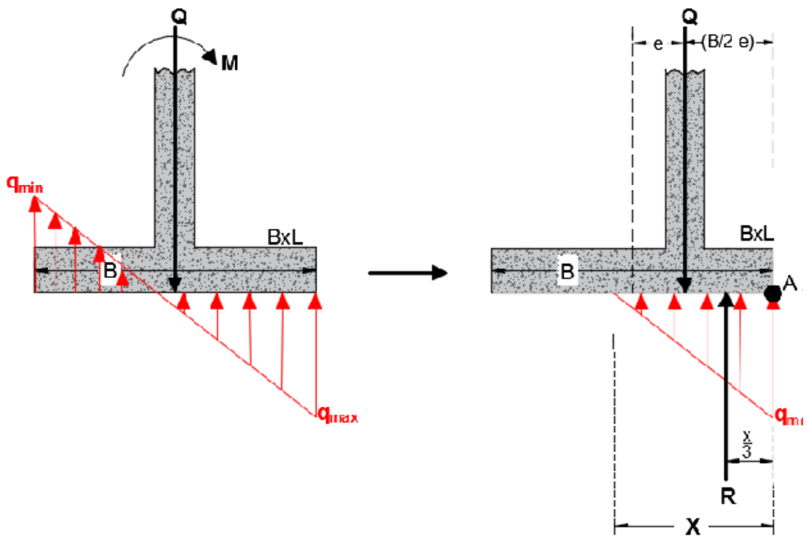
$$q_{min} = \frac{Q}{B \times L}(1 - 1) = 0$$

**Case III. (For e >B/6)**:

For $e > B/6$, $q_{min}$ will be negative, which means that tension will develop. Because soil cannot take any tension, there will then be a separation between the foundation and the soil underlying it. The nature of the pressure distribution on the soil will be as shown in Figure below. The value of $q_{max}$ is then

$$q_{max} = \frac{4Q}{3L(B - 2e)}$$

The exact distribution of pressure is difficult to estimate.




# 3.8 Ultimate Bearing Capacity under Eccentric Loading - One-Way Eccentricity

## Effective Area Method (Meyerhoff, 1953)

In 1953, Meyerhof proposed a theory that is generally referred to as the *effective area method*.

The following is a step-by-step procedure for determining the ultimate load that the soil can support and the factor of safety against bearing capacity failure:

***Step 1.*** Determine the effective dimensions of the foundation (**Fig. 3.10**):

$B'$ = effective width = $B$ - 2$e$

$L'$ = effective length = $L$

Note that if the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to L - 2e. The value of B' would equal B. The smaller of the two dimensions (i.e., L' and B') is the effective width of the foundation.

***Step 2.*** Use Eq. (3.15) for the ultimate bearing capacity:

$$q_u=cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + 0.5B'\,\gamma\, N_\gamma\, F_{\gamma s}\, F_{\gamma d}\, F_{\gamma i}$$

To evaluate $F_{cs}$, $F_{qs}$, and $F_{\gamma s}$, use the relationships given in Table 3.4 with *effective length* and *effective width* dimensions instead of $L$ and $B$, respectively. To determine $F_{cd}$, $F_{qd}$, and $F_{\gamma d}$, use the relationships given in Table 3.4. However, do not replace $B$ with $B'$.

***Step 3.*** The total ultimate load that the foundation can sustain is

$$Q_{ult} = q'_u \overbrace{(B')(L')}^{A'} \tag{3.19}$$

where $A'$= effective area.

***Step 4.*** The factor of safety against bearing capacity failure is

$$FS = \frac{Q_{ult}}{Q}$$

***Step 5.*** Check the factor of safety against $q_{max}$ , or FS= $q'_u$ / $q_{max}$.

It is important to note that $q'_u$ is the ultimate bearing capacity of a foundation of width $B' = B - 2e$ with a centric load (**Fig. 3.10a**). However, the actual distribution of soil reaction at ultimate load will be of the type shown in **Fig. 3.10b**. In Figure **3.10b**, $q_{u(e)}$ is the average load per unit area of the foundation. Thus,

$q_{u(e)}$ = [$q'_u$(B-2e)] / B

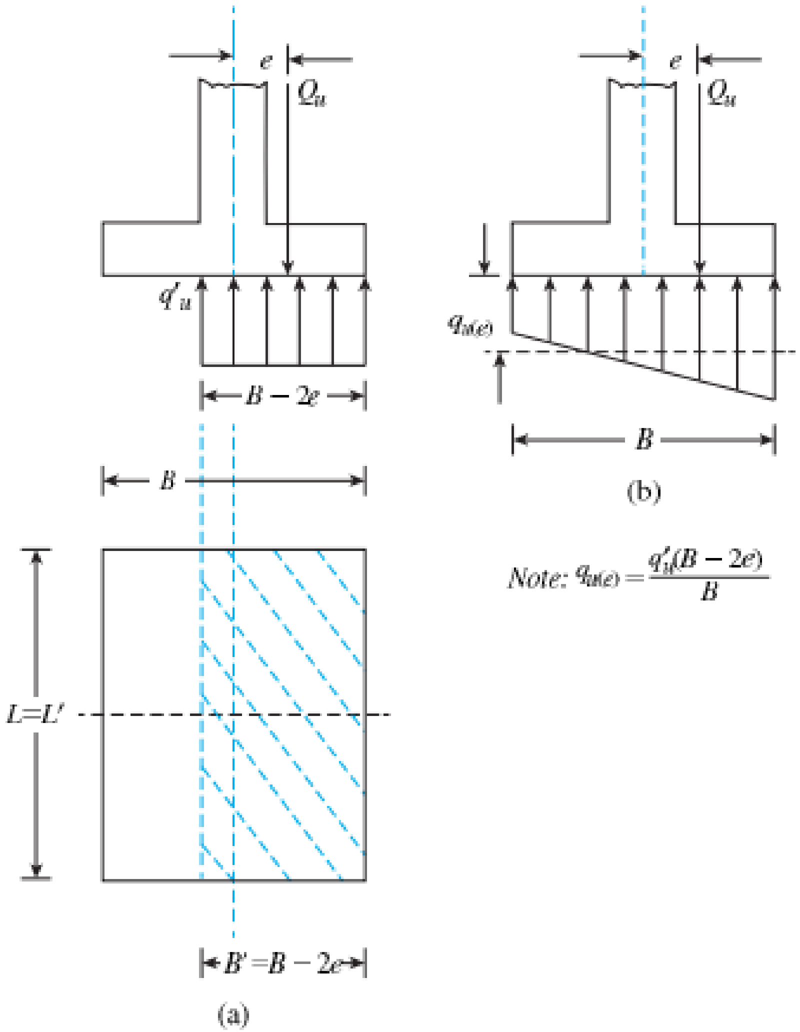



**Fig. 3.10 Definition of $q'_u$ and $q_{u(e)}$**

## 3.9 Bearing Capacity of Layered Soils: Stronger Soil Underlain by Weaker Soil (c'-ϕ' soil)

- The bearing capacity equations presented in preceding sections involve cases in which the soil supporting the foundation is homogeneous and extends to a considerable depth.
- The cohesion, angle of friction, and unit weight of soil were assumed to remain constant for the bearing capacity analysis. However, in practice, layered soil profiles are often encountered. In such instances, the failure surface at ultimate load may extend through two or more soil layers, and a determination of the ultimate bearing capacity in layered soils can be made in only a limited number of cases.
- This section features the procedure for estimating the bearing capacity for layered soils proposed by Meyerhof and Hanna (1978) and Meyerhof (1974) in c'-ϕ' soil.
- .Figure 3.11 shows a shallow, continuous foundation supported by a stronger soil layer, underlain by a weaker soil that extends to a great depth.
- For the two soil layers, the physical parameters are as follows:

| | Soil properties | | |
|---|---|---|---|
| **Layer** | **Unit weight** | **Friction angle** | **Cohesion** |
| Top | $\gamma_1$ | $\phi'_1$ | $c'_1$ |
| Bottom | $\gamma_2$ | $\phi'_2$ | $c'_2$ |

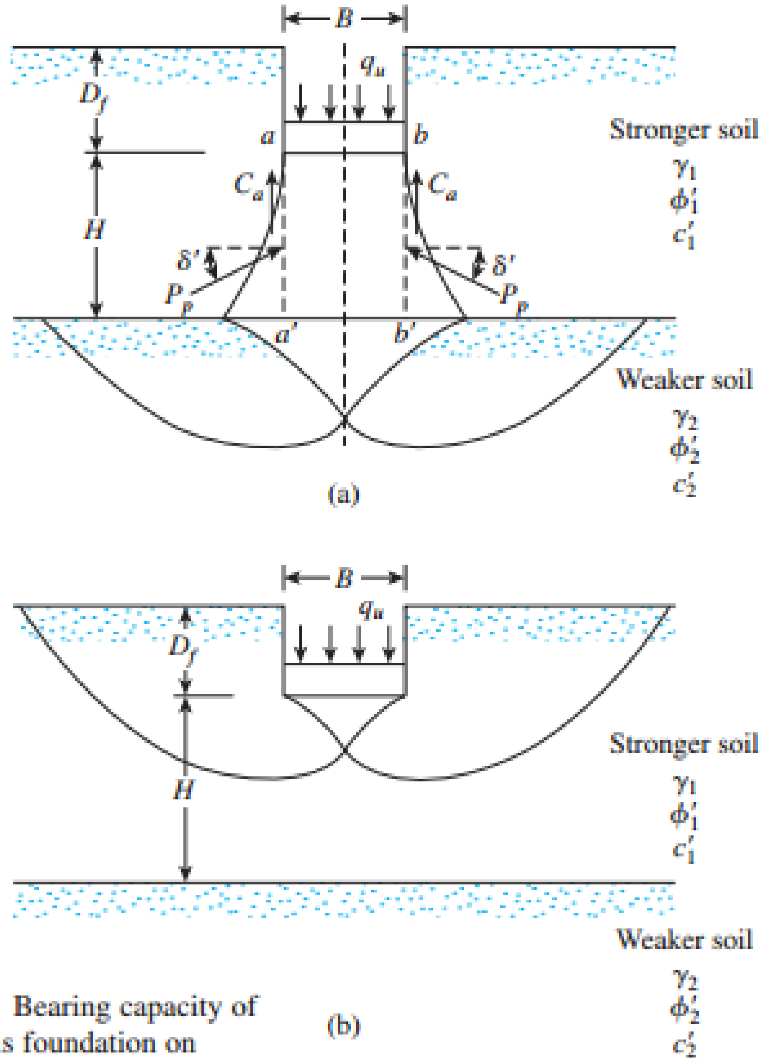



**Figure 3.11** Bearing capacity of a continuous foundation on layered soil

- At ultimate load per unit area ($q_u$ ) , the failure surface in soil will be as shown in the figure.
- If the depth H is relatively small compared with the foundation width B, a punching shear failure will occur in the top soil layer, followed by a general shear failure in the bottom soil layer. This is shown in Figure 3.11a.
- If the depth H is relatively large, then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity. This is shown in Figure 3.11b.

- The ultimate bearing capacity for this problem, as shown in Figure 3.11a, can be given as

$$q_u = q_b + \frac{2(C_a + P_p \sin \delta')}{B} - \gamma_1 H$$

where

$B$ = width of the foundation
$C_a$ = adhesive force
$P_p$ = passive force per unit length of the faces $aa'$ and $bb'$
$q_b$ = bearing capacity of the bottom soil layer
$\delta'$ = inclination of the passive force $P_p$ with the horizontal

**Note that, in the above Equation,**

$$C_a = c'_a H$$

where $c'_a$ = adhesion.

**Equation above can be simplified to the form**

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_{pH} \tan \delta'}{B} - \gamma_1 H$$

where $K_{pH}$ = horizontal component of passive earth pressure coefficient.

However, let

$$K_{pH} \tan \delta' = K_s \tan \phi'_1$$

where $K_s$ = punching shear coefficient. Then,

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan \phi'_1}{B} - \gamma_1 H$$

The punching shear coefficient, $K_s$, is a function of $q_2/q_1$ and $\phi'_1$, or, specifically,

$$K_s = f\left(\frac{q_2}{q_1}, \phi'_1\right)$$

Note that $q_1$ and $q_2$ are the ultimate bearing capacities of a continuous foundation of width B under vertical load on the surfaces of homogeneous thick beds of upper and lower soil, or

$$q_1 = c'_1 N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

and

$$q_2 = c'_2 N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

where

$N_{c(1)}, N_{\gamma(1)}$ = bearing capacity factors for friction angle $\phi'_1$ (Table 3.3)
$N_{c(2)}, N_{\gamma(2)}$ = bearing capacity factors for friction angle $\phi'_2$ (Table 3.3)

Observe that, for the top layer to be a stronger soil, $q_2/q_1$ should be less than unity.

The variation of $K_s$ with $q_2/q_1$ and $\phi'_1$ is shown in Figure 3.12. The variation of $c'_a/c'_1$ with $q_2/q_1$ is shown in Figure 3.13. If the height $H$ is relatively large, then the failure surface in soil will be completely located in the stronger upper-soil layer (Figure3.11b). For this case,

$$q_u = q_t = c'_1 N_{c(1)} + q N_{q(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}.$$

where $N_{c(1)}$, $N_{q(1)}$, and $N_{\gamma(1)}$ = bearing capacity factors for $\phi' = \phi'_1$ (Table 3.3) and $q = \gamma_1 D_f$.

**Combining Equations above yields**

$$q_u = q_b + \frac{2c'_a H}{B} + \gamma_1 H^2\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi'_1}{B} - \gamma_1 H \leq q_t$$

**For rectangular foundations, the preceding equation can be extended to the form**

$$q_u = q_b + \left(1 + \frac{B}{L}\right)\left(\frac{2c'_a H}{B}\right) + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan\phi'_1}{B}\right) - \gamma_1 H \leq q_t$$

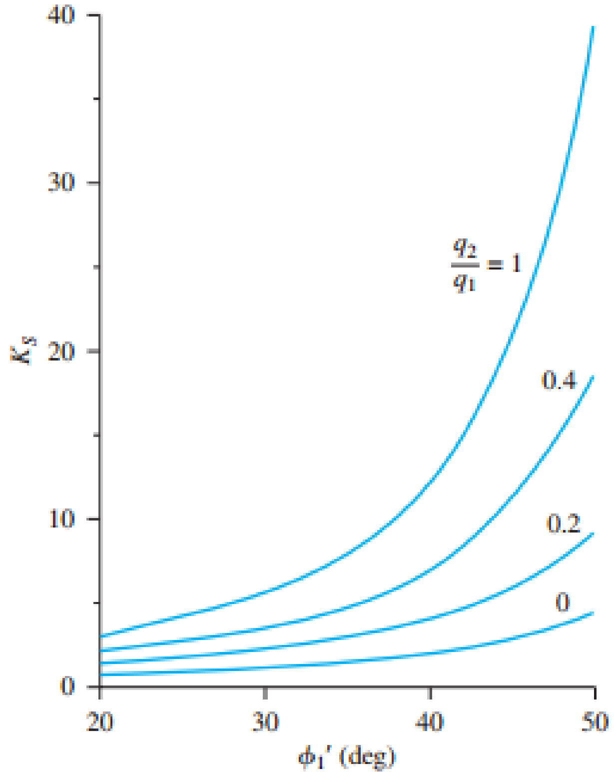



**Figure 3.12** Meyerhof and Hanna's punching shear coefficient $K_s$

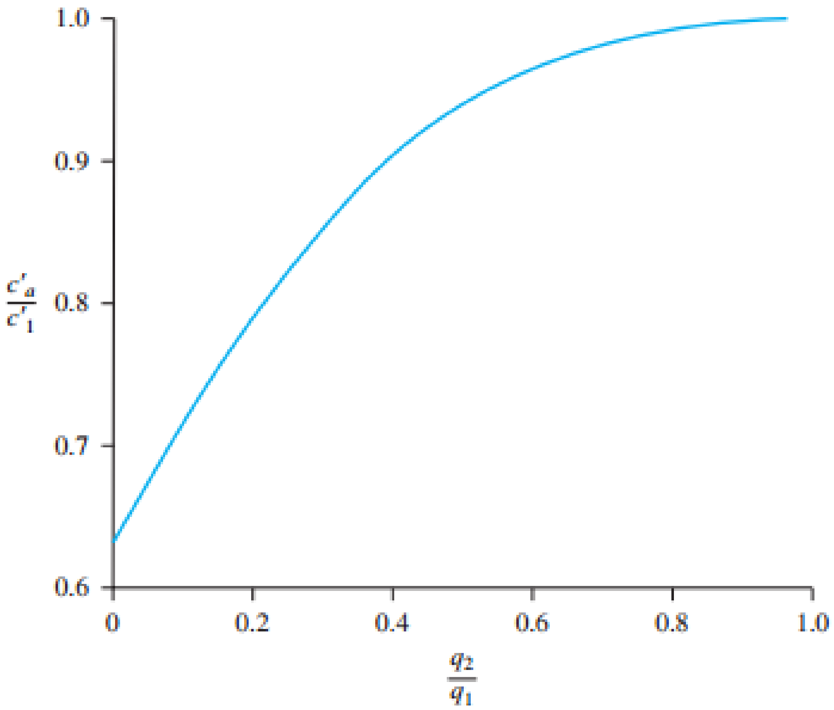



**Figure 3.13** Variation of $c'_a/c'_1$ with $q_2/q_1$ based on the theory of Meyerhof and Hanna (1978)

where

$$q_b = c_2' N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2} \gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}$$

and

$$q_t = c_1' N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2} \gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

in which

$F_{cs(1)}, F_{qs(1)}, F_{\gamma s(1)}$ = shape factors with respect to top soil layer

$F_{cs(2)}, F_{qs(2)}, F_{\gamma s(2)}$ = shape factors with respect to bottom soil layer

## Special Cases

1. *Top layer is strong sand and bottom layer is saturated soft clay* ($\phi_2 = 0$).

$$q_b = \left(1 + 0.2 \frac{B}{L}\right) 5.14 c_{u(2)} + \gamma_1 (D_f + H)$$

and

$$q_t = \gamma_1 D_f N_{q(1)} F_{qs(1)} + \tfrac{1}{2} \gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

Hence,

$$q_u = \left(1 + 0.2 \frac{B}{L}\right) 5.14 c_{u(2)} + \gamma_1 H^2 \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right) \frac{K_s \tan \phi_1'}{B} + \gamma_1 D_f \leq \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2} \gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}$$

For a determination of $K_s$ from Figure **3.12**

$$\frac{q_2}{q_1} = \frac{c_{u(2)} N_{c(2)}}{\frac{1}{2} \gamma_1 B N_{\gamma(1)}} = \frac{5.14 c_{u(2)}}{0.5 \gamma_1 B N_{\gamma(1)}}$$

2. *Top layer is stronger sand and bottom layer is weaker sand* ($c_1' = 0$, $c_2' = 0$). The ultimate bearing capacity can be given as

$$q_u = \left[\gamma_1(D_f + H)N_{q(2)}F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)}F_{\gamma s(2)}\right] + \gamma_1 H^2\left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\frac{K_s \tan\phi_1'}{B} - \gamma_1 H \leq q_t$$

where

$$q_t = \gamma_1 D_f N_{q(1)}F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)}F_{\gamma s(1)}$$

Then

$$\frac{q_2}{q_1} = \frac{\frac{1}{2}\gamma_2 B N_{\gamma(2)}}{\frac{1}{2}\gamma_1 B N_{\gamma(1)}} = \frac{\gamma_2 N_{\gamma(2)}}{\gamma_1 N_{\gamma(1)}}$$

3. *Top layer is stronger saturated clay* ($\phi_1 = 0$) *and bottom layer is weaker saturated clay* ($\phi_2 = 0$). The ultimate bearing capacity can be given as

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(2)} + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 D_f \leq q_t$$

where

$$q_t = \left(1 + 0.2\frac{B}{L}\right)5.14c_{u(1)} + \gamma_1 D_f$$

and $c_{u(1)}$ and $c_{u(2)}$ are undrained cohesions. For this case,

$$\frac{q_2}{q_1} = \frac{5.14c_{u(2)}}{5.14c_{u(1)}} = \frac{c_{u(2)}}{c_{u(1)}}$$

## 3.10 Foundations on Rock

- On some occasions, shallow foundations may have to be built on rocks, as shown in Figure 3.14.
- For estimation of the ultimate bearing capacity of shallow foundations on rock, we may use Terzaghi's bearing capacity equations with the bearing capacity factors given here (Stagg and Zienkiewicz, 1968; Bowles, 1996):

$$N_c = 5\tan^4\left(45 + \frac{\phi'}{2}\right)$$

$$N_q = \tan^6\left(45 + \frac{\phi'}{2}\right)$$

$$N_\gamma = N_q + 1$$

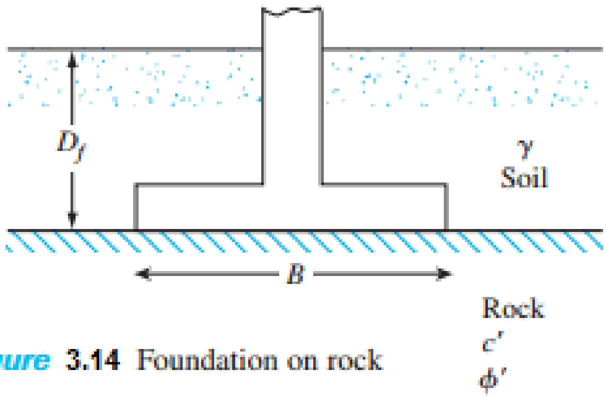



**Figure 3.14** Foundation on rock

For rocks, the magnitude of the cohesion intercept, $c'$, can be expressed as

$$q_{uc} = 2c'\tan\left(45 + \frac{\phi'}{2}\right)$$

where

$q_{uc}$ = unconfined compression strength of rock
$\phi'$ = angle of friction

- The unconfined compression strength and the friction angle of rocks can vary widely. Table 3.5 gives a general range of $q_{uc}$ for various types of rocks. It is important to keep in mind that the magnitude of $q_{uc}$ and $\phi'$ (hence $c'$) reported from laboratory tests are for intact rock specimens. It does not account for the effect of discontinuities.
- To account for discontinuities, Bowles (1996) suggested that the ultimate bearing capacity $q_u$ should be modified as

$$q_{u(\text{modified})} = q_u(\text{RQD})^2$$

where RQD = rock quality designation

In any case, the upper limit of the allowable bearing capacity should not exceed $f_c'$ (28-day compressive strength of concrete).

**Table 3.5** Range of the Unconfined Compression Strength of Various Types of Rocks

| Rock type | $q_{uc}$ MN/m² | $q_{uc}$ kip/in² | $\phi'$ (deg) |
|---|---|---|---|
| Granite | 65–250 | 9.5–36 | 45–55 |
| Limestone | 30–150 | 4–22 | 35–45 |
| Sandstone | 25–130 | 3.5–19 | 30–45 |
| Shale | 5–40 | 0.75–6 | 15–30 |

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# CHAPTER FOUR

# SETTLEMENT OF SHALLOW FOUNDATIONS

LECTURE
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2019 - 2020

## *VERTICAL STRESS INCREASE IN SOIL*

# 4.1. Introduction

It was mentioned in Chapter 3 that, in many foundation may control the allowable bear may be controlled by local building codes. the smaller of the following two conditions:

$$q_{\text{all}} = \begin{cases} \dfrac{q_u}{\text{FS}} \\ \text{or} \\ q_{\text{allowable settlement}} \end{cases}$$

For the calculation of foundation settlement, it is required that we estimate the vertical stress increase in the soil mass due to the net load applied on the foundation. Hence, in this chapter, we will discuss the general principles for estimating the increase of vertical stress at various depths in soil due to the application of (on the ground surface).

●A point load

●Circularly loaded area

●Vertical line load

●Strip load

●Rectangularly loaded area

# 4.2 Stress Due to a Concentrated Load

In 1885, Boussinesq developed the mathematical relationships for determining the normal and shear stresses at any point inside homogeneous, elastic, and isotropic mediums due to a concentrated point load located at the surface, as shown in Figure 4.1. According to his analysis, the vertical stress increase at point A caused by a point load of magnitude P is given by

$$\Delta\sigma = \frac{3P}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \tag{4.1}$$

where

$r = \sqrt{x^2 + y^2}$

$x, y, z$ = coordinates of the point $A$

.Note that Eq. (4.1) is not a function of Poisson's ratio of the soil

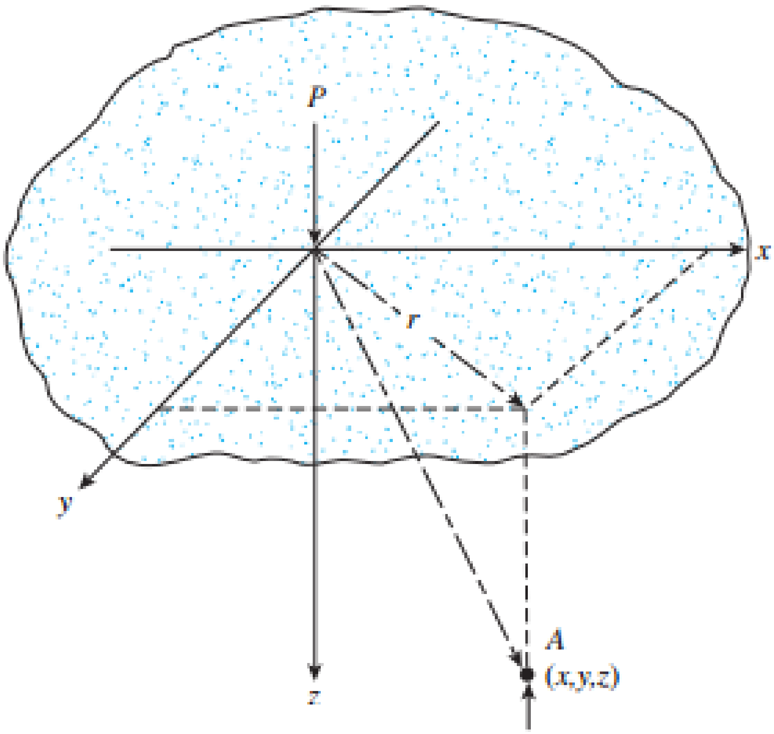



**Figure 4.1** Vertical stress at a point $A$ caused by a point load on the surface

## 4.3 Stress Due to a Circularly Loaded Area

The Boussinesq equation (4.1) can also be used to determine the vertical stress below the center of a flexible circularly loaded area, as shown in Figure 4.2. Let the radius of the loaded area be B/2, and let $q_o$ be the uniformly distributed load per unit area. To determine the stress increase at a point A, located at a depth z below the center of the circular area, consider an elemental area on the circle. The load on this elemental area may be taken to be a point load and expressed as $q_o\, r\, d\theta\, dr$. The stress increase at A caused by this load can be determined from Eq. (4.1) as

$$d\sigma = \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}} \qquad (4.2)$$

The total increase in stress caused by the entire loaded area may be obtained by integrating Eq. (4.2), or

$$\Delta\sigma = \int d\sigma = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=B/2} \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1 + \left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$= q_o\left\{1 - \frac{1}{\left[1 + \left(\frac{B}{2z}\right)^2\right]^{3/2}}\right\} \qquad (4.3)$$

Similar integrations could be performed to obtain the vertical stress increase at A', located a distance r from the center of the loaded area at a depth z (Ahlvin and Ulery, 1962). Table 4.1 gives the variation of $\Delta\sigma/q_o$ with $r/(B/2)$ and $z/(B/2)$ [for $0 \le r/(B/2) \le 1$]. Note that the variation of $\Delta\sigma/q_o$ with depth at $r/(B/2) = 0$ can be obtained from Eq. (4.3).

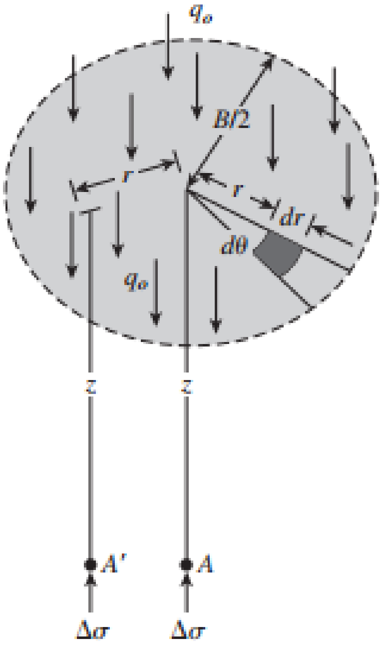



**Figure 4.2** Increase in pressure under a uniformly loaded flexible circular area

**Table 4.1** Variation of $\Delta\sigma/q_o$ for a Uniformly Loaded Flexible Circular Area

| | $r/(B/2)$ | | | | | |
|---|---|---|---|---|---|---|
| $z/(B/2)$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.1 | 0.999 | 0.999 | 0.998 | 0.996 | 0.976 | 0.484 |
| 0.2 | 0.992 | 0.991 | 0.987 | 0.970 | 0.890 | 0.468 |
| 0.3 | 0.976 | 0.973 | 0.963 | 0.922 | 0.793 | 0.451 |
| 0.4 | 0.949 | 0.943 | 0.920 | 0.860 | 0.712 | 0.435 |
| 0.5 | 0.911 | 0.902 | 0.869 | 0.796 | 0.646 | 0.417 |
| 0.6 | 0.864 | 0.852 | 0.814 | 0.732 | 0.591 | 0.400 |
| 0.7 | 0.811 | 0.798 | 0.756 | 0.674 | 0.545 | 0.367 |
| 0.8 | 0.756 | 0.743 | 0.699 | 0.619 | 0.504 | 0.366 |
| 0.9 | 0.701 | 0.688 | 0.644 | 0.570 | 0.467 | 0.348 |
| 1.0 | 0.646 | 0.633 | 0.591 | 0.525 | 0.434 | 0.332 |
| 1.2 | 0.546 | 0.535 | 0.501 | 0.447 | 0.377 | 0.300 |
| 1.5 | 0.424 | 0.416 | 0.392 | 0.355 | 0.308 | 0.256 |
| 2.0 | 0.286 | 0.286 | 0.268 | 0.248 | 0.224 | 0.196 |
| 2.5 | 0.200 | 0.197 | 0.191 | 0.180 | 0.167 | 0.151 |
| 3.0 | 0.146 | 0.145 | 0.141 | 0.135 | 0.127 | 0.118 |
| 4.0 | 0.087 | 0.086 | 0.085 | 0.082 | 0.080 | 0.075 |

## 4.4 Stress Due to a Line Load

Figure 4.3 shows a vertical flexible line load of infinite length that has an intensity q/unit length on the surface of a semi-infinite soil mass. The vertical stress increase, Δσ, inside the soil mass can be determined by using the principles of the theory of elasticity, or

$$\Delta\sigma = \frac{2qz^3}{\pi(x^2 + z^2)^2} \tag{4.4}$$

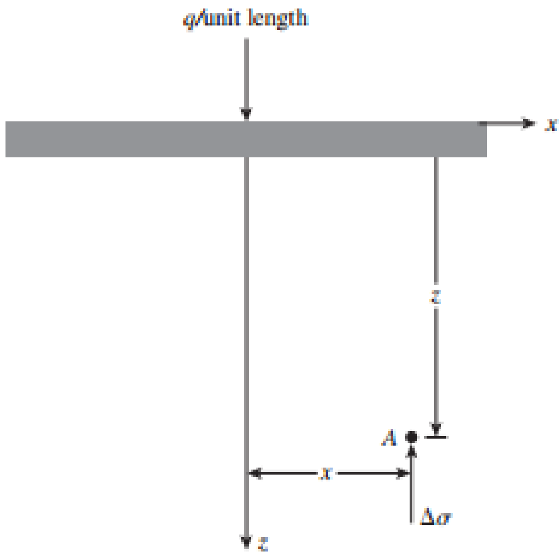



**Figure 4.3** Line load over the surface of a semi-infinite soil mass

This equation can be rewritten as

$$\Delta\sigma = \frac{2q}{\pi z[(x/z)^2 + 1]^2}$$

$$\frac{\Delta\sigma}{(q/z)} = \frac{2}{\pi[(x/z)^2 + 1]^2} \qquad (4.5)$$

Note that Eq. (4.5) is in a nondimensional form. Using this variation of Δσ/(q/z) with x/z. This is given in Table 4.2.

**Table 4.2** Variation of $\Delta\sigma/(q/z)$ with $x/z$ [Eq. (**4.5**)]

| *x/z* | $\Delta\sigma/(q/z)$ | *x/z* | $\Delta\sigma/(q/z)$ |
|---|---|---|---|
| 0 | 0.637 | 1.3 | 0.088 |
| 0.1 | 0.624 | 1.4 | 0.073 |
| 0.2 | 0.589 | 1.5 | 0.060 |
| 0.3 | 0.536 | 1.6 | 0.050 |
| 0.4 | 0.473 | 1.7 | 0.042 |
| 0.5 | 0.407 | 1.8 | 0.035 |
| 0.6 | 0.344 | 1.9 | 0.030 |
| 0.7 | 0.287 | 2.0 | 0.025 |
| 0.8 | 0.237 | 2.2 | 0.019 |
| 0.9 | 0.194 | 2.4 | 0.014 |
| 1.0 | 0.159 | 2.6 | 0.011 |
| 1.1 | 0.130 | 2.8 | 0.008 |
| 1.2 | 0.107 | 3.0 | 0.006 |

## 4.5 Stress below a Rectangular Area

The integration technique of Boussinesq's equation also allows the vertical stress at any point A below the corner of a flexible rectangular loaded area to be evaluated. (See Figure 4.5.) To do so, consider an elementary area dA= dx dy on the flexible loaded area. If the load per unit area is $q_o$, the total load on the elemental area is

$$dP = q_o\, dx\, dy \qquad (4.8)$$

The total stress increase $\Delta\sigma$ caused by the entire loaded area at point A may now be obtained by integrating the preceding equation:

$$\Delta\sigma = \int_{y=0}^{L}\int_{x=0}^{B} \frac{3q_o\,(dx\,dy)z^3}{2\pi(x^2+y^2+z^2)^{5/2}} = q_o I \qquad (4.9)$$

Here,

$$I = \text{influence factor} = \frac{1}{4\pi}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right) \qquad (4.10)$$

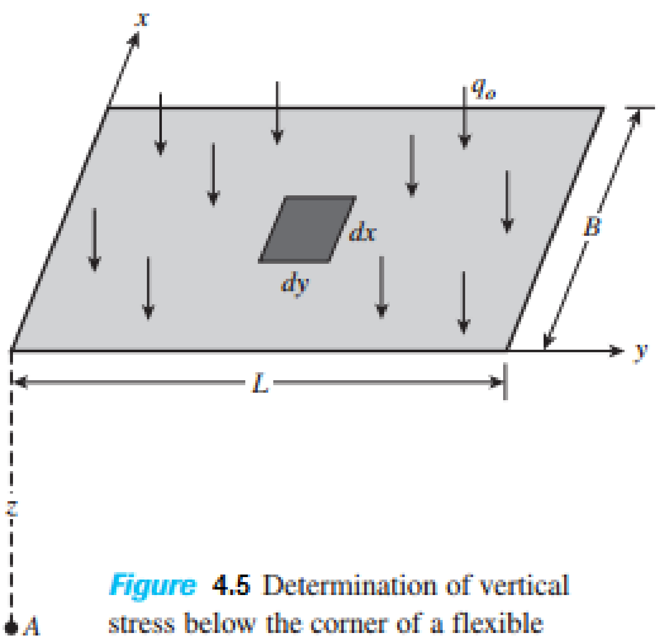



**Figure 4.5** Determination of vertical stress below the corner of a flexible rectangular loaded area

where

$$m = \frac{B}{z} \qquad (4.11)$$

and

$$n = \frac{L}{z} \qquad (4.12)$$

The arctangent term in Eq. **(4.10)** must be a positive angle in radians. When $m^2 + n^2 + 1 < m^2n^2$, it becomes a negative angle. So a term $\pi$ should be added to that angle. The variations of the influence values with $m$ and $n$ are given in Table **4.4.**

The stress increase at any point below a rectangular loaded area can also be found by using Eq. (4.9) in conjunction with Figure 4.6. To determine the stress at a depth z below point O, divide the loaded area into four rectangles, with O the corner common to each. Then use Eq. (6.9) to calculate the increase in stress at a depth z below O caused by each rectangular area. The total stress increase caused by the entire loaded area may now be expressed as

$$\Delta\sigma = q_o (I_1 + I_2 + I_3 + I_4) \qquad (4.13)$$

where $I_1$, $I_2$, $I_3$, and $I_4$ = the influence values of rectangles 1, 2, 3, and 4, respectively.

In most cases, the vertical stress below the center of a rectangular area is of importance. This can be given by the relationship

$$\Delta\sigma = q_o I_c \qquad (4.14)$$

where

$$I_c = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}} \frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right]$$

$$m_1 = \frac{L}{B}$$

$$n_1 = \frac{z}{\left(\frac{B}{2}\right)}$$

The variation of $I_c$ with $m_1$ and $n_1$ is given in Table 4.5.

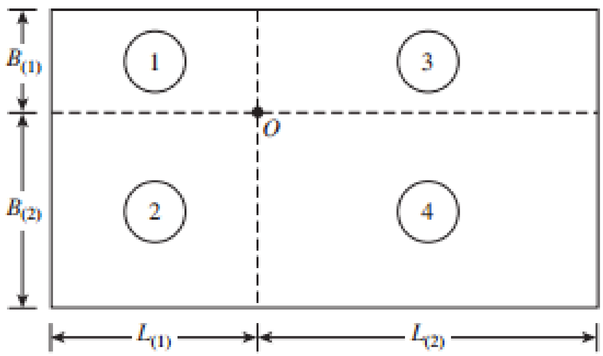



**Figure 4.6** Stress below any point of a loaded flexible rectangular area

**Table 4.4** Variation of Influence Value $I$ [Eq. (6.10)][a]

| | *n* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***m*** | **0.1** | **0.2** | **0.3** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **1.0** | **1.2** | **1.4** |
| 0.1 | 0.00470 | 0.00917 | 0.01323 | 0.01678 | 0.01978 | 0.02223 | 0.02420 | 0.02576 | 0.02698 | 0.02794 | 0.02926 | 0.03007 |
| 0.2 | 0.00917 | 0.01790 | 0.02585 | 0.03280 | 0.03866 | 0.04348 | 0.04735 | 0.05042 | 0.05283 | 0.05471 | 0.05733 | 0.05894 |
| 0.3 | 0.01323 | 0.02585 | 0.03735 | 0.04742 | 0.05593 | 0.06294 | 0.06858 | 0.07308 | 0.07661 | 0.07938 | 0.08323 | 0.08561 |
| 0.4 | 0.01678 | 0.03280 | 0.04742 | 0.06024 | 0.07111 | 0.08009 | 0.08734 | 0.09314 | 0.09770 | 0.10129 | 0.10631 | 0.10941 |
| 0.5 | 0.01978 | 0.03866 | 0.05593 | 0.07111 | 0.08403 | 0.09473 | 0.10340 | 0.11035 | 0.11584 | 0.12018 | 0.12626 | 0.13003 |
| 0.6 | 0.02223 | 0.04348 | 0.06294 | 0.08009 | 0.09473 | 0.10688 | 0.11679 | 0.12474 | 0.13105 | 0.13605 | 0.14309 | 0.14749 |
| 0.7 | 0.02420 | 0.04735 | 0.06858 | 0.08734 | 0.10340 | 0.11679 | 0.12772 | 0.13653 | 0.14356 | 0.14914 | 0.15703 | 0.16199 |
| 0.8 | 0.02576 | 0.05042 | 0.07308 | 0.09314 | 0.11035 | 0.12474 | 0.13653 | 0.14607 | 0.15371 | 0.15978 | 0.16843 | 0.17389 |
| 0.9 | 0.02698 | 0.05283 | 0.07661 | 0.09770 | 0.11584 | 0.13105 | 0.14356 | 0.15371 | 0.16185 | 0.16835 | 0.17766 | 0.18357 |
| 1.0 | 0.02794 | 0.05471 | 0.07938 | 0.10129 | 0.12018 | 0.13605 | 0.14914 | 0.15978 | 0.16835 | 0.17522 | 0.18508 | 0.19139 |
| 1.2 | 0.02926 | 0.05733 | 0.08323 | 0.10631 | 0.12626 | 0.14309 | 0.15703 | 0.16843 | 0.17766 | 0.18508 | 0.19584 | 0.20278 |
| 1.4 | 0.03007 | 0.05894 | 0.08561 | 0.10941 | 0.13003 | 0.14749 | 0.16199 | 0.17389 | 0.18357 | 0.19139 | 0.20278 | 0.21020 |
| 1.6 | 0.03058 | 0.05994 | 0.08709 | 0.11135 | 0.13241 | 0.15028 | 0.16515 | 0.17739 | 0.18737 | 0.19546 | 0.20731 | 0.21510 |
| 1.8 | 0.03090 | 0.06058 | 0.08804 | 0.11260 | 0.13395 | 0.15207 | 0.16720 | 0.17967 | 0.18986 | 0.19814 | 0.21032 | 0.21836 |
| 2.0 | 0.03111 | 0.06100 | 0.08867 | 0.11342 | 0.13496 | 0.15326 | 0.16856 | 0.18119 | 0.19152 | 0.19994 | 0.21235 | 0.22058 |
| 2.5 | 0.03138 | 0.06155 | 0.08948 | 0.11450 | 0.13628 | 0.15483 | 0.17036 | 0.18321 | 0.19375 | 0.20236 | 0.21512 | 0.22364 |
| 3.0 | 0.03150 | 0.06178 | 0.08982 | 0.11495 | 0.13684 | 0.15550 | 0.17113 | 0.18407 | 0.19470 | 0.20341 | 0.21633 | 0.22499 |
| 4.0 | 0.03158 | 0.06194 | 0.09007 | 0.11527 | 0.13724 | 0.15598 | 0.17168 | 0.18469 | 0.19540 | 0.20417 | 0.21722 | 0.22600 |
| 5.0 | 0.03160 | 0.06199 | 0.09014 | 0.11537 | 0.13737 | 0.15612 | 0.17185 | 0.18488 | 0.19561 | 0.20440 | 0.21749 | 0.22632 |
| 6.0 | 0.03161 | 0.06201 | 0.09017 | 0.11541 | 0.13741 | 0.15617 | 0.17191 | 0.18496 | 0.19569 | 0.20449 | 0.21760 | 0.22644 |
| 8.0 | 0.03162 | 0.06202 | 0.09018 | 0.11543 | 0.13744 | 0.15621 | 0.17195 | 0.18500 | 0.19574 | 0.20455 | 0.21767 | 0.22652 |
| 10.0 | 0.03162 | 0.06202 | 0.09019 | 0.11544 | 0.13745 | 0.15622 | 0.17196 | 0.18502 | 0.19576 | 0.20457 | 0.21769 | 0.22654 |
| ∞ | 0.03162 | 0.06202 | 0.09019 | 0.11544 | 0.13745 | 0.15623 | 0.17197 | 0.18502 | 0.19577 | 0.20458 | 0.21770 | 0.22656 |

**Table 4.4** (*Continued*)

| | *n* | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ***m*** | **1.6** | **1.8** | **2.0** | **2.5** | **3.0** | **4.0** | **5.0** | **6.0** | **8.0** | **10.0** | **∞** |
| 0.1 | 0.03058 | 0.03090 | 0.03111 | 0.03138 | 0.03150 | 0.03158 | 0.03160 | 0.03161 | 0.03162 | 0.03162 | 0.03162 |
| 0.2 | 0.05994 | 0.06058 | 0.06100 | 0.06155 | 0.06178 | 0.06194 | 0.06199 | 0.06201 | 0.06202 | 0.06202 | 0.06202 |
| 0.3 | 0.08709 | 0.08804 | 0.08867 | 0.08948 | 0.08982 | 0.09007 | 0.09014 | 0.09017 | 0.09018 | 0.09019 | 0.09019 |
| 0.4 | 0.11135 | 0.11260 | 0.11342 | 0.11450 | 0.11495 | 0.11527 | 0.11537 | 0.11541 | 0.11543 | 0.11544 | 0.11544 |
| 0.5 | 0.13241 | 0.13395 | 0.13496 | 0.13628 | 0.13684 | 0.13724 | 0.13737 | 0.13741 | 0.13744 | 0.13745 | 0.13745 |
| 0.6 | 0.15028 | 0.15207 | 0.15326 | 0.15483 | 0.15550 | 0.15598 | 0.15612 | 0.15617 | 0.15621 | 0.15622 | 0.15623 |
| 0.7 | 0.16515 | 0.16720 | 0.16856 | 0.17036 | 0.17113 | 0.17168 | 0.17185 | 0.17191 | 0.17195 | 0.17196 | 0.17197 |
| 0.8 | 0.17739 | 0.17967 | 0.18119 | 0.18321 | 0.18407 | 0.18469 | 0.18488 | 0.18496 | 0.18500 | 0.18502 | 0.18502 |
| 0.9 | 0.18737 | 0.18986 | 0.19152 | 0.19375 | 0.19470 | 0.19540 | 0.19561 | 0.19569 | 0.19574 | 0.19576 | 0.19577 |
| 1.0 | 0.19546 | 0.19814 | 0.19994 | 0.20236 | 0.20341 | 0.20417 | 0.20440 | 0.20449 | 0.20455 | 0.20457 | 0.20458 |
| 1.2 | 0.20731 | 0.21032 | 0.21235 | 0.21512 | 0.21633 | 0.21722 | 0.21749 | 0.21760 | 0.21767 | 0.21769 | 0.21770 |
| 1.4 | 0.21510 | 0.21836 | 0.22058 | 0.22364 | 0.22499 | 0.22600 | 0.22632 | 0.22644 | 0.22652 | 0.22654 | 0.22656 |
| 1.6 | 0.22025 | 0.22372 | 0.22610 | 0.22940 | 0.23088 | 0.23200 | 0.23236 | 0.23249 | 0.23258 | 0.23261 | 0.23263 |
| 1.8 | 0.22372 | 0.22736 | 0.22986 | 0.23334 | 0.23495 | 0.23617 | 0.23656 | 0.23671 | 0.23681 | 0.23684 | 0.23686 |
| 2.0 | 0.22610 | 0.22986 | 0.23247 | 0.23614 | 0.23782 | 0.23912 | 0.23954 | 0.23970 | 0.23981 | 0.23985 | 0.23987 |
| 2.5 | 0.22940 | 0.23334 | 0.23614 | 0.24010 | 0.24196 | 0.24344 | 0.24392 | 0.24412 | 0.24425 | 0.24429 | 0.24432 |
| 3.0 | 0.23088 | 0.23495 | 0.23782 | 0.24196 | 0.24394 | 0.24554 | 0.24608 | 0.24630 | 0.24646 | 0.24650 | 0.24654 |
| 4.0 | 0.23200 | 0.23617 | 0.23912 | 0.24344 | 0.24554 | 0.24729 | 0.24791 | 0.24817 | 0.24836 | 0.24842 | 0.24846 |
| 5.0 | 0.23236 | 0.23656 | 0.23954 | 0.24392 | 0.24608 | 0.24791 | 0.24857 | 0.24885 | 0.24907 | 0.24914 | 0.24919 |
| 6.0 | 0.23249 | 0.23671 | 0.23970 | 0.24412 | 0.24630 | 0.24817 | 0.24885 | 0.24916 | 0.24939 | 0.24946 | 0.24952 |
| 8.0 | 0.23258 | 0.23681 | 0.23981 | 0.24425 | 0.24646 | 0.24836 | 0.24907 | 0.24939 | 0.24964 | 0.24973 | 0.24980 |
| 10.0 | 0.23261 | 0.23684 | 0.23985 | 0.24429 | 0.24650 | 0.24842 | 0.24914 | 0.24946 | 0.24973 | 0.24981 | 0.24989 |
| ∞ | 0.23263 | 0.23686 | 0.23987 | 0.24432 | 0.24654 | 0.24846 | 0.24919 | 0.24952 | 0.24980 | 0.24989 | 0.25000 |

[a]Based on Saika, 2012

**Table 4.5** Variation of $I_c$ with $m_1$ and $n_1$

| $n_1$ | $m_1$ 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.20 | 0.994 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 |
| 0.40 | 0.960 | 0.976 | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 | 0.977 |
| 0.60 | 0.892 | 0.932 | 0.936 | 0.936 | 0.937 | 0.937 | 0.937 | 0.937 | 0.937 | 0.937 |
| 0.80 | 0.800 | 0.870 | 0.878 | 0.880 | 0.881 | 0.881 | 0.881 | 0.881 | 0.881 | 0.881 |
| 1.00 | 0.701 | 0.800 | 0.814 | 0.817 | 0.818 | 0.818 | 0.818 | 0.818 | 0.818 | 0.818 |
| 1.20 | 0.606 | 0.727 | 0.748 | 0.753 | 0.754 | 0.755 | 0.755 | 0.755 | 0.755 | 0.755 |
| 1.40 | 0.522 | 0.658 | 0.685 | 0.692 | 0.694 | 0.695 | 0.695 | 0.696 | 0.696 | 0.696 |
| 1.60 | 0.449 | 0.593 | 0.627 | 0.636 | 0.639 | 0.640 | 0.641 | 0.641 | 0.641 | 0.642 |
| 1.80 | 0.388 | 0.534 | 0.573 | 0.585 | 0.590 | 0.591 | 0.592 | 0.592 | 0.593 | 0.593 |
| 2.00 | 0.336 | 0.481 | 0.525 | 0.540 | 0.545 | 0.547 | 0.548 | 0.549 | 0.549 | 0.549 |
| 3.00 | 0.179 | 0.293 | 0.348 | 0.373 | 0.384 | 0.389 | 0.392 | 0.393 | 0.394 | 0.395 |
| 4.00 | 0.108 | 0.190 | 0.241 | 0.269 | 0.285 | 0.293 | 0.298 | 0.301 | 0.302 | 0.303 |
| 5.00 | 0.072 | 0.131 | 0.174 | 0.202 | 0.219 | 0.229 | 0.236 | 0.240 | 0.242 | 0.244 |
| 6.00 | 0.051 | 0.095 | 0.130 | 0.155 | 0.172 | 0.184 | 0.192 | 0.197 | 0.200 | 0.202 |
| 7.00 | 0.038 | 0.072 | 0.100 | 0.122 | 0.139 | 0.150 | 0.158 | 0.164 | 0.168 | 0.171 |
| 8.00 | 0.029 | 0.056 | 0.079 | 0.098 | 0.113 | 0.125 | 0.133 | 0.139 | 0.144 | 0.147 |
| 9.00 | 0.023 | 0.045 | 0.064 | 0.081 | 0.094 | 0.105 | 0.113 | 0.119 | 0.124 | 0.128 |
| 10.00 | 0.019 | 0.037 | 0.053 | 0.067 | 0.079 | 0.089 | 0.097 | 0.103 | 0.108 | 0.112 |

## Stress Increase Under a Rectangular Foundation- 2:1 Method

Foundation engineers often use an approximate method to determine the increase in stress with depth caused by the construction of a foundation. The method is referred to as the 2:1 method. (See Figure 4.7). According to this method, the increase in stress at depth z is

$$\Delta\sigma = \frac{q_o \times B \times L}{(B+z)(L+z)}$$

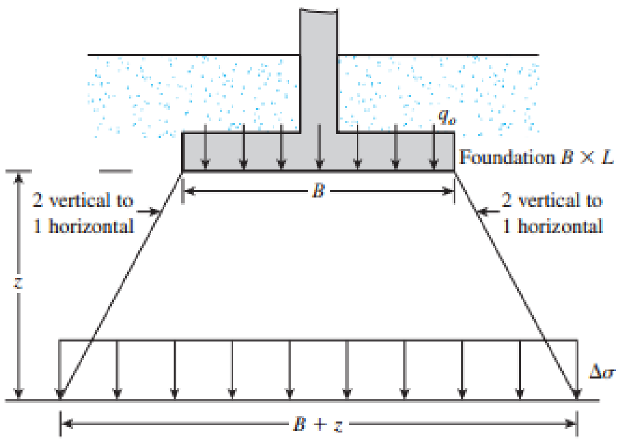



**Figure 4.7** 2:1 method of finding stress increase under a foundation

Note that Equation above is based on the assumption that the stress from the foundation spreads out along lines with a vertical-to-horizontal slope of 2:1.

# Settlement of Shallow Foundations

## 4.6 Type of Shallow Foundations

The settlement of a shallow foundation can be divided into two major categories:

**(a) elastic, or immediate, settlement and**

**(b) consolidation settlement.**

- Immediate, or elastic, settlement of a foundation takes place during or immediately after the construction of the structure.
- Consolidation settlement occurs over time. Pore water is extruded from the void spaces of saturated clayey soils submerged in water.
- The total settlement of a foundation is the sum of the elastic settlement and the consolidation settlement. Consolidation settlement comprises two phases: ***primary*** **and** ***secondary***. Primary consolidation settlement is more significant than secondary settlement in inorganic clays and silty soils. However, in organic soils, secondary consolidation settlement is more significant.

## 4.7 Elastic Settlement of Shallow Foundation on Saturated Clay ($\mu_s = 0.5$)

- Janbu et al. (1956) proposed an equation for evaluating the average settlement of flexible foundations on saturated clay soils (Poisson's ratio, (**$\mu_s$ = 0.5**). Referring to Figure 4.8, this relationship can be expressed as

$$S_e = A_1 A_2 \frac{q_o B}{E_s} \qquad (4.1)$$

where

$A_1 = f(H/B, L/B)$
$A_2 = f(D_f/B)$
$L$ = length of the foundation
$B$ = width of the foundation
$D_f$ = depth of the foundation
$H$ = depth of the bottom of the foundation to a rigid layer
$q_o$ = net load per unit area of the foundation

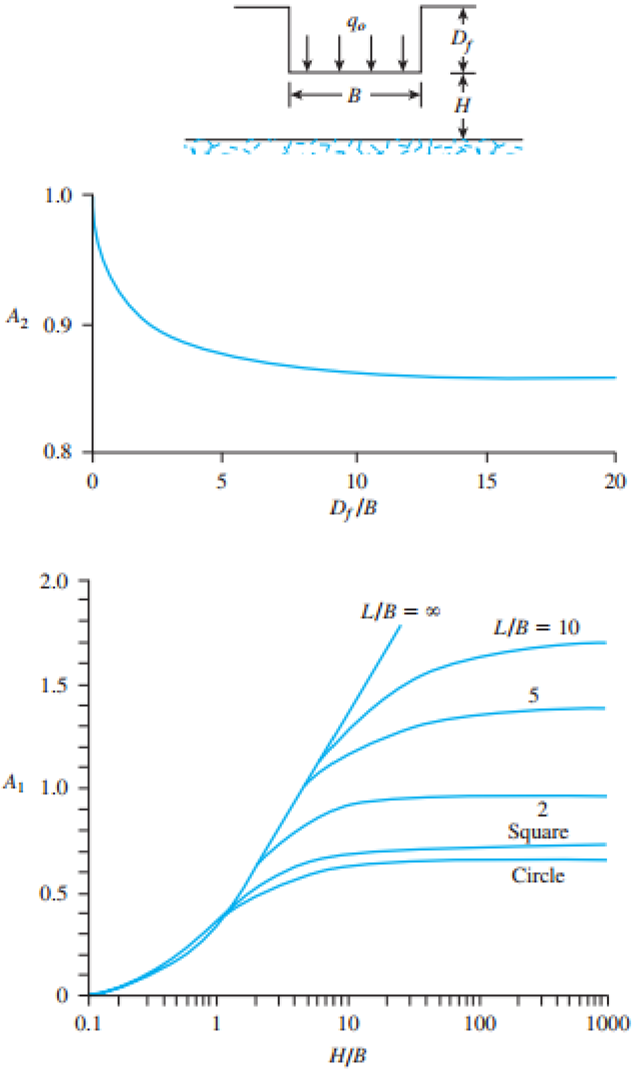



**Figure 4.8** Values of $A_1$ and $A_2$ for elastic settlement calculation–

- Christian and Carrier (1978) modified the values of $A_1$ and $A_2$ to some extent and is presented in Figure 4.8.

- The modulus of elasticity ($E_s$) for saturated clays can, in general, be given as

$$E_s = \beta c_u \qquad (4.2)$$

- The parameter $\beta$ is primarily a function of the plasticity index and overconsolidation ratio (OCR). Table 4.1 provides a general range for $\beta$based on that proposed by Duncan and Buchignani (1976). In any case, proper judgment should be used in selecting the magnitude of $\beta$.

**Table 4.1** Range of $\beta$ for Saturated Clay [Eq. (4.2)][a]

| Plasticity Index | $\beta$ | | | | |
|---|---|---|---|---|---|
| | OCR = 1 | OCR = 2 | OCR = 3 | OCR = 4 | OCR = 5 |
| <30 | 1500–600 | 1380–500 | 1200–580 | 950–380 | 730–300 |
| 30 to 50 | 600–300 | 550–270 | 580–220 | 380–180 | 300–150 |
| >50 | 300–150 | 270–120 | 220–100 | 180–90 | 150–75 |

[a]Based on Duncan and Buchignani (1976)

## Example 4.1

Consider a shallow foundation 2 m × 1 m in plan in a saturated clay layer. A rigid rock layer is located 8 m below the bottom of the foundation. Given:

Foundation: $D_f = 1$ m, $q_o = 120$ kN/m$^2$
Clay: $c_u = 150$ kN/m$^2$, OCR = 2, and Plasticity index, PI = 35

Estimate the elastic settlement of the foundation.

### Solution

From Eq. (7.1),

$$S_e = A_1 A_2 \frac{q_o B}{E_s}$$

Given:

$$\frac{L}{B} = \frac{2}{1} = 2$$

$$\frac{D_f}{B} = \frac{1}{1} = 1$$

$$\frac{H}{B} = \frac{8}{1} = 8$$

$$E_s = \beta c_u$$

For OCR = 2 and PI = 35, the value of $\beta \approx 480$ (Table 7.1). Hence,

$$E_s = (480)(150) = 72{,}000 \text{ kN/m}^2$$

Also, from Figure 7.1, $A_1 = 0.9$ and $A_2 = 0.92$. Hence,

$$S_e = A_1 A_2 \frac{q_o B}{E_s} = (0.9)(0.92)\frac{(120)(1)}{72{,}000} = 0.00138 \text{ m} = \mathbf{1.38\ mm}$$

## *Elastic Settlement in Granular Soil*

## 4.8 Settlement Based on the Theory of Elasticity

- The elastic settlement of a shallow foundation can be estimated by using the theory of elasticity. From Hooke's law, as applied to Figure 4.9, we obtain

$$S_e = \int_0^H \varepsilon_z dz = \frac{1}{E_s}\int_0^H (\Delta\sigma_z - \mu_s\Delta\sigma_x - \mu_s\Delta\sigma_y)dz \qquad (7.3)$$

where

$S_e$ = elastic settlement
$E_s$ = modulus of elasticity of soil
$H$ = thickness of the soil layer
$\mu_s$ = Poisson's ratio of the soil
$\Delta\sigma_x, \Delta\sigma_y, \Delta\sigma_z$ = stress increase due to the net applied foundation load in the $x$, $y$, and $z$ directions, respectively

Theoretically, if the foundation is perfectly flexible (see Figure 4.10 and Bowles, 1987), the settlement may be expressed as

$$S_e = q_o(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f \qquad (4.4)$$

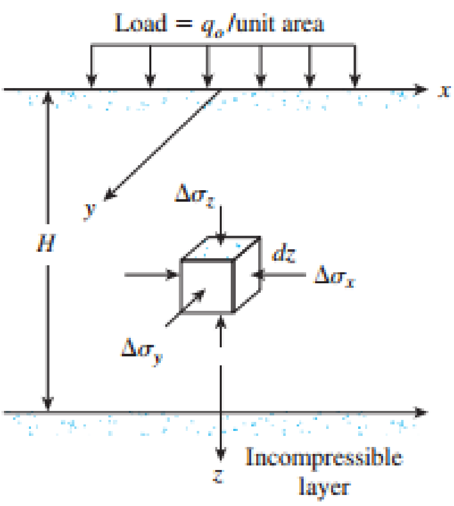



**Figure 4.9** Elastic settlement of shallow foundation

where

$q_o$ = net applied pressure on the foundation
$\mu_s$ = Poisson's ratio of soil
$E_s$ = average modulus of elasticity of the soil under the foundation, measured from $z = 0$ to about $z = 5B$
$B' = B/2$ for center of foundation
$= B$ for corner of foundation
$I_s$ = shape factor (Steinbrenner, 1934)

$$= F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2 \tag{7.5}$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1) \tag{7.6}$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}A_2 \tag{7.7}$$

$$A_0 = m'\ln\frac{\left(1 + \sqrt{m'^2 + 1}\right)\sqrt{m'^2 + n'^2}}{m'\left(1 + \sqrt{m'^2 + n'^2 + 1}\right)} \tag{7.8}$$

$$A_1 = \ln\frac{\left(m' + \sqrt{m'^2 + 1}\right)\sqrt{1 + n'^2}}{m' + \sqrt{m'^2 + n'^2 + 1}} \tag{7.9}$$

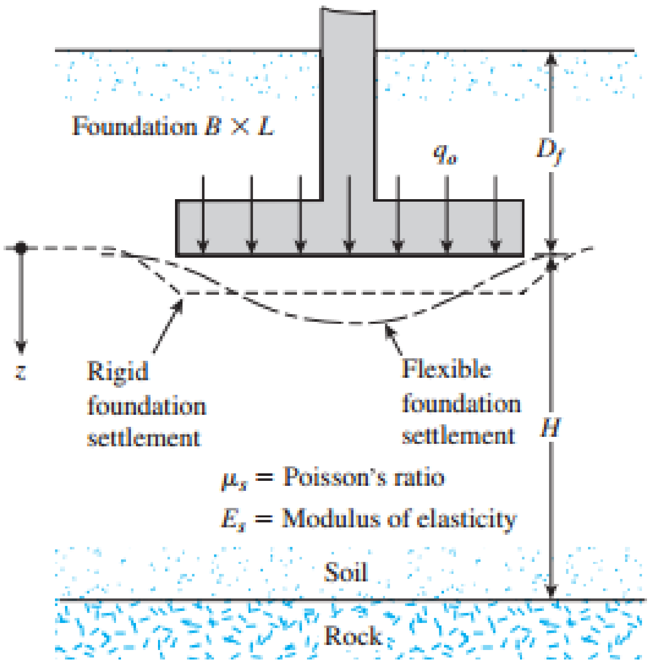



**Figure 4.10** Elastic settlement of flexible and rigid foundations

$$A_2 = \frac{m'}{n'\sqrt{m'^2 + n'^2 + 1}} \tag{7.10}$$

$$I_f = \text{depth factor (Fox, 1948)} = f\left(\frac{D_f}{B}, \mu_s, \text{and } \frac{L}{B}\right) \tag{7.11}$$

$\alpha$ = a factor that depends on the location on the foundation where settlement is being calculated

To calculate settlement at the *center* of the foundation, we use

$$\alpha = 4$$

$$m' = \frac{L}{B}$$

and

$$n' = \frac{H}{\left(\frac{B}{2}\right)}$$

To calculate settlement at a *corner* of the foundation,

$$\alpha = 1$$

$$m' = \frac{L}{B}$$

and

$$n' = \frac{H}{B}$$

The variations of $F_1$ and $F_2$ [see Eqs. (7.6) and (7.7)] with $m'$ and $n'$ are given in Tables 7.2 and 7.3. Also, the variation of $I_f$ with $D_f/B$ (for $\mu_s$ = 0.3, 0.4, and 0.5) is given in Table 7.4. These values are also given in more detailed form by Bowles (1987).

- The elastic settlement of a *rigid foundation* can be estimated as

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})} \tag{7.1}$$

Due to the nonhomogeneous nature of soil deposits, the magnitude of $E_s$ may vary with depth. For that reason, Bowles (1987) recommended using a weighted average of $E_s$ in Eq. (7.4), or

$$E_s = \frac{\Sigma E_{s(i)} \Delta z}{\bar{z}} \tag{7.13}$$

where

$E_{s(i)}$ = soil modulus of elasticity within a depth $\Delta z$

$\bar{z}$ = $H$ or $5B$, whichever is smaller

**Table 7.2** Variation of $F_1$ with $m'$ and $n'$

| | $m'$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n'$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| 0.25 | 0.014 | 0.013 | 0.012 | 0.011 | 0.011 | 0.011 | 0.010 | 0.010 | 0.010 | 0.010 |
| 0.50 | 0.049 | 0.046 | 0.044 | 0.042 | 0.041 | 0.040 | 0.038 | 0.038 | 0.037 | 0.037 |
| 0.75 | 0.095 | 0.090 | 0.087 | 0.084 | 0.082 | 0.080 | 0.077 | 0.076 | 0.074 | 0.074 |
| 1.00 | 0.142 | 0.138 | 0.134 | 0.130 | 0.127 | 0.125 | 0.121 | 0.118 | 0.116 | 0.115 |
| 1.25 | 0.186 | 0.183 | 0.179 | 0.176 | 0.173 | 0.170 | 0.165 | 0.161 | 0.158 | 0.157 |
| 1.50 | 0.224 | 0.224 | 0.222 | 0.219 | 0.216 | 0.213 | 0.207 | 0.203 | 0.199 | 0.197 |
| 1.75 | 0.257 | 0.259 | 0.259 | 0.258 | 0.255 | 0.253 | 0.247 | 0.242 | 0.238 | 0.235 |
| 2.00 | 0.285 | 0.290 | 0.292 | 0.292 | 0.291 | 0.289 | 0.284 | 0.279 | 0.275 | 0.271 |
| 2.25 | 0.309 | 0.317 | 0.321 | 0.323 | 0.323 | 0.322 | 0.317 | 0.313 | 0.308 | 0.305 |
| 2.50 | 0.330 | 0.341 | 0.347 | 0.350 | 0.351 | 0.351 | 0.348 | 0.344 | 0.340 | 0.336 |
| 2.75 | 0.348 | 0.361 | 0.369 | 0.374 | 0.377 | 0.378 | 0.377 | 0.373 | 0.369 | 0.365 |
| 3.00 | 0.363 | 0.379 | 0.389 | 0.396 | 0.400 | 0.402 | 0.402 | 0.400 | 0.396 | 0.392 |
| 3.25 | 0.376 | 0.394 | 0.406 | 0.415 | 0.420 | 0.423 | 0.426 | 0.424 | 0.421 | 0.418 |
| 3.50 | 0.388 | 0.408 | 0.422 | 0.431 | 0.438 | 0.442 | 0.447 | 0.447 | 0.444 | 0.441 |
| 3.75 | 0.399 | 0.420 | 0.436 | 0.447 | 0.454 | 0.460 | 0.467 | 0.458 | 0.466 | 0.464 |
| 4.00 | 0.408 | 0.431 | 0.448 | 0.460 | 0.469 | 0.476 | 0.484 | 0.487 | 0.486 | 0.484 |
| 4.25 | 0.417 | 0.440 | 0.458 | 0.472 | 0.481 | 0.484 | 0.495 | 0.514 | 0.515 | 0.515 |
| 4.50 | 0.424 | 0.450 | 0.469 | 0.484 | 0.495 | 0.503 | 0.516 | 0.521 | 0.522 | 0.522 |
| 4.75 | 0.431 | 0.458 | 0.478 | 0.494 | 0.506 | 0.515 | 0.530 | 0.536 | 0.539 | 0.539 |
| 5.00 | 0.437 | 0.465 | 0.487 | 0.503 | 0.516 | 0.526 | 0.543 | 0.551 | 0.554 | 0.554 |
| 5.25 | 0.443 | 0.472 | 0.494 | 0.512 | 0.526 | 0.537 | 0.555 | 0.564 | 0.568 | 0.569 |
| 5.50 | 0.448 | 0.478 | 0.501 | 0.520 | 0.534 | 0.546 | 0.566 | 0.576 | 0.581 | 0.584 |
| 5.75 | 0.453 | 0.483 | 0.508 | 0.527 | 0.542 | 0.555 | 0.576 | 0.588 | 0.594 | 0.597 |
| 6.00 | 0.457 | 0.489 | 0.514 | 0.534 | 0.550 | 0.563 | 0.585 | 0.598 | 0.606 | 0.609 |
| 6.25 | 0.461 | 0.493 | 0.519 | 0.540 | 0.557 | 0.570 | 0.594 | 0.609 | 0.617 | 0.621 |
| 6.50 | 0.465 | 0.498 | 0.524 | 0.546 | 0.563 | 0.577 | 0.603 | 0.618 | 0.627 | 0.632 |
| 6.75 | 0.468 | 0.502 | 0.529 | 0.551 | 0.569 | 0.584 | 0.610 | 0.627 | 0.637 | 0.643 |
| 7.00 | 0.471 | 0.506 | 0.533 | 0.556 | 0.575 | 0.590 | 0.618 | 0.635 | 0.646 | 0.653 |
| 7.25 | 0.474 | 0.509 | 0.538 | 0.561 | 0.580 | 0.596 | 0.625 | 0.643 | 0.655 | 0.662 |
| 7.50 | 0.477 | 0.513 | 0.541 | 0.565 | 0.585 | 0.601 | 0.631 | 0.650 | 0.663 | 0.671 |
| 7.75 | 0.480 | 0.516 | 0.545 | 0.569 | 0.589 | 0.606 | 0.637 | 0.658 | 0.671 | 0.680 |
| 8.00 | 0.482 | 0.519 | 0.549 | 0.573 | 0.594 | 0.611 | 0.643 | 0.664 | 0.678 | 0.688 |
| 8.25 | 0.485 | 0.522 | 0.552 | 0.577 | 0.598 | 0.615 | 0.648 | 0.670 | 0.685 | 0.695 |
| 8.50 | 0.487 | 0.524 | 0.555 | 0.580 | 0.601 | 0.619 | 0.653 | 0.676 | 0.692 | 0.703 |
| 8.75 | 0.489 | 0.527 | 0.558 | 0.583 | 0.605 | 0.623 | 0.658 | 0.682 | 0.698 | 0.710 |
| 9.00 | 0.491 | 0.529 | 0.560 | 0.587 | 0.609 | 0.627 | 0.663 | 0.687 | 0.705 | 0.716 |
| 9.25 | 0.493 | 0.531 | 0.563 | 0.589 | 0.612 | 0.631 | 0.667 | 0.693 | 0.710 | 0.723 |
| 9.50 | 0.495 | 0.533 | 0.565 | 0.592 | 0.615 | 0.634 | 0.671 | 0.697 | 0.716 | 0.719 |
| 9.75 | 0.496 | 0.536 | 0.568 | 0.595 | 0.618 | 0.638 | 0.675 | 0.702 | 0.721 | 0.735 |
| 10.00 | 0.498 | 0.537 | 0.570 | 0.597 | 0.621 | 0.641 | 0.679 | 0.707 | 0.726 | 0.740 |
| 20.00 | 0.529 | 0.575 | 0.614 | 0.647 | 0.677 | 0.702 | 0.756 | 0.797 | 0.830 | 0.858 |
| 50.00 | 0.548 | 0.598 | 0.640 | 0.678 | 0.711 | 0.740 | 0.803 | 0.853 | 0.895 | 0.931 |
| 100.00 | 0.555 | 0.605 | 0.649 | 0.688 | 0.722 | 0.753 | 0.819 | 0.872 | 0.918 | 0.956 |

*(Continued)*

**Table 7.2** *(Continued)*

| | $m'$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n'$ | **4.5** | **5.0** | **6.0** | **7.0** | **8.0** | **9.0** | **10.0** | **25.0** | **50.0** | **100.0** |
| 0.25 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 | 0.010 |
| 0.50 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 | 0.036 |
| 0.75 | 0.073 | 0.073 | 0.072 | 0.072 | 0.072 | 0.072 | 0.071 | 0.071 | 0.071 | 0.071 |
| 1.00 | 0.114 | 0.113 | 0.112 | 0.112 | 0.112 | 0.111 | 0.111 | 0.110 | 0.110 | 0.110 |
| 1.25 | 0.155 | 0.154 | 0.153 | 0.152 | 0.152 | 0.151 | 0.151 | 0.150 | 0.150 | 0.150 |
| 1.50 | 0.195 | 0.194 | 0.192 | 0.191 | 0.190 | 0.190 | 0.189 | 0.188 | 0.188 | 0.188 |
| 1.75 | 0.233 | 0.232 | 0.229 | 0.228 | 0.227 | 0.226 | 0.225 | 0.223 | 0.223 | 0.223 |
| 2.00 | 0.269 | 0.267 | 0.264 | 0.262 | 0.261 | 0.260 | 0.259 | 0.257 | 0.256 | 0.256 |
| 2.25 | 0.302 | 0.300 | 0.296 | 0.294 | 0.293 | 0.291 | 0.291 | 0.287 | 0.287 | 0.287 |
| 2.50 | 0.333 | 0.331 | 0.327 | 0.324 | 0.322 | 0.321 | 0.320 | 0.316 | 0.315 | 0.315 |
| 2.75 | 0.362 | 0.359 | 0.355 | 0.352 | 0.350 | 0.348 | 0.347 | 0.343 | 0.342 | 0.342 |
| 3.00 | 0.389 | 0.386 | 0.382 | 0.378 | 0.376 | 0.374 | 0.373 | 0.368 | 0.367 | 0.367 |
| 3.25 | 0.415 | 0.412 | 0.407 | 0.403 | 0.401 | 0.399 | 0.397 | 0.391 | 0.390 | 0.390 |
| 3.50 | 0.438 | 0.435 | 0.430 | 0.427 | 0.424 | 0.421 | 0.420 | 0.413 | 0.412 | 0.411 |
| 3.75 | 0.461 | 0.458 | 0.453 | 0.449 | 0.446 | 0.443 | 0.441 | 0.433 | 0.432 | 0.432 |
| 4.00 | 0.482 | 0.479 | 0.474 | 0.470 | 0.466 | 0.464 | 0.462 | 0.453 | 0.451 | 0.451 |
| 4.25 | 0.516 | 0.496 | 0.484 | 0.473 | 0.471 | 0.471 | 0.470 | 0.468 | 0.462 | 0.460 |
| 4.50 | 0.520 | 0.517 | 0.513 | 0.508 | 0.505 | 0.502 | 0.499 | 0.489 | 0.487 | 0.487 |
| 4.75 | 0.537 | 0.535 | 0.530 | 0.526 | 0.523 | 0.519 | 0.517 | 0.506 | 0.504 | 0.503 |
| 5.00 | 0.554 | 0.552 | 0.548 | 0.543 | 0.540 | 0.536 | 0.534 | 0.522 | 0.519 | 0.519 |
| 5.25 | 0.569 | 0.568 | 0.564 | 0.560 | 0.556 | 0.553 | 0.550 | 0.537 | 0.534 | 0.534 |
| 5.50 | 0.584 | 0.583 | 0.579 | 0.575 | 0.571 | 0.568 | 0.585 | 0.551 | 0.549 | 0.548 |
| 5.75 | 0.597 | 0.597 | 0.594 | 0.590 | 0.586 | 0.583 | 0.580 | 0.565 | 0.583 | 0.562 |
| 6.00 | 0.611 | 0.610 | 0.608 | 0.604 | 0.601 | 0.598 | 0.595 | 0.579 | 0.576 | 0.575 |
| 6.25 | 0.623 | 0.623 | 0.621 | 0.618 | 0.615 | 0.611 | 0.608 | 0.592 | 0.589 | 0.588 |
| 6.50 | 0.635 | 0.635 | 0.634 | 0.631 | 0.628 | 0.625 | 0.622 | 0.605 | 0.601 | 0.600 |
| 6.75 | 0.646 | 0.647 | 0.646 | 0.644 | 0.641 | 0.637 | 0.634 | 0.617 | 0.613 | 0.612 |
| 7.00 | 0.656 | 0.658 | 0.658 | 0.656 | 0.653 | 0.650 | 0.647 | 0.628 | 0.624 | 0.623 |
| 7.25 | 0.666 | 0.669 | 0.669 | 0.668 | 0.665 | 0.662 | 0.659 | 0.640 | 0.635 | 0.634 |
| 7.50 | 0.676 | 0.679 | 0.680 | 0.679 | 0.676 | 0.673 | 0.670 | 0.651 | 0.646 | 0.645 |
| 7.75 | 0.685 | 0.688 | 0.690 | 0.689 | 0.687 | 0.684 | 0.681 | 0.661 | 0.656 | 0.655 |
| 8.00 | 0.694 | 0.697 | 0.700 | 0.700 | 0.698 | 0.695 | 0.692 | 0.672 | 0.666 | 0.665 |
| 8.25 | 0.702 | 0.706 | 0.710 | 0.710 | 0.708 | 0.705 | 0.703 | 0.682 | 0.676 | 0.675 |
| 8.50 | 0.710 | 0.714 | 0.719 | 0.719 | 0.718 | 0.715 | 0.713 | 0.692 | 0.686 | 0.684 |
| 8.75 | 0.717 | 0.722 | 0.727 | 0.728 | 0.727 | 0.725 | 0.723 | 0.701 | 0.695 | 0.693 |
| 9.00 | 0.725 | 0.730 | 0.736 | 0.737 | 0.736 | 0.735 | 0.732 | 0.710 | 0.704 | 0.702 |
| 9.25 | 0.731 | 0.737 | 0.744 | 0.746 | 0.745 | 0.744 | 0.742 | 0.719 | 0.713 | 0.711 |
| 9.50 | 0.738 | 0.744 | 0.752 | 0.754 | 0.754 | 0.753 | 0.751 | 0.728 | 0.721 | 0.719 |
| 9.75 | 0.744 | 0.751 | 0.759 | 0.762 | 0.762 | 0.761 | 0.759 | 0.737 | 0.729 | 0.727 |
| 10.00 | 0.750 | 0.758 | 0.766 | 0.770 | 0.770 | 0.770 | 0.768 | 0.745 | 0.738 | 0.735 |
| 20.00 | 0.878 | 0.896 | 0.925 | 0.945 | 0.959 | 0.969 | 0.977 | 0.982 | 0.965 | 0.957 |
| 50.00 | 0.962 | 0.989 | 1.034 | 1.070 | 1.100 | 1.125 | 1.146 | 1.265 | 1.279 | 1.261 |
| 100.00 | 0.990 | 1.020 | 1.072 | 1.114 | 1.150 | 1.182 | 1.209 | 1.408 | 1.489 | 1.499 |

**Table 7.3** Variation of $F_2$ with $m'$ and $n'$

| | $m'$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n'$ | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| 0.25 | 0.049 | 0.050 | 0.051 | 0.051 | 0.051 | 0.052 | 0.052 | 0.052 | 0.052 | 0.052 |
| 0.50 | 0.074 | 0.077 | 0.080 | 0.081 | 0.083 | 0.084 | 0.086 | 0.086 | 0.0878 | 0.087 |
| 0.75 | 0.083 | 0.089 | 0.093 | 0.097 | 0.099 | 0.101 | 0.104 | 0.106 | 0.107 | 0.108 |
| 1.00 | 0.083 | 0.091 | 0.098 | 0.102 | 0.106 | 0.109 | 0.114 | 0.117 | 0.119 | 0.120 |
| 1.25 | 0.080 | 0.089 | 0.096 | 0.102 | 0.107 | 0.111 | 0.118 | 0.122 | 0.125 | 0.127 |
| 1.50 | 0.075 | 0.084 | 0.093 | 0.099 | 0.105 | 0.110 | 0.118 | 0.124 | 0.128 | 0.130 |
| 1.75 | 0.069 | 0.079 | 0.088 | 0.095 | 0.101 | 0.107 | 0.117 | 0.123 | 0.128 | 0.131 |
| 2.00 | 0.064 | 0.074 | 0.083 | 0.090 | 0.097 | 0.102 | 0.114 | 0.121 | 0.127 | 0.131 |
| 2.25 | 0.059 | 0.069 | 0.077 | 0.085 | 0.092 | 0.098 | 0.110 | 0.119 | 0.125 | 0.130 |
| 2.50 | 0.055 | 0.064 | 0.073 | 0.080 | 0.087 | 0.093 | 0.106 | 0.115 | 0.122 | 0.127 |
| 2.75 | 0.051 | 0.060 | 0.068 | 0.076 | 0.082 | 0.089 | 0.102 | 0.111 | 0.119 | 0.125 |
| 3.00 | 0.048 | 0.056 | 0.064 | 0.071 | 0.078 | 0.084 | 0.097 | 0.108 | 0.116 | 0.122 |
| 3.25 | 0.045 | 0.053 | 0.060 | 0.067 | 0.074 | 0.080 | 0.093 | 0.104 | 0.112 | 0.119 |
| 3.50 | 0.042 | 0.050 | 0.057 | 0.064 | 0.070 | 0.076 | 0.089 | 0.100 | 0.109 | 0.116 |
| 3.75 | 0.040 | 0.047 | 0.054 | 0.060 | 0.067 | 0.073 | 0.086 | 0.096 | 0.105 | 0.113 |
| 4.00 | 0.037 | 0.044 | 0.051 | 0.057 | 0.063 | 0.069 | 0.082 | 0.093 | 0.102 | 0.110 |
| 4.25 | 0.036 | 0.042 | 0.049 | 0.055 | 0.061 | 0.066 | 0.079 | 0.090 | 0.099 | 0.107 |
| 4.50 | 0.034 | 0.040 | 0.046 | 0.052 | 0.058 | 0.063 | 0.076 | 0.086 | 0.096 | 0.104 |
| 4.75 | 0.032 | 0.038 | 0.044 | 0.050 | 0.055 | 0.061 | 0.073 | 0.083 | 0.093 | 0.101 |
| 5.00 | 0.031 | 0.036 | 0.042 | 0.048 | 0.053 | 0.058 | 0.070 | 0.080 | 0.090 | 0.098 |
| 5.25 | 0.029 | 0.035 | 0.040 | 0.046 | 0.051 | 0.056 | 0.067 | 0.078 | 0.087 | 0.095 |
| 5.50 | 0.028 | 0.033 | 0.039 | 0.044 | 0.049 | 0.054 | 0.065 | 0.075 | 0.084 | 0.092 |
| 5.75 | 0.027 | 0.032 | 0.037 | 0.042 | 0.047 | 0.052 | 0.063 | 0.073 | 0.082 | 0.090 |
| 6.00 | 0.026 | 0.031 | 0.036 | 0.040 | 0.045 | 0.050 | 0.060 | 0.070 | 0.079 | 0.087 |
| 6.25 | 0.025 | 0.030 | 0.034 | 0.039 | 0.044 | 0.048 | 0.058 | 0.068 | 0.077 | 0.085 |
| 6.50 | 0.024 | 0.029 | 0.033 | 0.038 | 0.042 | 0.046 | 0.056 | 0.066 | 0.075 | 0.083 |
| 6.75 | 0.023 | 0.028 | 0.032 | 0.036 | 0.041 | 0.045 | 0.055 | 0.064 | 0.073 | 0.080 |
| 7.00 | 0.022 | 0.027 | 0.031 | 0.035 | 0.039 | 0.043 | 0.053 | 0.062 | 0.071 | 0.078 |
| 7.25 | 0.022 | 0.026 | 0.030 | 0.034 | 0.038 | 0.042 | 0.051 | 0.060 | 0.069 | 0.076 |
| 7.50 | 0.021 | 0.025 | 0.029 | 0.033 | 0.037 | 0.041 | 0.050 | 0.059 | 0.067 | 0.074 |
| 7.75 | 0.020 | 0.024 | 0.028 | 0.032 | 0.036 | 0.039 | 0.048 | 0.057 | 0.065 | 0.072 |
| 8.00 | 0.020 | 0.023 | 0.027 | 0.031 | 0.035 | 0.038 | 0.047 | 0.055 | 0.063 | 0.071 |
| 8.25 | 0.019 | 0.023 | 0.026 | 0.030 | 0.034 | 0.037 | 0.046 | 0.054 | 0.062 | 0.069 |
| 8.50 | 0.018 | 0.022 | 0.026 | 0.029 | 0.033 | 0.036 | 0.045 | 0.053 | 0.060 | 0.067 |
| 8.75 | 0.018 | 0.021 | 0.025 | 0.028 | 0.032 | 0.035 | 0.043 | 0.051 | 0.059 | 0.066 |
| 9.00 | 0.017 | 0.021 | 0.024 | 0.028 | 0.031 | 0.034 | 0.042 | 0.050 | 0.057 | 0.064 |
| 9.25 | 0.017 | 0.020 | 0.024 | 0.027 | 0.030 | 0.033 | 0.041 | 0.049 | 0.056 | 0.063 |
| 9.50 | 0.017 | 0.020 | 0.023 | 0.026 | 0.029 | 0.033 | 0.040 | 0.048 | 0.055 | 0.061 |
| 9.75 | 0.016 | 0.019 | 0.023 | 0.026 | 0.029 | 0.032 | 0.039 | 0.047 | 0.054 | 0.060 |
| 10.00 | 0.016 | 0.019 | 0.022 | 0.025 | 0.028 | 0.031 | 0.038 | 0.046 | 0.052 | 0.059 |
| 20.00 | 0.008 | 0.010 | 0.011 | 0.013 | 0.014 | 0.016 | 0.020 | 0.024 | 0.027 | 0.031 |
| 50.00 | 0.003 | 0.004 | 0.004 | 0.005 | 0.006 | 0.006 | 0.008 | 0.010 | 0.011 | 0.013 |
| 100.00 | 0.002 | 0.002 | 0.002 | 0.003 | 0.003 | 0.003 | 0.004 | 0.005 | 0.006 | 0.006 |

(*Continued*)

**Table 7.3** (*Continued*)

| | $m'$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n'$ | 4.5 | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 | 25.0 | 50.0 | 100.0 |
| 0.25 | 0.053 | 0.053 | 0.053 | 0.053 | 0.053 | 0.053 | 0.053 | 0.053 | 0.053 | 0.053 |
| 0.50 | 0.087 | 0.087 | 0.088 | 0.088 | 0.088 | 0.088 | 0.088 | 0.088 | 0.088 | 0.088 |
| 0.75 | 0.109 | 0.109 | 0.109 | 0.110 | 0.110 | 0.110 | 0.110 | 0.111 | 0.111 | 0.111 |
| 1.00 | 0.121 | 0.122 | 0.123 | 0.123 | 0.124 | 0.124 | 0.124 | 0.125 | 0.125 | 0.125 |
| 1.25 | 0.128 | 0.130 | 0.131 | 0.132 | 0.132 | 0.133 | 0.133 | 0.134 | 0.134 | 0.134 |
| 1.50 | 0.132 | 0.134 | 0.136 | 0.137 | 0.138 | 0.138 | 0.139 | 0.140 | 0.140 | 0.140 |
| 1.75 | 0.134 | 0.136 | 0.138 | 0.140 | 0.141 | 0.142 | 0.142 | 0.144 | 0.144 | 0.145 |
| 2.00 | 0.134 | 0.136 | 0.139 | 0.141 | 0.143 | 0.144 | 0.145 | 0.147 | 0.147 | 0.148 |
| 2.25 | 0.133 | 0.136 | 0.140 | 0.142 | 0.144 | 0.145 | 0.146 | 0.149 | 0.150 | 0.150 |
| 2.50 | 0.132 | 0.135 | 0.139 | 0.142 | 0.144 | 0.146 | 0.147 | 0.151 | 0.151 | 0.151 |
| 2.75 | 0.130 | 0.133 | 0.138 | 0.142 | 0.144 | 0.146 | 0.147 | 0.152 | 0.152 | 0.153 |
| 3.00 | 0.127 | 0.131 | 0.137 | 0.141 | 0.144 | 0.145 | 0.147 | 0.152 | 0.153 | 0.154 |
| 3.25 | 0.125 | 0.129 | 0.135 | 0.140 | 0.143 | 0.145 | 0.147 | 0.153 | 0.154 | 0.154 |
| 3.50 | 0.122 | 0.126 | 0.133 | 0.138 | 0.142 | 0.144 | 0.146 | 0.153 | 0.155 | 0.155 |
| 3.75 | 0.119 | 0.124 | 0.131 | 0.137 | 0.141 | 0.143 | 0.145 | 0.154 | 0.155 | 0.155 |
| 4.00 | 0.116 | 0.121 | 0.129 | 0.135 | 0.139 | 0.142 | 0.145 | 0.154 | 0.155 | 0.156 |
| 4.25 | 0.113 | 0.119 | 0.127 | 0.133 | 0.138 | 0.141 | 0.144 | 0.154 | 0.156 | 0.156 |
| 4.50 | 0.110 | 0.116 | 0.125 | 0.131 | 0.136 | 0.140 | 0.143 | 0.154 | 0.156 | 0.156 |
| 4.75 | 0.107 | 0.113 | 0.123 | 0.130 | 0.135 | 0.139 | 0.142 | 0.154 | 0.156 | 0.157 |
| 5.00 | 0.105 | 0.111 | 0.120 | 0.128 | 0.133 | 0.137 | 0.140 | 0.154 | 0.156 | 0.157 |
| 5.25 | 0.102 | 0.108 | 0.118 | 0.126 | 0.131 | 0.136 | 0.139 | 0.154 | 0.156 | 0.157 |
| 5.50 | 0.099 | 0.106 | 0.116 | 0.124 | 0.130 | 0.134 | 0.138 | 0.154 | 0.156 | 0.157 |
| 5.75 | 0.097 | 0.103 | 0.113 | 0.122 | 0.128 | 0.133 | 0.136 | 0.154 | 0.157 | 0.157 |
| 6.00 | 0.094 | 0.101 | 0.111 | 0.120 | 0.126 | 0.131 | 0.135 | 0.153 | 0.157 | 0.157 |
| 6.25 | 0.092 | 0.098 | 0.109 | 0.118 | 0.124 | 0.129 | 0.134 | 0.153 | 0.157 | 0.158 |
| 6.50 | 0.090 | 0.096 | 0.107 | 0.116 | 0.122 | 0.128 | 0.132 | 0.153 | 0.157 | 0.158 |
| 6.75 | 0.087 | 0.094 | 0.105 | 0.114 | 0.121 | 0.126 | 0.131 | 0.153 | 0.157 | 0.158 |
| 7.00 | 0.085 | 0.092 | 0.103 | 0.112 | 0.119 | 0.125 | 0.129 | 0.152 | 0.157 | 0.158 |
| 7.25 | 0.083 | 0.090 | 0.101 | 0.110 | 0.117 | 0.123 | 0.128 | 0.152 | 0.157 | 0.158 |
| 7.50 | 0.081 | 0.088 | 0.099 | 0.108 | 0.115 | 0.121 | 0.126 | 0.152 | 0.156 | 0.158 |
| 7.75 | 0.079 | 0.086 | 0.097 | 0.106 | 0.114 | 0.120 | 0.125 | 0.151 | 0.156 | 0.158 |
| 8.00 | 0.077 | 0.084 | 0.095 | 0.104 | 0.112 | 0.118 | 0.124 | 0.151 | 0.156 | 0.158 |
| 8.25 | 0.076 | 0.082 | 0.093 | 0.102 | 0.110 | 0.117 | 0.122 | 0.150 | 0.156 | 0.158 |
| 8.50 | 0.074 | 0.080 | 0.091 | 0.101 | 0.108 | 0.115 | 0.121 | 0.150 | 0.156 | 0.158 |
| 8.75 | 0.072 | 0.078 | 0.089 | 0.099 | 0.107 | 0.114 | 0.119 | 0.150 | 0.156 | 0.158 |
| 9.00 | 0.071 | 0.077 | 0.088 | 0.097 | 0.105 | 0.112 | 0.118 | 0.149 | 0.156 | 0.158 |
| 9.25 | 0.069 | 0.075 | 0.086 | 0.096 | 0.104 | 0.110 | 0.116 | 0.149 | 0.156 | 0.158 |
| 9.50 | 0.068 | 0.074 | 0.085 | 0.094 | 0.102 | 0.109 | 0.115 | 0.148 | 0.156 | 0.158 |
| 9.75 | 0.066 | 0.072 | 0.083 | 0.092 | 0.100 | 0.107 | 0.113 | 0.148 | 0.156 | 0.158 |
| 10.00 | 0.065 | 0.071 | 0.082 | 0.091 | 0.099 | 0.106 | 0.112 | 0.147 | 0.156 | 0.158 |
| 20.00 | 0.035 | 0.039 | 0.046 | 0.053 | 0.059 | 0.065 | 0.071 | 0.124 | 0.148 | 0.156 |
| 50.00 | 0.014 | 0.016 | 0.019 | 0.022 | 0.025 | 0.028 | 0.031 | 0.071 | 0.113 | 0.142 |
| 100.00 | 0.007 | 0.008 | 0.010 | 0.011 | 0.013 | 0.014 | 0.016 | 0.039 | 0.071 | 0.113 |

**Table 7.4** Variation of $I_f$ with $D_f/B$, $B/L$, and $\mu_s$

| | | B/L | | |
|---|---|---|---|---|
| $\mu_s$ | $D_f/B$ | 0.2 | 0.5 | 1.0 |
| 0.3 | 0.2 | 0.95 | 0.93 | 0.90 |
| | 0.4 | 0.90 | 0.86 | 0.81 |
| | 0.6 | 0.85 | 0.80 | 0.74 |
| | 1.0 | 0.78 | 0.71 | 0.65 |
| 0.4 | 0.2 | 0.97 | 0.96 | 0.93 |
| | 0.4 | 0.93 | 0.89 | 0.85 |
| | 0.6 | 0.89 | 0.84 | 0.78 |
| | 1.0 | 0.82 | 0.75 | 0.69 |
| 0.5 | 0.2 | 0.99 | 0.98 | 0.96 |
| | 0.4 | 0.95 | 0.93 | 0.89 |
| | 0.6 | 0.92 | 0.87 | 0.82 |
| | 1.0 | 0.85 | 0.79 | 0.72 |

## Example 4.2

A rigid shallow foundation 1 m × 2 m is shown in Figure 7.4. Calculate the elastic settlement at the center of the foundation.

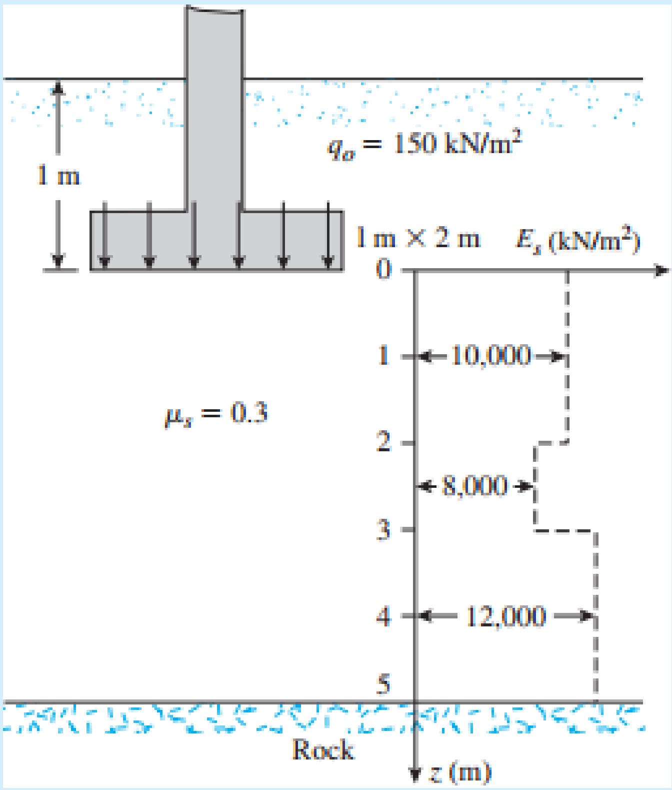



**Figure 7.4** Elastic settlement below the center of a foundation

**Solution**

We are given that $B = 1$ m and $L = 2$ m. Note that $\bar{z} = 5$ m $= 5B$. From Eq. (7.13)

$$E_s = \frac{\Sigma E_{s(i)} \Delta z}{\bar{z}}$$

$$= \frac{(10{,}000)(2) + (8{,}000)(1) + (12{,}000)(2)}{5} = 10{,}400 \text{ kN/m}^2$$

For the *center of the foundation,*

$$\alpha = 4$$

$$m' = \frac{L}{B} = \frac{2}{1} = 2$$

and

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{5}{\left(\frac{1}{2}\right)} = 10$$

From Tables 7.2 and 7.3, $F_1 = 0.641$ and $F_2 = 0.031$. From Eq. (7.5),

$$I_s = F_1 + \frac{2 - \mu_s}{1 - \mu_s} F_2$$

$$= 0.641 + \frac{2 - 0.3}{1 - 0.3}(0.031) = 0.716$$

Again, $D_f/B = 1/1 = 1$, $B/L = 0.5$, and $\mu_s = 0.3$. From Table 7.4, $I_f = 0.71$.

Hence,

$$S_{e(\text{flexible})} = q_0(\alpha B') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

$$= (150)\left(4 \times \frac{1}{2}\right)\left(\frac{1 - 0.3^2}{10{,}400}\right)(0.716)(0.71) = 0.0133 \text{ m} = 13.3 \text{ mm}$$

Since the foundation is rigid, from Eq.(7.12) we obtain

$$S_{e(\text{rigid})} = (0.93)(13.3) = \mathbf{12.4 \text{ mm}}$$

## 4.9 Settlement of Sandy Soil: Use of Strain Influence Factor Solution of Schmertmann et al. (1978)

- The settlement of granular soils can also be evaluated by the use of a semiempirical ***strain influence factor*** proposed by Schmertmann et al. (1978). According to this method (Figure 7.9), the settlement is

$$S_e = C_1 C_2(\bar{q} - q)\sum_{0}^{z_2}\frac{I_z}{E_s}\Delta z \qquad (7.20)$$

where

$I_z$ = strain influence factor
$C_1$ = a correction factor for the depth of foundation embedment = $1 - 0.5\,[q/(\bar{q} - q)]$
$C_2$ = a correction factor to account for creep in soil
= $1 + 0.2 \log(\text{time in years}/0.1)$
$\bar{q}$ = stress at the level of the foundation
$q = \gamma D_f$ = effective stress at the base of the foundation
$E_s$ = modulus of elasticity of soil

The variation of the strain influence factor with the depth below the foundation is shown in Figure 7.9. Note that,

For square or circular foundations,

$I_z = 0.1$ at $z = 0$

$I_z = 0.5$ at $z = z_1 = 0.5B$

and

$I_z = 0$ at $z = z_2 = 2B$

Similarly, for foundations with $L/B \geq 10$,

$I_z = 0.2$ at $z = 0$

$I_z = 0.5$ at $z = z_1 = B$

and

$I_z = 0$ at $z = z_2 = 4B$

Where B = width of the foundation and L = length of the foundation. Values of L/B between 1 and 10 can be interpolated.

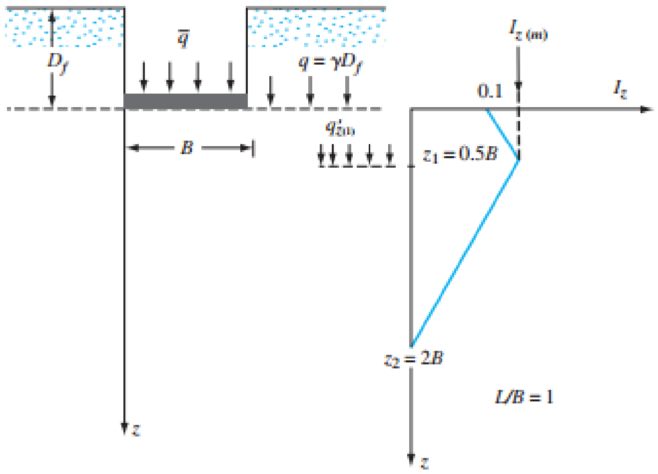



*Figure 7.9* Variation of strain influence factor with depth and $L/B$

The procedure for calculating elastic settlement using Eq. (7.20) is given here (Figure 7.10).

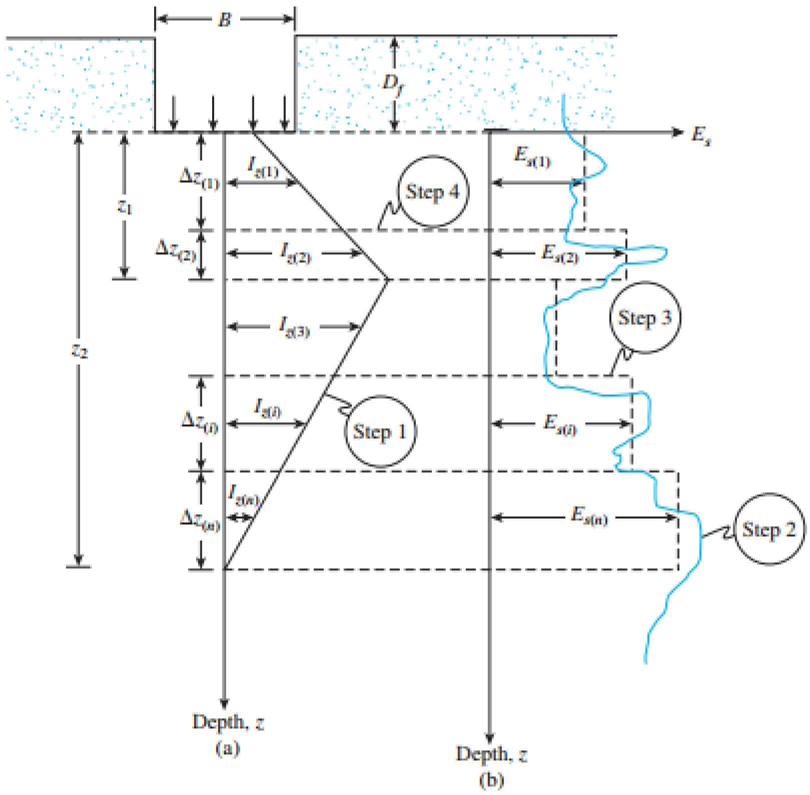



*Figure 7.10* Procedure for calculation of $S_e$ using the strain influence factor

**Table 7.5** Calculation of $\Sigma \frac{I_z}{E_s} \Delta z$

| Layer no. | $\Delta z$ | $E_s$ | $I_z$ at the middle of the layer | $\frac{I_z}{E_s} \Delta z$ |
|---|---|---|---|---|
| 1 | $\Delta z_{(1)}$ | $E_{s(1)}$ | $I_{z(1)}$ | $\frac{I_{z(1)}}{E_{s(1)}} \Delta z_1$ |
| 2 | $\Delta z_{(2)}$ | $E_{s(2)}$ | $I_{z(2)}$ | |
| ⋮ | ⋮ | ⋮ | ⋮ | |
| $i$ | $\Delta z_{(i)}$ | $E_{s(i)}$ | $I_{z(i)}$ | $\frac{I_{z(i)}}{E_{s(i)}} \Delta z_i$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $n$ | $\Delta z_{(n)}$ | $E_{s(n)}$ | $I_{z(n)}$ | $\frac{I_{z(n)}}{E_{s(n)}} \Delta z_n$ |
| | | | | $\Sigma \frac{I_z}{E_s} \Delta z$ |

*Step 1.* Plot the foundation and the variation of $I_z$ with depth to scale (Figure 7.10a).

*Step 2.* Using the correlation from standard penetration resistance ($N_{60}$) or cone penetration resistance ($q_c$), plot the actual variation of $E_s$ with depth (Figure 7.10b).

*Step 3.* Approximate the actual variation of $E_s$ into a number of layers of soil having a constant $E_s$, such as $E_{s(1)}, E_{s(2)}, \ldots, E_{s(i)}, \ldots E_{s(n)}$ (Figure 7.10b).

*Step 4.* Divide the soil layer from $z = 0$ to $z = z_2$ into a number of layers by drawing horizontal lines. The number of layers will depend on the break in continuity in the $I_z$ and $E_s$ diagrams.

*Step 5.* Prepare a table (such as Table 7.5) to obtain $\Sigma \frac{I_z}{E_s} \Delta z$.

*Step 6.* Calculate $C_1$ and $C_2$.

*Step 7.* Calculate $S_e$ from Eq. (7.20).

Example 4.3

Figure 3.19a shows a shallow foundation on a deposit of sandy soil that is 3 m × 3 m in plan. The actual variation of the values of the modulus of elasticity with depth determined by using the standard penetration numbers are also shown in Figure 3.19a. Using the strain influence factor method, estimate the elastic settlement of the foundation after five years of construction.

**Solution**

By observing the actual variation of the modulus of elasticity with depth one can plot an estimated idealized form of the variation of $E_s$, as shown in Figure 3.19a. Figure 3.19b shows the plot of the strain influence factor. The following table can

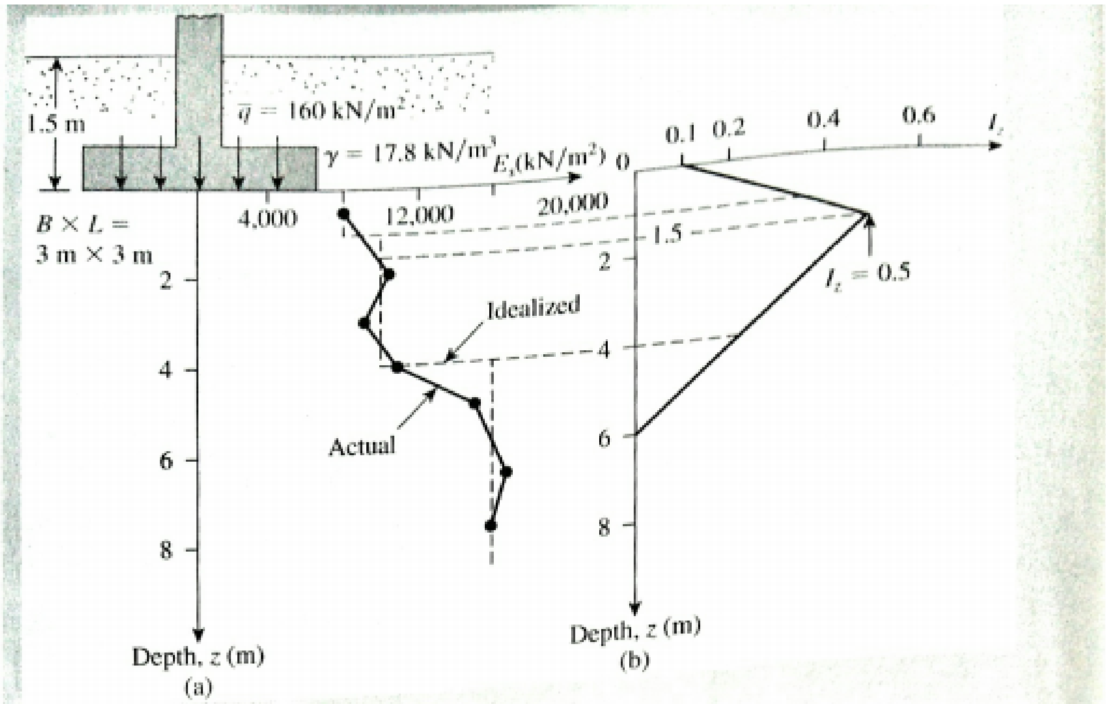



*Figure 3.19*

| Depth (m) | $\Delta z$ (m) | $E_s$ (kN/m²) | Average $I_z$ | $\frac{I_z}{E_s}\cdot\Delta z$ (m³/kN) |
|---|---|---|---|---|
| 0–1 | 1 | 8,000 | 0.233 | $0.291 \times 10^{-4}$ |
| 1.0–1.5 | 0.5 | 10,000 | 0.433 | $0.217 \times 10^{-4}$ |
| 1.5–4 | 2.5 | 10,000 | 0.361 | $0.903 \times 10^{-4}$ |
| 4.0–6 | 2 | 16,000 | 0.111 | $0.139 \times 10^{-4}$ |
| | | | | $\sum = 1.55 \times 10^{-4}$ |

$$C_1 = 1 - 0.5\left(\frac{q}{\bar{q} - q}\right) = 1 - 0.5\left[\frac{17.8 \times 1.5}{160 - (17.8 \times 1.5)}\right] = 0.9$$

$$C_2 = 1 + 0.2 \log\left(\frac{5}{0.1}\right) = 1.34$$

Hence,

$$S_e = C_1, C_2(\bar{q} - q)\sum_0^{2B}\frac{I_z}{E_s}\cdot\Delta z$$

$$= (0.9)(1.34)[160 - (17.8 \times 1.5)](1.55 \times 10^{-4})$$

$$= 249.2 \times 10^{-4}\ \text{m} \approx \mathbf{24.9\ mm}$$

# 4.10 Settlement of Foundation on Sand Based on Standard Penetration Resistance

## Meyerhof's Method

- Meyerhof (1956) proposed a correlation for the *net bearing pressure* for foundations with the standard penetration resistance, $N_{60}$. The net pressure has been defined as

$$q_{\text{net}} = \overline{q} - \gamma D_f$$

where $\overline{q}$ = stress at the level of the foundation.

$$q_{\text{net}}(\text{kip/ft}^2) = \frac{N_{60}}{4} \quad (\text{for } B \leq 4 \text{ ft}) \tag{7.32}$$

and

$$q_{\text{net}}(\text{kip/ft}^2) = \frac{N_{60}}{6}\left(\frac{B+1}{B}\right)^2 \quad (\text{for } B > 4 \text{ ft}) \tag{7.33}$$

- Bowles (1977) proposed that the modified form of the bearing equations be expressed as

$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22 \text{ m}) \tag{7.38}$$

and

$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m}) \tag{7.39}$$

where $B$ is in meters and $S_e$ is in mm. Hence,

$$S_e(\text{mm}) = \frac{1.25 q_{\text{net}}(\text{kN/m}^2)}{N_{60} F_d} \quad (\text{for } B \leq 1.22 \text{ m}) \tag{7.40}$$

and

$$S_e(\text{mm}) = \frac{2 q_{\text{net}}(\text{kN/m}^2)}{N_{60} F_d}\left(\frac{B}{B+0.3}\right)^2 \quad (\text{for } B > 1.22 \text{ m}) \tag{7.41}$$

- The $N_{60}$ referred to in the preceding equations is the standard penetration resistance between the bottom of the foundation and $2B$ below the bottom.

# Consolidation Settlement

## 4.11 Primary Consolidation Settlement Relationships

As mentioned before, consolidation settlement occurs over time in saturated clayey soils subjected to an increased load caused by construction of the foundation. (See Figure 4.20.) On the basis of the one-dimensional consolidation settlement equations, we write

$$S_{c(p)} = \int \varepsilon_z dz$$

where

$\varepsilon_z$ = vertical strain

$= \dfrac{\Delta e}{1 + e_o}$

$\Delta e$ = change of void ratio

$= f(\sigma'_o, \sigma'_c, \text{ and } \Delta\sigma')$

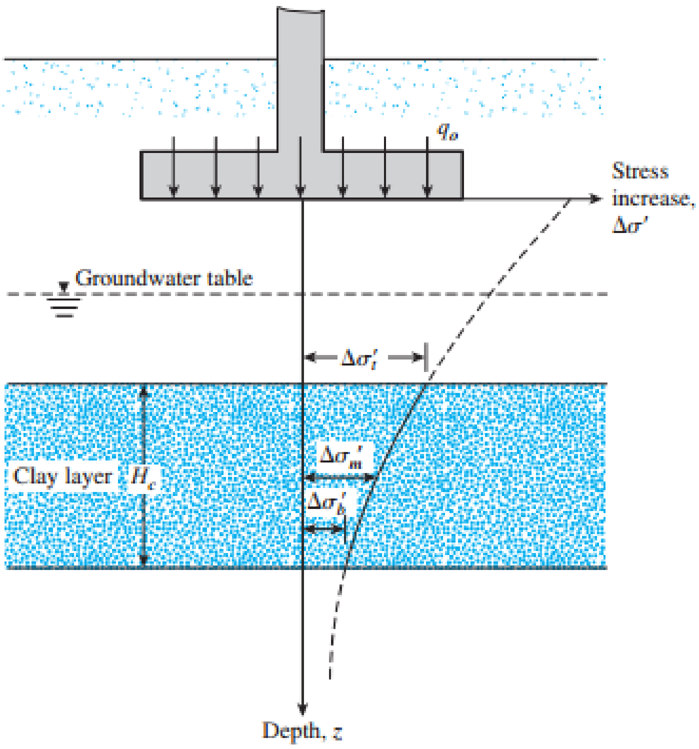



Figure 4.20 Consolidation settlement calculation

So,

$$S_{c(p)} = \frac{C_c H_c}{1+e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \qquad \text{(for normally consolidated clays)}$$

$$S_{c(p)} = \frac{C_s H_c}{1+e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \qquad \text{(for overconsolidated clays with } \sigma'_o + \Delta\sigma'_{av} < \sigma'_c\text{)}$$

$$S_{c(p)} = \frac{C_s H_c}{1+e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1+e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c} \qquad \text{(for overconsolidated clays with } \sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'_{av}\text{)}$$

where

$\sigma'_o$ = average effective pressure on the clay layer before the construction of the foundation
$\Delta\sigma'_{av}$ = average increase in effective pressure on the clay layer caused by the construction of the foundation
$\sigma'_c$ = preconsolidation pressure
$e_o$ = initial void ratio of the clay layer
$C_c$ = compression index
$C_s$ = swelling index
$H_c$ = thickness of the clay layer

- Note that the increase in effective pressure, $\Delta\sigma'$, on the clay layer is not constant with depth: The magnitude of $\Delta\sigma'$ will decrease with the increase in depth measured from the bottom of the foundation. However, the average increase in pressure may be approximated by

$$\Delta\sigma'_{av} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

where $\Delta\sigma'_t$, $\Delta\sigma'_m$, and $\Delta\sigma'_b$ are, respectively, the effective pressure increases at the *top, middle,* and *bottom* of the clay layer that are caused by the construction of the foundation.

The method of determining the pressure increase caused by various types of foundation load using Boussinesq's solution is discussed in previous Sections

## Example 4.4

A plan of a foundation 1 m × 2 m is shown in Figure 4.23. Estimate the consolidation settlement of the foundation.

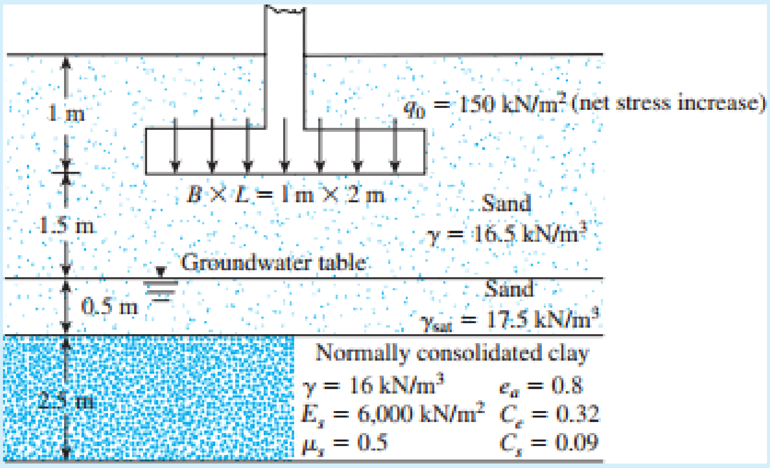



*Figure 4.23* Calculation of primary consolidation settlement for a foundation

### Solution

The clay is normally consolidated. Thus,

$$S_{c(p)-\text{oed}} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{\text{av}}}{\sigma'_o}$$

so

$$\sigma'_o = (2.5)(16.5) + (0.5)(17.5 - 9.81) + (1.25)(16 - 9.81)$$
$$= 41.25 + 3.85 + 7.74 = 52.84 \text{ kN/m}^2$$

From Eq. (6.29),

$$\Delta\sigma'_{\text{av}} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

Now the following table can be prepared (*Note:* $L = 2$ m; $B = 1$ m):

| $m_1 = L/B$ | $z$(m) | $z/(B/2) = n_1$ | $I_c$[a] | $\Delta\sigma' = q_o I_c$[b] |
|---|---|---|---|---|
| 2 | 2 | 4 | 0.190 | $28.5 = \Delta\sigma'_t$ |
| 2 | $2 + 2.5/2 = 3.25$ | 6.5 | $\approx 0.085$ | $12.75 = \Delta\sigma'_m$ |
| 2 | $2 + 2.5 = 4.5$ | 9 | 0.045 | $6.75 = \Delta\sigma'_b$ |

[a]Table 6.5
[b]Eq. (6.14)

Now,

$$\Delta\sigma'_{\text{av}} = \tfrac{1}{6}(28.5 + 4 \times 12.75 + 6.75) = 14.38 \text{ kN/m}^2$$

so

$$S_{c(p)-\text{oed}} = \frac{(0.32)(2.5)}{1 + 0.8} \log\left(\frac{52.84 + 14.38}{52.84}\right) = 0.0465 \text{ m}$$
$$= \mathbf{46.5 \text{ mm}}$$

## 4.12 Settlement Due to Secondary Consolidation

At the end of primary consolidation (i.e., after the complete dissipation of excess pore water pressure) some settlement is observed that is due to the plastic adjustment of soil fabrics. This stage of consolidation is called *secondary consolidation.* A plot of deformation against the logarithm of time during secondary consolidation is practically linear as shown in Figure 4.24. From the figure, the secondary compression index can be defined as

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log (t_2/t_1)}$$

where

$C_\alpha$ = secondary compression index
$\Delta e$ = change of void ratio
$t_1, t_2$ = time

The magnitude of the secondary consolidation can be calculated as

$$S_{c(s)} = C'_\alpha H_c \log(t_2/t_1)$$

where

$C'_\alpha = C_\alpha/(1 + e_p)$
$e_p$ = void ratio at the end of primary consolidation
$H_c$ = thickness of clay layer

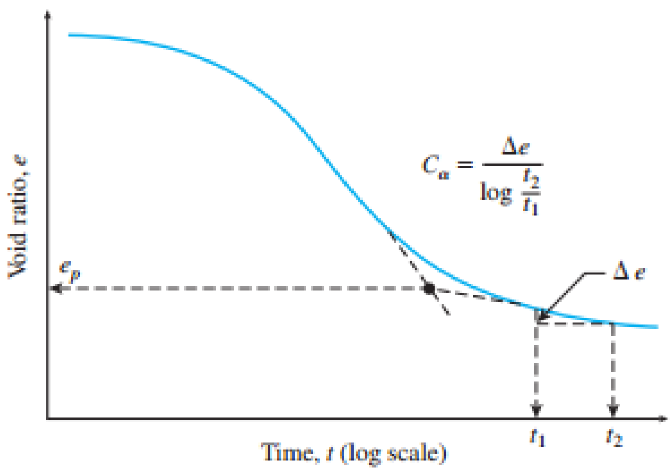



**Figure 4.24** Variation of $e$ with log $t$ under a given load increment, and definition of secondary compression index

Mesri (1973) correlated $C'_\alpha$ with the natural moisture content ($w$) of several soils, from which it appears that

$$C'_\alpha \approx 0.0001w$$

where $w$ = natural moisture content, in percent. For most overconsolidated soils, $C'_\alpha$ varies between 0.0005 to 0.001.

Secondary consolidation settlement is more important in the case of all organic and highly compressible inorganic soils. In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

There are several factors that might affect the magnitude of secondary consolidation, some of which are not yet very clearly understood (Mesri, 1973). The ratio of secondary to primary compression for a given thickness of soil layer is dependent on the ratio of the stress increment, $\Delta\sigma'$, to the initial effective overburden stress, $\sigma'_o$ . For small $\Delta\sigma' / \sigma'_o$ ratios, the secondary-to-primary compression ratio is larger.

**Example 4.5**

Refer to Example 7.10. Given for the clay layer: $C_\alpha = 0.02$. Estimate the total consolidation settlement five years after the completion of the primary consolidation settlement. (*Note:* Time for completion of primary consolidation settlement is 1.3 years).

**Solution**

From Eq. (2.53),

$$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)}$$

For this problem, $e_1 - e_2 = \Delta e$.

Referring to Example 4.4, we have

$$\sigma_2' = \sigma_o' + \Delta\sigma' = 52.84 + 14.38 = 67.22 \text{ kN/m}^2$$

$$\sigma_1' = \sigma_o' = 52.84 \text{ kN/m}^2$$

$$C_c = 0.32$$

Hence,

$$\Delta e = C_c \log\left(\frac{\sigma_o' + \Delta\sigma}{\sigma_o'}\right) = 0.32 \log\left(\frac{67.22}{52.84}\right) = 0.0335$$

Given: $e_o = 0.8$. Hence,

$$e_p = e_o - e = 0.8 - 0.0335 = 0.7665$$

From Eq. (7.71),

$$C_\alpha' = \frac{C_\alpha}{1 + e_p} = \frac{0.02}{1 + 0.7665} = 0.0113$$

From Eq. (7.70),

$$S_{c(s)} = C_\alpha' H_c \log\left(\frac{t_2}{t_1}\right)$$

*Note*: $t_1 = 1.3$ years; $t_2 = 1.3 + 5 = 6.3$ years.
Thus,

$$S_{c(s)} = (0.0113)(2.5 \text{ m}) \log\left(\frac{6.3}{1.3}\right) = 0.0194 \text{ m} = 19.4 \text{ mm}$$

Total consolidation settlement is

$$\underbrace{36.3 \text{ mm}}_{\substack{\uparrow \\ \text{Example 7.10} \\ \text{(Primary} \\ \text{consolidation} \\ \text{settlement)}}} + 19.4 = \mathbf{55.7\ m}$$

## 4.13 Field Load Test

The ultimate load-bearing capacity of a foundation, as well as the allowable bearing capacity based on tolerable settlement considerations, can be effectively determined from the field load test, generally referred to as the *plate load test.* The plates that are used for tests in the field are usually made of steel and are 25 mm (1 in.) thick and 150 mm to 762 mm

(6 in. to 30 in.) in diameter. Occasionally, square plates that are 305 mm × 305 mm (12 in. × 12 in.) are also used.

To conduct a plate load test, a hole is excavated with a minimum diameter of 4*B* (*B* is the diameter of the test plate) to a depth of $D_f$ , the depth of the proposed foundation. The plate is placed at the center of the hole, and a load that is about one-fourth to one-fifth of the estimated ultimate load is applied to the plate in steps by means of a jack. A schematic diagram of the test arrangement is shown in Figure 4.25a. During each step of the application of the load, the settlement of the plate is observed on dial gauges. At least one hour is allowed to elapse between each application. The test should be conducted until failure, or at least until the plate has gone through 25 mm (1 in.) of settlement. Figure 4.25b shows the nature of the load–settlement curve obtained from such tests, from which the ultimate load per unit area can be determined. Figure 4.26 shows a plate load test conducted in the field.

For tests in clay,

$$q_{u(F)} = q_{u(P)}$$

where

$q_{u(F)}$ = ultimate bearing capacity of the proposed foundation
$q_{u(P)}$ = ultimate bearing capacity of the test plate

Equation above implies that the ultimate bearing capacity in clay is virtually independent of the size of the plate.

For tests in sandy soils,

$$q_{u(F)} = q_{u(P)} \frac{B_F}{B_P}$$

where

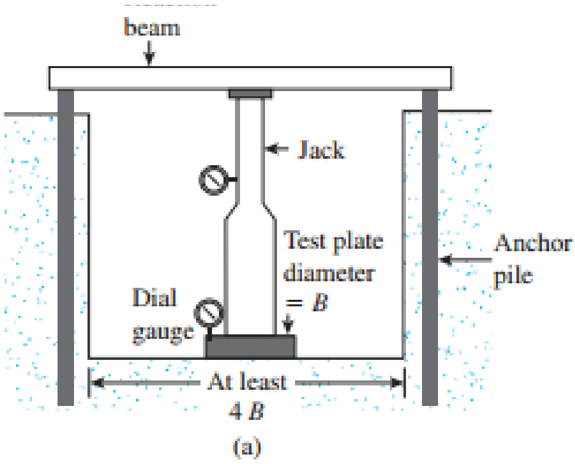
$B_F$ = width of the foundation
$B_P$ = width of the test plate

The allowable bearing capacity of a foundation, based on settlement considerations and for a given intensity of load, $q_o$, is

$$S_F = S_P \frac{B_F}{B_P} \quad \text{(for clayey soil)}$$

and

$$S_F = S_P\left(\frac{2B_F}{B_F + B_P}\right)^2 \quad \text{(for sandy soil)}$$



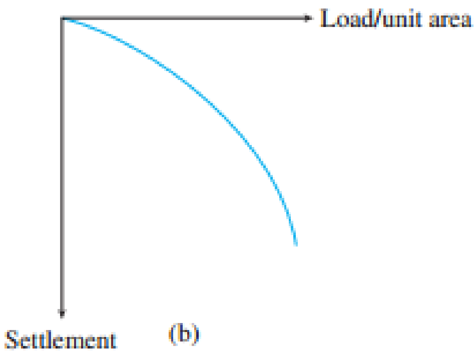




**Figure 4.25** Plate load test: (a) test arrangement; (b) nature of load–settlement curve

**Figure 4.26** Plate load test in the field (*Courtesy of Braja M. Das, Henderson, Nevada*)

## 4.14 Tolerable Settlement of Buildings

In most instances of construction, the subsoil is not homogeneous and the load carried by various shallow foundations of a given structure can vary widely. As a result, it is reasonable to expect varying degrees of settlement in different parts of a given building. The ***differential settlement*** of the parts of a building can lead to damage of the superstructure. Hence, it is important to define certain parameters that quantify differential settlement and to develop limiting values for those parameters in order that the resulting structures be safe.

Burland and Wroth (1970) summarized the important parameters relating to differential settlement. Figure 4.27 shows a structure in which various foundations, at *A, B, C, D,* and *E,* have gone through some settlement. The settlement at *A* is $AA'$, at *B* is $BB'$, etc. Based on this figure, the definitions of the various parameters are as follows:

$S_T$ = total settlement of a given point
$\Delta S_T$ = difference in total settlement between any two points
$\alpha$ = gradient between two successive points

$$\beta = \text{angular distortion} = \frac{\Delta S_{T(ij)}}{l_{ij}}$$

(*Note*: $l_{ij}$ = distance between points $i$ and $j$)
$\omega$ = tilt
$\Delta$ = relative deflection (i.e., movement from a straight line joining two reference points)

$$\frac{\Delta}{L} = \text{deflection ratio}$$

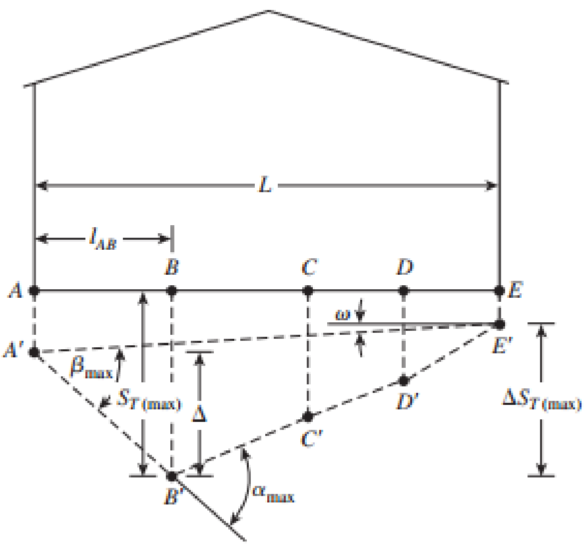



**Figure 4.27** Definition of parameters for differential settlement

- In 1956, Skempton and McDonald proposed the following limiting values for maximum settlement and maximum angular distortion, to be used for building purposes:

| | |
|---|---|
| Maximum settlement, $S_{T(max)}$ | |
| In sand | 32 mm |
| In clay | 45 mm |
| Maximum differential settlement, $\Delta S_{T(max)}$ | |
| Isolated foundations in sand | 51 mm |
| Isolated foundations in clay | 76 mm |
| Raft in sand | 51–76 mm |
| Raft in clay | 76–127 mm |
| Maximum angular distortion, $\beta_{max}$ | 1/300 |

- Polshin and Tokar (1957) suggested the following allowable deflection ratios for buildings as a function of $L / H$, the ratio of the length to the height of a building:

$$\Delta/L = 0.0003 \text{ for } L/H \leq 2$$
$$\Delta/L = 0.001 \text{ for } L/H = 8$$

- The 1955 Soviet Code of Practice allowable values are given in Table 4.10.
- Bjerrum (1963) recommended the following limiting angular distortion, $\beta_{max}$ for various structures, as shown in Table 4.11.

**Table 4.10**

| Type of building | $L/H$ | $\Delta/L$ |
|---|---|---|
| Multistory buildings and civil dwellings | ≤3 | 0.0003 (for sand)<br>0.0004 (for clay) |
| | ≥5 | 0.0005 (for sand)<br>0.0007 (for clay) |
| One-story mills | | 0.001 (for sand and clay) |

**Table 4.11**

| Category of potential damage | $\beta_{max}$ |
|---|---|
| Safe limit for flexible brick wall ($L/H > 4$) | 1/150 |
| Danger of structural damage to most buildings | 1/150 |
| Cracking of panel and brick walls | 1/150 |
| Visible tilting of high rigid buildings | 1/250 |
| First cracking of panel walls | 1/300 |
| Safe limit for no cracking of building | 1/500 |
| Danger to frames with diagonals | 1/600 |

If the maximum allowable values of $\beta_{max}$ are known, the magnitude of the allowable $S_{T(max)}$ can be calculated with the use of the foregoing correlations.

- The European Committee for Standardization has also provided limiting values for serviceability and the maximum accepted foundation movements. (See Table 4.12.)

**Table 4.12** Recommendations of European Committee for Standardization on Differential Settlement Parameters

| Item | Parameter | Magnitude | Comments |
|---|---|---|---|
| Limiting values for serviceability | $S_T$ | 25 mm | Isolated shallow foundation |
| | | 50 mm | Raft foundation |
| (European Committee | $\Delta S_T$ | 5 mm | Frames with rigid cladding |
| for Standardization, | | 10 mm | Frames with flexible cladding |
| 1994a) | | 20 mm | Open frames |
| | $\beta$ | 1/500 | — |
| Maximum acceptable | $S_T$ | 50 | Isolated shallow foundation |
| foundation movement | $\Delta S_T$ | 20 | Isolated shallow foundation |
| (European Committee | $\beta$ | ≈1/500 | — |
| for Standardization, 1994b) | | | |

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# CHAPTER SIX

# PILE FOUNDATIONS

**LECTURE**
**DR. AHMED H. ABDULKAREEM**
**2019 - 2020**

# 6.1. Introduction

Piles are structural members that are made of steel, concrete, or timber. They are used to build pile foundations, which are deep and which cost more than shallow foundations. Despite the cost, the use of piles often is necessary to ensure structural safety. The following list identifies some of the conditions that require pile foundations (Vesic, 1977):

**1.** When one or more upper soil layers are highly compressible and too weak to support the load transmitted by the superstructure, piles are used to transmit the load to underlying bedrock or a stronger soil layer, as shown in Figure 9.1a. When bedrock is not encountered at a reasonable depth below the ground surface, piles are used to transmit the structural load to the soil gradually. The resistance to the applied structural load is derived mainly from the frictional resistance developed at the soil–pile interface. (See Figure 9.1b.)

**2.** When subjected to horizontal forces (see Figure 9.1c), pile foundations resist by bending, while still supporting the vertical load transmitted by the superstructure. This type of situation is generally encountered in the design and construction of earth-retaining structures and foundations of tall structures that are subjected to high wind or to earthquake forces.

**3.** In many cases, expansive and collapsible soils may be present at the site of a proposed structure. These soils may extend to a great depth below the ground surface. Expansive soils swell and shrink as their moisture content increases and decreases, and the pressure of the swelling can be considerable. If shallow foundations are used in such circumstances, the structure may suffer considerable damage. However, pile foundations may be considered as an alternative when piles are extended beyond the active zone, which is where swelling and shrinking occur. (See Figure 9.1d.) Soils such as loess are collapsible in nature. When the moisture content of these soils increases, their structures may break down. A sudden decrease in the void ratio of soil induces large settlements of structures supported by shallow foundations. In such cases, pile foundations may be used in which the piles are extended into stable soil layers beyond the zone where moisture will change.

4. The foundations of some structures, such as transmission towers, offshore platforms, and basement mats below the water table, are subjected to uplifting forces. Piles are sometimes used for these foundations to resist the uplifting force. (See Figure 9.1e.)
5. Bridge abutments and piers are usually constructed over pile foundations to avoid the loss of bearing capacity that a shallow foundation might suffer because of soil erosion at the ground surface. (See Figure 9.1f.)

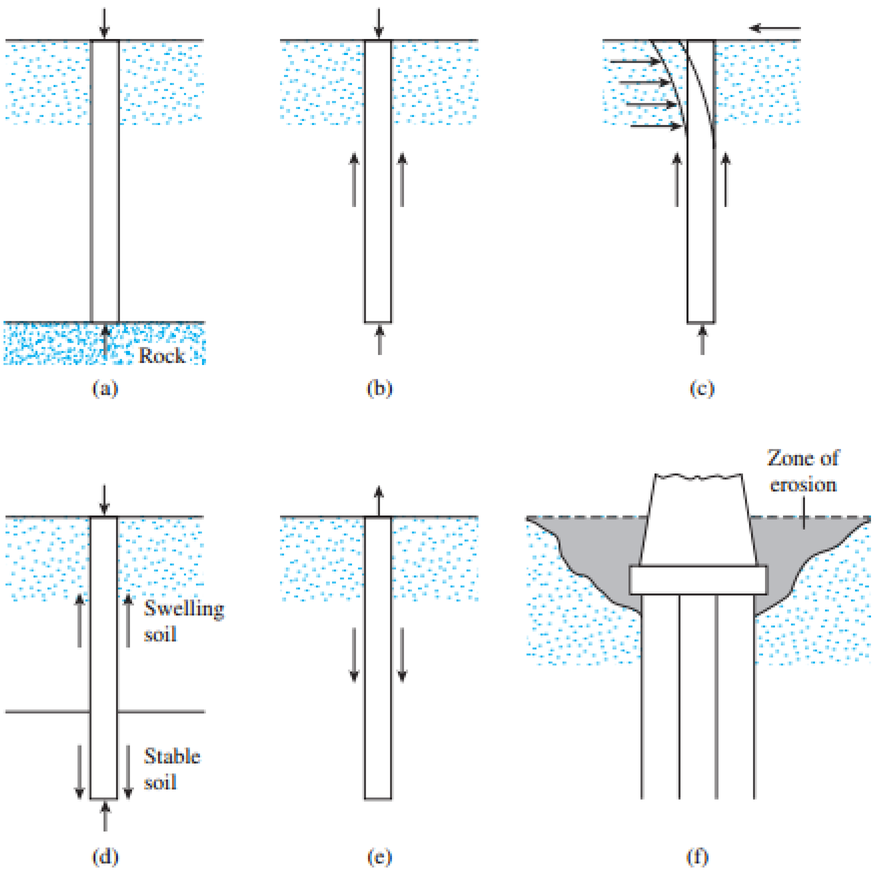



**Figure 9.1** Conditions that require the use of pile foundations

## 9.2 Types of Piles and Their Structural Characteristics

- Different types of piles are used in construction work, depending on the type of load to be carried, the subsoil conditions, and the location of the water table.
- Piles can be divided into the following categories with the general descriptions for conventional steel, concrete, timber, and composite piles.

## Steel Piles

- *Steel piles* generally are either *pipe piles* or *rolled steel* H-*section piles.*
- Pipe piles can be driven into the ground with their ends open or closed.
- Wide-flange and I-section steel beams can also be used as piles. However, H-section piles are usually preferred because their web and flange thicknesses are equal. (In wide-flange and I-section beams, the web thicknesses are smaller than the thicknesses of the flange.) Table 9.1 gives the dimensions of some standard H-section steel piles used in the United States.
- Table 9.2 shows selected pipe sections frequency used for piling purposes.
- In many cases, the pipe piles are filled with concrete after they have been driven.
- The allowable structural capacity for steel piles is

$$Q_{all} = A_s f_s \tag{9.1}$$

where

$A_s$ = cross-sectional area of the steel
$f_s$ = allowable stress of steel ($\approx 0.33\text{–}0.5\, f_y$)

When hard driving conditions are expected, such as driving through dense gravel, shale, or soft rock, steel piles can be fitted with driving points or shoes. Figures 9.2d and 9.2e are diagrams of two types of shoe used for pipe piles.

Here are some general facts about steel piles:

- Usual length: 15 m to 60 m (50 ft to 200 ft)
- Usual load: 300 kN to 1200 kN (67 kip to 265 kip)

- Advantages:
  a. Easy to handle with respect to cutoff and extension to the desired length
  b. Can stand high driving stresses
  c. Can penetrate hard layers such as dense gravel and soft rock
  d. High load-carrying capacity
- Disadvantages:
  a. Relatively costly
  b. High level of noise during pile driving
  c. Subject to corrosion
  d. H-piles may be damaged or deflected from the vertical during driving through hard layers or past major obstructions

***Table 9.1a*** Common H-Pile Sections used in the United States (SI Units)

| Designation, size (mm) × weight (kg/m) | Depth $d_1$ (mm) | Section area ($m^2 \times 10^{-3}$) | Flange and web thickness $w$ (mm) | Flange width $d_2$ (mm) | Moment of inertia ($m^4 \times 10^{-6}$) | |
|---|---|---|---|---|---|---|
| | | | | | $I_{xx}$ | $I_{yy}$ |
| HP 200 × 53 | 204 | 6.84 | 11.3 | 207 | 49.4 | 16.8 |
| HP 250 × 85 | 254 | 10.8 | 14.4 | 260 | 123 | 42 |
| × 62 | 246 | 8.0 | 10.6 | 256 | 87.5 | 24 |
| HP 310 × 125 | 312 | 15.9 | 17.5 | 312 | 271 | 89 |
| × 110 | 308 | 14.1 | 15.49 | 310 | 237 | 77.5 |
| × 93 | 303 | 11.9 | 13.1 | 308 | 197 | 63.7 |
| × 79 | 299 | 10.0 | 11.05 | 306 | 164 | 62.9 |
| HP 330 × 149 | 334 | 19.0 | 19.45 | 335 | 370 | 123 |
| × 129 | 329 | 16.5 | 16.9 | 333 | 314 | 104 |
| × 109 | 324 | 13.9 | 14.5 | 330 | 263 | 86 |
| × 89 | 319 | 11.3 | 11.7 | 328 | 210 | 69 |
| HP 360 × 174 | 361 | 22.2 | 20.45 | 378 | 508 | 184 |
| × 152 | 356 | 19.4 | 17.91 | 376 | 437 | 158 |
| × 132 | 351 | 16.8 | 15.62 | 373 | 374 | 136 |
| × 108 | 346 | 13.8 | 12.82 | 371 | 303 | 109 |

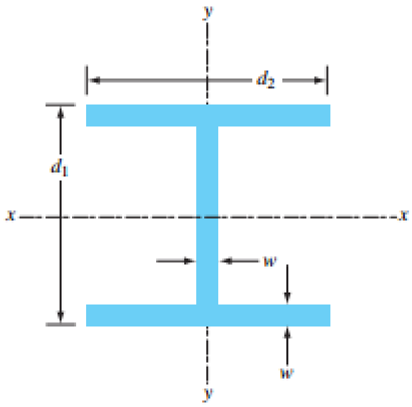


**Table 9.2a** Selected Pipe Pile Sections (SI Units)

| Outside diameter (mm) | Wall thickness (mm) | Area of steel ($cm^2$) |
|---|---|---|
| 219 | 3.17 | 21.5 |
| | 4.78 | 32.1 |
| | 5.56 | 37.3 |
| | 7.92 | 52.7 |
| 254 | 4.78 | 37.5 |
| | 5.56 | 43.6 |
| | 6.35 | 49.4 |
| 305 | 4.78 | 44.9 |
| | 5.56 | 52.3 |
| | 6.35 | 59.7 |
| 406 | 4.78 | 60.3 |
| | 5.56 | 70.1 |
| | 6.35 | 79.8 |
| 457 | 5.56 | 80 |
| | 6.35 | 90 |
| | 7.92 | 112 |
| 508 | 5.56 | 88 |
| | 6.35 | 100 |
| | 7.92 | 125 |
| 610 | 6.35 | 121 |
| | 7.92 | 150 |
| | 9.53 | 179 |
| | 12.70 | 238 |

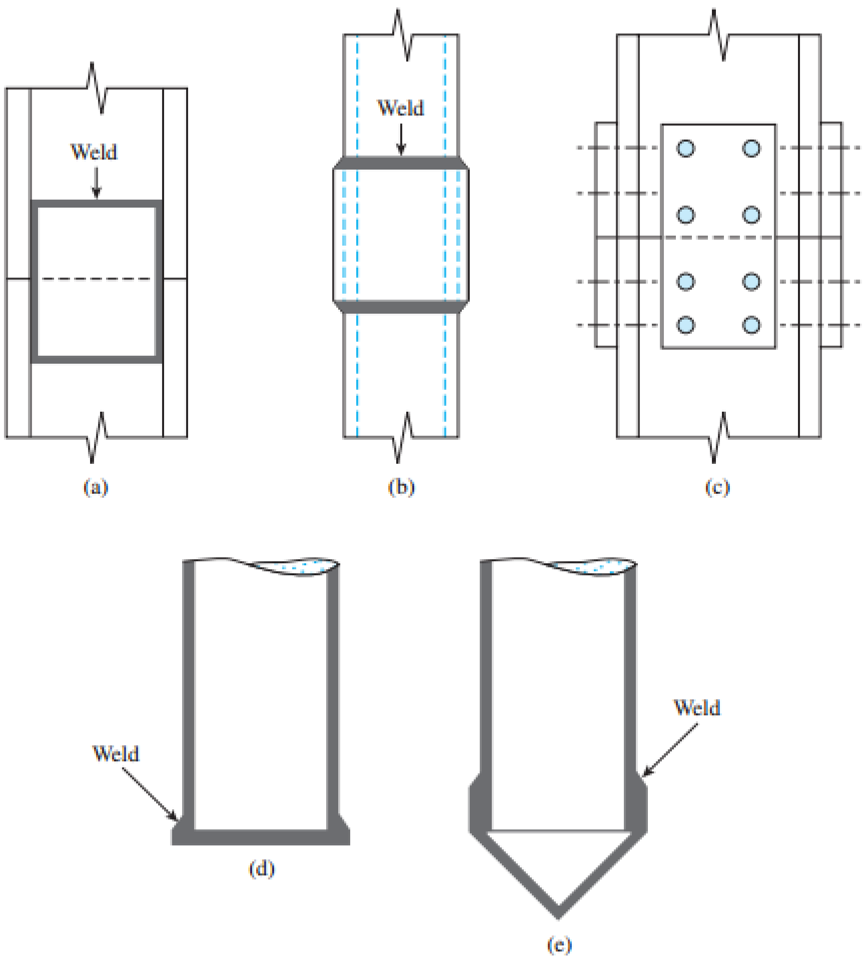



**Figure 9.2** Steel piles: (a) splicing of H-pile by welding; (b) splicing of pipe pile by welding; (c) splicing of H-pile by rivets and bolts; (d) flat driving point of pipe pile; (e) conical driving point of pipe pile

## Concrete Piles

Concrete piles may be divided into two basic categories:

(a) precast piles and

(b) cast-in-situ piles.

Precast piles can be prepared by using ordinary reinforcement, and they can be square or octagonal in cross section. (See Figure 9.3.) Reinforcement is provided to enable the pile to resist the bending moment developed during pickup and transportation, the vertical load, and the bending moment caused by a lateral load. The piles are cast to desired lengths and cured before being transported to the work sites.

**Some general facts about concrete piles are as follows:**

- Usual length: 10 m to 15 m (30 ft to 50 ft)
- Usual load: 300 kN to 3000 kN (67 kip to 675 kip)
- Advantages:
  a. Can be subjected to hard driving
  b. Corrosion resistant
  c. Can be easily combined with a concrete superstructure
- Disadvantages:
  a. Difficult to achieve proper cutoff
  b. Difficult to transport

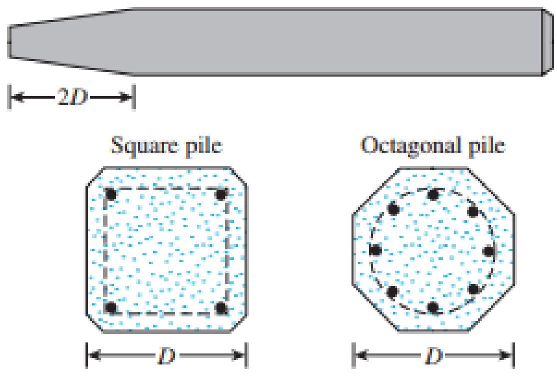



*Figure 9.3* Precast piles with ordinary reinforcement

Precast piles can also be prestressed by the use of high-strength steel prestressing cables. The ultimate strength of these cables is about 1800 MN/m2. During casting of the piles, the cables are pretensioned to about

900 to 1300 MN/m2, and concrete is poured around them. After curing, the cables are cut, producing a compressive force on the pile section. Table 9.3 gives additional information about prestressed concrete piles with square and octagonal cross sections.

**Some general facts about precast prestressed piles are as follows:**

- Usual length: 10 m to 45 m (30 ft to 150 ft)
- Maximum length: 60 m (200 ft)
- Maximum load: 7500 kN to 8500 kN (1700 kip to 1900 kip)

The advantages and disadvantages are the same as those of precast piles.

*Cast-in-situ,* or *cast-in-place, piles* are built by making a hole in the ground and then filling it with concrete. Various types of cast-in-place concrete piles are currently used in construction, and most of them have been patented by their manufacturers. These piles may be divided into two broad categories: (a) cased and (b) uncased. Both types may have a pedestal at the bottom.

*Cased piles* are made by driving a steel casing into the ground with the help of a mandrel placed inside the casing. When the pile reaches the proper depth the mandrel is withdrawn and the casing is filled with concrete. Figures 9.4a, 9.4b, 9.4c, and 9.4d show some examples of cased piles without a pedestal. Figure 9.4e shows a cased pile with a pedestal. The pedestal is an expanded concrete bulb that is formed by dropping a hammer on fresh concrete.

Some general facts about cased cast-in-place piles are as follows:

- Usual length: 5 m to 15 m (15 ft to 50 ft)
- Maximum length: 30 m to 40 m (100 ft to 130 ft)

- Usual load: 200 kN to 500 kN (45 kip to 115 kip)
- Approximate maximum load: 800 kN (180 kip)
- Advantages:
  a. Relatively cheap
  b. Allow for inspection before pouring concrete
  c. Easy to extend
- Disadvantages:
  a. Difficult to splice after concreting
  b. Thin casings may be damaged during driving
- Allowable load:

$$Q_{all} = A_s f_s + A_c f_c \tag{9.2}$$

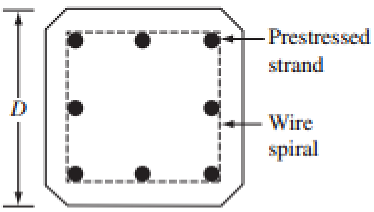
where

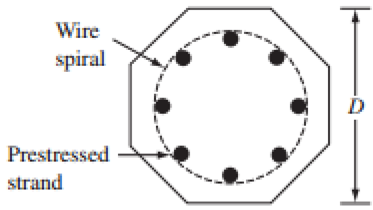
$A_s$ = area of cross section of steel
$A_c$ = area of cross section of concrete
$f_s$ = allowable stress of steel
$f_c$ = allowable stress of concrete

**Table 9.3:** Typical Prestressed Concrete Pile in Use (SI Units)

| Pile shape[a] | D (mm) | Area of cross section (cm²) | Perimeter (mm) | Number of strands: 12.7-mm diameter | Number of strands: 11.1-mm diameter | Minimum effective prestress force (kN) | Section modulus ($m^3 \times 10^{-3}$) | Design bearing capacity (kN), Strength of concrete (MN/m²): 34.5 | Design bearing capacity (kN), Strength of concrete (MN/m²): 41.4 |
|---|---|---|---|---|---|---|---|---|---|
| S | 254 | 645 | 1016 | 4 | 4 | 312 | 2.737 | 556 | 778 |
| O | 254 | 536 | 838 | 4 | 4 | 258 | 1.786 | 462 | 555 |
| S | 305 | 929 | 1219 | 5 | 6 | 449 | 4.719 | 801 | 962 |
| O | 305 | 768 | 1016 | 4 | 5 | 369 | 3.097 | 662 | 795 |
| S | 356 | 1265 | 1422 | 6 | 8 | 610 | 7.489 | 1091 | 1310 |
| O | 356 | 1045 | 1168 | 5 | 7 | 503 | 4.916 | 901 | 1082 |
| S | 406 | 1652 | 1626 | 8 | 11 | 796 | 11.192 | 1425 | 1710 |
| O | 406 | 1368 | 1346 | 7 | 9 | 658 | 7.341 | 1180 | 1416 |
| S | 457 | 2090 | 1829 | 10 | 13 | 1010 | 15.928 | 1803 | 2163 |
| O | 457 | 1729 | 1524 | 8 | 11 | 836 | 10.455 | 1491 | 1790 |
| S | 508 | 2581 | 2032 | 12 | 16 | 1245 | 21.844 | 2226 | 2672 |
| O | 508 | 2136 | 1677 | 10 | 14 | 1032 | 14.355 | 1842 | 2239 |
| S | 559 | 3123 | 2235 | 15 | 20 | 1508 | 29.087 | 2694 | 3232 |
| O | 559 | 2587 | 1854 | 12 | 16 | 1250 | 19.107 | 2231 | 2678 |
| S | 610 | 3658 | 2438 | 18 | 23 | 1793 | 37.756 | 3155 | 3786 |
| O | 610 | 3078 | 2032 | 15 | 19 | 1486 | 34.794 | 2655 | 3186 |

[a] S = square section; O = octagonal section



Figures 9.4f and 9.4g are two types of uncased pile, one with a pedestal and the other without. The uncased piles are made by first driving the casing to the desired depth and then filling it with fresh concrete. The casing is then gradually withdrawn. Following are some general facts about uncased cast-in-place concrete piles:

- Usual length: 5 m to 15 m (15 ft to 50 ft)
- Maximum length: 30 m to 40 m (100 ft to 130 ft)
- Usual load: 300 kN to 500 kN (67 kip to 115 kip)
- Approximate maximum load: 700 kN (160 kip)
- Advantages:
  a. Initially economical
  b. Can be finished at any elevation
- Disadvantages:
  a. Voids may be created if concrete is placed rapidly
  b. Difficult to splice after concreting
  c. In soft soils, the sides of the hole may cave in, squeezing the concrete
- Allowable load:

$$Q_{all} = A_c f_c \tag{9.3}$$

where

$A_c$ = area of cross section of concrete
$f_c$ = allowable stress of concrete

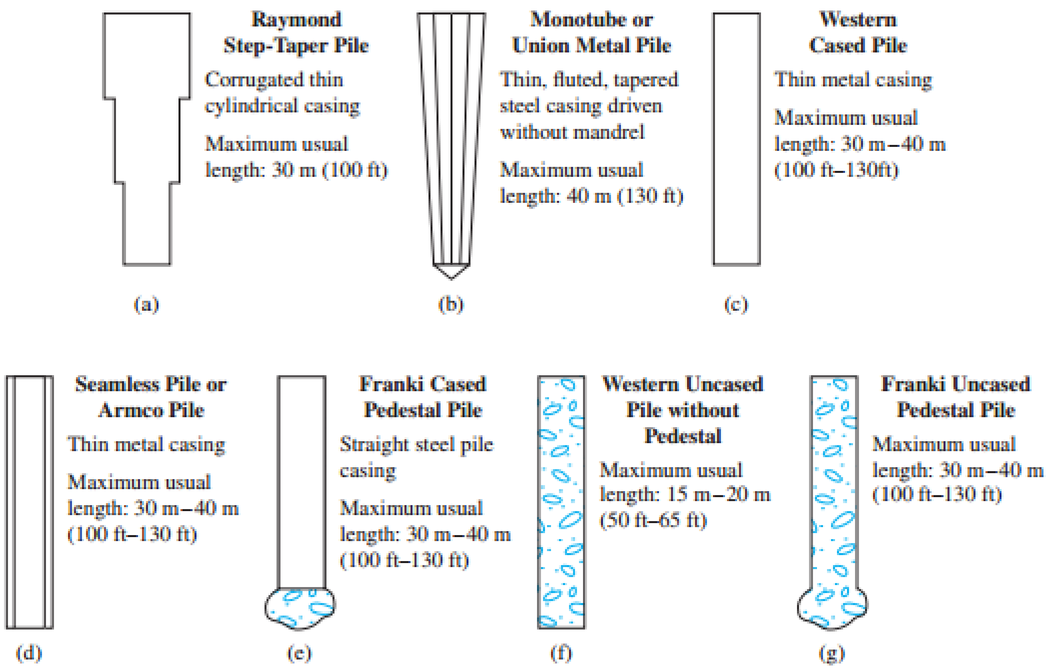



**Figure 9.4** Cast-in-place concrete piles

## Timber Piles

*Timber piles* are tree trunks that have had their branches and bark carefully trimmed off. The maximum length of most timber piles is 10 to 20 m (30 to 65 ft). To qualify for use as a pile, the timber should be straight, sound, and without any defects. The American Society of Civil Engineers' *Manual of Practice*, No. 17 (1959), divided timber piles into three classes:

**1.** *Class A piles* carry heavy loads. The minimum diameter of the butt should be 356 mm (14 in.).

**2.** *Class B piles* are used to carry medium loads. The minimum butt diameter should be 305 to 330 mm (12 to 13 in.).

**3.** *Class C piles* are used in temporary construction work. They can be used permanently for structures when the entire pile is below the water table. The minimum butt diameter should be 305 mm (12 in.).

In any case, a pile tip should not have a diameter less than 150 mm.

Timber piles cannot withstand hard driving stress; therefore, the pile capacity is generally limited. Steel shoes may be used to avoid damage at the pile tip (bottom). The tops of timber piles may also be damaged during the driving operation. The crushing of the wooden fibers caused by the impact of the hammer is referred to as *brooming*. To avoid damage to the top of the pile, a metal band or a cap may be used.

### Composite Piles

The upper and lower portions of *composite piles* are made of different materials. For example, composite piles may be made of steel and concrete or timber and concrete. Steel-and-concrete piles consist of a lower portion of steel and an upper portion of cast-inplace concrete. This type of pile is used when the length of the pile required for adequate bearing exceeds the capacity of simple cast-in-place concrete piles. Timber-and-concrete piles usually consist of a lower portion of timber pile below the permanent water table and an upper portion of concrete. In any case, forming proper joints between two dissimilar materials is difficult, and for that reason, composite piles are not widely used.

## 6.4 Estimating Pile Length

Selecting the type of pile to be used and estimating its necessary length are fairly difficult tasks that require good judgment. In addition to being broken down into the classification given in Section 6.2, piles can be divided into three major categories, depending on their lengths and the mechanisms of load transfer to the soil:

(a) point bearing piles,

(b) friction piles, and

(c) compaction piles.

## a. Point Bearing Piles

If soil-boring records establish the presence of bedrock or rocklike material at a site within a reasonable depth, piles can be extended to the rock surface. (See Figure 9.6a.) In this case, the ultimate capacity of the piles depends entirely on the load-bearing capacity of the underlying material; thus, the piles are called *point bearing piles*. In most of these cases, the necessary length of the pile can be fairly well established.

If, instead of bedrock, a fairly compact and hard stratum of soil is encountered at a reasonable depth, piles can be extended a few meters into the hard stratum. (See Figure 9.6b.) Piles with pedestals can be constructed on the bed of the hard stratum, and the ultimate pile load may be expressed as

$$Q_u = Q_p + Q_s \tag{9.5}$$

where

$Q_p$ = load carried at the pile point
$Q_s$ = load carried by skin friction developed at the side of the pile (caused by shearing resistance between the soil and the pile)

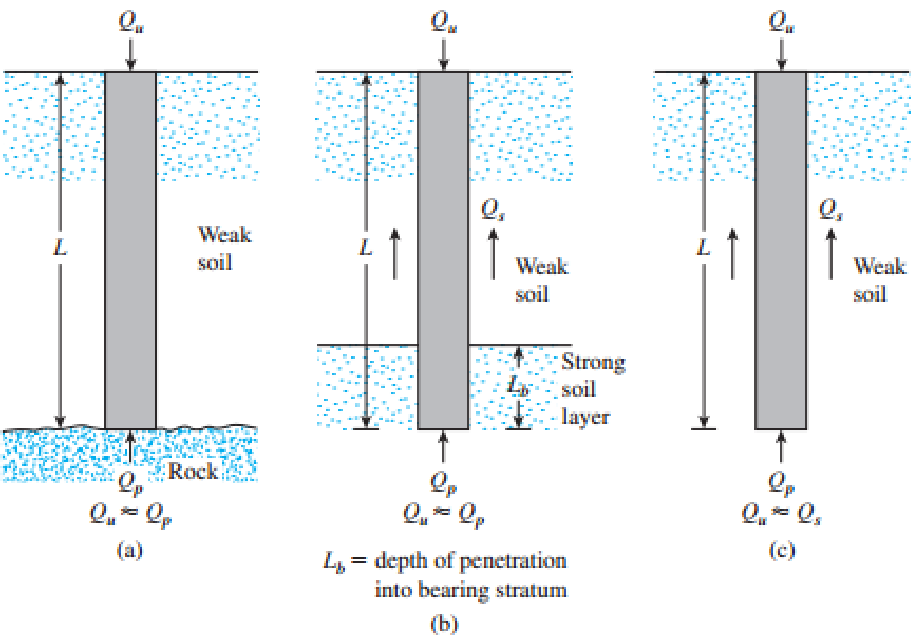



**Figure 9.6** (a) and (b) Point bearing piles; (c) friction piles

If $Q_s$ is very small,

$$Q_s \approx Q_p \tag{9.6}$$

In this case, the required pile length may be estimated accurately if proper subsoil exploration records are available.

## b- Friction Piles

When no layer of rock or rocklike material is present at a reasonable depth at a site, point bearing piles become very long and uneconomical. In this type of subsoil, piles are driven through the softer material to specified depths. (See Figure 9.6c.) The ultimate load of the piles may be expressed by Eq. (9.5). However, if the value of $Q_p$ is relatively small, then

$$Q_u \approx Q_s \qquad (9.7)$$

These piles are called *friction piles,* because most of their resistance is derived from skin friction. However, the term *friction pile,* although used often in the literature, is a misnomer: In clayey soils, the resistance to applied load is also caused by *adhesion.*

The lengths of friction piles depend on the shear strength of the soil, the applied load, and the pile size. To determine the necessary lengths of these piles, an engineer needs a good understanding of soil–pile interaction, good judgment, and experience.

## c- Compaction Piles

Under certain circumstances, piles are driven in granular soils to achieve proper compaction of soil close to the ground surface. These piles are called *compaction piles.* The lengths of compaction piles depend on factors such as

(a) the relative density of the soil before compaction,

(b) the desired relative density of the soil after compaction, and

(c) the required depth of compaction.

These piles are generally short; however, some field tests are necessary to determine a reasonable length.

## 6.5 Installation of Piles

Most piles are driven into the ground by means of *hammers* or *vibratory drivers*. In special circumstances, piles can also be inserted by *jetting* or *partial augering*. The types of hammer used for pile driving include

(a) the drop hammer,

(b) the single-acting air or steam hammer,

(c) the double-acting and differential air or steam hammer, and

(d) the diesel hammer.

In the driving operation, a cap is attached to the top of the pile. A cushion may be used between the pile and the cap. The cushion has the effect of reducing the impact force and spreading it over a longer time; however, the use of the cushion is optional. A hammer cushion is placed on the pile cap. The hammer drops on the cushion.

Figure 9.7 illustrates various hammers.

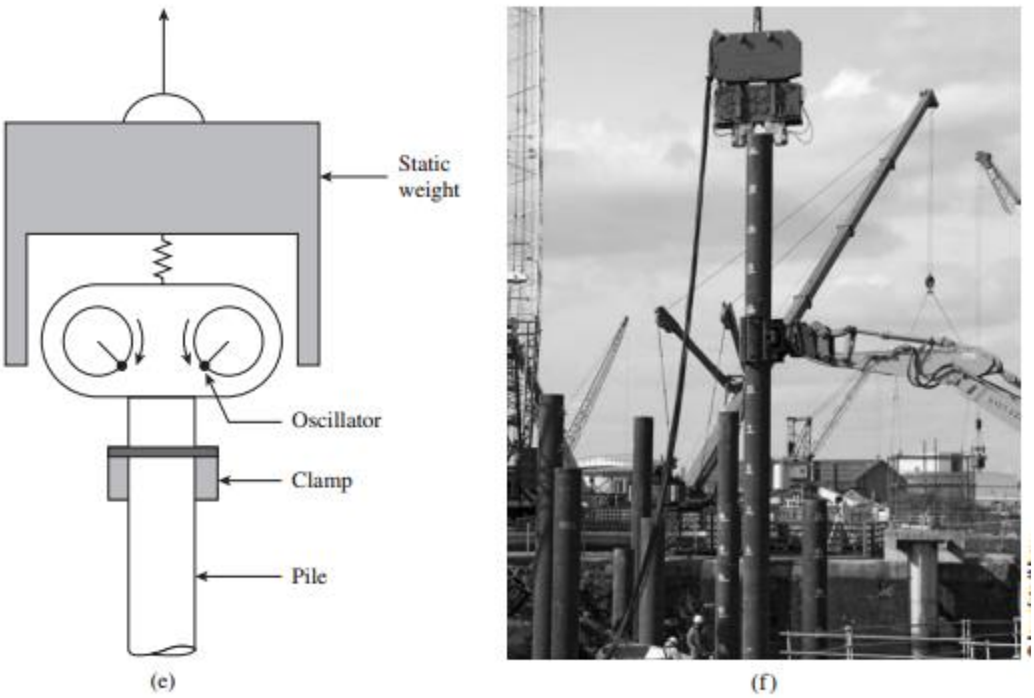



**Figure 9.7** (*continued*) Pile-driving equipment: (e) vibratory pile driver; (f) photograph of a vibratory pile driver (*Courtesy of Reinforced Earth Company, Reston, Virginia*)

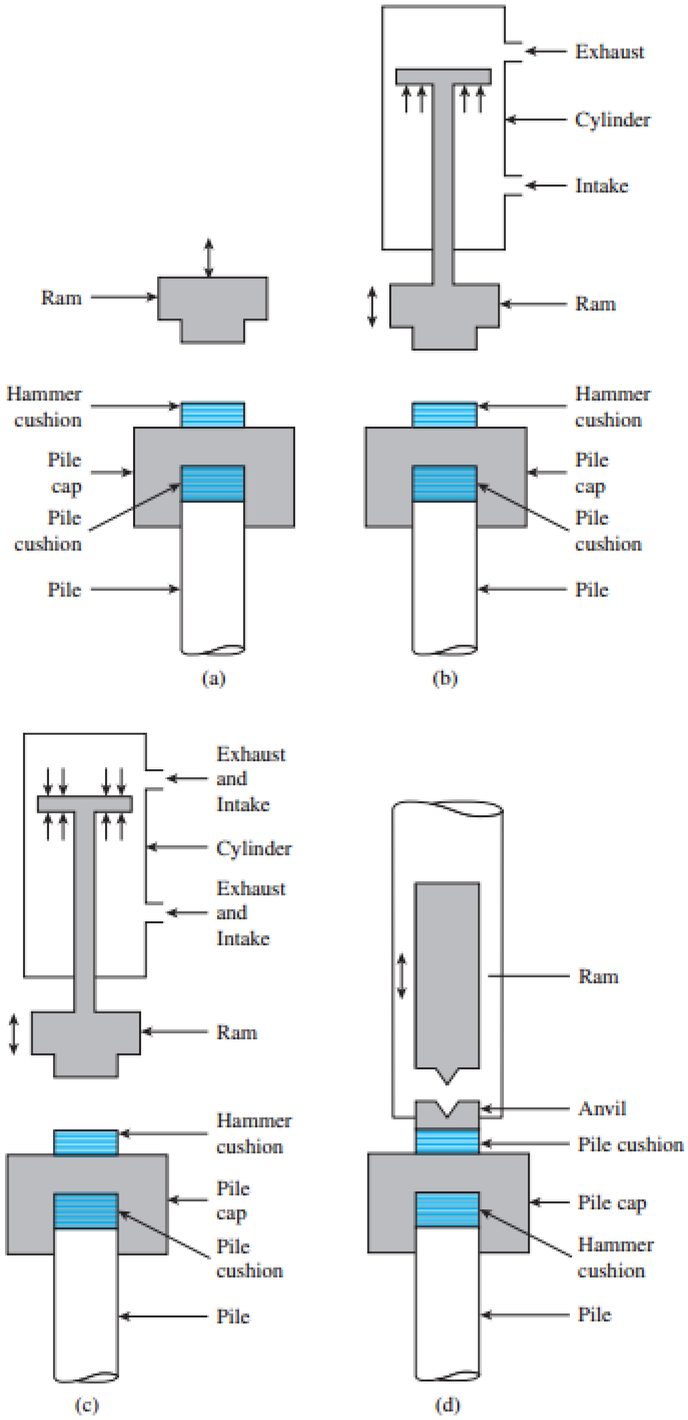



*Figure 9.7* Pile-driving equipment: (a) drop hammer; (b) single-acting air or steam hammer; (c) double-acting and differential air or steam hammer; (d) diesel hammer

## 6.6 Load Transfer Mechanism

The load transfer mechanism from a pile to the soil is complicated. To understand it, consider a pile of length $L$, as shown in Figure 9.9a. The load on the pile is gradually increased from zero to $Q_{(z=0)}$ at the ground surface. Part of this load will be resisted by the side friction developed along the shaft, $Q_1$ , and part by the soil below the tip of the pile, $Q_2$ . Now, how are $Q_1$ and $Q_2$ related to the total load? If measurements are made to obtain the load carried by the pile shaft, $Q_{(z)}$ , at any depth $z$, the nature of the variation found will be like that shown in curve 1 of Figure 9.9b. The *frictional resistance per unit area* at any depth $z$ may be determined as

$$f_{(z)} = \frac{\Delta Q_{(z)}}{(p)(\Delta z)} \qquad (9.8)$$

Where

$p$ = perimeter of the cross section of the pile. Figure 9.9c shows the variation of $f_{(z)}$ with depth.

If the load $Q$ at the ground surface is gradually increased, maximum frictional resistance along the pile shaft will be fully mobilized when the relative displacement between the soil and the pile is about 5 to 10 mm ,irrespective of the pile size and length $L$. However, the maximum point resistance $Q_2 = Q_p$ will not be mobilized until the tip of the pile has moved about 10 to 25% of the pile width (or diameter). (The lower limit applies to driven piles and the upper limit to bored piles). At ultimate load (Figure 9.9d and curve 2 in Figure 9.9b), $Q_{(z=0)} = Q_u$ . Thus,

$$Q_1 = Q_s$$

and

$$Q_2 = Q_p$$

The preceding explanation indicates that $Q_s$ (or the unit skin friction, $f$, along the pile shaft) is developed at a *much smaller pile displacement compared with the point resistance*, $Q_p$ .

At ultimate load, the failure surface in the soil at the pile tip (a bearing capacity failure caused by $Q_p$) is like that shown in Figure 9.9e. Note that pile foundations are deep foundations and that the soil fails mostly in a *punching mode*. That is, a *triangular zone*, I, is developed at the pile tip, which is pushed downward without producing any other visible slip surface. In dense sands and stiff clayey soils, a *radial shear zone*, II, may partially develop.

## 9.7 Equations for Estimating Pile Capacity

The ultimate load-carrying capacity $Q_u$ of a pile is given by the equation

$$Q_u = Q_p + Q_s \tag{9.9}$$

where

$Q_p$ = load-carrying capacity of the pile point
$Q_s$ = frictional resistance (skin friction) derived from the soil–pile interface (see Figure 9.11)

Numerous published studies cover the determination of the values of $Q_p$ and $Q_s$ . Excellent reviews of many of these investigations have been provided by Vesic (1977), Meyerhof (1976), and Coyle and Castello (1981). These studies afford an insight into the problem of determining the ultimate pile capacity.

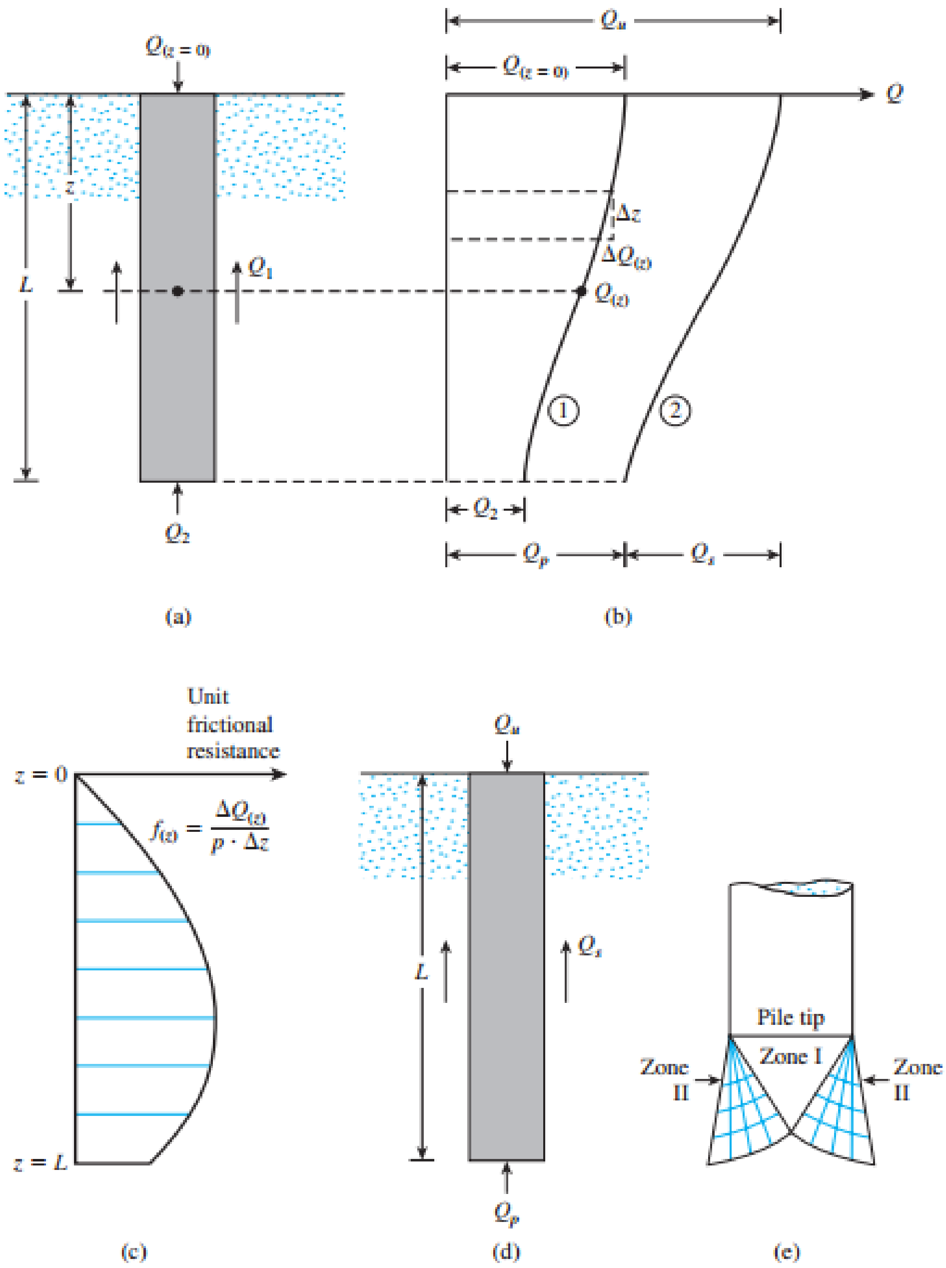



**Figure 9.9** Load transfer mechanism for piles

## Point Bearing Capacity, $Q_p$

The ultimate bearing capacity of shallow foundations was discussed in Chapter 3. According to Terzaghi's equations,

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma BN_\gamma \qquad \text{(for shallow square foundations)}$$

and

$$q_u = 1.3c'N_c + qN_q + 0.3\gamma BN_\gamma \qquad \text{(for shallow circular foundations)}$$

Similarly, the general bearing capacity equation for shallow foundations was given in Chapter 4 (for vertical loading) as

$$q_u = c'N_cF_{cs}F_{cd} + qN_qF_{qs}F_{qd} + \tfrac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}$$

Hence, in general, the ultimate load-bearing capacity may be expressed as

$$q_u = c'N_c^* + qN_q^* + \gamma BN_\gamma^* \tag{9.10}$$

where $N_c^*$, $N_q^*$, and $N_\gamma^*$ are the bearing capacity factors that include the necessary shape and depth factors.

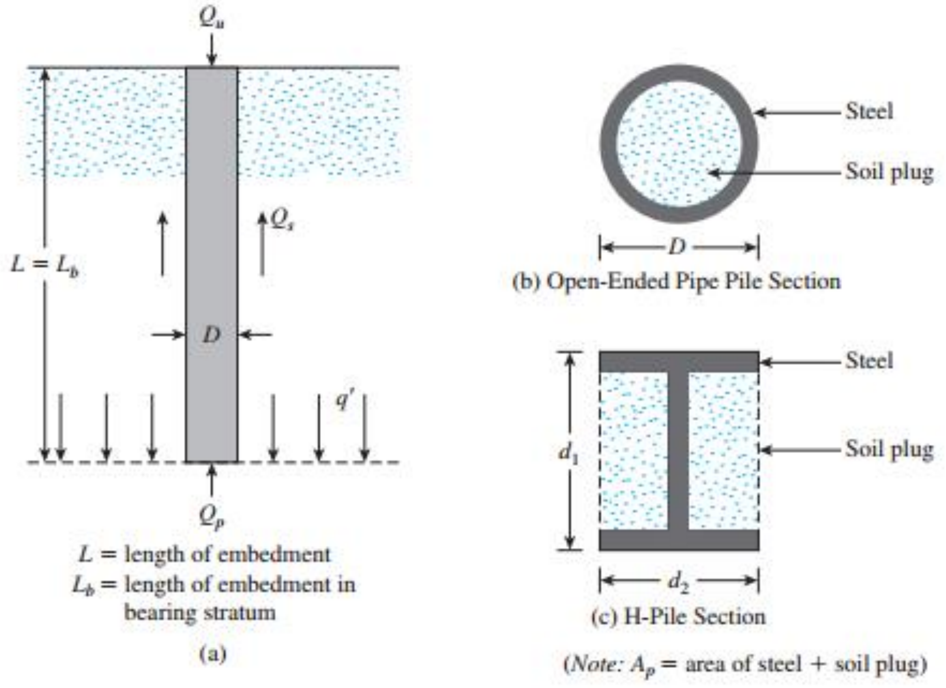



**Figure 9.11** Ultimate load-carrying capacity of pile

Pile foundations are deep. However, the ultimate resistance per unit area developed at the pile tip, $q_p$ , may be expressed by an equation similar in form to Eq. (9.10), although the values of $N^*c$ , $N^*q$ , and $N^*\gamma$ will change. The notation used in this chapter for the width of a pile is *D*. Hence, substituting *D* for *B* in Eq. (9.10) gives

$$q_u = q_p = c'N_c^* + qN_q^* + \gamma DN_\gamma^* \tag{9.11}$$

Because the width $D$ of a pile is relatively small, the term $\gamma DN_\gamma^*$ may be dropped from the right side of the preceding equation without introducing a serious error; thus, we have

$$q_p = c'N_c^* + q'N_q^* \tag{9.12}$$

Note that the term $q$ has been replaced by $q'$ in Eq. (9.12), to signify effective vertical stress. Thus, the point bearing of piles is

$$Q_p = A_p q_p = A_p(c'N_c^* + q'N_q^*) \tag{9.13}$$

where

$A_p$ = area of pile tip
$c'$ = cohesion of the soil supporting the pile tip
$q_p$ = unit point resistance
$q'$ = effective vertical stress at the level of the pile tip
$N_c^*, N_q^*$ = the bearing capacity factors

## Frictional Resistance, $Q_s$

The frictional, or skin, resistance of a pile may be written as

$$Q_s = \Sigma\, p\, \Delta L f \tag{9.14}$$

where

$p$ = perimeter of the pile section
$\Delta L$ = incremental pile length over which $p$ and $f$ are taken to be constant
$f$ = unit friction resistance at any depth $z$

The various methods for estimating $Q_p$ and $Q_s$ are discussed in the next several sections. It needs to be reemphasized that, in the field, for full mobilization of the point resistance ($Q_p$), the pile tip must go through a displacement of 10 to 25% of the pile width (or diameter).

## Allowable Load, $Q_{all}$

After the total ultimate load-carrying capacity of a pile has been determined by summing the point bearing capacity and the frictional (or skin) resistance, a reasonable factor of safety should be used to obtain the total allowable load for each pile, or

$$Q_{all} = \frac{Q_u}{FS}$$

where

$Q_{all}$ = allowable load-carrying capacity for each pile
FS = factor of safety

The factor of safety generally used ranges from 2.5 to 4, depending on the uncertainties surrounding the calculation of ultimate load.

## 6.8 Meyerhof's Method for Estimating $Q_p$

### Sand

The point bearing capacity, $q_p$ , of a pile in sand generally increases with the depth of embedment in the bearing stratum and reaches a maximum value at an embedment ratio of $L_b/D = (L_b/D)_{cr}$ . Note that in a homogeneous soil $L_b$ is equal to the actual embedment length of the pile, $L$. However, where a pile has penetrated into a bearing stratum, $L_b < L$. Beyond the critical embedment ratio, $(L_b/D)_{cr}$ , the value of $q_p$ remains constant $(q_p = q_l)$. That is, as shown in Figure 9.12 for the case of a homogeneous soil, $L = L_b$ .

For piles in sand, $c' = 0$, and Eq. (9.13) simplifies to

$$Q_p = A_p q_p = A_p q' N_q^* \qquad (9.15)$$

The variation of $N^*q$ with soil friction angle $\phi'$ is shown in Figure 9.13. The interpolated values of $N^*q$ for various friction angles are also given in Table 9.5. However, $Q_p$ should not exceed the limiting value $A_p q_l$ ; that is,

$$Q_p = A_p q' N_q^* \leq A_p q_l \qquad (9.16)$$

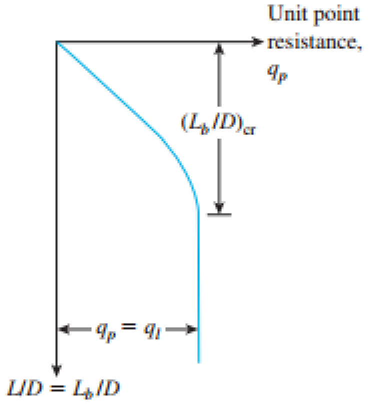



**Figure 9.12** Nature of variation of unit point resistance in a homogeneous sand

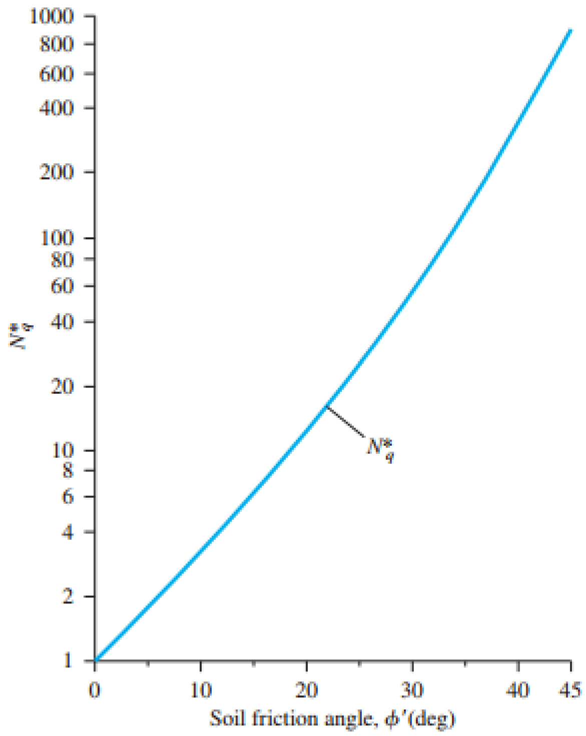



**Figure 9.13** Variation of the maximum values of $N_q^*$ with soil friction angle $\phi'$ (Based on Meyerhof, G. G. (1976). "Bearing Capacity and Settlement of Pile Foundations," *Journal of the Geotechnical Engineering Division*, American Society of Civil Engineers, Vol. 102, No. GT3, pp. 197–228.)

**Table 9.5** Interpolated Values of $N_q^*$ Based on Meyerhof's Theory

| Soil friction angle, $\phi$ (deg) | $N_q^*$ |
|---|---|
| 20 | 12.4 |
| 21 | 13.8 |
| 22 | 15.5 |
| 23 | 17.9 |
| 24 | 21.4 |
| 25 | 26.0 |
| 26 | 29.5 |
| 27 | 34.0 |
| 28 | 39.7 |
| 29 | 46.5 |
| 30 | 56.7 |
| 31 | 68.2 |
| 32 | 81.0 |
| 33 | 96.0 |
| 34 | 115.0 |
| 35 | 143.0 |
| 36 | 168.0 |
| 37 | 194.0 |
| 38 | 231.0 |
| 39 | 276.0 |
| 40 | 346.0 |
| 41 | 420.0 |
| 42 | 525.0 |
| 43 | 650.0 |
| 44 | 780.0 |
| 45 | 930.0 |

The limiting point resistance is

$$q_l = 0.5\, p_a N_q^* \tan \phi' \tag{9.17}$$

where

$p_a$ = atmospheric pressure (=100 kN/m$^2$ or 2000 lb/ft$^2$)
$\phi'$ = effective soil friction angle of the bearing stratum

### Clay ($\phi = 0$)

For piles in *saturated clays* under undrained conditions ($\phi = 0$), the net ultimate load can be given as

$$Q_p \approx N_c^* c_u A_p = 9 c_u A_p \tag{9.18}$$

where $c_u$ = undrained cohesion of the soil below the tip of the pile.

## 6.9 Coyle and Castello's Method for Estimating *Qp* in Sand

Coyle and Castello (1981) analyzed 24 large-scale field load tests of driven piles in sand. On the basis of the test results, they suggested that, in sand,

$$Q_p = q' N_q^* A_p \tag{9.36}$$

where

$q'$ = effective vertical stress at the pile tip
$N_q^*$ = bearing capacity factor

Figure 9.15 shows the variation of $N_q^*$ with $L/D$ and the soil friction angle $\phi'$.

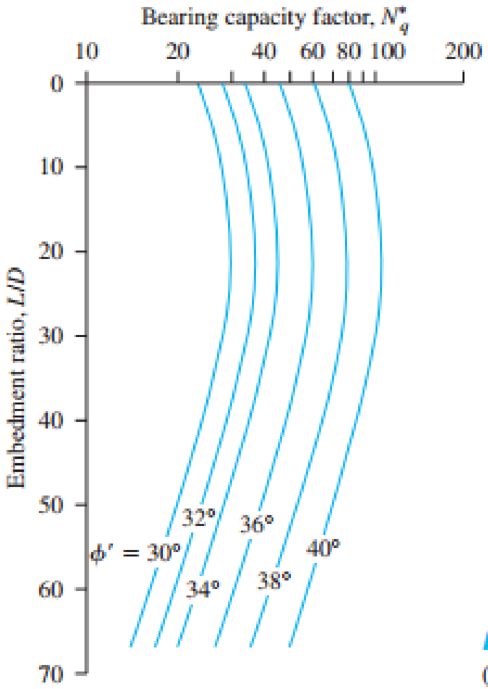



**Figure 9.15** Variation of $N_q^*$ with $L/D$ (Based on Coyle and Costello, 1981)

## Example 6.1

Consider a 20-m-long concrete pile with a cross section of 0.407 m × 0.407 m fully embedded in sand. For the sand, given: unit weight, $\gamma = 18$ kN/m$^3$; and soil friction angle, $\phi' = 35°$. Estimate the ultimate point $Q_p$ with each of the following:

**a.** Meyerhof's method

**c.** The method of Coyle and Castello

**d.** Based on the results of parts a, b, and c, adopt a value for $Q_p$

### Solution

Part a

From Eqs. (9.16) and (9.17),

$$Q_p = A_p q' N_q^* \leq A_p (0.5 p_a N_q^* \tan \phi')$$

For $\phi' = 35°$, the value of $N_q^* \approx 143$ (Table 9.5). Also, $q' = \gamma L = (18)(20) = 360$ kN/m$^2$. Thus,

$$A_p q' N_q^* = (0.407 \times 0.407)(360)(143) \approx 8528 \text{ kN}$$

Again,

$$A_p(0.5 p_a N_q^* \tan \phi') = (0.407 \times 0.407)[(0.5)(100)(143)(\tan 35)] \approx 829 \text{ kN}$$

Hence, $Q_p =$ **829 kN**.

Part c

From Eq. (9.36),

$$Q_p = q' N_q^* A_p$$

$$\frac{L}{D} = \frac{20}{0.407} = 49.1$$

For $\phi' = 35°$ and $L/D = 49.1$, the value of $N_q^*$ is about 34 (Figure 9.15). Thus,

$$Q_p = q' N_q^* A_p = (20 \times 18)(34)(0.407 \times 0.407) \approx \mathbf{2028\ kN}$$

Part d

It appears that $Q_p$ obtained from the method of Coyle and Castello is too large. Thus, the average of the results from parts a and b is

$$\frac{829 + 1731}{2} = 1280 \text{ kN}$$

$$\text{Use } Q_p = \mathbf{1280\ kN}.$$

## Example 6.2

Consider a pipe pile (flat driving point—see Figure 9.2d) having an outside diameter of 457 mm. The embedded length of the pile in layered saturated clay is 20 m.
The following are the details of the subsoil:

| Depth from ground surface (m) | Saturated unit weight, $\gamma$(kN/m$^3$) | $c_u$(kN/m$^2$) |
|---|---|---|
| 0–3 | 16 | 25 |
| 3–10 | 17 | 40 |
| 10–30 | 18 | 90 |

The groundwater table is located at a depth of 3 m from the ground surface. Estimate $Q_p$ by using

**a.** Meyerhof's method

### Solution

Part a

From Eq. (9.18),

$$Q_p = 9 c_u A_p$$

The tip of the pile is resting on a clay with $c_u = 90$ kN/m$^2$. So,

$$Q_p = (9)(90)\left[\left(\frac{\pi}{4}\right)\left(\frac{457}{1000}\right)^2\right] = \mathbf{132.9\ kN}$$

## 6.11 Correlations for Calculating $Q_p$ with SPT and CPT Results in Granular Soil

On the basis of field observations, Meyerhof (1976) also suggested that the ultimate point resistance $q_p$ in a homogeneous granular soil ($L = L_b$) may be obtained from standard penetration numbers as

$$q_p = 0.4 p_a N_{60} \frac{L}{D} \leq 4 p_a N_{60} \tag{9.37}$$

where

$N_{60}$ = the average value of the standard penetration number near the pile point (about $10D$ above and $4D$ below the pile point)
$p_a$ = atmospheric pressure ($\approx$ 100 kN/m$^2$ or 2000 lb/ft$^2$)

Briaud et al. (1985) suggested the following correlation for $q_p$ in granular soil with the standard penetration resistance $N_{60}$.

$$q_p = 19.7 p_a (N_{60})^{0.36} \tag{9.38}$$

Meyerhof (1956) also suggested that

$$q_p \approx q_c \tag{9.39}$$

where $q_c$ = cone penetration resistance.

**Example 6.3**

Consider a concrete pile that is 0.305 m × 0.305 m in cross section in sand. The pile is 12 m long. The following are the variations of $N_{60}$ with depth.

| Depth below ground surface (m) | $N_{60}$ |
|---|---|
| 1.5 | 8 |
| 3.0 | 10 |
| 4.5 | 9 |
| 6.0 | 12 |
| 7.5 | 14 |
| 9.0 | 18 |
| 10.5 | 11 |
| 12.0 | 17 |
| 13.5 | 20 |
| 15.0 | 28 |
| 16.5 | 29 |
| 18.0 | 32 |
| 19.5 | 30 |
| 21.0 | 27 |

**a.** Estimate $Q_p$ using Eq. (9.37).
**b.** Estimate $Q_p$ using Eq. (9.38).

**Solution**

Part a

The tip of the pile is 12 m below the ground surface. For the pile, $D = 0.305$ m. The average of $N_{60}$ $10D$ above and about $5D$ below the pile tip is

$$N_{60} = \frac{18 + 11 + 17 + 20}{4} = 16.5 \approx 17$$

From Eq. (9.37)

$$Q_p = A_p(q_p) = A_p\left[0.4p_a N_{60}\left(\frac{L}{D}\right)\right] \le A_p(4p_a N_{60})$$

$$A_p\left[0.4p_a N_{60}\left(\frac{L}{D}\right)\right] = (0.305 \times 0.305)\left[(0.4)(100)(17)\left(\frac{12}{0.305}\right)\right] = 2488.8 \text{ kN}$$

$$A_p(4p_a N_{60}) = (0.305 \times 0.305)[(4)(100)(17)] = 632.6 \text{ kN} \approx 633 \text{ kN}$$

Thus, $Q_p$ = **633 kN**

Part b
From Eq. (9.38),

$$Q_p = A_p q_p = A_p[19.7p_a(N_{60})^{0.36}] = (0.305 \times 0.305)[(19.7)(100)(17)^{0.36}]$$

$$= \mathbf{508.2\ kN}$$

\

*Figure 3.5* A small enclosure with steel sheet piles for an excavation work (*Courtesy of N. Sivakugan, James Cook University, Australia*)

***Table 3.1*** Properties of Some Sheet-Pile Sections Production by Bethlehem Steel Corporation

**Example 3.4:**

*University of Anbar*
*Engineering College*
*Civil Engineering Department*

# CHAPTER FIVE

# GEOMETRIC DESIGN OF SHALLOW FOUNDATIONS

LECTURE
DR. AHMED H. ABDULKAREEM
2019 - 2020

## 5.1. Introduction

The foundations are considered to be shallow if [Df ≤ (3→4)B].

Shallow foundations have several advantages:

- minimum cost of materials and construction,
- easy in construction "labor don't need high experience to construct shallow foundations".

On the other hand, the main disadvantage of shallow foundations that if the bearing capacity of the soil supporting the foundation is small, the amount of settlement will be large.

The types of sallow foundations is the following:

1. Isolated Footings (spread footings).
2. Combined Footings.
3. Strap Footings.
4. Mat "Raft" Foundations.

# SPREAD FOOTING DESIGN

### 5.2 Geometric Design of Isolated Footings

- A footing carrying a single column is called a *spread footing*, since its function is to "spread" the column load laterally to the soil so that the stress intensity is reduced to a value that the soil can safely carry.
- These members are sometimes called single or isolated footings.
- Wall footings serve a similar purpose of spreading the wall load to the soil. Often, however, wall footing widths are controlled by factors other than the allowable soil pressure since wall loads (including wall weight) are usually rather low.
- Spread footings with tension reinforcing may be called two-way or one-way depending on whether the steel used for bending runs both

ways (usual case) or in one direction (as is common for wall footings).

- Single footings may be of constant thickness or either stepped or sloped. Stepped or sloped footings are most commonly used to reduce the quantity of concrete away from the column where the bending moments are small and when the footing is not reinforced. When labor costs are high relative to material, it is usually more economical to use constant-thickness reinforced footings.
- Figure 5-1 illustrates several spread footings.
- A pedestal (Fig. 5-Ie) may be used to interface metal columns with spread or wall footings that are located at the depth in the ground.

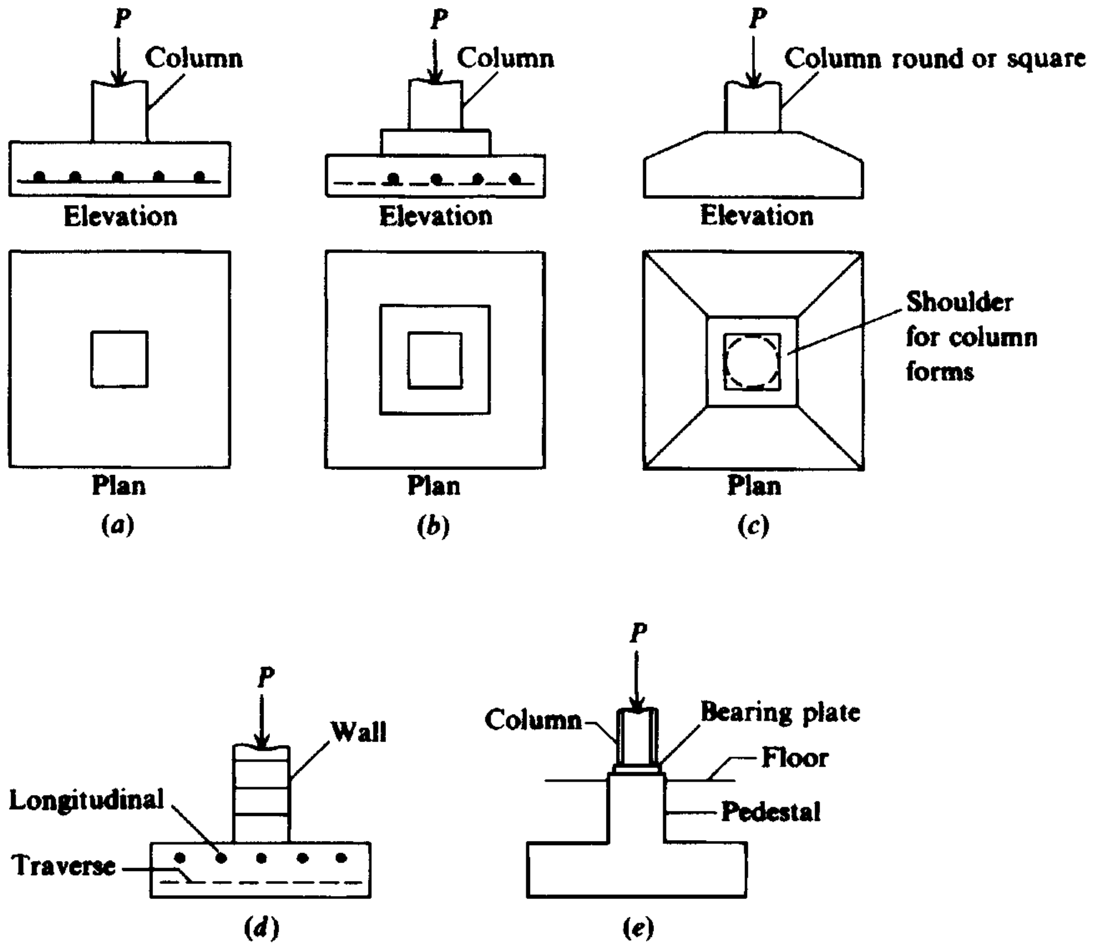



**Figure 5-1** Typical footings, *(a)* Single or spread footings; *(Jb)* stepped footing; *(c)* sloped footing; *(d)* wall footing; *(e)* footing with pedestal.

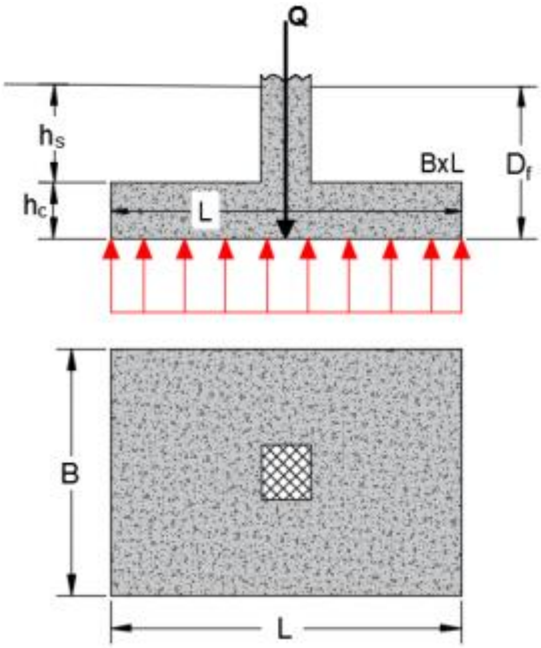
### 5.2.1 Design Procedures:

### 1. Calculate the net allowable bearing capacity:

The first step for geometric design of foundations is to calculate the allowable bearing capacity of the foundations as we discussed in previous chapters as shown in Fig. 5.2.

$$q_{all,net} = \frac{q_{u,net}}{FS}$$

$$q_{u,net} = q_{u,gross} - \gamma_c h_c - \gamma_s h_s$$




### 2. Calculate the required area of the footing:

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = B \times L$$

Assume B or L then find the other dimension.

If the footing is square:

$$A_{req} = B^2 \rightarrow B = \sqrt{A_{req}}$$

$$Q_{service} = P_D + P_L$$

**Why we use $Q_{service}$:**

$$A_{req} = \frac{Q_{service}}{q_{all,net}} = A_{req} = \frac{Q_{service}}{\frac{q_{u,net}}{FS}}$$

$$= \frac{FS \times Q_{service}}{q_{u,net}}$$

**Note:**

The equation of calculating the required area $\left(A_{req} = \frac{Q_{service}}{q_{all,net}}\right)$ is valid only if the pressure under the base of the foundation is uniform.

## 5.3 STRUCTURAL DESIGN OF SPREAD FOOTINGS

- The allowable soil pressure controls the plan *(B* X L) dimensions of a spread footing.
- Structural (such as a basement) and environmental factors locate the footing vertically in the soil.

- Shear stresses usually control the footing thickness *D*.
- Two-way action shear always controls the depth for centrally loaded square footings.
- Wide-beam shear may control the depth for rectangular footings when the *L/B* ratio is greater than about 1.2 and may control for other *L/B* ratios when there are overturning or eccentric loadings.

The depth of footing for two-way action produces a quadratic equation that is developed from Fig. 5-4b, c using

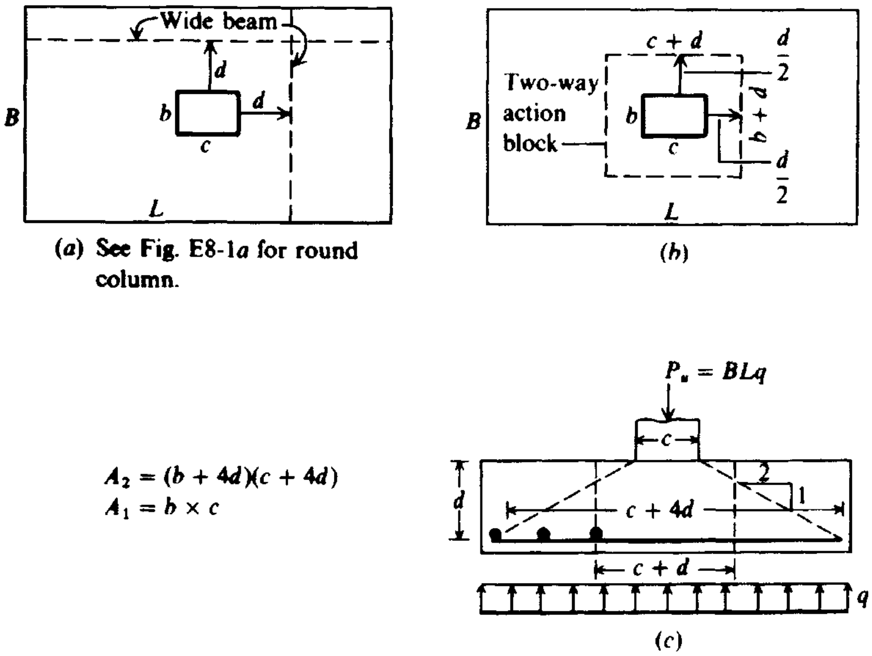



**Figure 8-4** (*a*) Section for wide-beam shear; (*b*) section for diagonal-tension shear; (*c*) method of computing area $A_2$ for allowable column bearing stress.

$\sum F_v = 0$

on the two-way action zone shown. Noting the footing block weight cancels, we have:

$$P_u = 2dv_c(b + d) + 2dv_c(c + d) + (c + d)(b + d)q$$

Substitution of $P_u$ or $P_d = BLq$ and using shear stress $v_c$ gives

$$d^2(4v_c + q) + d(2v_c + q)(b + c) = (BL - cb)q$$

For a square column $c = b = w$ we obtain

$$d^2\left(v_c + \frac{q}{4}\right) + d\left(v_c + \frac{q}{2}\right)w = (BL - w^2)\frac{q}{4}$$

For a round column, $a =$ diameter, the expression is

$$d^2\left(v_c + \frac{q}{4}\right) + d\left(v_c + \frac{q}{2}\right)a = (BL - A_{\text{col}})\frac{q}{\pi}$$

- If we neglect the upward soil pressure on the diagonal tension block, an *approximate* effective concrete depth $d$ can be obtained for rectangular and round columns as

Rectangular: $$4d^2 + 2(b + c)d = \frac{BLq}{v_c} = \frac{P_v}{v_c}$$

Round: $$d^2 + ad = \frac{P_u}{\pi v_c}$$

- Steps in square or rectangular spread footing design with a centrally loaded column and no moments are as follows:

1. Compute the footing plan dimensions $B \times L$ or $B$ using the allowable soil pressure:

   Square: $$B = \sqrt{\frac{\text{Critical load combination}}{q_a}} = \sqrt{\frac{P}{q_a}}$$

   Rectangular: $$BL = \frac{P}{q_a}$$

   A rectangular footing may have a number of satisfactory solutions unless either $B$ or $L$ is fixed.

2. Convert the allowable soil pressure $q_a$ to an ultimate value $q_{ult} = q$ for footing depth

$$\frac{P_u}{BL} = q = \frac{P_{\text{ult}}}{P_{\text{design}}}q_a$$

   Obtain $P_u$ by applying appropriate load factors to the given design loading.

3. Obtain the allowable two-way action shear stress $v_c$ and using the Equations above to compute the effective footing depth $d$.

4. If the footing is rectangular, immediately check wide-beam shear. Use the larger $d$ from two-way action (step 3) or wide-beam.
5. Compute the required steel for bending, and use the same amount each way for square footings. Use the effective $d$ to the intersection of the two bar layers for square footings and if $d$ > 305 mm or 12 in. For $d$ less than this and for rectangular footings use the actual $d$ for the two directions. The bending moment is computed at the critical section shown in Fig. 5-5. For the length $l$ shown the ultimate bending moment/unit width is

$$M_u = \frac{ql^2}{2}$$

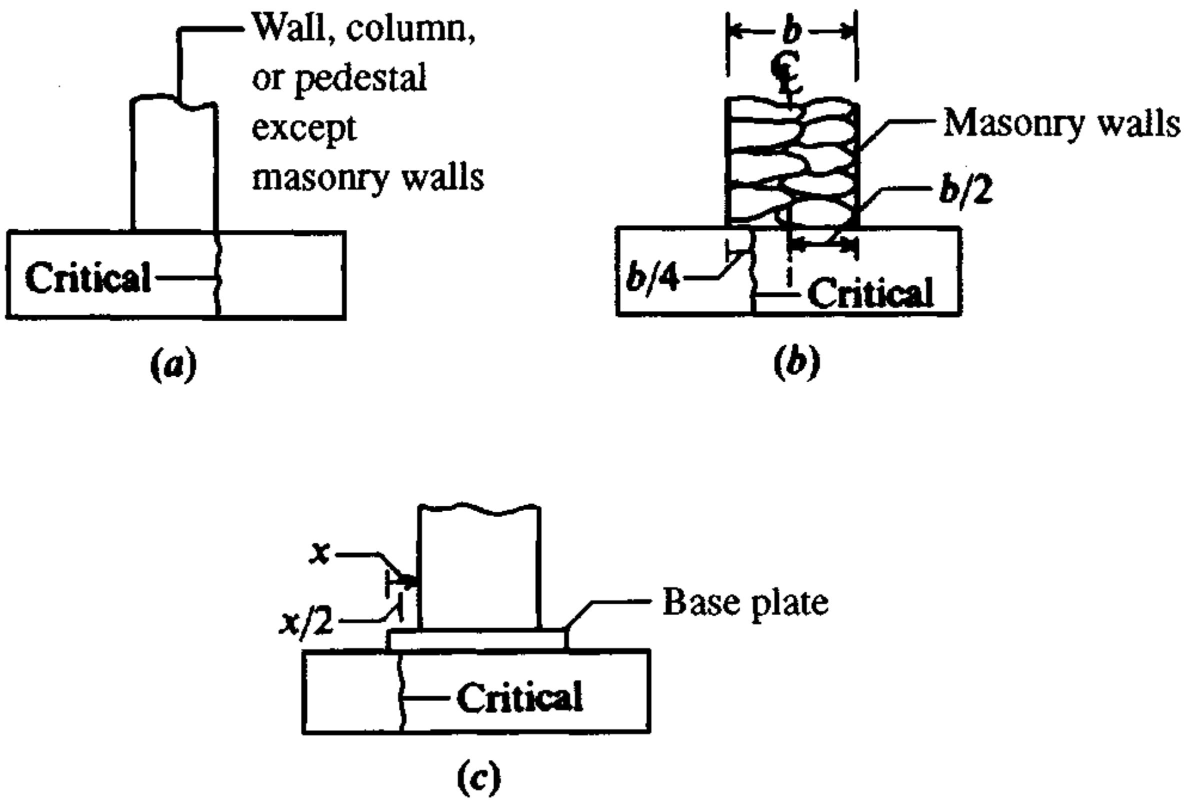



Fig. 5.5 Sections for computing bending moment.

6. Compute column bearing and use dowels for bearing if the allowable bearing stress is exceeded. In that case, compute the required dowels based on the difference between actual and allowable stresses X column area. This force, divided by *fy*, is the required area of dowels for bearing.

# APPENDIX A
# Reinforced Concrete Design of Shallow Foundations

## A.1 Fundamentals of Reinforced Concrete Design

At the present time, most reinforced concrete designs are based on the recommendations of the building code prepared by the American Concrete Institute—that is, ACI 318-11. The basis for this code is the *ultimate strength design* or *strength design*. Some of the fundamental recommendations of the code are briefly summarized in the following sections.

### Load Factors

According to ACI Code Section 9.2, depending on the type, the ultimate load-carrying capacity of a structural member should be one of the following:

$$U = 1.4D \tag{A.1a}$$
$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \tag{A.1b}$$
$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \tag{A.1c}$$
$$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \tag{A.1d}$$
$$U = 1.2D + 1.0E + 1.0L + 0.2S \tag{A.1e}$$
$$U = 0.9D + 1.0W \tag{A.1f}$$

or

$$U = 0.9D + 1.0E \tag{A.1g}$$

where

$U$ = ultimate load-carrying capacity of a member
$D$ = dead loads
$E$ = effects of earthquake
$L$ = live loads
$L_r$ = roof live loads
$R$ = rain load
$S$ = snow load
$W$ = wind load

## Strength Reduction Factor

The design strength provided by a structural member is equal to the nominal strength times a strength reduction factor, $\phi$, or

Design strength = $\phi$ (nominal strength)

The reduction factor, $\phi$, takes into account the inaccuracies in the design assumptions, changes in property or strength of the construction materials, and so on. Following are some of the recommended values of $\phi$ (ACI Code Section 9.3):

| Condition | Value of $\phi$ |
|---|---|
| a. Axial tension; flexure with or without axial tension | 0.9 |
| b. Shear or torsion | 0.75 |
| c. Axial compression with spiral reinforcement | 0.75 |
| d. Axial compression without spiral reinforcement | 0.65 |
| e. Bearing on concrete | 0.65 |
| f. Flexure in plain concrete | 0.65 |

## Design Concepts for a Rectangular Section in Bending

Figure A.1a shows a section of a concrete beam having a width *b* and a depth *h*. The assumed stress distribution across the section at ultimate load is shown in Figure A.1b. The following notations have been used in this figure:

$f'_c$ = compressive strength of concrete at 28 days
$A_s$ = area of steel tension reinforcement
$f_y$ = yield stress of reinforcement in tension
$d$ = effective depth
$l$ = location of the neutral axis measured from the top of the compression face
$a = \beta l$

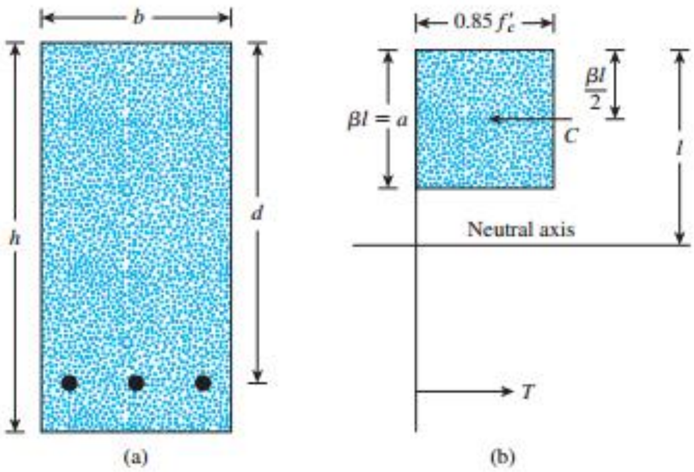



**Figure A.1** Rectangular section in heading

$\beta$ = 0.85 for $f'_c$ of 28 MN/m² (4000 lb/in.²) of less and decreases at the rate of 0.05 for every 7 MN/m² (1000 lb/in.²) increase of $f'_c$. However, it cannot be less than 0.65 in any case (ACI Code Section 10.2.7).

From the principles of statics, for the section

$$\Sigma \text{ compressive force, } C = \Sigma \text{ tensile force, } T$$

Thus,

$$0.85 f'_c ab = A_s f_y$$

or

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{A.2}$$

Also, for the beam section, the nominal ultimate moment can be given as

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) \tag{A.3}$$

where $M_n$ = theoretical ultimate moment.

The design ultimate moment, $M_u$, can be given as

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) \tag{A.4}$$

Combining Eqs. (A.2) and (A.4)

$$M_u = \phi A_s f_y \left[ d - \left( \frac{1}{2} \right) \frac{A_s f_y}{0.85 f'_c b} \right] = \phi A_s f_y \left( d - \frac{0.59\, A_s f_y}{f'_c b} \right) \tag{A.5}$$

The steel percentage is defined by the equation

$$s = \frac{A_s}{bd} \tag{A.6}$$

In a balanced beam, failure would occur by sudden simultaneous yielding of tensile steel and crushing of concrete. The balanced percentage of steel (for Young's modulus) of steel, $E_s$ = 200 MN/m²) can be given as

$$s_b = \frac{0.85 f'_c}{f_y} (\beta) \left( \frac{600}{600 + f_y} \right) \tag{A.7a}$$

where $f'_c$ and $f_y$ are in MN/m².

In conventional English units (with $E_s = 29 \times 10^6$ lb/in.²)

$$s_b = \frac{0.85 f'_c}{f_y} (\beta) \left( \frac{87{,}000}{87{,}000 + f_y} \right) \tag{A.7b}$$

where $f'_c$ and $f_y$ and in lb/in.²

To avoid sudden failure without warning, ACI Code Section 10.3.5 recommends that the maximum steel percentage ($s_{max}$) should be limited to a net tensile strain ($\epsilon_t$) of 0.004. For all practical purposes,

$$s_{max} \approx 0.75\, s_b \tag{A.8}$$

The nominal or theoretical shear strength of a section, $V_n$, can be given as

$$V_n = V_c + V_s \tag{A.9}$$

where $V_c$ = nominal shear strength of concrete
$V_s$ = nominal shear strength of reinforcement

The permissible shear strength, $V_u$, can be given by

$$V_u = \phi V_n = \phi(V_c + V_s) \tag{A.10}$$

The values of $V_c$ can be given by the following equations (ACI Code Sections 11.2 and 11.11).

$$V_c = 0.17\lambda \sqrt{f'_c}\, bd \quad \text{(for member subjected to shear and flexure)} \tag{A.11a}$$

and

$$V_c = 0.33\lambda \sqrt{f'_c}\, bd \quad \text{(for member subjected to diagonal tension)} \tag{A.11b}$$

where $f'_c$ is in MN/m$^2$, $V_c$ is in MN, $b$ and $d$ are in m, and $\lambda = 1$ for normal weight concrete.

In conventional English units, Eqs. (A.11a) and (A.11b) take the following form:

$$V_c = 2\lambda \sqrt{f'_c}\, bd \tag{A.12a}$$

$$V_c = 4\lambda \sqrt{f'_c}\, bd \tag{A.12b}$$

where $V_c$ is in lb, $f'_c$ is in lb/in.$^2$, and $b$ and $d$ are in inches.

Note that

$$v_c = \frac{V_c}{bd} \tag{A.13}$$

where $v_c$ is the shear stress.

Now, combining Eqs. (A.11a), and (A.13), one obtains

$$\text{Permissible shear stress} = v_u = \frac{V_u}{bd} = 0.17\,\phi\lambda\sqrt{f'_c} \tag{A.14a}$$

Similarly, from Eqs. (A.11b), and (A.13),

$$v_u = 0.33\lambda\phi\sqrt{f'_c} \tag{A.14b}$$

## A.2 Reinforcing Bars

The nominal sizes of reinforcing bars commonly used in the United States are given in Table A.1.

**Table A.1** Nominal Sizes of Reinforcing Bars Used in the United States

| | Diameter | | Area of cross section | |
|---|---|---|---|---|
| Bar No. | (mm) | (in.) | ($mm^2$) | ($in.^2$) |
| 3 | 9.52 | 0.375 | 71 | 0.11 |
| 4 | 12.70 | 0.500 | 129 | 0.20 |
| 5 | 15.88 | 0.625 | 200 | 0.31 |
| 6 | 19.05 | 0.750 | 284 | 0.44 |
| 7 | 22.22 | 0.875 | 387 | 0.60 |
| 8 | 25.40 | 1.000 | 510 | 0.79 |
| 9 | 28.65 | 1.128 | 645 | 1.00 |
| 10 | 32.26 | 1.270 | 819 | 1.27 |
| 11 | 35.81 | 1.410 | 1006 | 1.56 |
| 14 | 43.00 | 1.693 | 1452 | 2.25 |
| 18 | 57.33 | 2.257 | 2580 | 4.00 |

Reinforcing-bar sizes in the metric system have been recommended by UNESCO (1971). (Bars in Europe will be specified to comply with the standard EN 100080).

| Bar diameter, mm | Area, $mm^2$ |
|---|---|
| 6 | 28 |
| 8 | 50 |
| 10 | 79 |
| 12 | 113 |
| 14 | 154 |
| 16 | 201 |
| 18 | 254 |
| 20 | 314 |
| 22 | 380 |
| 25 | 491 |
| 30 | 707 |
| 32 | 804 |
| 40 | 1256 |
| 50 | 1963 |
| 60 | 2827 |

This appendix uses the standard bar diameters recommended by UNESCO.

## A.3 Development Length

The development length, $L_d$ , is the length of embedment required to develop the yield stress in the tension reinforcement for a section in flexure. ACI Code Section 12.2 lists the basic development lengths for tension reinforcement.

## A.4 Design Example of a Continuous Wall Foundation

Let it be required to design a load-bearing wall with the following data:

Dead load $= D = 43.8$ kN/m
Live load $= L = 17.5$ kN/m
Gross allowable bearing capacity of soil $= 94.9$ kN/m$^2$
Depth of the top of foundation from the ground surface $= 1.2$ m
$f_y = 413.7$ MN/m$^2$
$f'_c = 20.68$ MN/m$^2$
Unit weight of soil $= \gamma = 17.27$ kN/m$^3$
Unit weight of concrete $= \gamma_c = 22.97$ kN/m$^3$

## General Considerations

For this design, assume the foundation thickness to be 0.3 m. Refer to ACI Code Section 7.7.1, which recommends a minimum cover of 76 mm over steel reinforcement, and assume that the steel bars to be used are 12 mm in diameter (Figure A.2a). Thus,

$$d = 300 - 76 - \frac{12}{2} = 218 \text{ mm}$$

Also,

$$\text{Weight of the foundation} = (0.3)\gamma_c = (0.3)(22.97) = 6.89 \text{ kN/m}^2$$

$$\text{Weight of soil above the foundation} = (1.2)\gamma = (1.2)(17.27) = 20.72 \text{ kN/m}^2$$

So, the net allowable soil bearing capacity is

$$q_{\text{net(all)}} = 94.9 - 6.89 - 20.72 = 67.29 \text{ kN/m}^2$$

Hence, the required width of foundation is

$$B = \frac{D + L}{q_{\text{net(all)}}} = \frac{43.8 + 17.5}{67.29} = 0.91 \text{ m}$$

So, assume $B = 1$ m.
According to ACI Code Section 9.2,

$$U = 1.2D + 1.6L = (1.2)(43.8) + (1.6)(17.5) = 80.56 \text{ kN/m}$$

Converting the net allowable soil pressure to an ultimate (factored) value,

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{(1)(1)} = 80.56 \text{ kN/m}^2$$

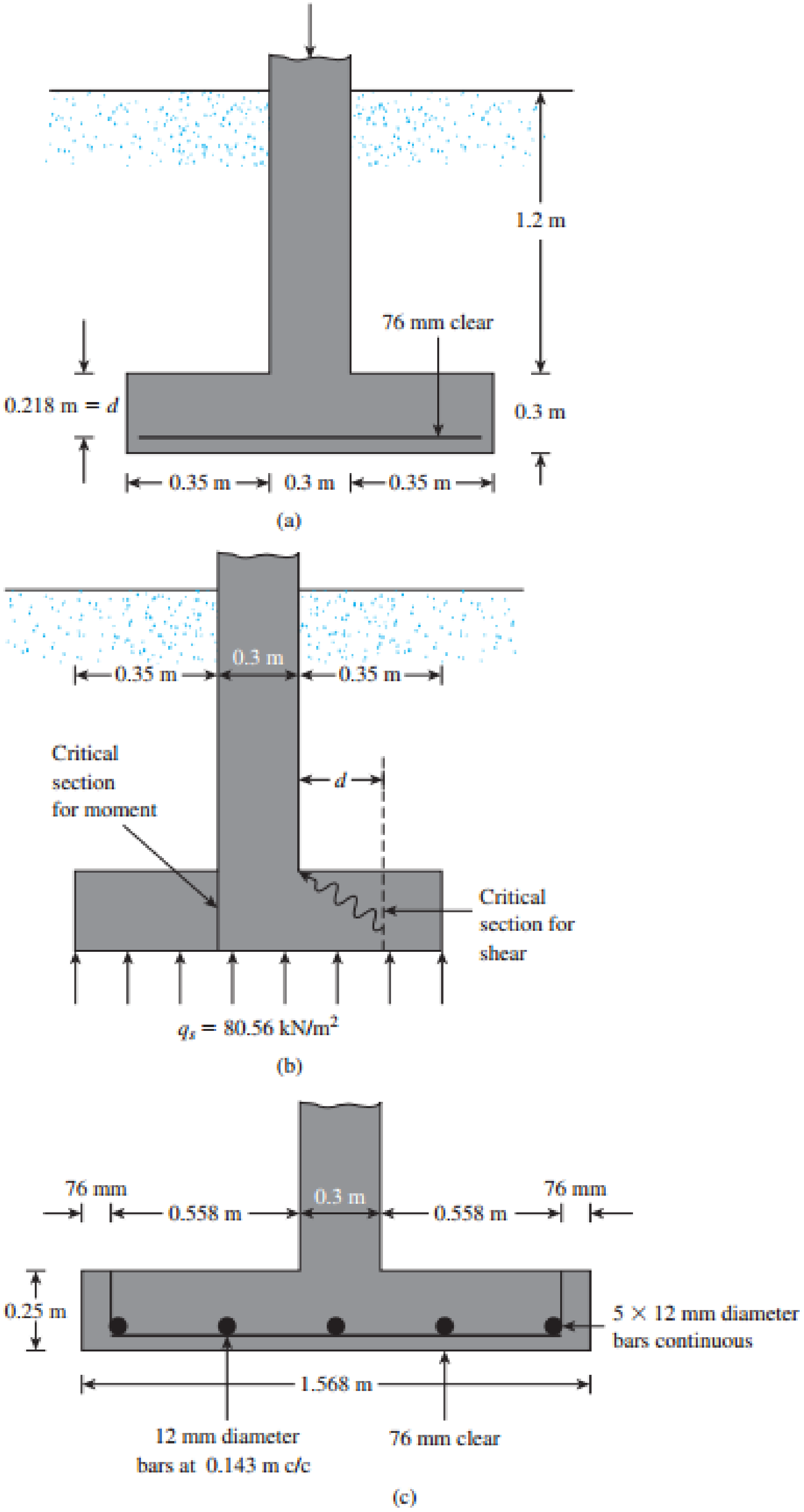



**Figure A.2** Continuous wall foundation

## Investigation of Shear Strength of the Foundation

The critical section for shear occurs at a distance *d* from the face of the wall (ACI Code Section 11.11.3), as shown in Figure A.2b. So, shear at critical section

$$V_u = (0.35 - d)q_s = (0.35 - 0.218)(80.56) = 10.63 \text{ kN/m}$$

From Eq. (A.11a) with $\lambda = 1$,

$$V_c = 0.17\sqrt{f_c'}\,bd = 0.17\sqrt{20.68}\,(1)(0.218) = 0.1685 \text{ MN/m} \approx 168 \text{ kN/m}$$

Also,

$$\phi V_c = (0.75)(168) = 126 \text{ kN/m} > V_u = 10.63 \text{ kN/m—O.K.}$$

(*Note:* $\phi = 0.75$ for shear—ACI Code Section 9.3.2.3.)

Because $V_u < \phi V_c$, the total thickness of the foundations could be reduced to 250 mm. So, the modified

$$d = 250 - 76 - \frac{12}{2} = 168 \text{ mm} > 152 \text{ mm} = d_{\min} \text{ (ACI Code Section 15.7)}$$

Neglecting the small difference in footing weight, if $d = 168$ mm,

$$\phi V_c = (0.75)(0.17)\sqrt{20.68}\,(1)(0.168) = 0.0974 \text{ MN}$$

$$= 97.4 \text{ kN} > V_u\text{—O.K.}$$

## Flexural Reinforcement

For steel reinforcement, factored moment at the face of the wall has to be determined (ACI Code Section 15.4.2). The bending of the foundation will be in one direction only. So, according to Figure A.2b, the design ultimate moment

$$M_u = \frac{q_s l^2}{2}$$

$$l = 0.35 \text{ m}$$

So,

$$M_u = \frac{(80.56)(0.35)^2}{2} = 4.93 \text{ kN-m/m}$$

From Eqs. (A.2) and (A.3),

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1)} = 23.5351 A_s$$

Thus,

$$M_n = (A_s)(413.7)\left(0.168 - \frac{23.5351}{2} A_s\right)$$

or

$$M_n(\text{MN-m/m}) = 69.5 A_s - 4868.24 A_s^2$$

Again, from Eq. (A.4)

$$M_u \leq \phi M_n$$

where $\phi = 0.9$.

Thus,

$$4.93 \times 10^{-3}(\text{MN-m/m}) = 0.9\,(69.5 A_s - 4868.24 A_s^2)$$

Solving for $A_s$, one gets

$$A_{s(1)} = 0.0128\ \text{m}^2;\ A_{s(2)} = 0.0001\ \text{m}^2$$

Hence, steel percentage with $A_{s(1)}$ is

$$s_1 = \frac{A_{s(1)}}{bd} = \frac{0.0128}{(1)(0.168)} = 0.0762$$

Similarly, steel percentage with $A_{s(2)}$ is

$$s_2 = \frac{A_{s(2)}}{bd} = \frac{0.0001}{(1)(0.168)} = 0.0006 < s_{\min} = 0.0018\ (\text{ACI Code Section 7.12.2.1})$$

The maximum steel percentage that can be provided is given in Eqs. (A.7a) and (A.8). Thus,

$$s_{\max} = (0.75)(0.85)\frac{f_c'}{f_y}\beta\left(\frac{600}{600 + f_y}\right)$$

Note that $\beta = 0.85$. Substituting the proper values of $\beta$, $f_c'$, and $f_y$ in the preceding equation, one obtains

$$s_{\max} = 0.016$$

Note that $s_1 = 0.0762 > s_{\max} = 0.016$. So use $s = s_{\min} = 0.0018$. So,

$$A_s = (s_{\min})(b)(d) = (0.0018)(1)(0.168) = 0.000302\ \text{m}^2 = 302\ \text{mm}^2$$

Use 12-mm diameter bars @ 350 mm c/c. Hence,

$$A_s(\text{provided}) = \frac{1000}{350}\left(\frac{\pi}{4}\right)(12)^2 = 323\ \text{mm}^2$$

**Development Length of Reinforcement Bars (*Ld*)**

According to ACI Code Section 12.2, the minimum development length *Ld* for 12 mm diameter bars is about 558 mm (approximately equivalent to No. 4 U.S. bar). Assuming a 76-mm cover to be on both sides of the footing, the minimum footing width should be [2(558 + 76) + 300] mm = 1568 mm = 1.568 m. Hence, the revised calculations are

$$q_s = \frac{U}{(B)(1)} = \frac{80.56}{1.568} = 51.38\,\text{kN/m}^2$$

$$M_u = \frac{q_s l^2}{2} = \frac{1}{2}(51.38)(0.558 + 0.076)^2$$

$$= 10.326\ \text{kN} \cdot \text{m/m} = 10.326 \times 10^{-3}\,\text{MN} \cdot \text{m/m}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{A_s(413.7)}{(0.85)(20.68)(1.568)} = 15.01A_s$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) = A_s(413.7)\left(0.168 - \frac{15.01A_s}{2}\right)$$

$$\phi M_n \geqslant M_u$$

$$10.326 \times 10^{-3} = 0.9A_s(413.7)\left(0.168 - \frac{15.01A_s}{2}\right)$$

and

$$A_s = 0.00016\ \text{m}^2$$

The steel percentage is $s = \dfrac{A_s}{bd} = \dfrac{0.00016}{(1.568)(0.25)} < 0.0018.$

(*Note:* Use gross area when $s_{min} = 0.0018$ is used.)

Use $A_s = (0.0018)(1.568)(0.25) = 0.000706\ \text{m}^2 = 706\ \text{mm}^2$. Provide 7 × 12 mm bars ($A_s = 565\ \text{mm}^2$).

Minimum reinforcement should be furnished in the long direction to offset shrinkage and temperature effects (ACI Code Section 7.12.). So,

$$A_s = (0.0018)(b)(d) = (0.0018)\,[(0.558 + 0.076)(2) + 0.3](0.168)$$
$$= 0.000474\ \text{m}^2 = 474\ \text{mm}^2$$

Provide 5 × 12 mm bars ($A_s = 565\ \text{mm}^2$).

The final design sketch is shown in Figure A.2c.

## A.5 Design Example of a Square Foundation for a Column

Figure A.3a shows a square column foundation with the following conditions:

Live load= $L$ = 675 kN

Dead load = $D$ = 1125 kN

Allowable gross soil-bearing capacity = $q_{all}$ = 145 kN/m²

Column size = 0.5 m × 0.5 m

$f'_c$ = 20.68 MN/m²

$f_y$ = 413.7 MN/m²

Let it be required to design the column foundation.

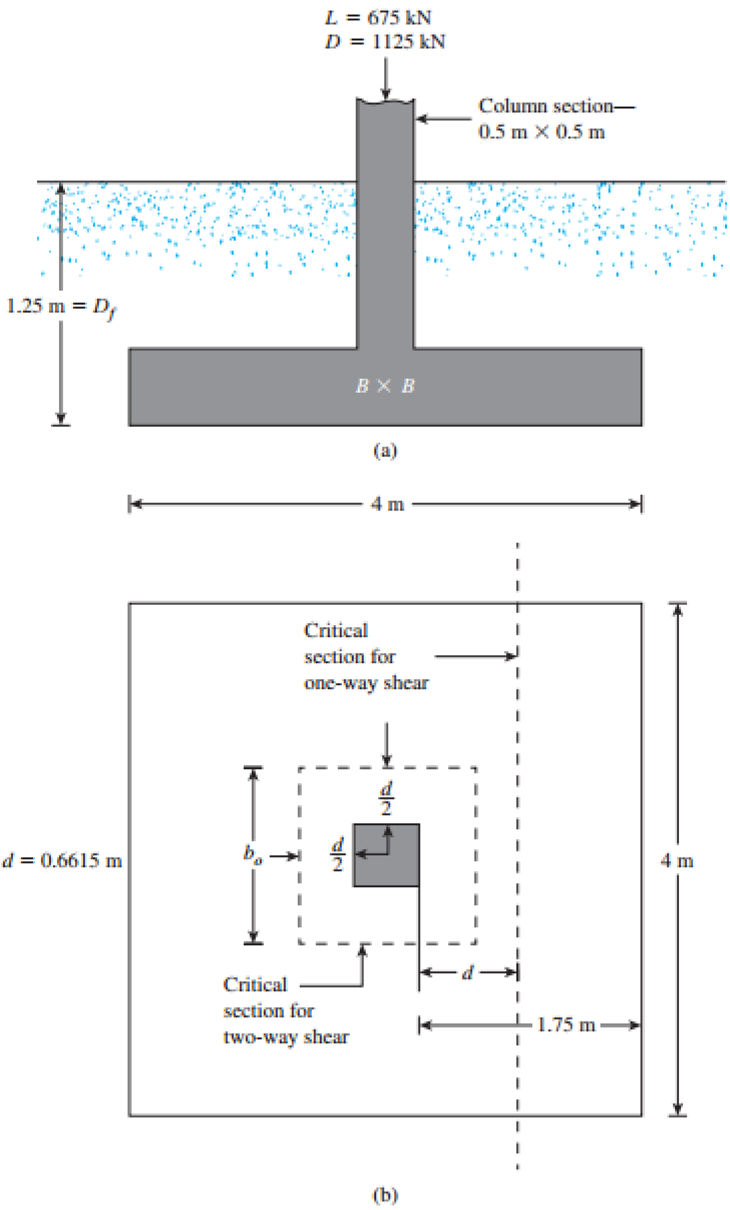



**Figure A.3** Square foundation for a column

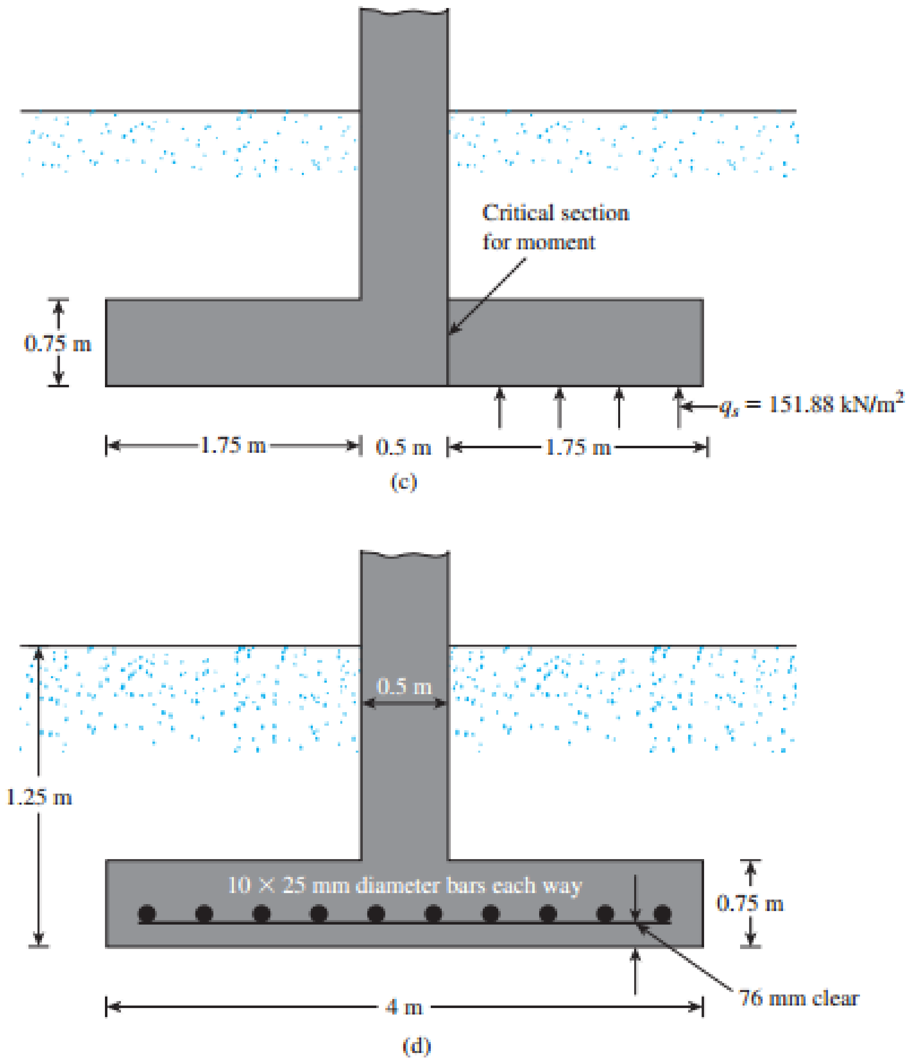



*Figure A.3* (*continued*)

## General Considerations

Let the average unit weight of concrete and soil above the base of the foundation be 21.97 kN/m³. So, the net allowable soil-bearing capacity

$$q_{\text{all(net)}} = 145 - (D_f)(21.97) = 145 - (1.25)(21.97) = 117.54 \text{ kN/m}^2$$

Hence, the required foundation area is

$$A = B^2 = \frac{D + L}{q_{\text{all(net)}}} = \frac{675 + 1125}{117.54} = 15.31 \text{ m}^2$$

Use a foundation with dimensions ($B$) of 4 m × 4 m.

The factored load for the foundation is

$$U = 1.2D + 1.6L = (1.2)(1125) + (1.6)(675) = 2430 \text{ kN}$$

Hence, the factored soil pressure is

$$q_s = \frac{U}{B^2} = \frac{2430}{16} = 151.88\,\text{kN/m}^2$$

Assume the thickness of the foundation to be equal to 0.75 m. With a clear cover of 76 m over the steel bars and an assumed bar diameter of 25 mm, we have

$$d = 0.75 - 0.076 - \frac{0.025}{2} = 0.6615\,\text{m}$$

## Check for Shear

As we have seen in Section A.4, $V_u$ should be equal to or less than $\phi V_c$. For one-way shear [with $\lambda = 1$ in Eq. (A.11a)],

$$V_u \leq \phi(0.17)\sqrt{f'_c}\,bd$$

The critical section for one-way shear is located at a distance $d$ from the edge of the column (ACI Code Section 11.1.3) as shown in Figure A.3b. So

$$V_u = q_s \times \text{critical area} = (151.88)(4)(1.75 - 0.6615) = 661.3 \text{ kN}$$

Also (with $\lambda = 1$),

$$\phi V_c = (0.75)(0.17)(\sqrt{20.68})(4)(0.6615)(1000) = 1534.2 \text{ kN}$$

So,

$$V_u = 661.3 \text{ kN} \leq \phi V_c = 1534.2 \text{ kN—O.K.}$$

For two-way shear, the critical section is located at a distance of $d/2$ from the edge of the column (ACI Code Section 11.11.1.2). This is shown in Figure A.3b. For this case, [with $\lambda = 1$ in Eq. (A.11b)]

$$\phi V_c = \phi(0.33)\sqrt{f'_c}\,b_o d$$

The term $b_o$ is the perimeter of the critical section for two-way shear. Or for this design,

$$b_o = 4[0.5 + 2(d/2)] = 4[0.5 + 2(0.3308)] = 4.65 \text{ m}$$

Hence,

$$\phi V_c = (0.75)(0.33)(\sqrt{20.68})(4.65)(0.6615) = 3.462\,\text{MN} = 3462 \text{ kN}$$

Also,

$$V_u = (q_s)(\text{critical area})$$
$$\text{Critical area} = (4 \times 4) - (0.5 + 0.6615)^2 = 14.65 \text{ m}^2$$

So,

$$V_u = (151.88)\,(14.65) = 2225.18 \text{ kN}$$
$$V_u = 2225.18 \text{ kN} < \phi V_c = 3462 \text{ kN—O.K.}$$

The assumed depth of foundation is more than adequate.

## Flexural Reinforcement

According to Figure A.3c, the moment at critical section (ACI Code Section 15.4.2) is

$$M_u = (q_s B)\left(\frac{1.75}{2}\right)^2 = \frac{[(151.88)(4)](1.75)^2}{2} = 930.27 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_v}{0.85 f_o' b} \qquad (Note:\ b = B)$$

or

$$A_s = \frac{0.85 f_c' B a}{f_y} = \frac{(0.85)(20.68)(4)a}{413.7} = 0.17a$$

From Eq. (A.4),

$$M_u \le \phi A_s f_v \left(d - \frac{a}{2}\right)$$

With $\phi = 0.9$ and $A_s = 0.17a$,

$$M_u = 930.27 = (0.9)(0.17a)(413700)\left(0.6615 - \frac{a}{2}\right)$$

Solution of the preceding equation given $a = 0.0226$ m. Hence,

$$A_s = 0.17a = (0.17)(0.0226) = 0.0038 \text{ m}^2$$

The percentage of steel is

$$s = \frac{A_s}{bd} = \frac{A_s}{Bd} = \frac{0.0038}{(4)(0.6615)} = 0.0015 < s_{\min}$$
$$= 0.0018 \text{ (ACI Code Section 7.12)}$$

So,

$$A_{s(\min)} = (0.0018)(B)(d) = (0.0018)(4)(0.6615)$$
$$= 0.004762 \text{ m}^2 = 47.62 \text{ cm}^2$$

Provide 10 × 25-mm diameter bars each way [$A_s = (4.91)(10) = 49.1 \text{ cm}^2$].

## Check for Development Length ($L_d$)

From ACI Code Section 12.2.2, for 25 mm diameter bars, $L_d \approx 1338$ mm. Actual $L_d$ provided is $(4 - 0.5/2) - 0.076$ (cover) = 1.674 m > 1338 mm—O.K.

## Check for Bearing Strength

ACI Code Section 10.14 indicates that the bearing strength should be at least $0.85\phi f_c' A_1 \sqrt{A_2/A_1}$ with a limit of $\sqrt{A_2/A_1} \le 2$. For this problem, $\sqrt{A_2/A_1} = \sqrt{(4 \times 4)/(0.5 \times 0.5)} = 8$. So, use $\sqrt{A_2/A_1} = 2$. Also $\phi = 0.7$. Hence, the design bearing strength = (0.85)(0.65)(20.68)(0.5 × 0.5)(2) = 5.713 MN = 5713 kN. However, the factored column load $U$ = 2430 kN < 5713 kN—O.K.

The final design section is shown in Figure A.3d.

# A.6 Design Example of a Rectangular Foundation for a Column

This section describes the design of a rectangular foundation to support a column having dimensions of 0.4 m × 0.4 m in cross section. Other details are as follows:

Dead load = $D$ = 290 kN
Live load = $L$ = 110 kN
Depth from the ground surface to the top of the foundation = 1.2 m
Allowable gross soil-bearing capacity = 120 kN/m$^2$
Maximum width of foundation = $B$ = 1.5 m
$f_y$ = 413.7 MN/m$^2$
$f_c'$ = 20.68 MN/m$^2$
Unit weight of soil = $\gamma$ = 17.27 kN/m$^3$
Unit weight of concrete = $\gamma_c$ = 22.97 kN/m$^3$

## General Considerations

For this design, let us assume a foundation thickness of 0.45 m (Figure A.4a). The weight of foundation/m$^2$ = 0.45 $\gamma_c$ = (0.45) (22.97) = 10.34 kN/m$^2$, and the weight of soil above the foundation/m$^2$ = (1.2)$\gamma$ = (1.2) (17.27) = 20.72 kN/m$^2$. Hence, the net allowable soil-bearing capacity [$q_{net(all)}$] = 120 − 10.34 − 20.72 = 88.94 kN/m$^2$.

The required area of the foundation = $(D + L)/q_{net(all)}$ = (290 + 110)/88.94 = 4.5 m$^2$. Hence, the length of the foundation is 4.5 m$^2$/$B$ = 4.5/1.5 = 3 m.

The factored column load = 1.2$D$ + 1.6$L$ = 1.2(290) + 1.6(110) = 524 kN.

The factored soil-bearing capacity, $q_s$ = factored load/foundation area = 524/4.5 = 116.44 kN/m$^2$.

## Shear Strength of Foundation

Assume that the steel bars to be used have a diameter of 16 mm. So, the effective depth $d$ = 450 − 76 − 16/2 = 366 mm. (Note that the assumed clear cover is 76 mm.)

Figure A.4a shows the critical section for one-way shear (ACI Code Section 11.11.1.1). According to this figure

$$V_u = \left(1.5 - \frac{0.4}{2} - 0.366\right)Bq_s = (0.934)(1.5)(116.44) = 163.13 \text{ kN}$$

The nominal shear capacity of concrete for one-way beam action [with $\lambda$ = 1 in Eq. (11.a)]

$$V_c = 0.17\sqrt{f_c'}Bd = 0.17(\sqrt{20.68})(1.5)(0.366) = 0.4244 \text{ MN} = 424.4 \text{ kN}$$

The critical section for two-way shear is also shown in Figure A.4a. This is based on the recommendations given by ACI Code Section 11.11.1.2. For this section

$$V_u = q_s[(1.5)(3) - 0.766^2] = 455.66 \text{ kN}$$

The nominal shear capacity of the foundation can be given as (ACI Code Section 11.11.2)

$$V_c = \upsilon_c b_o d = 0.33\lambda\sqrt{f'_c}\, b_o d$$

where $b_o$ = perimeter of the critical section

or

$$V_c = (0.33)(1)(\sqrt{20.68})(4 \times 0.766)(0.366) = 1.683 \text{ MN}$$

So, for two-way shear condition

$$V_u = 455.66 \text{ kN} < \phi V_c = (0.75)(1683) = 1262.25 \text{ kN}$$

Therefore, the section is adequate.

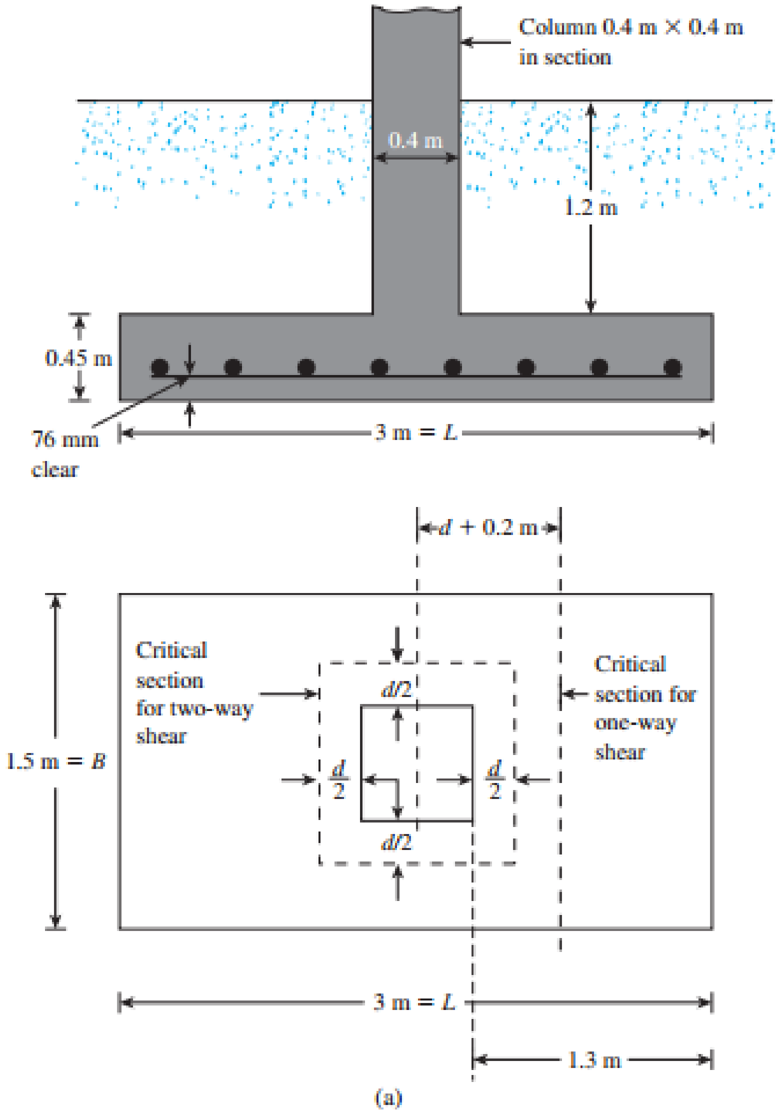



**Figure A.4** Rectangular foundation for a column

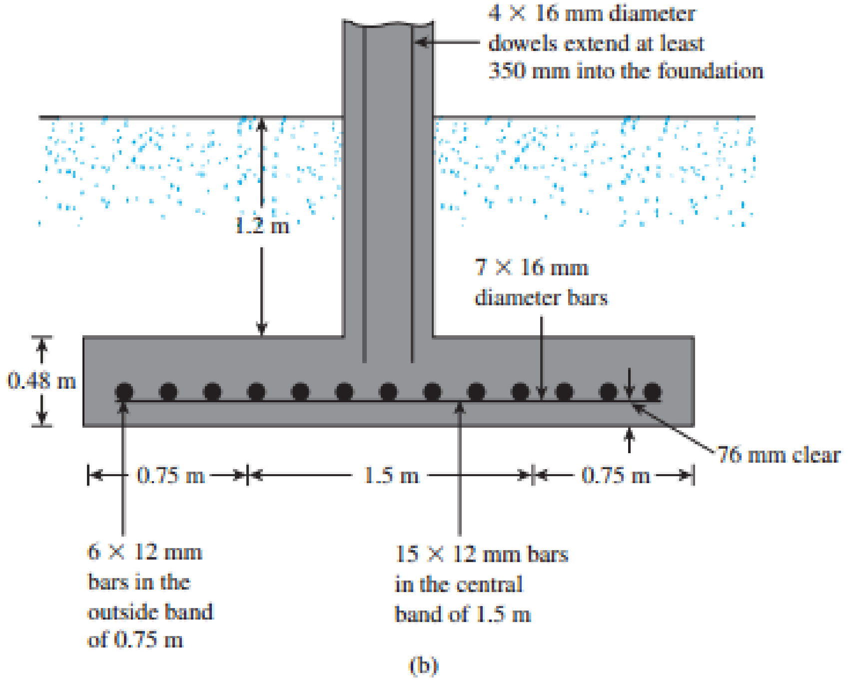



**Figure A.4** (*continued*)

## Check for Bearing Capacity of Concrete Column at the interface with Foundation

According to ACI Code Section 10.14.1, the bearing strength is equal to 0.85 $\phi f_c' A_1$ ($\phi$ = 0.65). For this problem, $U$ = 524 kN < bearing strength = (0.85)(0.65)(20.68)$(0.4)^2$ = 1.828 MN.

So, a minimum area of dowels should be provided across the interface of the column and the foundation (ACI Code Section 15.8.2). Based on ACI Code Section 15.8.2.1

$$\text{Minimum area of steel} = (0.005)\ (\text{area of column}) = (0.005)\ (400^2) = 800\ \text{mm}^2$$

So use 4 × 16-mm diameter bars as dowels.

The minimum required length of development ($L_d$) of dowels in the foundation is $(0.24 f_y d_b)/\lambda\sqrt{f_c'}$, but not less than 0.043 $f_y d_b$ (ACI Code Section 12.3.2). So,

$$L_d = \frac{0.24 f_y d_b}{\lambda\sqrt{f_c'}} = \frac{(0.24)(413.7)(16)}{(1)(\sqrt{20.68})} = 349.33\ \text{mm}$$

Also,

$$L_d = 0.043\ f_y d_b = (0.043)(413.7)(16) = 284.6\ \text{mm}$$

Hence, $L_d$ = 349.33-mm controls.

Available depth for the dowels (Figure A.4a) is 450 − 76 − 16 − 16 = 342 mm. Since hooks cannot be used, the foundation depth must be increased. Let the new depth be equal to 480 mm to accommodate the required $L_d$ = 349.33 mm. Hence, the new value of $d$ is equal to 480 − 76 − 16 − 16 = 372 mm.

## Flexural Reinforcement in the Long Direction

According to Figure A.4a, the design moment about the column face is

$$M_u = \frac{(q_s B)1.3^2}{2} = \frac{(116.44)(1.5)(1.3)^2}{2} = 147.59 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(1.5)} = 15.69 A_s$$

Again, from Eq. (A.4),

$$M_u = \phi M_n = \phi A_s f_y \left( d - \frac{a}{2} \right)$$

or

$$147.59 = (0.9)(A_s)(413.7 \times 10^3) \left[ 0.396 - \frac{15.69}{2}(A_s) \right]$$

$$147.59 = 147{,}444.7\, A_s - 2{,}920{,}928 A_s^2$$

(*Note:* $d = 0.396$ m, assuming that these bars are placed as the bottom layer.)

The solution of the preceding equation gives

$$A_s = 0.00102 \text{ m}^2 \left[ \text{that is, steel percentage} = \frac{A_s}{Bd} = \frac{0.00102}{(1.5)(0.396)} = 0.0017 \right]$$

Also, from ACI Code Section 7.12.2, $s_{min} = 0.0018$. Hence, provide 7 × 16-mm diameter bars ($A_s$ provided is 0.001407 m$^2$).

## Flexural Reinforcement in the Short Direction

According to Figure A.4a, the moment at the face of the column is

$$M_u = \frac{(q_s L)(0.55)^2}{2} = \frac{(116.44)(3)(0.55)^2}{2} = 52.83 \text{ kN-m}$$

From Eq. (A.2),

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(A_s)(413.7)}{(0.85)(20.68)(3)} = 7.845 A_s$$

From Eq. (A.4),

$$M_u = \phi A_s f_y \left( d - \frac{a}{2} \right)$$

or

$$52.83 = (0.9)(A_s)\,(413.7 \times 10^3) \left[ 0.380 - \frac{7.845}{2}(A_s) \right]$$

(*Note:* $d = 480 - 76 - 16 - \frac{16}{2} = 380$ mm for short bars in the upper layer.)

The solution of the preceding equation gives

$$A_s = 0.0004 \text{ m}^2 \qquad (\text{thus } s < s_{\min})$$

So, use $s = s_{\min}$, or

$$A_s = s_{\min}\, bd = (0.0018)(3)(0.48) \approx 0.0026 \text{ m}^2$$

(*Note* : Use gross area when $s_{\min} = 0.0018$ is used.)

Use 13 × 16-mm diameter bars.

According to ACI Code Section 12.2, the development length $L_d$ for 16 mm diameter bars is about 693 mm. For such a case, the footing width needs to be [2(0.693 + 0.076) + 0.4] = 1.938 m. Since the footing width is limited to 1.5 m, we should use 12-mm diameter bars.

So, use 23 × 12 mm diameter bars.

## Final Design Sketch

According to ACI Code Section 15.4.4, a portion of the reinforcement in the short direction shall be distributed uniformly over a bandwidth equal to the smallest dimension of the foundation. The remainder of the reinforcement should be distributed uniformly outside the central band of the foundation. The reinforcement in the central band can be given to the equal to $2/(\beta_c + 1)$ (where $\beta_c = L/B$). For this problem, $\beta_c = 2$. Hence, $2/3$ of the reinforcing bars (that is, 15 bars) should be placed in the center band of the foundation. The remaining bars should be placed outside the central band. However, one needs to check the steel percentage in the outside band, or

$$s = \frac{A_s}{bd} = \frac{(2)(113\,\text{mm}^2)}{\left(\dfrac{3000 - 1500}{2}\right)(380)} = 0.00079 < s_{\min} = 0.0018$$

So, use $A_s = (s_{\min})(b)(d) = (0.0018)(750)(480) = 648 \text{ mm}^2$. Hence, 6 × 12-mm diameter bars on each side of the central band will be sufficient.

The final design sketch is shown in Figure A.4b.

# Geometric Design of Combined Footings

## 5.4 Combined Footings

**Types:**

1. Rectangular Combined Footing (two columns).
2. Trapezoidal Combined Footing (two columns).
3. Strip Footing (more than two columns and may be rectangular or trapezoidal).

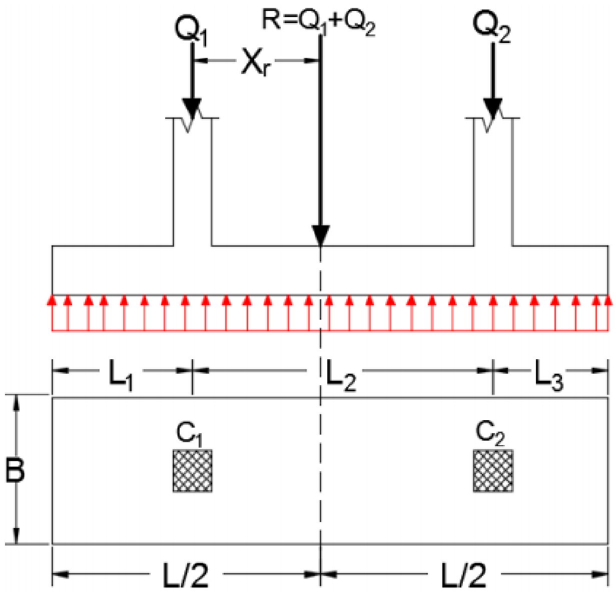
**Usage:**

1. Used when the loads on the columns are heavy and the distance between these columns is relatively small (i.e. when the distance between isolated footings is less than 30 cm).
2. Used as an alternative to neighbor footing which is an eccentrically loaded footing and it's danger if used when the load on the column is heavy.

### 5.4.1 Design of Rectangular Combined Footings:

**There are three cases:**

1. Extension is permitted from both side of the footing:



- The resultant force R is more closed to the column which have largest load.

- To keep the pressure under the foundation uniform, the resultant force of all columns loads (R) must be at the center of the footing, and since the footing is rectangular, R must be at the middle of the footing (at distance L/2) from each edge to keep uniform pressure.

$$A_{req} = \frac{\sum Q_{service}}{q_{all,net}} = \frac{Q_1+Q_2}{q_{all,net}} = B \times L$$

How we can find L:

$L_2 =$ distance between centers of the two columns and it's always known

$X_r =$ distance between the resultant force and column (1)

OR column (2) as u like ☺.

$L_1$ and $L_3 =$ extensions from left and right " usually un knowns"

Now take summation moments at $C_1$ equals zero to find $X_r$:

$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_2 + (W_{footing} + W_{soil}) \times X_r = R \times X_r \rightarrow X_r = \checkmark.$$

$(W_{footing} + W_{soil})$ are located at the center of the footing

If we are not given any information about $(W_{footing} + W_{soil}) \rightarrow$

$Q_2 L_2 = R \times X_r \rightarrow X_r = \checkmark.$

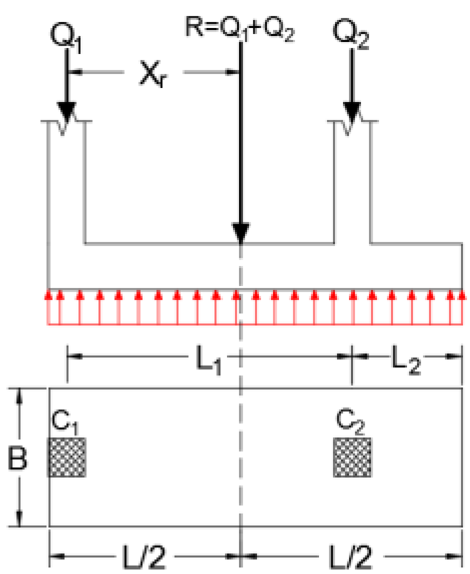
Now, to keep uniform pressure under the foundation:

$$X_r + L_1 = \frac{L}{2} \quad \text{(Two unknowns "}L_1\text{" and "}L\text{")}$$

The value of $L_1$ can be assumed according the permitted extension in site.

$$\rightarrow L = \checkmark \quad \rightarrow\rightarrow B = \frac{A_{req}}{L} = \checkmark.$$

## 2. Extension is permitted from one side and prevented from other side:




- The only difference between this case and the previous case that the extension exists from one side and when we find $X_r$ we can easily find L:
To keep the pressure uniform

$$X_r + \frac{\text{column width}}{2} = \frac{L}{2} \rightarrow L = \checkmark.$$

### 3. Extension is not permitted from both sides of the footing:

- In this case the resultant force R doesn't in the center of rectangular footing because $Q_1$ and $Q_2$ are not equals and no extensions from both sides. So the pressure under the foundation is not uniform and we design the footing in this case as following:

$L = L_1 + W_1 + W_2 = \checkmark.$

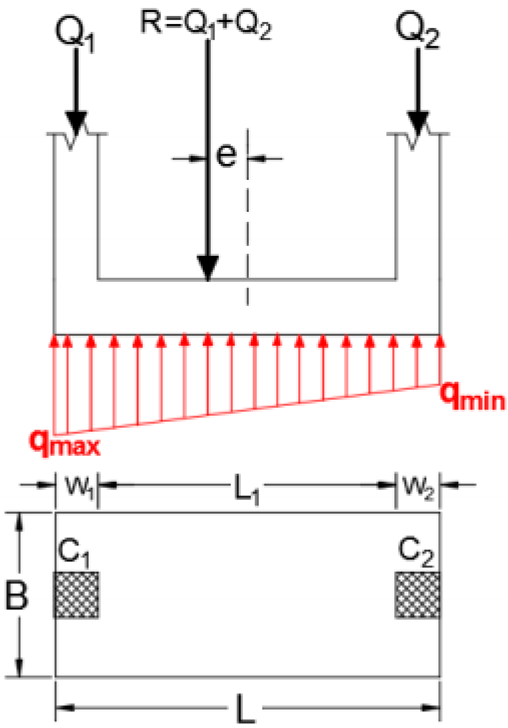
How we can find e:

$$\sum M_{\text{foundation center}} = 0.0$$

$$Q_1 \times \left(\frac{L}{2} - \frac{W_1}{2}\right) - Q_2 \times \left(\frac{L}{2} - \frac{W_2}{2}\right) = R \times e$$

$\rightarrow e = \checkmark.$

Note: the moment of $W_f + W_s$ is zero because they located at the center of footing.




The eccentricity in the direction of L:

Usually $e < \frac{L}{6}$ (because L is large)

$$q_{max} = \frac{R}{B \times L}\left(1 + \frac{6e}{L}\right)$$

$q_{all,gross} \geq q_{max} \rightarrow q_{all,gross} = q_{max}$ (critical case)

$$q_{all,gross} = \frac{R}{B \times L}\left(1 + \frac{6e}{L}\right) \rightarrow B = \checkmark.$$

**Check for B:**

$$q_{min} = \frac{R}{B \times L}\left(1 - \frac{6e}{L}\right) \text{ must be } \geq 0.0$$

If this condition doesn't satisfied, use the modified equation for $q_{max}$ to find B:

$$q_{max,modified} = \frac{4R}{3B(L - 2e)} \rightarrow B = \checkmark.$$

## 5.4.2 Design of Trapezoidal Combined Footings:

### Advantages:

1. More economical than rectangular combined footing if the extension is not permitted from both sides especially if there is a large difference between columns loads.

2. We can keep uniform contact pressure in case of “extension is not permitted from both sides” if we use trapezoidal footing because the resultant force “R” can be located at the centroid of trapezoidal footing.

**Design:**

$Q_1 > Q_2 \rightarrow B_1 \text{at } Q_1 \text{ and } B_2 \text{at } Q_2$

$L = L_1 + W_1 + W_2 = \checkmark.$

$$A_{req} = \frac{\sum Q_{service}}{q_{all,net}} = \frac{Q_1 + Q_2}{q_{all,net}}$$

$$\frac{Q_1 + Q_2}{q_{all,net}} = \frac{L}{2}(B_1 + B_2) \rightarrow\rightarrow \text{Eq. (1)}$$

Now take summation moments at $C_1$ equals zero to find $X_r$:

$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_1 +$$

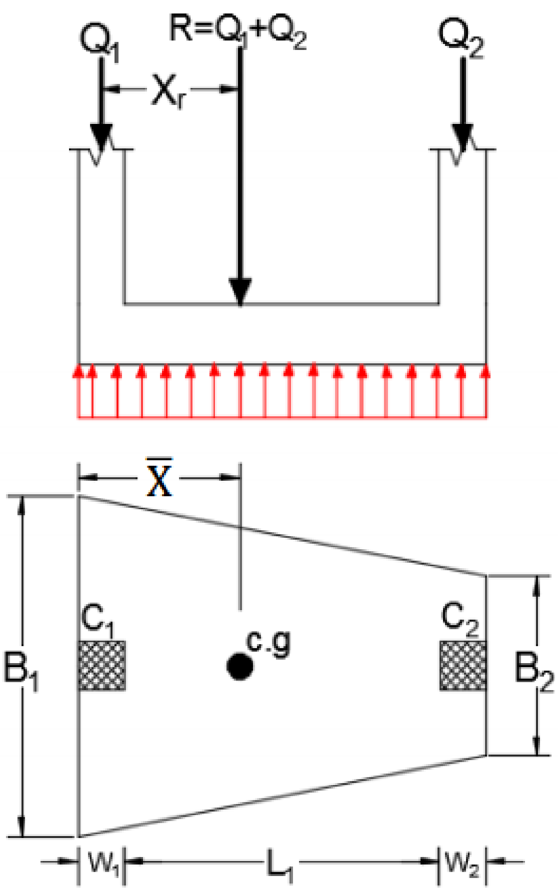
$$(W_f + W_s) \times X_r = R \times X_r \rightarrow X_r = \checkmark.$$

$$X_r + \frac{W_1}{2} = \bar{X} = \checkmark.$$

$$\bar{X} = \frac{L}{3}\left(\frac{B_1 + 2B_2}{B_1 + B_2}\right) \rightarrow\rightarrow \text{Eq. (2)}$$

Solve Eq. (1)and Eq. (2) $\rightarrow\rightarrow$

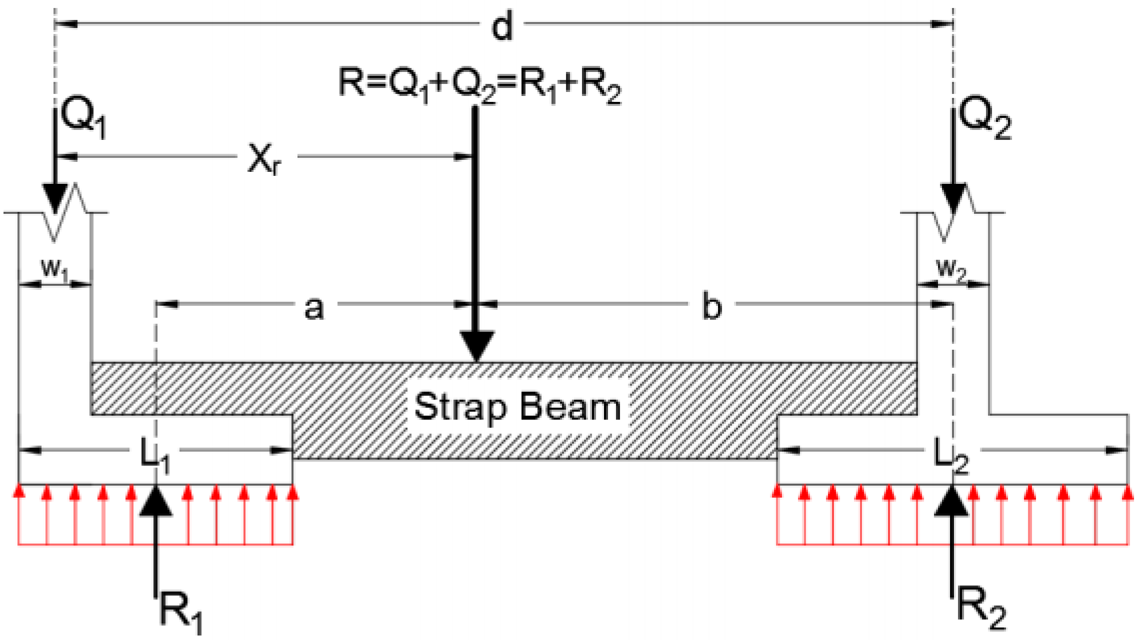
$B_1 = \checkmark$ and $B_2 = \checkmark.$



### 5.4.3 Geometric Design of Strap Footing (Cantilever Footing)

**Usage:**

1. Used when there is a **property line** which prevents the footing to be extended beyond the face of the edge column. In addition to that the edge column is relatively far from the interior column so that the rectangular and trapezoidal combined footings will be too narrow and long which increases the cost. And may be used to connect between two interior foundations one of them have a large load require a large area but this area not available, and the other foundation have a small load and there is available area to enlarge this footing, so we use strap beam to connect between these two foundations to transfer the load from largest to the smallest foundation.
2. There is a “strap beam” which connects two separated footings. The edge Footing is usually eccentrically loaded and the interior footing is centrically loaded. The **purpose of the beam is to prevent overturning of the eccentrically loaded footing and to keep uniform pressure under this foundation** as shown in figure below**.**



- Note that the strap beam doesn't touch the ground (i.e. there is no contact between the strap beam and soil, so no bearing pressure applied on it).
- This footing also called "cantilever footing" because the overall moment on the strap beam is negative moment.

**Design:**

$R = Q_1 + Q_2 = R_1 + R_2 \quad \text{but, } Q_1 \neq R_1 \text{ and } Q_2 \neq R_2$

$Q_1$ and $Q_2$ are knowns but $R_1$ and $R_2$ are unknowns

**Finding $X_r$:**

$\sum M_{Q_1} = 0.0$ (before use of strap beam) $\rightarrow R \times X_r = Q_2 \times d \rightarrow X_r = \checkmark.$

$a = X_r + \frac{w_1}{2} - \frac{L_1}{2}$ ($L_1$ should be assumed "if not given")

$b = d - X_r$

**Finding $R_1$:**

$\sum M_{R_2} = 0.0$(after use of strap beam) $\rightarrow R_1 \times (a + b) = R \times b \rightarrow R_1 = \checkmark.$

**Finding $R_2$:**

$R_2 = R - R_1$

**Design:**

$$A_1 = \frac{R_1}{q_{all,net}} \quad , \quad A_2 = \frac{R_2}{q_{all,net}}$$