

Syllabus:

1. Introduction of Power System.
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12. Newton Raphson Method.

References

- 1- power system analysis John J.Grainger , William D. Stevenson , JR.
- 2- power system analysis Hadi Saadat .
- 3- power system analysis and Design J. Duncan Glover

1- Introduction of Power System

Electric energy is the most popular form of energy. Because it can be transported easily at high efficiency and reasonable cost.

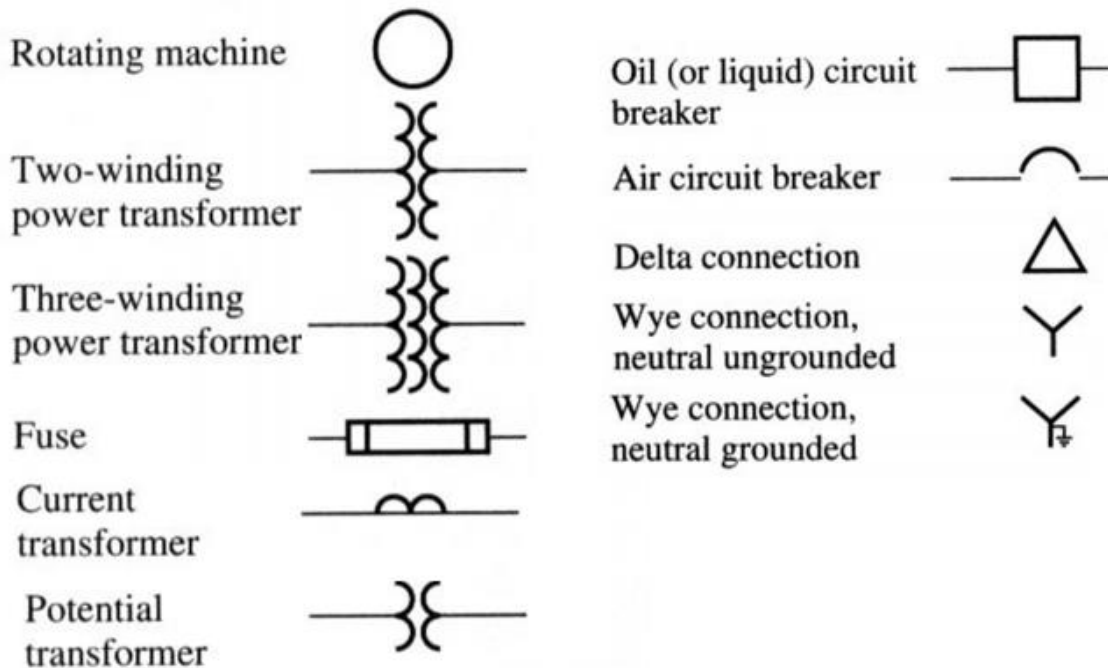
The first electric network in the united states was established in 1882 at the Peral Street station in New York city by Thomas Edison. The station supplied dc power for lighting the lower Manhattan area. The power was generated by dc generators and distributed by underground cables. In the same year the first water-wheel driven generator was installed in Appleton, Wisconsin. Within a few years many companies were established producing energy for lighting all operated under Edison's patents. Because of the excessive power loss, I^2R at low voltage Edison's companies could deliver energy only a short distance from their stations.

With the invention of transformer (William Stanley 1885) to raise the level of ac voltage for transmission and distribution and the invention of the induction motor (Nikola Tesla 1888) to replace the dc motor, the advantages of ac system become apparent, and made the ac system prevalent. Another advantage of the ac system is that due to lack of commutators in the generator, more power can be produced conveniently at higher voltage.

An interconnected power system consists of generation units, transmission lines and distribution network. The function of an integrated power system is to generate electrical power from different sources of energy, transmit the generated power to the load centers and distribute it to the end consumers. Different types of generators and loads are an integral part of the power system network. Source of generation may be thermal, hydel, nuclear, wind or solar and the load may be inductive, capacitive and resistive type, depending upon the nature of utilization of electrical power. In a power system load demand keeps changing all the time; therefore, proper power balance between load and generation have to be maintained continuously by the power system operators. Therefore, proper planning is required to operate an interconnected power system to ensure uninterrupted quality power supply to the consumers.

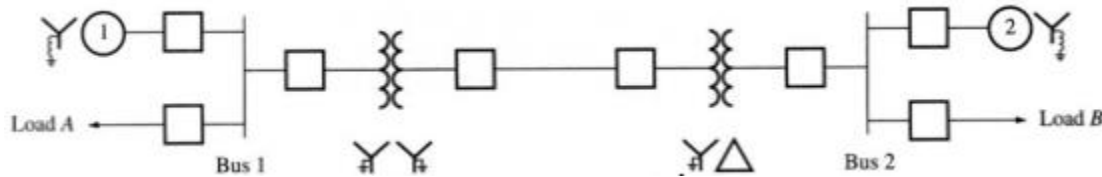
2-Power System Representation

Almost all modern power systems are three-phase systems with the phases of equal amplitude and shifted by 120° . Since phases are similar, it is customary to sketch power system in a simple form with a single line representing all three phases of the real system. Combined with a standard set of symbols for electrical components, such one-line diagrams provide compact way to represent information.



2-1 One-Line (Single-Line) Diagrams:

As example a power system containing two synchronous machines, two loads, two busses, two transformers, and a transmission line to connect busses together.



All devices are protected by oil circuit breakers (OCBs). We notice that the diagram indicates the type of connection for each machine and transformer, and also the points in the system connected to the ground.

The ground connections are important since they affect the current flowing in nonsymmetrical faults. These connection can be direct or through a resistor or inductor (they help reducing the fault current that flows in unsymmetrical faults, while having no impact on the steady-state operation of the system since the current through them will be zero). Machine ratings, impedances, and/or consumed (or supplied) powers are usually included in the diagrams.

3-PER UNIT SYSTEM

Because of the large amount of power transmitted (KW or MW and KVA or MVA). These quantities as well as ampere and voltages are often expressed as a percent or per unit of a reference or base value specified for each.

3-1 Advantages of Per Unit System

1. While performing calculations, referring quantities from one side of the transformer to the other side serious errors may be committed. This can be avoided by using per unit system.
2. Voltages, currents and impedances expressed in per unit do not change when they are referred from one side of transformer to the other side. This is a great advantage'.
3. Per unit impedances of electrical equipment of similar type usually lie within a narrow range, when the equipment ratings are used as base values.
4. Transformer connections do not affect the per unit values.

5. Manufacturers usually specify the impedances of machines and transformers in per unit or percent of name plate ratings.

In power systems there are four base quantities required to define a per unit system. These are: power S_b , voltage V_b , current I_b , and impedance Z_b . In single phase systems, the relationships among these quantities are:

$$S_b = V_b I_b$$

$$V_b = I_b Z_b$$

It is necessary to specify two base values (usually power and voltage). The other two bases value (current and impedance) are computed from the above equations. as follows.

$$I_b = \frac{S_b}{V_b}$$

$$Z_b = \frac{V_b \text{ in } V_{LN}}{I_b \text{ in Amp}} = \frac{(V_b \text{ in } KV_{LN})^2 \times 1000}{S_b (KVA_{1\phi})} = \frac{(KV_{b(LN)})^2}{MVA_{b(1\phi)}}$$

The formulas above relate the various quantities in a single phase system and 3-ph system where the current is line current, voltage is voltage to neutral and S is S per phase.

But normally we select a three-phase power base (S_b or MVA_b) and a line-to-line voltage base (V_b or kV_b). From these two the other bases can be computed using circuit laws thus:

$$I_{b-Line} = \frac{S_{b-3\phi}}{3} \frac{\sqrt{3}}{V_{b-LL}} = \frac{S_{b-3\phi} (KVA)}{\sqrt{3} V_{b-LL} (KV)}$$

$$Z_b = \frac{(V_{b-LN})^2}{S_{b-1\phi}} = \frac{(V_{b-LL})^2}{S_{b-3\phi}} = \frac{(KV_b)^2}{MVA_b}$$

In general for any quantity:

$$\text{quantity per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

The specified power base is applicable to all parts of the power system. The voltage base varies across a transformer and so do the current base and impedance base. The pu electrical quantities are calculated as follows:

$$S_{pu} = \frac{S}{S_b} = \frac{P + jQ}{S_b} = \frac{P + jQ}{S_b} = P_{pu} + jQ_{pu} \quad pu$$

$$V_{pu} = \frac{V}{V_b} \quad pu$$

$$I_{pu} = \frac{I}{I_b} \quad pu$$

$$Z_{pu} = \frac{Z}{Z_b} \quad pu$$

$$Z_{pu} = Z_{ohm} \frac{MVA_b}{(KV_b)^2}$$

It's clear that the base voltage change as the voltage level of the system is changed by transformers, but the voltage and impedance expressed in pu remain unaffected on both sides of transformers.

Different parts of a system may have different base values under such conditions the conversion of pu impedance from one base to another is perform as:

$$Z_{pu2(new)} = Z_{pu1(old)} \times \frac{MVA_{b2(new)}}{MVA_{b1(old)}} \times \left(\frac{KV_{b1(old)}}{KV_{b2(new)}} \right)^2$$

Note

It can be seen by inspection of any power system diagram that:

- a.** Several voltage levels exist in a system
- b.** It is common practice to refer to plant MVA in terms of per unit or percentage values
- c.** Transmission line and cable constants are given in ohms/km.

3-2 Analysis Of A Power System In Per Unit: Steps :

- 1) Convert all three phase bus load MW and MVAR, generator MW and MVAR, to three phase per unit quantities using the 3phase system base.
- 2) Convert all actual resistance, inductive and capacitive reactance values to per unit using the Zbase for each particular part of the power system where each r, x etc. is located.
- 3) Carry out all calculations in per unit, solving for the per unit bus voltage magnitudes and bus phase angles, line currents, and transmission losses.

4) Convert from per unit back to actual MW, MVAR, MVA, and KV.

Example 1: Prove that the pu equivalent impedance of a two winding transformer is the same whether referred to the H.T side or L.T side.

Solution:

$$Z_{L\Omega} = Z_{H\Omega} \left(\frac{KV_L}{KV_H} \right)^2$$

$$Z_{L(\text{pu})} = Z_{L\Omega} \frac{MVA}{(KV_L)^2}$$

$$Z_{L(\text{pu})} = Z_{H\Omega} \left(\frac{KV_L}{KV_H} \right)^2 \frac{MVA}{(KV_L)^2}$$

$$Z_{L(\text{pu})} = Z_{H\Omega} \frac{MVA}{(KV_H)^2}$$

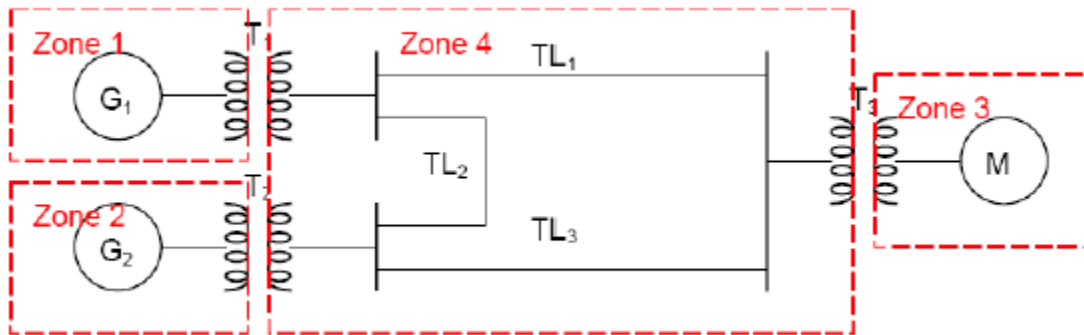
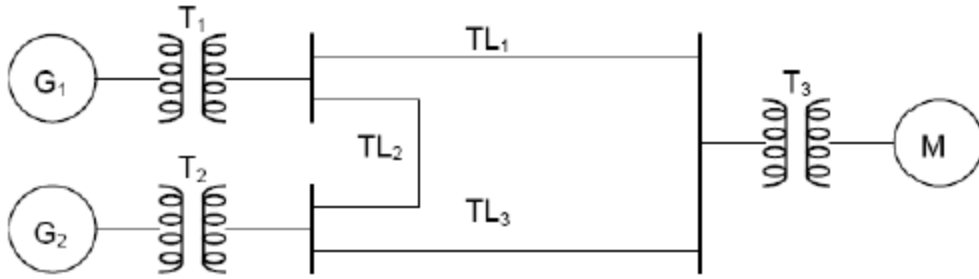
$$\therefore Z_{L(\text{pu})} = Z_{H(\text{pu})}$$

Example 2:

A one-line diagram of a three-phase power system is shown. Draw the impedance diagram of the power system, and mark all impedances in per unit. Use a base of 100 MVA and 138 kV for the transmission lines. All transformers are connected to step up the voltage of the generators to the transmission line voltages. Calculate the terminal voltage of G_2 (in pu) if G_1 is out of service and the motor draws 50 MW of power with 1 pu voltage at its terminals.

Equipment Ratings:

item	MVA	kV	X_{pu}	item	MVA	kV	
G_1	45	13.2	0.15	T_3	70	138 / 11.6	0.10
G_2	55	18	0.12	Line 1	$Z_{\text{TL}} = j40 \Omega$		
Motor	75	11.6	0.23	Line 2	$Z_{\text{TL}} = j20 \Omega$		
T_1	50	13.8 / 138	0.10	Line 3	$Z_{\text{TL}} = j15 \Omega$		
T_2	60	19.05 / 138	0.10				



Voltage zones:

Zone 1: $V_b = 13.8 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Zone 2: $V_b = 19.05 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Zone 3: $V_b = 11.6 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Zone 4: $V_b = 138 \text{ Kv}$, $S_b = 100 \text{ MVA}$

Impedances:

$$Z_{G1} = j0.15 \left(\frac{100 \text{ MVA}}{45 \text{ MVA}} \right) \left(\frac{13.2 \text{ KV}}{13.8 \text{ KV}} \right)^2 = j0.305 \text{ pu}$$

$$Z_{G2} = j0.12 \left(\frac{100 \text{ MVA}}{55 \text{ MVA}} \right) \left(\frac{18 \text{ KV}}{19.05 \text{ KV}} \right)^2 = j0.195 \text{ pu}$$

$$Z_M = j0.23 \left(\frac{100 \text{ MVA}}{75 \text{ MVA}} \right) \left(\frac{11.6 \text{ KV}}{11.6 \text{ KV}} \right)^2 = j0.307 \text{ pu}$$

$$Z_{T1} = j0.1 \left(\frac{100 \text{ MVA}}{50 \text{ MVA}} \right) \left(\frac{138 \text{ KV}}{138 \text{ KV}} \right)^2 = j0.2 \text{ pu}$$

$$Z_{T2} = j0.1 \left(\frac{100 \text{ MVA}}{60 \text{ MVA}} \right) \left(\frac{138 \text{ KV}}{138 \text{ KV}} \right)^2 = j0.167 \text{ pu}$$

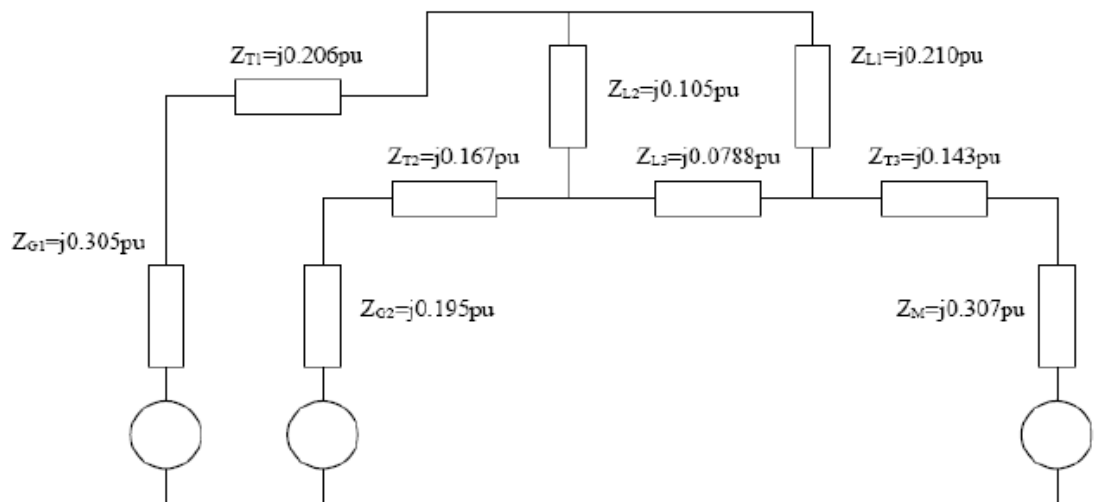
$$Z_{T3} = j0.1 \left(\frac{100 \text{ MVA}}{70 \text{ MVA}} \right) \left(\frac{138 \text{ KV}}{138 \text{ KV}} \right)^2 = j0.143 \text{ pu}$$

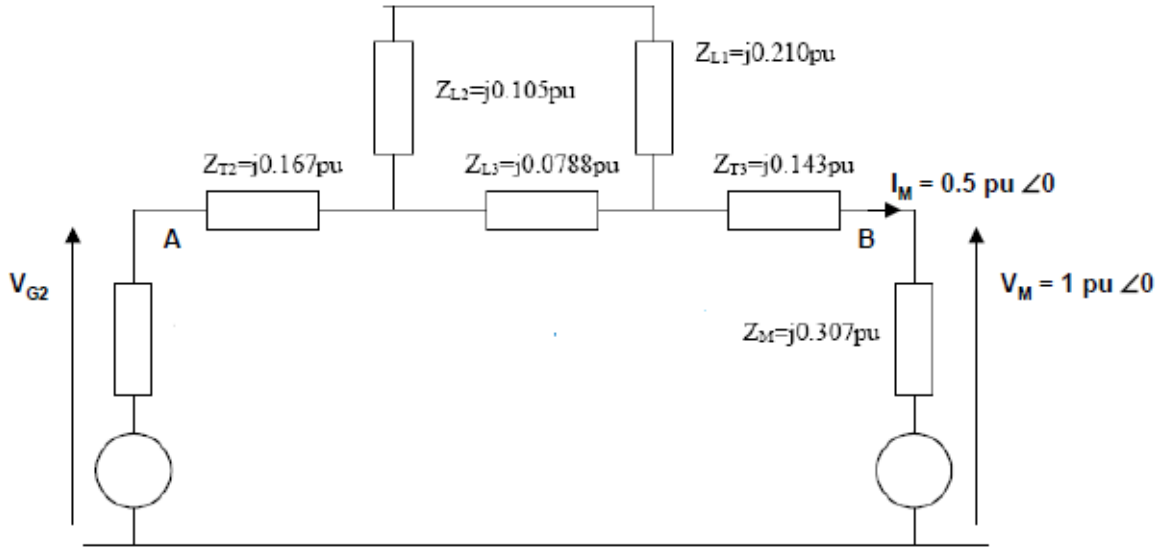
$$Z_b = \frac{(138 \text{ KV})^2}{100 \text{ MVA}} = 190.4 \Omega$$

$$Z_{TL1} = \frac{j40 \Omega}{190.4 \Omega} = j0.21 \text{ pu}$$

$$Z_{TL2} = \frac{j20 \Omega}{190.4 \Omega} = j0.105 \text{ pu}$$

$$Z_{TL3} = \frac{j15 \Omega}{190.4 \Omega} = j0.0788 \text{ pu}$$





$$Z_L = \frac{j0.315 \cdot j0.0788}{j0.315 + j0.0788} = j0.063$$

$$Z_{AB} = j0.063 + j0.167 + j0.143 = j0.373$$

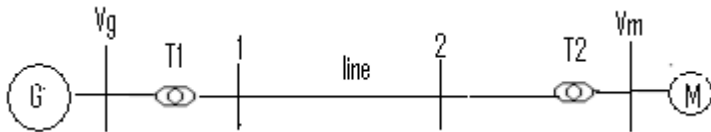
$$V_{AB} = I_M \cdot Z_{AB} = 0.5 \cdot j0.373 = 0.187 \angle 90$$

$$V_{G2} = V_{AB} + V_m = 1 + 0.187 \angle 90 = 1.017 \angle 10.6$$

Example 3: The three-phase power and line-line ratings of the electric power system shown in Figure are given below.

G:	60 MVA	20 kV	X = 9%
T1 :	50 MVA	20/200 kV	X = 10%
T2 :	50 MVA	200/20 kV	X = 10%
M :	43.2 MVA	18 kV	X = 8%
Line:		200 kV	Z = 120 + j200 Ω

Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.



Solution:

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is:

$$V_{B1} = 20 \left(\frac{200}{20} \right) = 200 \text{ KV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{Bm} = 200 \left(\frac{20}{200} \right) = 20 \text{ KV}$$

The generator and transformer reactances in per unit on a 100 MVA base,

$$G: \quad X = 0.09 \left(\frac{100}{60} \right) = 0.15 \text{ pu}$$

$$T_1: \quad X = 0.1 \left(\frac{100}{50} \right) = 0.2 \text{ pu}$$

$$T_2: \quad X = 0.1 \left(\frac{100}{50} \right) = 0.2 \text{ pu}$$

$$M: \quad X = 0.08 \left(\frac{100}{43.2} \right) \left(\frac{18}{20} \right)^2 = 0.15 \text{ pu}$$

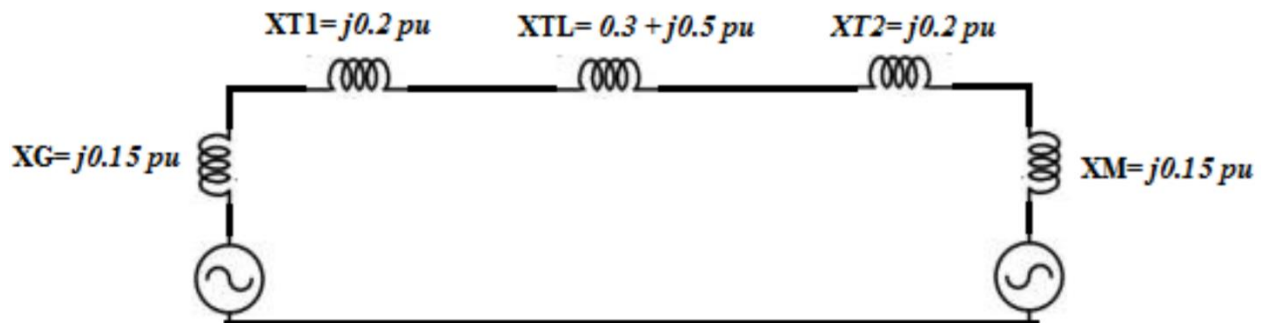
The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \Omega$$

The per unit line impedance is

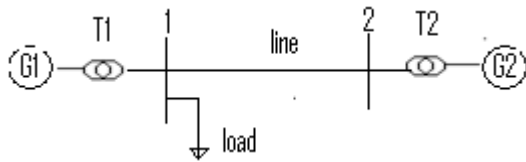
$$\text{line:} \quad Z_{line} = \left(\frac{120 + j200}{400} \right) = 0.3 + j0.5 \text{ pu}$$

The per unit equivalent circuit is shown in Figure



Example 4: Draw an impedance diagram for the electric power system shown in Figure showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below

G1 :90 MVA	20 kV	$X = 9\%$
T1 :80 MVA	20/200 kV	$X = 16\%$
T2 :80 MVA	200/20 kV	$X = 20\%$
G2 :90 MVA	18 kV	$X = 9\%$
Line:	200 kV	$X = 120 \Omega$
Load:	200kv	$Z = 300 + j400 \Omega$



Solution:

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is:

$$V_{B1} = 20 \left(\frac{200}{20} \right) = 200 \text{ KV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{BG2} = 200 \left(\frac{20}{200} \right) = 20 \text{ KV}$$

The generator and transformer reactance in per unit on a 100 MVA base,

$$G_1: \quad X = 0.09 \left(\frac{100}{90} \right) = 0.1 \text{ pu}$$

$$T_1: \quad X = 0.16 \left(\frac{100}{80} \right) = 0.2 \text{ pu}$$

$$T_2: \quad X = 0.2 \left(\frac{100}{80} \right) = 0.25 \text{ pu}$$

$$G_2: \quad X = 0.09 \left(\frac{100}{90} \right) \left(\frac{18}{20} \right)^2 = 0.081 \text{ pu}$$

The base impedance for the transmission line is

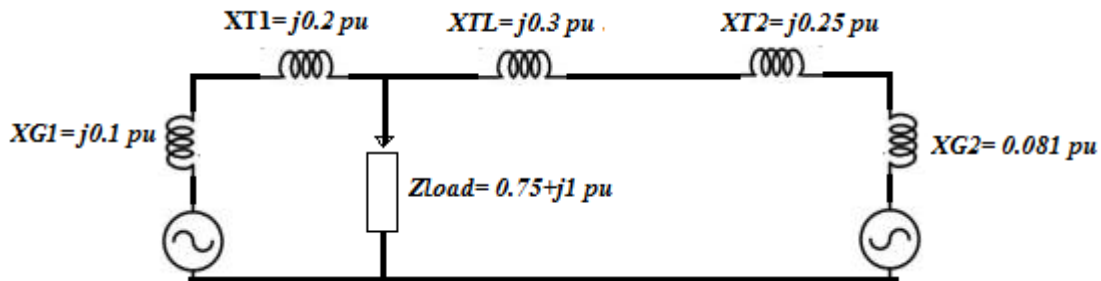
$$Z_{BL} = \frac{(200)^2}{100} = 400\Omega$$

The per unit line impedance is

$$Z_{line} = \frac{120}{400} = 0.3 pu$$

$$\text{load: } Z_{load} = \left(\frac{300 + j400}{400} \right) = 0.75 + j1 pu$$

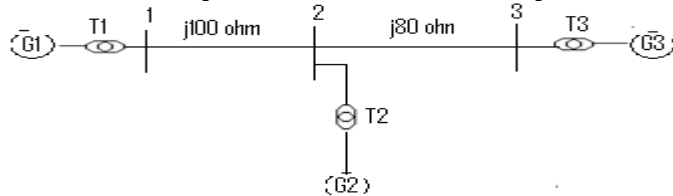
The per unit equivalent circuit is shown in Figure



Example 5: Draw an impedance diagram for the electric power system shown in Figure showing all impedances in per unit on a 30MVA base. Choose 6.9 kV as the voltage base for generator. The three-phase power and line-line ratings are given below

G1 :20 MVA	6.9 kV	X = 15%
G2 :10 MVA	6.9 kV	X = 15%
G3:30 MVA	13.8 kV	X = 15%
T1:25 MVA	6.9/115 kV	X = 10%
T2 :12.5 MVA	6.9/115 kV	X = 10%
T3:10 MVA	7.5/75 kV	X = 10%

Line 1-2: $X = j100 \Omega$, Line 1-3: $X = j80 \Omega$



Solution:

$$G_1: \quad X = 0.15 \left(\frac{6.9}{6.9} \right)^2 \left(\frac{30}{20} \right) = 0.225 \text{ pu}$$

$$\text{Base voltage in } L_{1-2} = 6.9 \left(\frac{115}{6.9} \right) = 115 \text{ KV}$$

$$\text{Base impedance} = \frac{(115)^2}{30} = 440 \Omega$$

$$L_{1-2}X = \frac{100}{440} = 0.2268 \text{ pu}$$

$$L_{2-3}X = \frac{80}{440} = 0.1814 \text{ pu}$$

$$T_1: \quad X = 0.1 \left(\frac{6.9}{6.9} \right)^2 \left(\frac{30}{25} \right) = 0.12 \text{ pu}$$

$$\text{Base KV in } G_2 = 115 \left(\frac{6.9}{115} \right) = 6.9 \text{ KV}$$

$$G_2: \quad X = 0.15 \left(\frac{30}{10} \right) \left(\frac{6.9}{6.9} \right)^2 = 0.45 \text{ pu}$$

$$T_2: \quad X = 0.1 \left(\frac{30}{12.5} \right) \left(\frac{115}{115} \right)^2 = 0.25 \text{ pu}$$

$$\text{Base voltage} = 115 \left(\frac{7.5}{75} \right) = 11.5 \text{ KV}$$

$$G_3: \quad X = 0.15 \left(\frac{13.8}{11.5} \right)^2 \left(\frac{30}{30} \right) = 0.216 \text{ pu}$$

$$T_3: \quad X = 0.1 \left(\frac{30}{10} \right) \left(\frac{75}{115} \right)^2 = 0.127 \text{ pu}$$

HW.

Q1/ A 100 MVA, 13.8 KV, 3-phase generator has a reactance of 20%. The generator is connected to a 3-phase transformer T I rated 100 MVA 12.5 KV 1110 KV with 10% reactance. The h.v. side of the transformer is connected to a transmission line of reactance 100 ohm. The far end of the line is connected to a step down transformer T 2' made of three single-phase transformers each rated 30 MVA, 60 KV / 10 KV with 10% reactance the generator supplies two motors connected on the l.v. side T2 as shown in Fig. E.6.2. The motors are rated at 25 MVA and 50 MVA both at 10 KV with 15% reactance. Draw the reactance diagram showing all the values in per unit. Take generator rating as base.

Q2/ Figure below shows single-line diagram of a power system. The ratings of the generators and transformers are given below:

G1: 25 MVA, 6.6 kV, $x_{g1}=0.20$ pu

G2: 15 MVA, 6.6 kV, $x_{g2} = 0.15$ pu

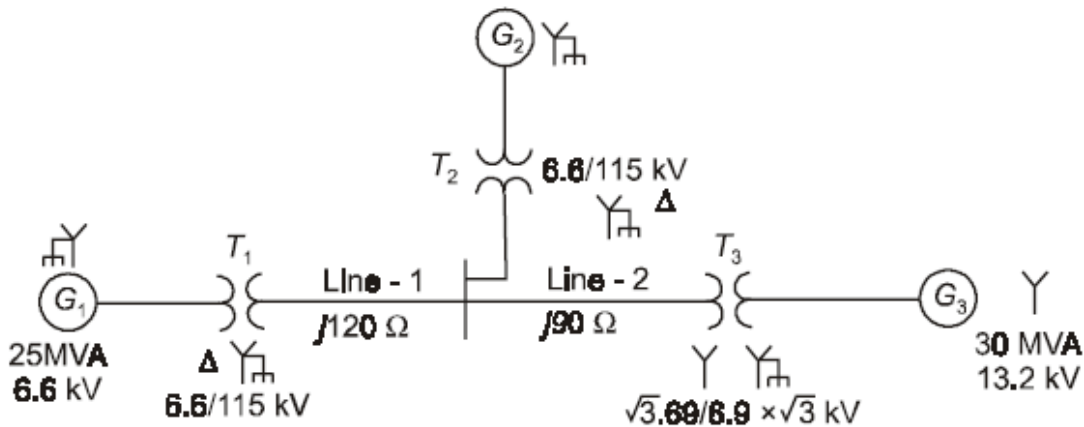
G3: 30 MVA, 13.2 kV, $x_{g3} = 0.15$ pu

T1 : 30 MVA, 6.6 fl - 115 Y kV, $x_{T1} = 0.10$ pu

T2 : 15 MVA, 6.6 fl - 115 Y kV, $x_{T2} = 0.10$ pu

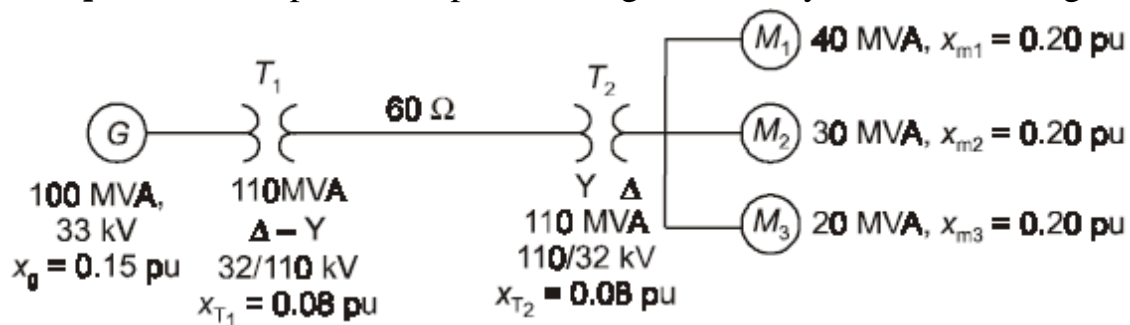
T3 : Single-phase unit each rated 10 MVA, 6.9/69 kV, $x_{T3} = 0.10$ pu.

Draw per-unit circuit diagram using base values of 30 MVA and 6.6 kV in the circuit of generator-1.

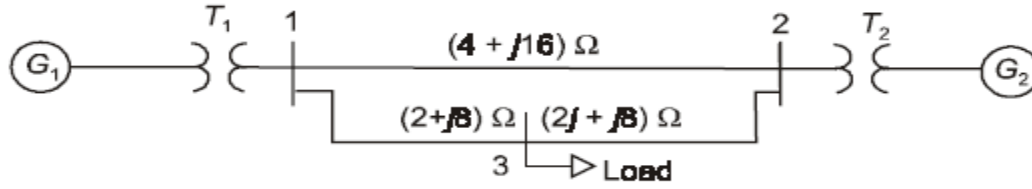


Q3: A 100 MVA, 33 kV, three phase generator has a reactance of 15%. The generator is connected to the motors through a transmission line and transformers as shown in Fig. below. Motors have rated inputs of 40 MVA, 30 MVA and 20 MVA at 30 kV with 20% reactance-each. Draw the per-unit circuit diagram.

Example: Draw the per-unit impedance diagram of the system shown in Fig.



Assumed base values are 100 MVA and 100 kV.



G_1 50 MVA, 12.2 kV, $x_{g1} = 0.10$ pu

G_2 20 MVA, 13.8 kV, $x_{g2} = 0.10$ pu

T_1 80 MVA, 12.2/132 kV, $x_{T1} = 0.10$ pu

T_2 40 MVA, 13.8/132 kV, $x_{T2} = 0.10$ pu

Load 50 MVA, 0.80 pf lagging operating at 124 kV.

4- Nodal Analysis

In per-phase analysis the components of the power transmission system are modeled and represented by passive impedances or equivalent admittances accompanied, where appropriate, by active voltage or current sources. In the steady state, for example, a generator can be represented by the circuit of either Fig.1 (a) or Fig.1 (b). The circuit having the constant emf E_s ' series impedance Z_a ' and terminal voltage V has the voltage equation

$$E_s = IZ_a + V$$

Dividing across by Z_a gives the current equation for Fig. 1 (b)

$$I_s = \frac{E_s}{Z_a} = I + VY_a$$

where $Y_a = 1/Z_a$ ' Thus, the emf E_s and its series impedance Z_a can be interchanged with the current source I_s and its shunt admittance Y_a , provided

$$I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a}$$

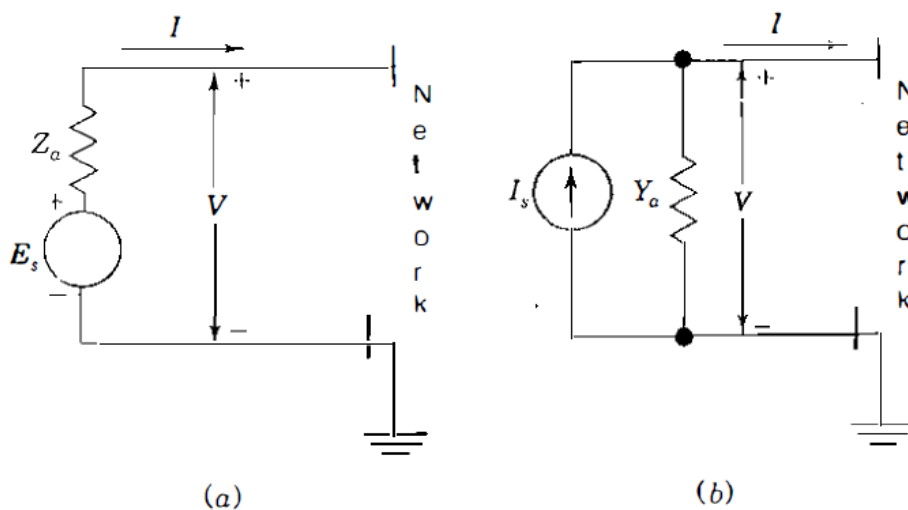
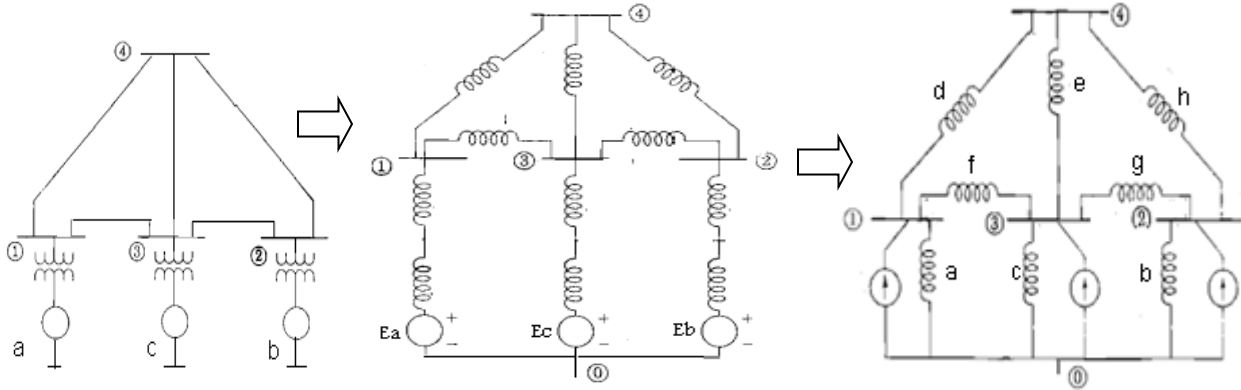


Figure 1 Circuits illustrating the equivalenc of sources when $I_s = E_s/Z_a$ and $Y_a = 1/Z_a$

Systematic formulation of equations determined at nodes of a circuit by applying Kirchoff current law. In electrical power networks, the reference node is the ground (major node), then replace the e.m.f,s and series impedance by equivalent current source and shunt admittances.



For node (1):

$$I_1 = V_1 Y_a + (V_1 - V_3) Y_f + (V_1 - V_4) Y_d$$

$$I_1 = V_1 (Y_a + Y_f + Y_d) - V_3 Y_f - V_4 Y_d \quad \dots\dots\dots 1$$

$$0 = (V_4 - V_1) Y_d + (V_4 - V_2) Y_h + (V_4 - V_3) Y_e$$

$$0 = -V_1 Y_d - V_2 Y_h - V_3 Y_e + V_4 (Y_d + Y_e + Y_h) \quad \dots\dots\dots 2$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

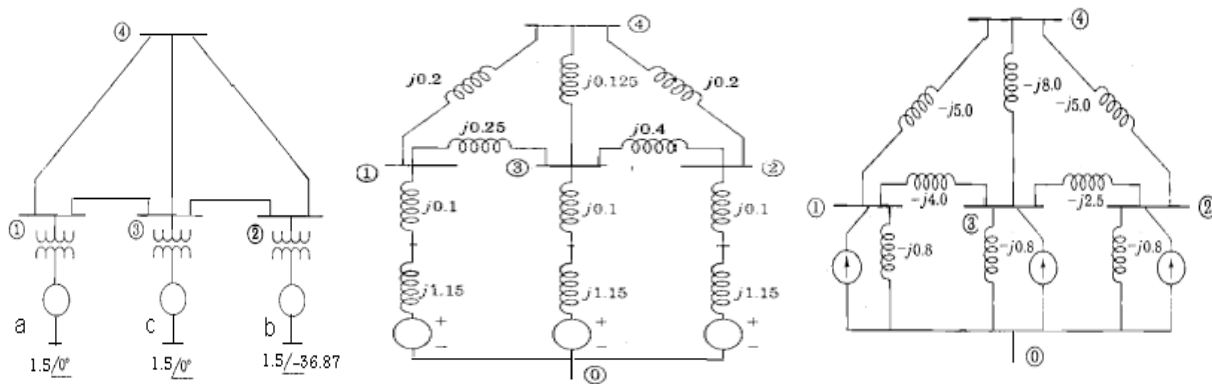
The (Y) matrix is designated (Y_{bus}) and called (the bus admittance matrix)

- 1- Each diagonal element in the nodal admittance matrix ($Y_{11} Y_{22} Y_{33} Y_{44}$) is called self admittance and it is the sum of the admittances of the branches terminating in each node.
- 2- Each off-diagonal element of the nodal admittance matrix (all the other admittances) called the mutual admittances and each equal the negative

- of the sum of all admittances connected between the nodes identified by the double subscript.
- 3- If no direct connection exists between any two nodes the corresponding off- diagonal elements will be zero.

Example:

Write in matrix form the node equations necessary to solve for the voltages of the numbered buses of the figure below.



Solution:

$$I_1 = I_3 = \frac{1.5 \angle 0}{j1.25} = 1.2 \angle -90 = -j1.2 \text{ p.u}$$

$$I_2 = \frac{1.5 \angle -36.87}{j1.25} = 1.2 \angle -126.87 = -0.72 - j0.96 \text{ p.u}$$

Self admittances

$$Y_{11} = -j5 - j4 - j0.8 = -j9.8$$

$$Y_{22} = -j5 - j2.5 - j0.8 = -j8.3$$

$$Y_{33} = -j4 - j2.5 - j8 - j0.8 = -j15.3$$

$$Y_{44} = -j5 - j5 - j8 = -j18$$

Mutual admittances

$$Y_{12} = Y_{21} = 0, \quad Y_{23} = Y_{32} = j2.5, \quad Y_{13} = Y_{31} = j4$$

$$Y_{14} = Y_{41} = j5, \quad Y_{24} = Y_{42} = j5, \quad Y_{34} = Y_{43} = j8$$

The node equations in matrix form

$$\begin{array}{c|c|c|c|c|c} -j1.2 & & -j9.8 & j0 & j4 & j5 & V_1 \\ -0.72-j0.96 & & j0 & -j8.3 & j2.5 & j5 & V_2 \\ -j1.2 & & j4 & j2.5 & -j15.3 & j8 & V_3 \\ 0 & & j5 & j5 & j8 & -j18 & V_4 \end{array}$$

5- Fault

5-1 Introduction

A fault is any abnormal condition in a power system. The steady state operating mode of a power system is balanced 3-phase a.c. .However, due to sudden external or internal changes in the system, this condition is disrupted. When the insulation of the system fails at one or more points or a conducting object comes into contact with a live point, a short circuit or a fault occurs.

Causes of Power System Faults

The causes of faults are numerous, e.g.

- Lightning
- Heavy winds
- Trees falling across lines
- Vehicles colliding with towers or poles
- Birds shorting lines
- Aircraft colliding with lines
- Vandalism
- Small animals entering switchgear
- Line breaks due to excessive loading

5-2 Common Power System Faults

Power system faults may be categorized as one of four types; in order of frequency of occurrence, they are:

- Single line to ground fault
- Line to line fault
- Double line to ground fault
- Balanced three phase faults.

The first three types constitute severe unbalanced operating conditions which involves only one or two phases hence referred to as unsymmetrical faults. In the fourth type, a fault involving all the three phases occurs therefore referred to as symmetrical (balanced) fault.

5-3 Effects Of Power System Faults

Faults may lead to fire breakout that consequently results into loss of property, loss of life and destruction of a power system network. Faults also leads to cut of supply in areas beyond the fault point in a transmission and distribution network leading to power blackouts; this interferes with industrial and commercial activities that supports economic growth, stalls learning activities in institutions, work in offices, domestic applications and creates insecurity at night. All the above results into retarded development due to low gross domestic product realised. It is important therefore to determine the values of system voltages and currents during faulted conditions, so that protective devices may be set to detect and minimize the harmful effects of such contingencies.

5-4 Balanced Fault

Short circuit occur in power systems when equipment insulation fails, due to system over voltages caused by lightning or switching surges, to insulation contamination, or to other mechanical causes.

Fault studies form an important part of power system analysis. The problem consists of determining bus voltages and line currents during various type of

faults. Faults in power systems are divided into *three-phase balanced* faults and *unbalanced faults*. The information gained from fault studies are used for proper relay setting and coordination, and used to obtain the rating of the protective switchgears.

The fault current is determined by the internal voltages of the synchronous machines and by the impedances between the machine voltages and the fault. It may be several orders of magnitude larger than normal operation currents and, if allowed to persist, may cause thermal damage to equipments. Winding and busbars may also suffer mechanical damage due to high magnetic forces during faults. It is therefore necessary to remove faulted sections of a power system from service as soon as possible (for EHV, the fault is clear with in 3 cycles (50ms at 60Hz), and for lower voltage more slowly, with in 5 to 20 cycles.

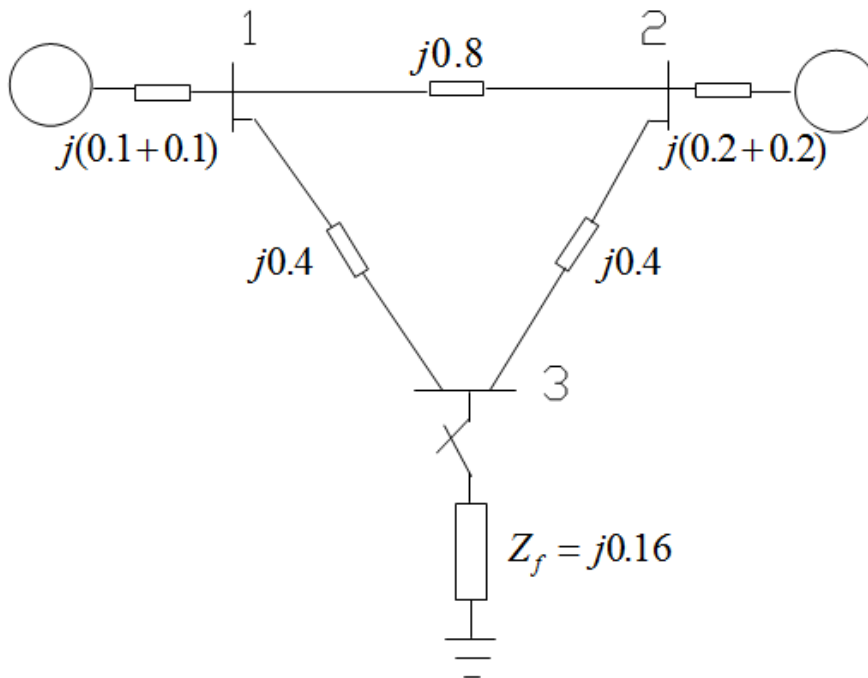
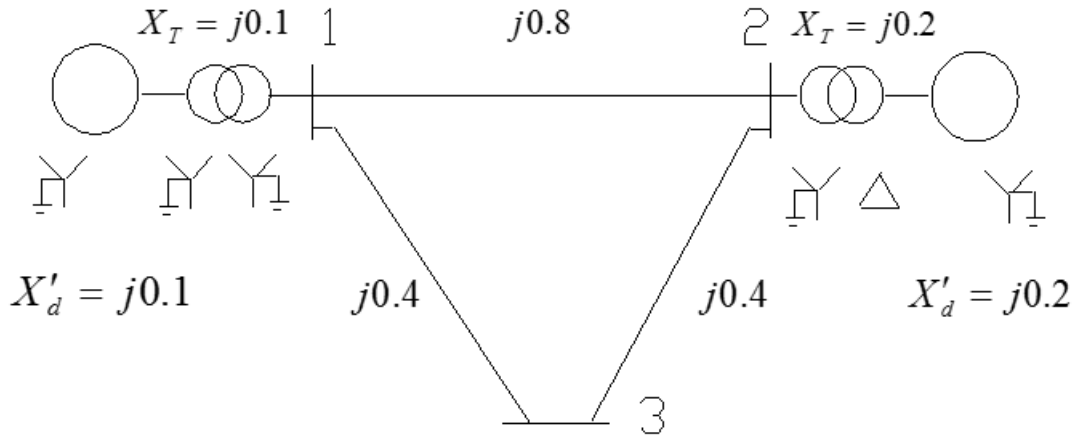
The magnitude of the fault currents depends on the internal impedance of the generators plus the impedance of the intervening circuit.

In course of power system stability, it was shown that the reactance of a generator under short circuit condition is not constant (is a time-varying quantity) and for network analysis three reactances were defined. The sub transient reactance X_d'' , for the first few cycles of the short circuit current, transient reactance X_d' , for the next (say) 30 cycles, and the synchronous reactance X_d , thereafter. Since the duration of the short circuit depends on the time of operation of the protective system, it is not always easy to decide which reactance to use. Generally, the sub transient reactance is used for determining the interrupting capacity of the circuit breakers. Transient reactance is used for relay setting and coordination, and for transient stability studies.

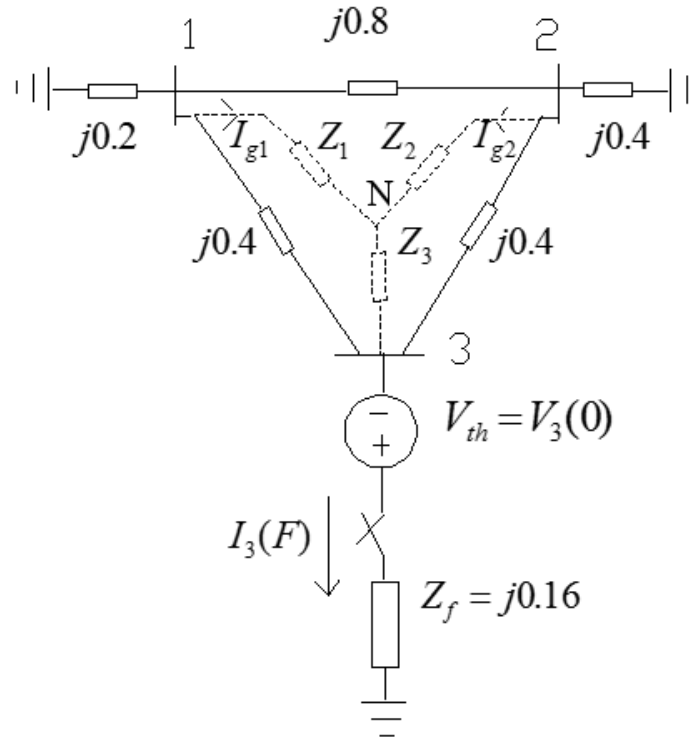
If the fault impedance is zero, the fault is referred to as the *bolted fault* or the *solid fault*. The faulted network can be solved by the Thevenin's method. The procedure is demonstrated in the following example.

In the power system shown in fig. All impedance in per unit on a common 100MVA, the resistances and shunt capacitances are neglected. The system is considered on no-load and the generators are running at their rated voltage and frequency with their emfs in phase. Find the fault current, the bus voltages, and

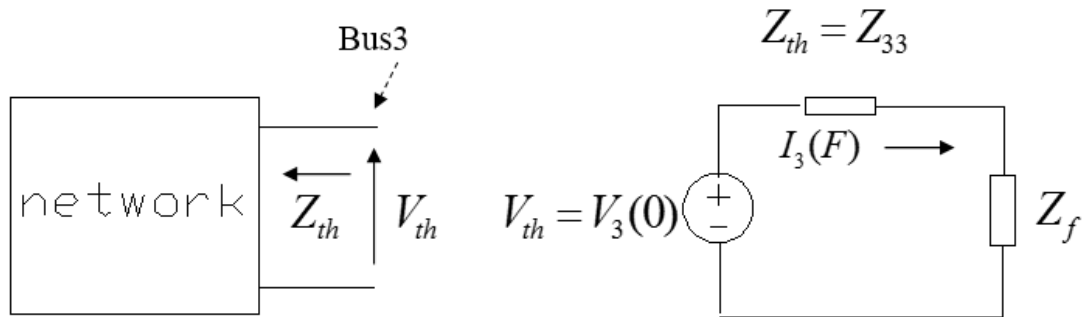
the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = 0.16$ per unit, occurs on bus 3.



The impedance network for fault at bus 3



Thevenin's equivalent network



$V_{th} = V_3(0) =$ pre fault bus voltage (obtain from the results of the power flow solution)

$$Z_{th} = Z_{33}$$

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f}$$

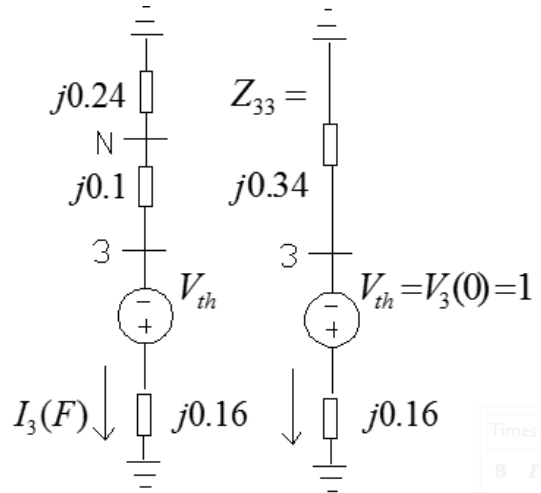
$$Z_{\Delta} \rightarrow Z_Y$$

$$Z_1 = Z_2 = \frac{(j0.4)(j0.8)}{j0.16} = j0.2$$

$$Z_3 = \frac{(j0.4)(j0.4)}{j0.16} = j0.1$$

$$Z_{33} = [(Z_1 + j0.2) \parallel (Z_2 + j0.4)] + Z_3$$

$$= \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1 = j0.24 + j0.1 = j0.34$$



The load are neglected, and the generators emfs are assumed equal to the rated value, therefore all the pre-fault bus voltages are equal to 1.0 pu, i.e.:

$$V_1(0) = V_2(0) = V_3(0) = 1.0 \text{ pu}$$

$$\therefore I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \text{ pu}$$

$$I_{g1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2 \text{ pu}$$

$$I_{g2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8 \text{ pu}$$

For the bus voltage changes:

$$\Delta V_1 = 0 - (I_{g1} \times j0.2) = 0 - (-j1.2)(j0.2) = -0.24 \text{ pu}$$

$$\Delta V_2 = 0 - (I_{g2} \times j0.4) = 0 - (-j0.8)(j0.4) = -0.32 \text{ pu}$$

$$\Delta V_3 = I_3(F) \times Z_f - V_3(0) = (-j2)(j0.16) - 1.0 = -0.68 \text{ pu}$$

The bus voltages during the fault are obtained by superposition of the pre-fault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus $V_3(F)$.

$$V_1(F) = V_1(0) + \Delta V_1 = 1 - 0.24 = 0.76 \text{ pu}$$

$$V_2(F) = V_2(0) + \Delta V_2 = 1 - 0.32 = 0.68 \text{ pu}$$

$$V_3(F) = V_3(0) + \Delta V_3 = 1 - 0.68 = 0.32 \text{ pu}$$

The short circuit-currents in the lines are:

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{Z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \text{ pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{Z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \text{ pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{Z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \text{ pu}$$

The procedure is summarized in the following steps:

- 1- The pre-fault bus voltages are obtained from the results of the power flow solution.
- 2- In order to preserve the linearity feature of the network, loads are converted to constant admittances using the pre-fault bus voltages.
- 3- The faulted network is reduced into a Thevenin's equivalent circuit as viewed from the faulted bus. Applying Thevenin's theorem, changes in the bus voltages are obtained.
- 4- Bus voltages during the fault are obtained by superposition of the pre-fault bus voltages and the changes in the bus voltages computed in the previous step.
- 5- The current during the fault in all branches of the network are then obtained.

5-4-1 Short-circuit Capacity (SCC) or Short-circuit MVA:

SCC at a bus is a common measured of the strength of a bus. The SCC at bus K is defined as the product of the magnitudes of the rated bus voltage and the fault current $I(F)$. The short circuit MVA is used for determining the dimension of a bus bar, and the interrupting capacity of a circuit breaker.

The SCC or the short-circuit MVA at bus K is given by:

$$SCC = \sqrt{3} V_{LK} I_K(F) \times 10^{-3} \text{ MVA} \quad \dots\dots\dots (1)$$

Where V_{LK} - Line to line voltage in KV

$I_K(F)$ - Fault current at node K in amperes.

For 3-phase fault:

$$I_K(F)_{pu} = \frac{V_K(0)}{X_{KK}} \quad \dots\dots\dots (2)$$

Where $V_K(0)$ - pre-fault bus voltage in pu.

X_{KK} - The per unit reactance to the point of fault.

(where system resistance is neglected)

System resistance is neglected and only the inductive reactance of the system is allowed for. This gives minimum impedance and maximum fault current and pessimistic answer.

The base current is:

$$I_B = \frac{S_B \times 10^3}{\sqrt{3} V_B} \quad \dots\dots\dots (3)$$

Where S_B - base MVA

V_B - line-to-line base voltage in KV

$$I_K(F) \text{ in amp.} = I_K(F)_{pu} \times I_B$$

$$= \frac{V_K(0)}{X_{KK}} \frac{S_B \times 10^3}{\sqrt{3} V_B} \dots\dots\dots (4)$$

Substituting eq.(4) in eq(1)

$$SCC = \frac{V_K(0) S_B}{X_{KK}} \frac{V_L}{V_B} \dots\dots\dots (5)$$

If the base voltage is equal to the rated voltage ($V_L = V_B$)

$$\therefore SCC = \frac{V_K(0) S_B}{X_{KK}} \dots\dots\dots (6)$$

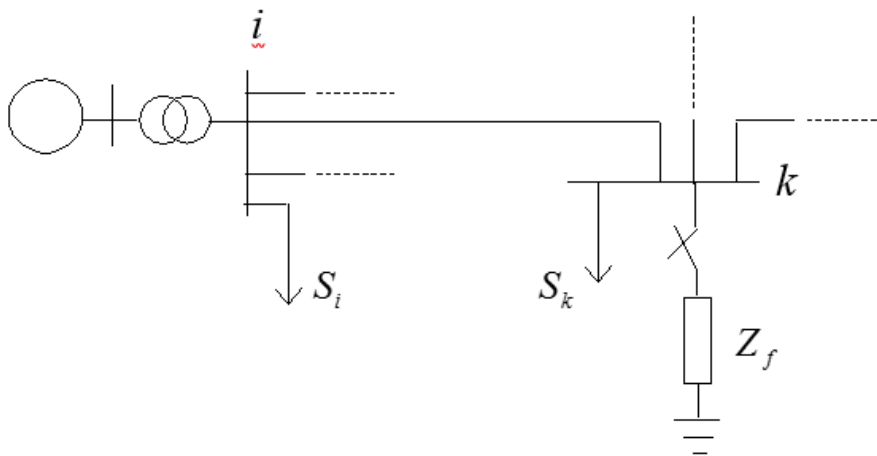
The prefault bus voltage $V_K(0)$ is usually assumed to 1.0 pu

$$\therefore SCC \text{ (short – circuit MVA)} = \frac{S_B}{X_{KK}} \text{ MVA} \dots (7)$$

5-5 Systematic Fault Analysis Using Z_{bus} :

For fault circuit analysis in large networks, nodal method is used. By utilizing the elements of Z_{bus} , the fault current and the bus voltages during fault are readily and easily calculated.

In the fig. shown below, the generator is represented by a constant voltage source behind proper reactances which may be $X''_d, X'_d, \text{ or } X_d$. Transmission lines are represented by their equivalent π model and all impedances are in per unit on a common MVA base. A balance 3-phase fault is to be applied at bus K through a fault impedance Z_f .



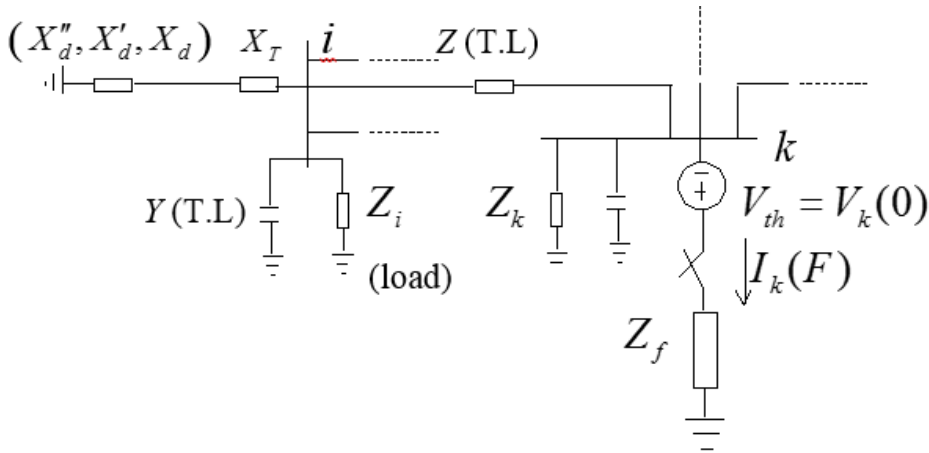
The pre fault bus voltages are obtained from the power flow solution and are represented by the column vector.

$$V_{bus}(0) = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} \dots\dots\dots(8)$$

The bus load by a constant impedance evaluated at the pre fault bus voltage, i.e.:

$$Z_{iL} = \frac{|V_i(0)|^2}{S_L^*} \dots\dots\dots(9)$$

Thevenin's circuit shown in fig. below.



The bus voltage changes caused by the fault in this circuit are represented by the column vector :

$$\Delta V_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} \dots\dots\dots(10)$$

By applying Thevenin's theorem, bus voltages during the fault are obtained by superposition of the pre fault bus voltages and the changes in the bus voltages i.e.:

$$V_{bus}(F) = V_{bus}(0) + \Delta V_{bus} \dots\dots\dots(11)$$

The node-voltage equation for an *n*-bus network:

$$I_{bus} = Y_{bus} V_{bus} \dots\dots\dots(12)$$

Where *I_{bus}* is the injected bus currents.

Y_{bus} is the bus admittance matrix.

Also , we know :

$$Y_{ii} = \sum_{j=0}^m y_{ij} \quad j \neq i \quad , \text{ and } Y_{ij} = Y_{ji} = -y_{ij} \dots\dots (13)$$

In the Thevenin's circuit of fig. above, current entering every bus is zero except at the faulted bus. Since the current at faulted bus is leaving the bus, it is taken as a negative current entering bus k .

Thus the nodal equation applied to the Thevenin's circuit in fig. above becomes:

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & \dots & y_{1k} & \dots & y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{k1} & \dots & y_{kk} & \dots & y_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{n1} & \dots & y_{nk} & \dots & y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} \dots\dots\dots (14)$$

$$\text{or} \quad I_{bus}(F) = Y_{bus} \Delta V_{bus}$$

$$\text{or} \quad \Delta V_{bus} = Z_{bus} I_{bus}(F)$$

where $Z_{bus} = Y_{bus}^{-1}$ is known as the bus impedance matrix

The bus voltage vector during the fault becomes:

$$V_{bus}(F) = V_{bus}(0) + Z_{bus} I_{bus}(F)$$

The above matrix equation, can be writing in terms of its elements as shown:

$$\begin{bmatrix} V_1(F) \\ \vdots \\ V_k(F) \\ \vdots \\ V_n(F) \end{bmatrix} = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} + \begin{bmatrix} Z_{11} & \dots & Z_{1k} & \dots & Z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \dots & Z_{kk} & \dots & Z_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1} & \dots & Z_{nk} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} \dots\dots (15)$$

Since we have only one single nonzero element in the current vector , the *k*th equation in eq. 15 becomes :

$$V_k(F) = V_k(0) + Z_{kk} I_k(F) \dots\dots\dots (16)$$

From the Thevenin's circuit shown in fig. above:

$$V_k(F) = Z_f I_k(F) \dots\dots\dots (17)$$

From eq. (16) and eq. (17) , the fault current becomes :

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f} \dots\dots\dots (18)$$

[For bolted fault , $Z_f = 0$ and $V_k(F) = 0$] , therefore the fault current is :

$$I_k(F) = \frac{V_k(0)}{Z_{kk}}$$

Thus, for a fault at bus *k* we need only the Z_{kk} element of the bus impedance matrix. This element is indeed the Thevenin's impedance as viewed from the faulted bus.

For *i* th equation in eq. (15):

$$V_i(F) = V_i(0) + Z_{ik} I_k(F) \dots\dots\dots (19)$$

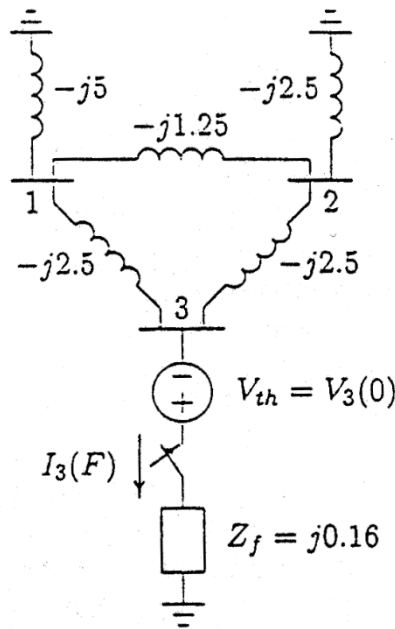
$$V_i(F) = V_i(0) - \frac{Z_{ik}}{Z_{kk} + Z_f} V_k(0) \dots\dots\dots (20)$$

Where $V_i(F)$ is bus voltage during the fault at bus i

The fault current in all the lines:

$$I_{ij}(F) = \frac{V_i(F) - V_j(F)}{Z_{ij}} \dots\dots\dots (21)$$

For previous example, to compute the fault current, the bus voltages , and the line currents during the fault at bus 3 , by using the bus impedance matrix method , the Thevenin's equivalent network is redraw with impedances converted to admittances as shown in fig. below , in order to find the bus admittance matrix .



The admittance diagram for system of previous example.

Referring to above fig., the bus admittance matrix by inspection is:

$$Y_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix}$$

$$Z_{bus} = Y_{bus}^{-1} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

From eq. (18), for a fault at bus 3 with fault impedance $Z_f = 0.16 \text{ pu}$:

$$I_3(F) \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \text{ pu}$$

From eq. (19), bus voltages during the fault are:

$$V_1(F) = V_1(0) - Z_{13} I_3(F) = 1.0 - (j0.12)(-j2.0) = 0.76 \text{ pu}$$

$$V_2(F) = V_2(0) - Z_{23} I_3(F) = 1.0 - (j0.16)(-j2.0) = 0.68 \text{ pu}$$

$$V_3(F) = V_3(0) - Z_{33} I_3(F) = 1.0 - (j0.34)(-j2.0) = 0.32 \text{ pu}$$

From eq. (21), the short circuit currents in the lines are:

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \text{ pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \text{ pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \text{ pu}$$

The results are exactly the same as the values found previously.

The values of the diagonal elements in the bus impedance matrix are equal to the thevenin's impedances for other fault locations , i.e. if the fault occurs at bus2, the thevenin's impedance is $j0.24$, and equal to $j0.16$ if the fault occurs at bus1, then can be used eq. (18) to find directly the fault current at buses 1 and 2 , as following :

$$I_1(F) \frac{V_1(0)}{Z_{11} + Z_f} = \frac{1.0}{j0.16 + j0.16} = -j3.125 \text{ pu}$$

$$I_2(F) \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.24 + j0.16} = -j2.5 \text{ pu}$$

Furthermore, the Off-diagonal elements are utilized in eq. (20) to obtain bus voltages during the fault.

Therefore, the bus impedance matrix method is an indispensable tool for fault studies.

The matrix inversion for a large power system with a large number of buses is not feasible. A computationally attractive and efficient method for finding Z_{bus} matrix is (building) or (assembling) the impedance matrix by adding one network element at a time. In effect, this is an indirect matrix inversion of the bus admittance matrix.

Example 1/ A 25 MVA, 11 kV generator with $X_l = 20\%$ is connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown in Fig. Each motor has $X_d'' = 25\%$ and $X_d' = 30\%$ on a base of 5 MVA, 6.6 kV. The three-phase rating of the step-up transformer is 25 MVA, 11/66 kV with a leakage reactance of 10% and that of the step-down transformer is 25 MVA, 66/6.6 kV with a leakage reactance of 10%. The bus voltage at the motors is 6.6 kV when a three-phase fault occurs at the point F. For the specified fault, calculate

- the sub transient current in the fault,
- the sub transient current in the breaker B.

Given: Reactance of the transmission line = 15% on a base of 25 MVA, 66 kV. Assume that the system is operating on no load when the fault occurs.



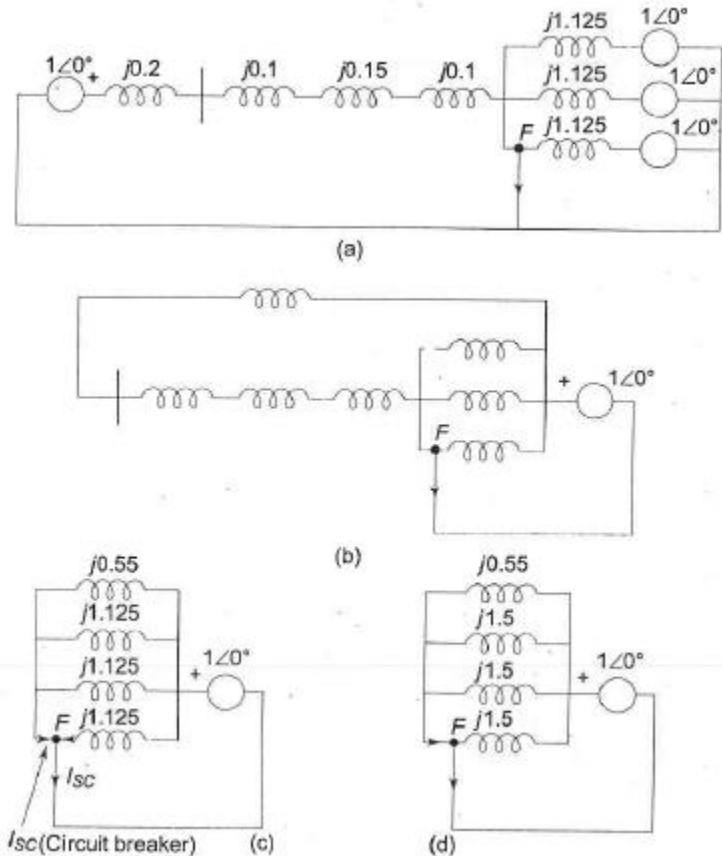
Solution

Choose a system base of 25 MVA.
 For a generator voltage base of 11 kV, line voltage base is 66 kV and motor voltage base is 6.6 kV.

(a) For each motor

$$X''_{dm} = j0.25 \times \frac{25}{5} = j1.25 \text{ pu}$$

Line, transformer and generator reactances are already given on proper base values. The circuit model of the system for fault calculations is given in Fig.a. The system being initially on no load, the generator and motor induced emfs are identical. The circuit can therefore be reduced to that of Fig. b and then to Fig.c. Now



$$I_{SC} = 3 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j4.22 \text{ pu}$$

$$\text{Base current in 6.6 kV circuit} = \frac{25 \times 1,000}{\sqrt{3} \times 6.6} = 2,187 \text{ A}$$

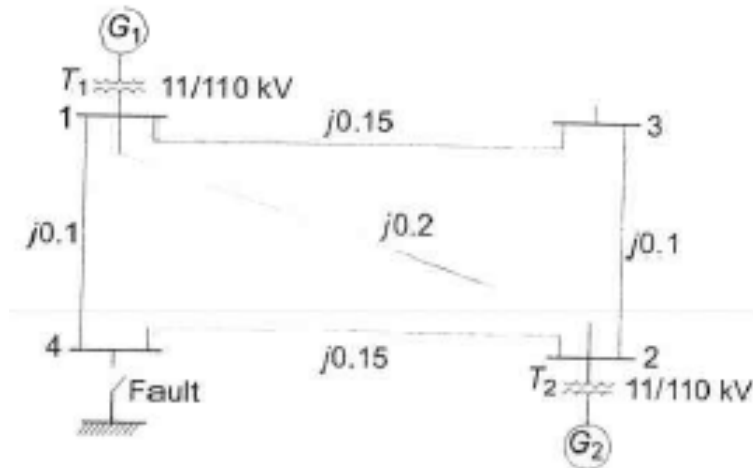
$$\therefore I_{SC} = 4.22 \times 2,187 = 9,229 \text{ A}$$

(b) From Fig.1c, current through circuit breaker *B* is

$$\begin{aligned} I_{SC(B)} &= 2 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j3.42 \\ &= 3.42 \times 2,187 = 7,479.5 \text{ A} \end{aligned}$$

Example 2/ Consider the 4-bus system of Fig. Buses 1 and 2 are generator buses and 3 and 4 are load buses. The generators are rated 11kV, 100 MVA, with transient reactance of 10% each. Both the transformers are 11/110 kV, 100 MVA with a leakage reactance of 5%. The reactances of the lines to a base of 100 MVA, 110 kV are indicated on the figure. obtain the short circuit solution for a three-phase solid fault on bus 4 (load bus).

Assume pre-fault voltages to be 1 pu and pre-fault currents to be zero.



Solution

Changes in voltages and currents caused by a short circuit can be calculated from the circuit model of Fig.1. Fault current I_f is calculated by systematic network reduction as in Fig.2,

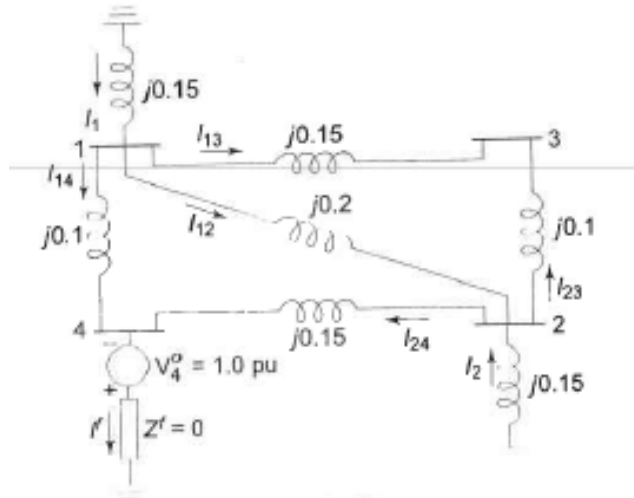


Fig. 1

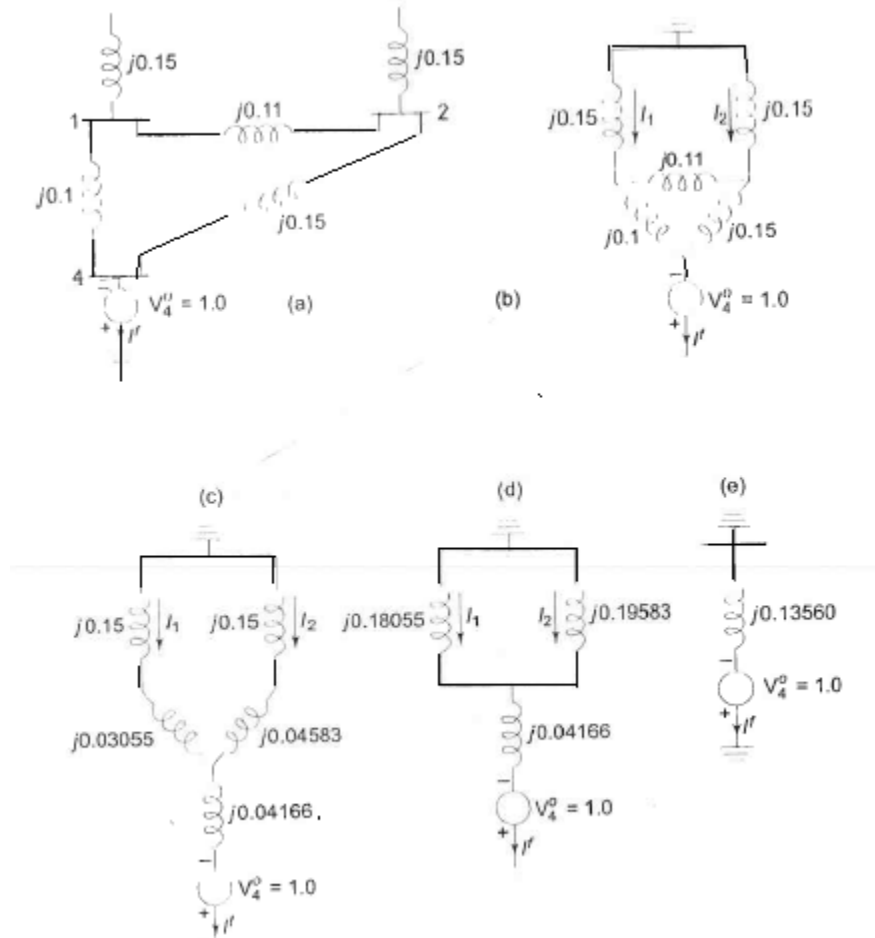


Fig 2

From Fig.e, we get directly the fault current as

$$I^f = \frac{1.0}{j0.13560} = -j7.37463 \text{ pu}$$

From Fig. d, it is easy to see that

$$I_1 = I_f \times \frac{j0.19583}{j0.37638} = -j3.83701 \text{ pu}$$

$$I_2 = I_f \times \frac{j0.18055}{j0.37638} = -j3.53762 \text{ pu}$$

Let us now compute the voltage changes for buses 1, 2 and 3. From Fig b, we give

$$\Delta V_1 = 0 - (j0.15) (-j3.83701) = -0.57555 \text{ pu}$$

$$\Delta V_2 = 0 - (j0.15) (-j3.53762) = -0.53064 \text{ pu}$$

Now

$$V_1^f = 1 + \Delta V_1 = 0.42445 \text{ pu}$$

$$V_2^f = 1 + \Delta V_2 = 0.46936 \text{ pu}$$

$$\therefore I_{13} = \frac{V_1^f - V_2^f}{j0.15 + j0.1} = j0.17964 \text{ pu}$$

$$\text{Now } \Delta V_3 = 0 - [(j0.15) (-j3.83701) + (j0.15) (j0.17964)] \\ = -0.54860 \text{ pu}$$

$$V_3^f = 1 - 0.54860 = 0.4514 \text{ pu}$$

$$V_4^f = 0$$

The determination of currents in the remaining lines is left as an exercise to The reader. Short circuit study is complete with the computation of SC MVA at bus 4.

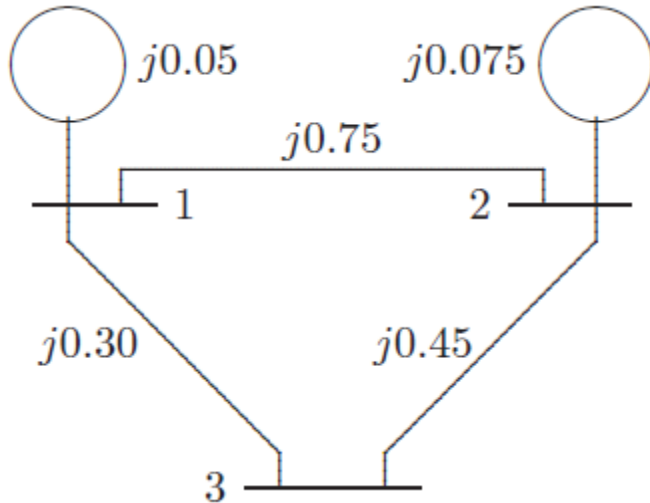
$$(\text{SC MVA})_4 = 7.37463 \times 100 = 737.463 \text{ MVA}$$

HW.

Q1/ The one-line diagram of a simple three-bus power system is shown in Figure. Each generator is represented by an emf behind the sub transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z^f = j0.19$ per unit.

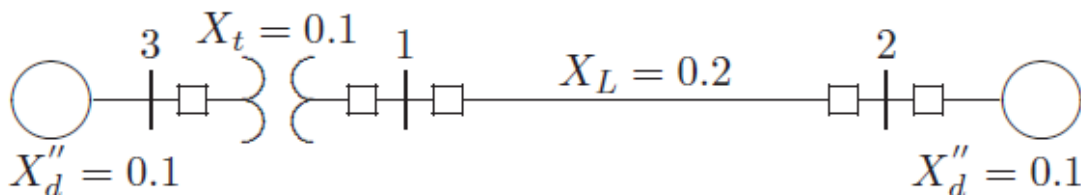
(a) Using Thevenin's theorem obtain the impedance to the point of fault and the fault current in per unit.

(b) Determine the bus voltages and line currents during fault.



Q2/ The one-line diagram of a simple power system is shown in Figure. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.

- (a) Using Thevenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
 (b) Determine the bus voltages and line currents during fault.



Q3/ Equipment ratings for the four-bus power system shown in Figure are as follows:

Generator G1: 500 MVA, 13.8 kV, $X'' = 0.20$ per unit

Generator G2: 750 MVA, 18 kV, $X'' = 0.18$ per unit

Generator G3: 1000 MVA, 20 kV, $X'' = 0.17$ per unit

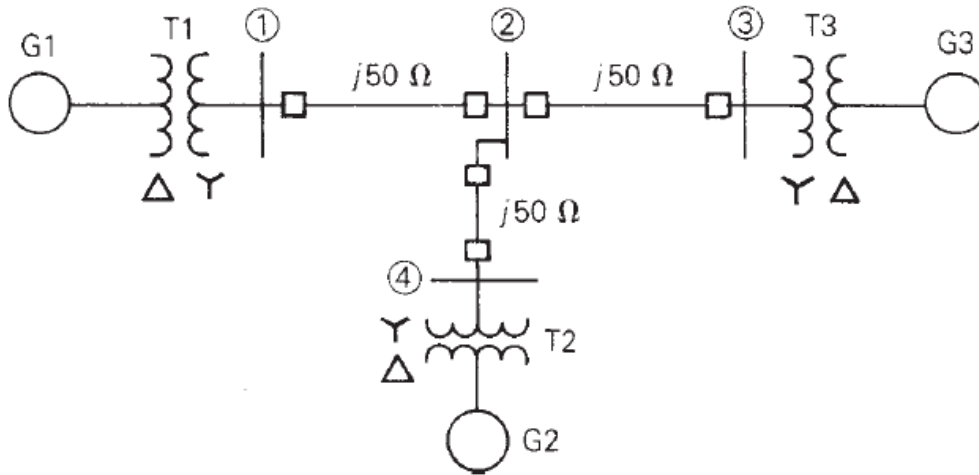
Transformer T1: 500 MVA, 13.8 Δ /500Y kV, $X = 0.12$ per unit

Transformer T2: 750 MVA, 18 Δ /500Y kV, $X = 0.10$ per unit

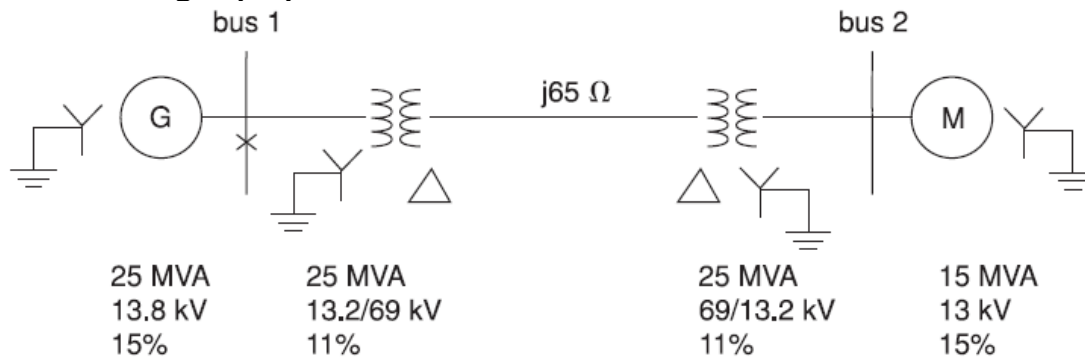
Transformer T3: 1000 MVA, 20 Δ /500Y kV, $X = 0.10$ per unit

Each 500-kV line: $X_1 = 50 \Omega$

A three-phase short circuit occurs at bus 1, where the pre fault voltage is 525 kV. Pre fault load current is neglected. Draw the positive-sequence reactance diagram in per-unit on a 1000-MVA, 20-kV base in the zone of generator G3. Determine (a) the Thevenin reactance in per-unit at the fault, (b) the sub transient fault current in per unit and in kA rms, and (c) contributions to the fault current from generator G1 and from line 1–2.



Q4/A three-phase short circuit occurs at the generator bus (bus 1) for the system shown in Figure. Neglecting pre fault currents and assuming that the generator is operating at its rated voltage, determine the subtransient fault current using superposition.



Symmetrical Components

The Solution of unsymmetrical fault problems can be obtained by either (a) Kirchoff's laws or (b) Symmetrical components method. The latter method is preferred because of the following reasons:

(i) it is a simple method and gives more generality to be given to fault performance studies.

(ii) It provides a useful tool for protection engineers, particularly in connection with tracing out of fault currents

In 1918, Dr. C.L. Fortescue, an American scientist, showed that any unbalanced system of 3-phase currents (or voltages) may be regarded as being composed of three separate sets of balanced vectors

(1) A balanced * system of 3-phase currents having a positive (or normal) phase sequence. These are called positive phase sequence components.

(2) A balanced system of 3-phase currents having the opposite or negative phase sequence. These are called negative phase sequence components.

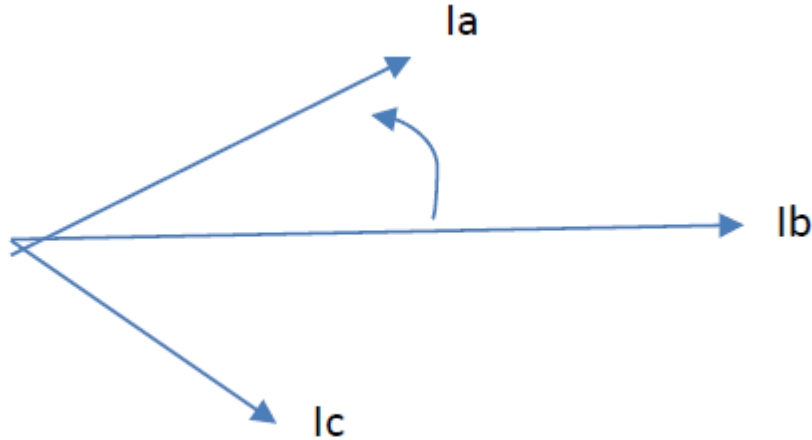
(3) A system of three currents equal in magnitude and having zero phase displacement. These are called zero phase sequence components.

The positive, negative and zero phase sequence components are called the symmetrical components of the original unbalanced system. the term symmetrical is appropriate because the unbalanced 3-phase system has been resolved into three sets of balanced (or symmetrical) components.

The subscripts 1,2 and 0 are generally used to indicate positive, negative and zero phase sequence components respectively.

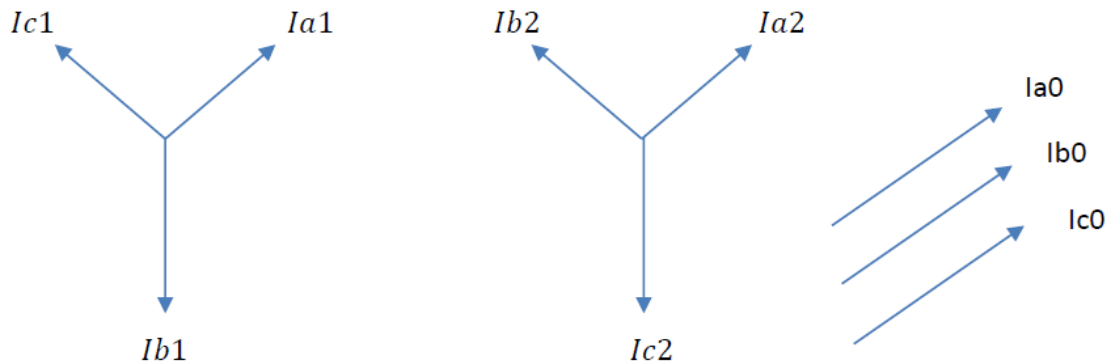
For instance, I_{a0} indicates the zero phase sequence components of the current in the first phase similarly I_{b1} implies the positive phase sequence components of current in the second phase.

Let us now apply the symmetrical components theory to an unbalanced 3-phase system. Suppose an unsymmetrical fault occurs on a 3-phasesystems having phase sequence ABC. According to symmetrical components theory, the resulting unbalanced currents I_a, I_b and I_c (see Fig.1) can be resolved into:



Fig(1)

1. A balanced system of 3-phase currents I_a , I_b and I_c having positive phase sequence (i.e. ABC) as shown in Fig.(2i) these are the positive phase sequence components .




2. A balanced system of 3-phase currents I_{a2} , I_{b2} and I_{c2} having negative phase sequence (i. e. ACB) as shown in Fig. (2ii). These are the negative phase sequence components.


3. A system of three currents I_{a0} , I_{b0} and I_{c0} equal in magnitude with zero phase displacement from each other as shown in Fig. (2iii). These are the zero phase sequence components.

The current in any phase is equal to the vector sum of positive, negative and zero phase sequence currents in that phase as shown in Fig. 3

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

A 

$$I_b = I_{b1} + I_{b2} + I_{b0}$$

B 

$$I_c = I_{c1} + I_{c2} + I_{c0}$$


C 

Fig. 3

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_b = I_{b1} + I_{b2} + I_{b0}$$

$$I_c = I_{c1} + I_{c2} + I_{c0}$$

Operator (a): It is the vector which is a unit vector at an angle of 120 degrees.

$$a = 1\angle 120^\circ, a^2 = 1\angle 240^\circ = 1\angle -120^\circ \text{ and } a^3 = 1.$$

apply operator (a) on the symmetrical components:

positive sequence equation:

$$I_a^1 = I_a^1 \angle 0^\circ = I_a^1$$

$$I_b^1 = I_a^1 \angle 240^\circ = a^2 I_a^1$$

$$I_c^1 = I_a^1 \angle 120^\circ = a I_a^1$$

negative sequence equations

$$Ia^2 = Ia^2 \angle 0^\circ = Ia^2$$

$$Ib^2 = Ia^2 \angle 120^\circ = aIa^2$$

$$Ic^2 = Ia^2 \angle 240^\circ = a^2Ia^2$$

zero sequence equations

$$Ia^0 = Ib^0 = Ic^0$$

“abc” currents can be expressed in terms of operator (a)

$$Ia = Ia^0 + Ia^1 + Ia^2$$

$$Ib = Ia^0 + a^2Ia^1 + aIa^2$$

$$Ic = Ia^0 + aIa^1 + a^2Ia^2$$

In matrix

$$\begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} Ia^0 \\ Ia^1 \\ Ia^2 \end{bmatrix} \quad A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}, \quad A^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$\begin{bmatrix} Ia0 \\ Ia1 \\ Ia2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix}$$

$$Ia^0 = \frac{1}{3} \{Ia + Ib + Ic\}$$

$$Ia^1 = \frac{1}{3} \{Ia + aIb + a^2Ic\}$$

$$Ia^2 = \frac{1}{3\{Ia + a^2Ib + aIc\}}$$

Also for voltages :

$$Va = Va^0 + Va^1 + Va^2$$

$$Vb = Va^0 + a^2Va^1 + aVa^2$$

$$Vc = Va^0 + aVa^1 + a^2Va^2$$

$$Va^0 = \frac{1}{3} [Va + Vb + Vc]$$

$$Va^1 = \frac{1}{3\{Va + aVb + a^2Vc\}}$$

$$Va^2 = \frac{1}{3} \{Va + a^2Vb + aVc\}$$

Often it understand that the quantities with superscript “0”, “1”, or “2” will have subscript “a” It is noted that Ia^0 is the sum of the three “abc” component’s .thus if the abc Components are balanced their sum is zero hence the zero –sequence component’s is also zero And we know that in 3–ph sys. $Ia+Ib+Ic=In$

Comparing with above equations **$In=3Ia^0$**

Example Obtain the symmetrical components of unbalanced current $I_a=1.6\angle 25^\circ$, $I_b=1.0\angle 180^\circ$, and $I_c=0.9\angle 132^\circ$

Solution:

$$I_a^0 = \frac{(1.6\angle 25^\circ) + (1.0\angle 180^\circ) + (0.9\angle 132^\circ)}{3} = 0.45\angle 96.5^\circ$$

$$I_a^1 = \frac{(1.6\angle 25^\circ) + a(1.0\angle 180^\circ) + a^2(0.9\angle 132^\circ)}{3} = 0.94\angle -0.1^\circ$$

$$I_a^2 = \frac{(1.6\angle 25^\circ) + a^2(1.0\angle 180^\circ) + a(0.9\angle 132^\circ)}{3} = 0.6\angle 22.3^\circ$$

Example :

The symmetrical components of a set of unbalanced 3 – ph voltage are $V^0 = 0.6\angle 25^\circ$, $V^1 = 1\angle 30^\circ$, and $V^2 = 0.8\angle -30^\circ$ obtain the original unbalanced phasors.

$$\text{Solution: } V_a = (0.6\angle 90^\circ) + (1.0\angle 30^\circ) + (0.8\angle -30^\circ) = 1.7088\angle 24.2^\circ$$

$$V_b = (0.6\angle 90^\circ) + a^2(1.0\angle 30^\circ) + a(0.8\angle -30^\circ) = 0.4\angle 90^\circ$$

$$V_c = (0.6\angle 90^\circ) + a(1.0\angle 30^\circ) + a^2(0.8\angle -30^\circ) = 1.7088\angle 155.8^\circ$$

Power in terms of symmetrical components: For 3–ph unbalanced network ,the total power fed given by

$P = \text{real of } \{V_a I_a^* + V_b I_b^* + V_c I_c^*\}$ Where V_a, V_b, V_c phase voltage

I_a^*, I_b^*, I_c^* conjugate of currents I_a, I_b, I_c But

$$Va = Va^0 + Va^1 + Va^2$$

$$Vb = Va^0 + a^2Va^1 + aVa^2$$

$$Vc = Va^0 + aVa^1 + a^2Va^2$$

And

$$Ia = Ia^0 + Ia^1 + Ia^2$$

$$Ib = Ia^0 + a^2Ia^1 + aIa^2$$

$$Ic = Ia^0 + aIa^1 + a^2Ia^2$$

$$Ia^* = Ia^{0*} + Ia^{1*} + Ia^{2*}$$

$$Ib^* = Ia^{0*} + aIa^{1*} + a^2Ia^{2*}$$

$$Ic^* = Ia^{0*} + a^2Ia^{1*} + aIa^{2*}$$

$$p = \text{real} [(Va^0 + Va^1 + Va^2)(Ia^{0*} + Ia^{1*} + Ia^{2*} \\ + (Va^0 + a^2Va^1 + aVa^2)(Ia^{0*} + aIa^{1*} + a^2Ia^{2*} \\ + (Va^0 + aVa^1 + a^2Va^2)(Ia^{0*} + a^2Ia^{1*} + aIa^{2*})$$

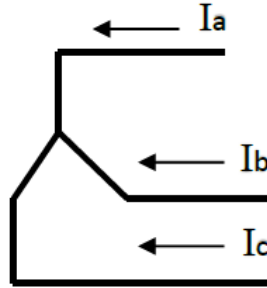
$$p = \text{Real}[3Va^0Ia^{0*} + 3Va^1Ia^{1*} + 3Va^2Ia^{2*}]$$

Identically, it may be mentioned that the complex power may be related as

$$s = p + jQ = 3[Va^0Ia^{0*} + Va^1Ia^{1*} + Va^2Ia^{2*}]$$

Example: prove that for any 3-ph, 3-wire system, star connected and for (abc0

Sequence



$$Ib^2 = \frac{1}{\sqrt{3}} [Ia\angle 30 + Ic\angle 90]\angle 120$$

Solution:

$$\begin{bmatrix} Ia^0 \\ Ia^1 \\ Ia^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix}$$

$$Ia^2 = \frac{1}{3} [Ia + a^2Ib + aIc]$$

For 3-wire system

$$I_a + I_b + I_c = 0$$

$$I_b = -I_a - I_c$$

$$I_a^2 = \frac{1}{3} [I_a + a^2(-I_a - I_c) + aI_c]$$

$$= \frac{1}{3} [(1 - a^2)I_a + (a - a^2)I_c]$$

$$I_a^2 = \frac{1}{3} [\sqrt{3} I_a \angle 30 + \sqrt{3} I_c \angle 90]$$

$$= \frac{1}{\sqrt{3}} [I_a \angle 30 + I_c \angle 90]$$

$$\text{since } I_b^2 = aI_a^2 = 1 \angle 120 I_a^2$$

$$\therefore I_b^2 = \frac{1}{\sqrt{3}} [I_a \angle 30 + I_c \angle 90] \angle 120$$

6- Sequence Impedances and Sequence Networks

Each element of power system will offer impedance to different phase sequence components of current which may not be the same. For example, the impedance which any piece of equipment offers to positive sequence current will not necessarily be the same as offered to negative sequence current or zero sequence current. Therefore, in unsymmetrical fault calculations, each piece of equipment will have three values of impedance—one corresponding to each sequence current.

- (i) Positive sequence impedance (Z_1).
- (ii) Negative sequence impedance (Z_2).
- (iii) Zero sequence impedance (Z_0).

The impedance offered by an equipment or circuit to positive sequence current is called positive sequence impedance and is represented by Z_1 . Similarly, impedances offered by any circuit or equipment to negative and zero sequence currents are respectively called negative sequence impedance (Z_2) and zero sequence impedance (Z_0).

The Following Points May Be Noted:

- (a) In a 3-phase balanced system, each piece of equipment or circuit offers only one impedance— the one offered to positive or normal sequence current. This is expected because of the absence of negative and zero sequence currents in the 3-phase balanced system.
- (b) In a 3-phase unbalanced system, each piece of equipment or circuit will have three values of impedance *viz.* positive sequence impedance, negative sequence impedance and zero sequence impedance.
- (c) The positive and negative sequence impedances of linear, symmetrical and static circuits (*e.g.* transmission lines, cables, transformers and static loads) are equal and are the same as those used in the analysis of balanced conditions. This is due to the fact that impedance of such circuits is independent of the phase order, provided the applied voltages are balanced. It may be noted that positive and negative sequence impedances of rotating machines (*e.g.* synchronous and induction motors) are normally different.
- (d) The zero sequence impedance depends upon the path taken by the zero sequence current. As this path is generally different from the path taken by the positive and negative sequence currents, therefore, zero sequence

impedance is usually different from positive or negative sequence impedance.

6-1 Sequence Impedances of Power System Elements

The concept of impedances of various elements of power system (*e.g.* generators, transformers, transmission lines *etc.*) to positive, negative and zero sequence currents is of considerable importance in determining the fault currents in a 3-phase unbalanced system. A complete consideration of this topic does not fall within the scope of this book, but a short preliminary explanation may be of interest here. The following three main pieces of equipment will be considered:

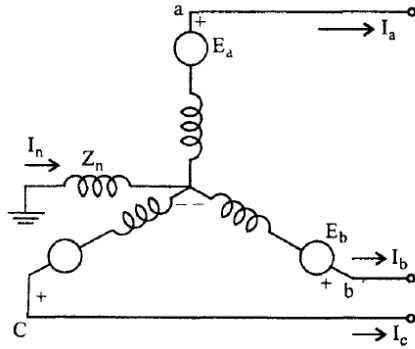
- (i) Synchronous generators
- (ii) Transformers
- (iii) Transmission lines

6- 1-1 Sequence Impedances of Synchronous Generator

Figure below depicts an unloaded synchronous machine (generator or motor) grounded through a reactor (impedance Z_n). E_a , E_b and E_c are the induced EMFs of the three phases

when a fault (not shown in the figure) takes place at machine terminals currents I_a , I_b and I_c flow to neutral from ground via Z_n . unbalance line current can be resolved into three symmetrical components, I_{a1} , I_{a2} , and I_{a0}

before we can proceed with fault analyses we must know the equivalent circuits presented by the machine to the flow of positive negative and zero sequence currents, respectively. Because of winding symmetry currents of a particular sequence produce voltage drops of that sequence only. Therefore, there is a no coupling between the equivalent circuits of various sequences.

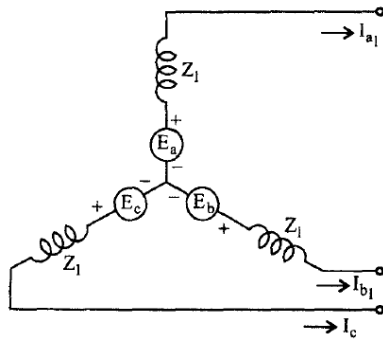


1. Positive Sequence Impedance and Network

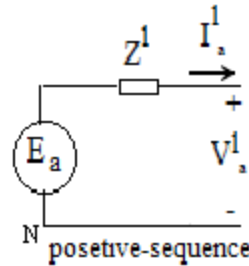
Since asynchronous machine is Designed with symmetrical windings, it induces emfs of positive sequence only, i.e. no negative or zero sequence voltages are induced in it ' When the machine carries positive sequence current. only, this mode of operation is the balanced mode. The armature reaction field caused by positive sequence currents rotates at synchronous speed in the same direction as the rotor i.e., it is stationary with respect to field excitation. The machine equivalently offers a direct axis reactance whose value reduces from sub transient reactance (X''_d) to transient reactance (X'_d) and finally to steady state (synchronous) reactance (X_d), as the short circuit transient progresses in time. If armature resistance is assumed negligible, the positive sequence impedance of the machine is

$$\begin{aligned} Z_1 &= jX''_d && \text{(if 1 cycle transient is of interest)} \\ &= jX'_d && \text{(if 3-4 cycle transient is of interest)} \\ &= jX_d && \text{(if steady state value is of interest)} \end{aligned}$$

If the machine short circuit takes place from unloaded conditions, the Terminal voltages constitutes the positive sequence voltage; on the other hand. If the short circuit occurs from loaded condition the voltage behind an appropriate reactance (sub transient, transient or synchronous) Constitutes the positive sequence voltage.



a- 3-phase model



b- single phase model

With reference to Fig, the positive sequence voltage of terminal a with respect to the reference bus is given by

$$V_{a1} = E_a - Z_{a1} I_{a1}$$

2. Negative Sequence Impedance and Network

It has already been said that a synchronous machine has zero, negative sequence induced voltages. With the flow of negative sequence currents in the stator a rotating field is created which rotates in the opposite direction to that of the positive sequence field and, therefore, at double synchronous speed with respect to rotor. Currents at double the stator frequency are therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the negative sequence MMF is alternately presented with reluctances of direct and quadrature axes. The negative sequence impedance presented by the machine with consideration given to the damper windings, is often defined as

$$Z_2 = \frac{x_q'' + x_d''}{2} \quad |Z_2| < |Z_1|$$

Negative sequence network models of a synchronous machine, on a three phase and single-phase basis are shown in Figs. a and b, respectively The reference bus is of course at neutral potential which is the same as ground potential. From Fig b the negative sequence voltage of terminal a with respect to reference bus is

$$V_{a2} = - Z_2 I_{a2}$$

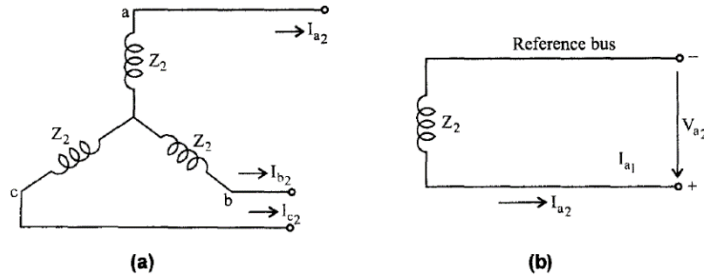


FIG. Negative sequence network of a synchronous machine

3. Zero Sequence Impedance and Network

We state once again that no zero sequence voltages are induced in a synchronous machine. The flow of zero sequence currents creates three mmfs which are in time phase but are distributed in space phase by 120°. The resultant air gap field caused by zero sequence currents is therefore zero. Hence, the rotor windings present leakage reactance only to the flow of zero sequence currents ($Z_{og} < Z_2 < Z_1$).

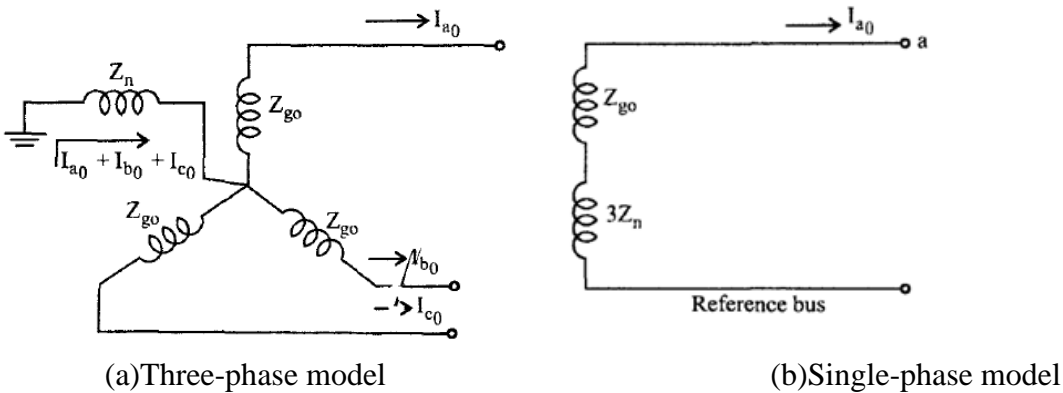


Fig. Zero sequence network of a synchronous machine

Zero sequence network models on a three- and single-phase basis are shown in Figs. A and b. In Fig. a, the current flowing in the impedance Z_n between neutral and ground is $I_n = 3I_{a0}$. The zero-sequence voltage of terminal a with respect to ground, the reference bus. is therefore

$$V_{a0} = -3Z_n I_{a0} - Z_{og} I_{a0} = -(3Z_n + Z_{og})I_{a0}$$

Where Z_o , is the zero-sequence impedance per phase of the machine.

Since the single-phase zero sequence network of Fig. b carries only per Phase zero sequence current, its total zero sequence impedance must be

$$Z_o = 3Z_n + Z_{og}$$

in order for it to have the same voltage from a to reference bus. The reference bus here is, of course at ground potential. From Fig. b zero sequence voltage of point a with respect to the reference bus is

$$V_{a0} = -Z_0 I_{a0}$$

Order of Values of Sequence Impedances of a Synchronous Generator:

Typical values of sequence impedances of a turbo-generator rated 5 MVA, 6.6 kV, 3;000 rpm are:

$$Z_1 = 12\% \text{ (sub transient)}$$

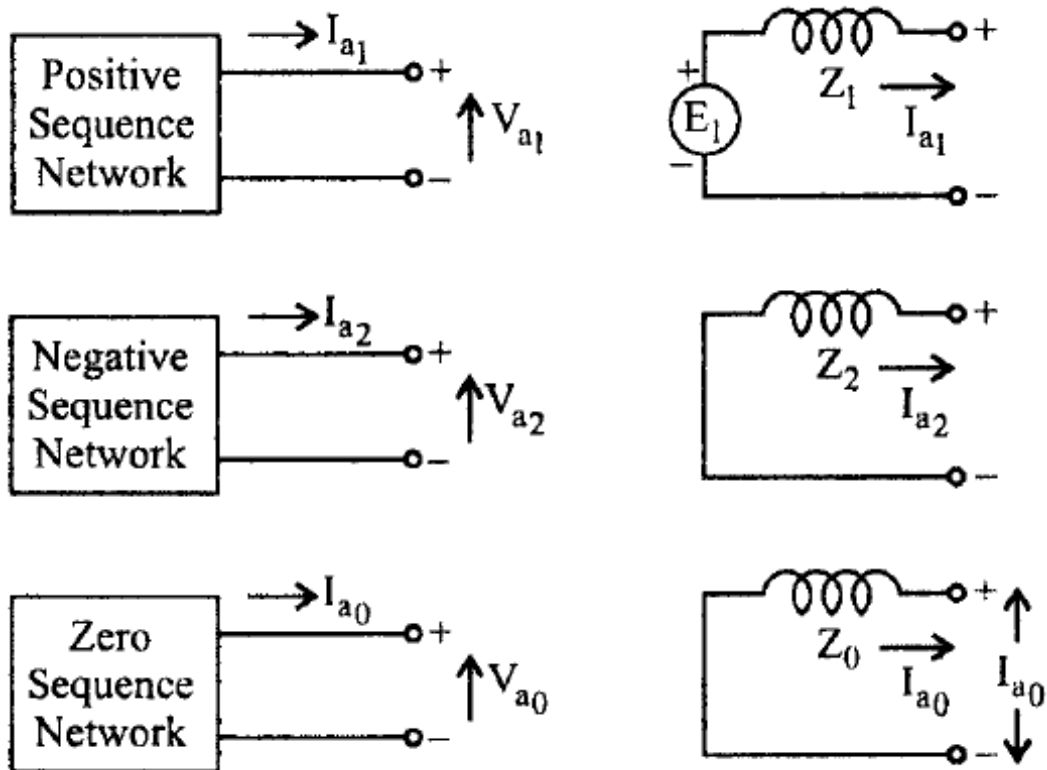
$$Z_1 = 20\% \text{ (transient)}$$

$$Z_1 = 110\% \text{ (synchronous)}$$

$$Z_2 = 12\%$$

$$Z_0 = 5\%$$

thus, it is possible to represent the sequence networks for a power system differently as different sequence currents flow as summarized in Fig

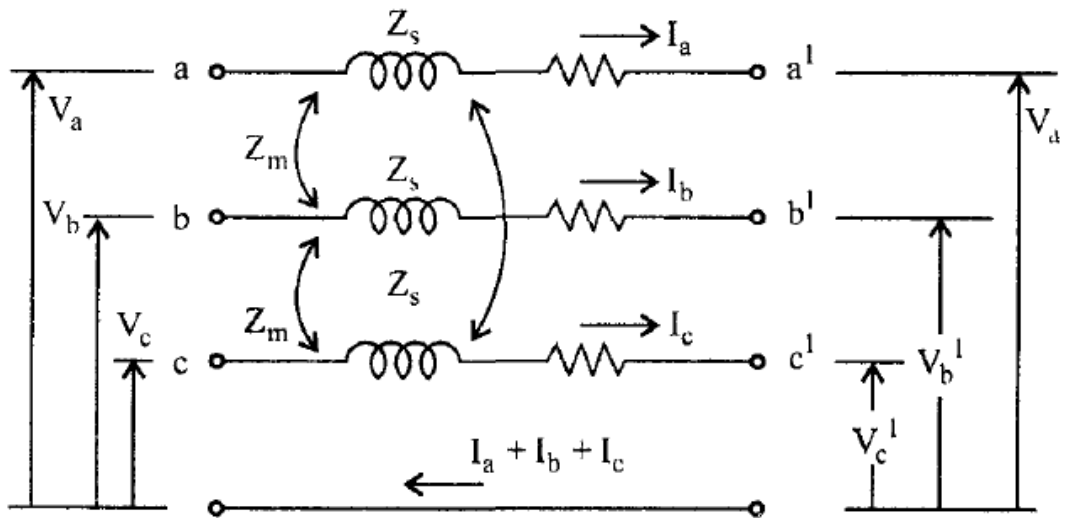


6-1-2 Sequence Impedances of Transmission Lines

Transmission lines are static components in a power system. Phase sequence has thus, no effect on the impedance. The geometry of the lines is fixed whatever may be the phase sequence. Hence, for transmission lines

$$Z_1 = Z_2$$

we can proceed in the same way as for the balanced 3-phase load for 3-phase transmission lines also



$$V_a - V_{a'} = Z_s I_a + Z_m I_b + Z_m I_c$$

$$V_b - V_{b'} = Z_m I_a + Z_s I_b + Z_m I_c$$

$$V_c - V_{c'} = Z_m I_a + Z_m I_b + Z_s I_c$$

$$\begin{bmatrix} V_a - V_{a'} \\ V_b - V_{b'} \\ V_c - V_{c'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[V_{abc}] = [V_{abc}] - [V_{abc}'] = [Z_{abc}] [I_{abc}]$$

$$[Z^{0,1,2}] = [A^{-1}] [Z_{abc}] [A]$$

$$= \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

The zero sequence currents are in phase and flow through the line conductors only if a return conductor is provided. The zero-sequence impedance is different from positive and negative sequence impedances.

6- 1-3 Sequence Impedances and Networks of Transformer

It is well known that almost all present day installations have three-phase transformers since they entail lower initial cost, have smaller space requirements and higher efficiency.

The positive sequence series impedance of a transformer equals its leakage impedance. Since a transformer is a static device, the leakage impedance does not change with alteration of phase sequence of balanced applied voltages. The transformer negative sequence impedance is also therefore equal to its leakage reactance. Thus. for a transformer

$$Z_1 = Z_2 = Z_{\text{leakage}}$$

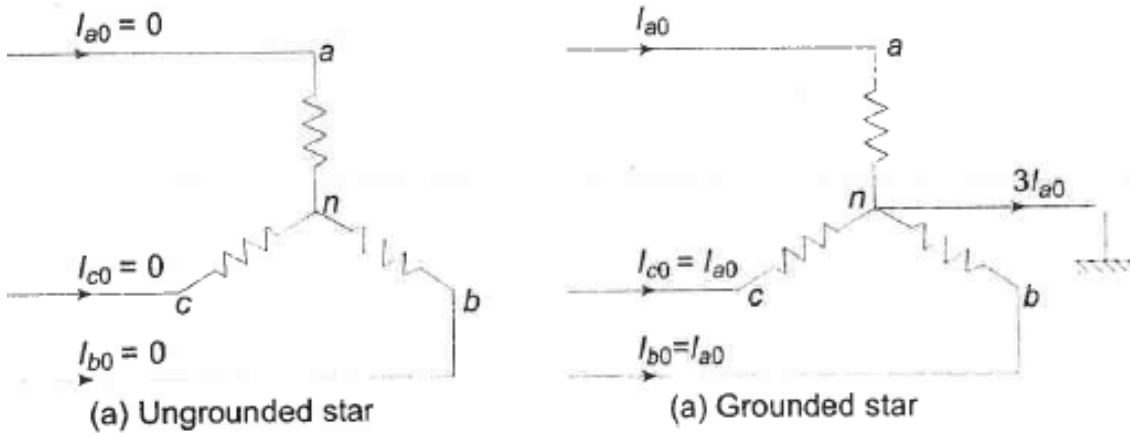
Assuming such transformer connections that zero sequence currents can flow on both sides, a transformer offers a zero-sequence impedance which may differ slightly from the corresponding positive and negative sequence values. It is, however, normal practice to assume that the series impedances of all sequences are equal regardless of the type of transformer.

The zero-sequence magnetizing current is somewhat higher in a core type than in a shell type transformer. This difference does not matter as the magnetizing current of a transformer is always neglected in short circuit analysis.

1. Zero-Sequence Networks of Transformers

Before consider in the zero sequence networks of various types of transformer connections three important observations are made:

1. when magnetizing current is neglected transformer primary would carry current only if there is current flow on the secondary side.
2. Zero sequence currents can flow in the legs of a star connection only if the star point is grounded which provides the necessary return path for zero sequence currents. This fact is illustrated by Figs. a and b



3. No zero sequence currents can flow in the lines connected to a delta connection as no return path is available for these currents. Zero sequence currents can, however, flow in the legs of a delta—such currents are caused by the presence of zero sequence voltages in the delta connection. This fact is illustrated by Fig.

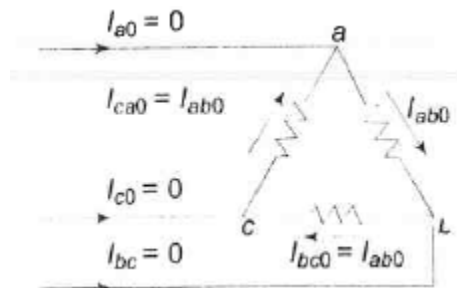


Fig. Flow of zero Sequence currents in delta connection

Let us now consider various types of transformer connections

Case I; Y-Y transformer bank with any one neutral grounded.

If any one of the two neutrals of a Y-Y transformer is ungrounded, zero sequence currents cannot flow in the ungrounded star and consequently these cannot flow in the grounded star. Hence, an open circuit exists in the zero sequence network between H and L, i.e. between the two parts of the system connected by the transformer as shown in Fig. 10.17.

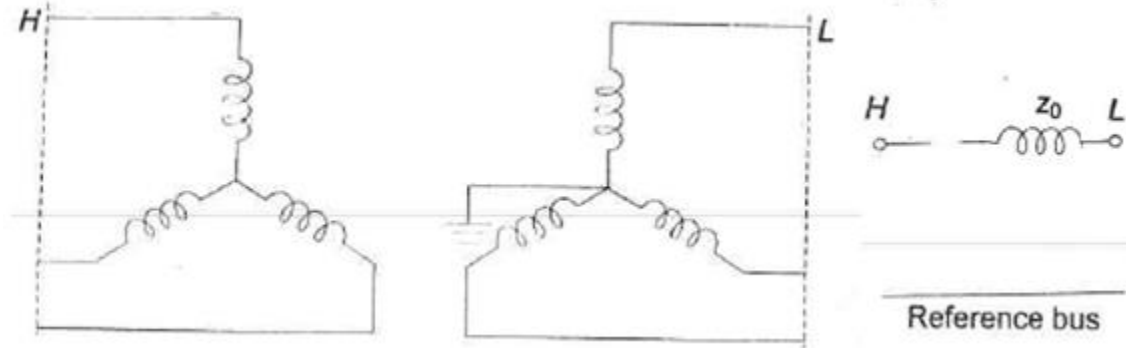


Fig. Y-Y transformer bank with one neutral grounded and its zero Sequence network

Case 2: Y-Y Transformer Bank Both Neutrals Grounded

When both the neutrals of a Y-Y transformer are grounded, a path through the transformer exists for zero sequence currents in both windings via the two grounded neutrals. Hence, in the zero sequence network H and L are connected by the zero sequence impedance of the transformer as shown in Fig.

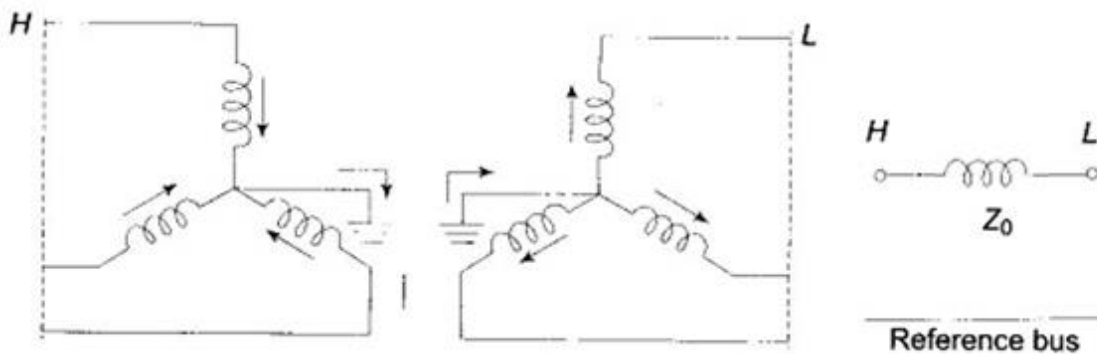
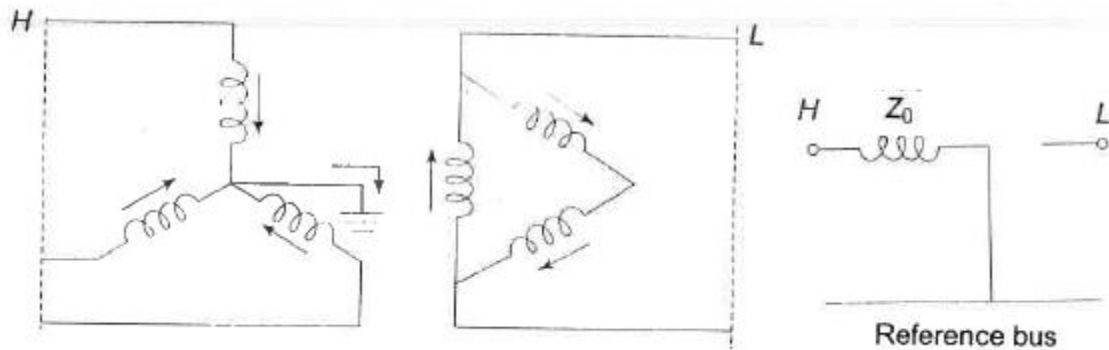


Fig. Y-Y transformer bank with sequence network neutrals grounded and its zero

Case 3: Y-Δ Transformer Bank With Grounded Y Neutral

If the neutral of stars ide is grounded, zero sequence currents can flow in star Because a path is available to ground and the balancing zero sequence currents can flow in delta. Of course, no zero sequence currents can flow in the line on the delta side. The zero-sequence network must therefore have a path from the line H on the star side through the zero sequence impedance of the transformer to the reference bus, while an open circuit must exist on the line L side of delta

(see Fig.). If the star neutral is grounded through Z_n an impedance $3Z_n$ appears in series with Z_o in the sequence network



Cas 4: Y-Δ Transformer Bank With Ungrounded Star

This is the special case of Case 3 where the neutral is grounded through $Z_n = \infty$. There are no zero-sequence current can flow in the transformer windings. The zero-sequence network then modifies to that shown in Fig.

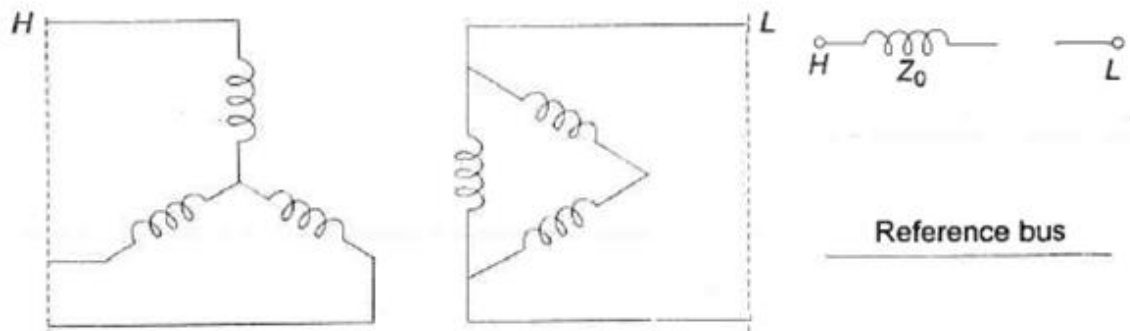
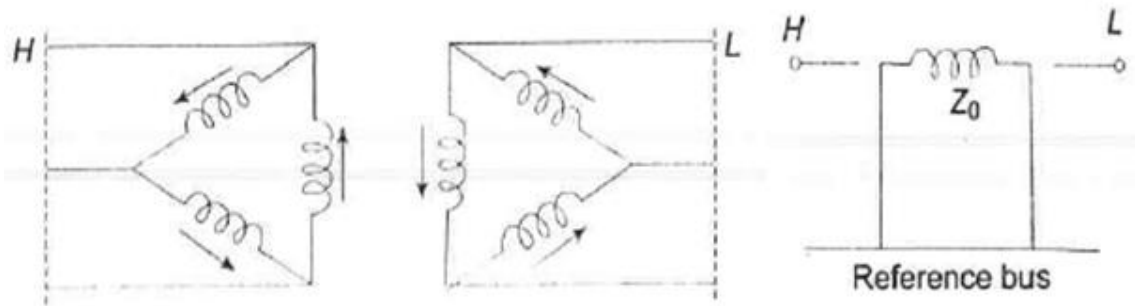


Fig. Y-Δ transformer bank with ungrounded star and its zero sequence network

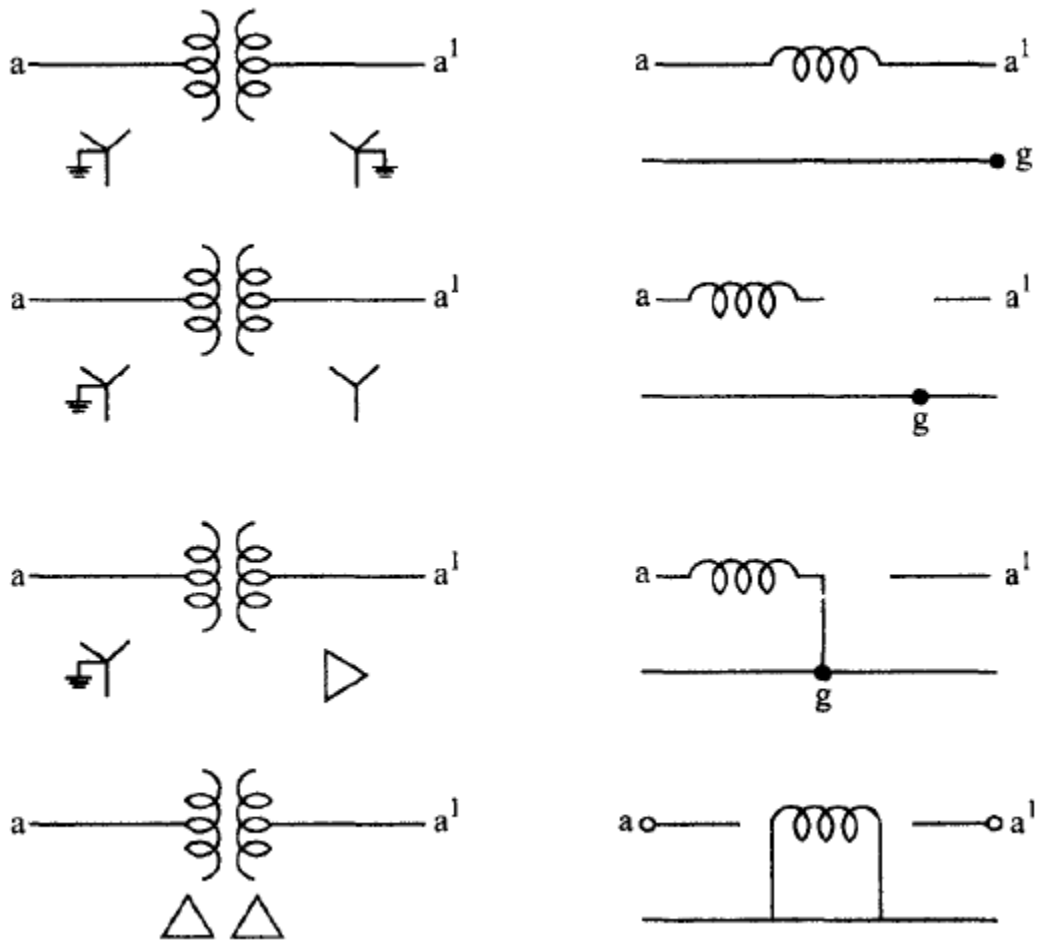
Case 5: Δ-Δ Transformer Bank

Since a delta circuit provides no return path, the zero sequence currents cannot flow in or out of Δ-Δ transformer; however, it can circulate in the delta windings*. Therefore, there is an open circuit between H and L and Z_o is connected to the reference bus on both ends to account for any circulating zero sequence current in the two deltas (see Fig.).



Δ-Δ transformer bank and its zero-sequence network

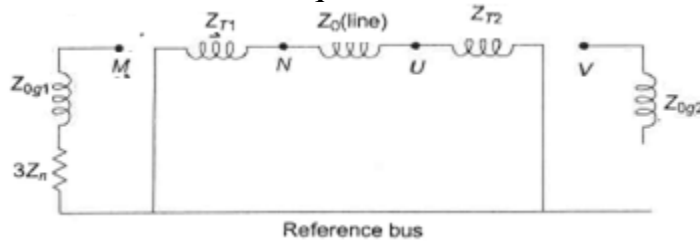
SUMMARY



EX1. For the power system whose one-line diagram is shown in Fig. sketch the zero sequence network



Solution The zero sequence network is drawn in Fig.



Ex2. A 25 MVA, 11 kV, three-phase generator has a sub transient reactance of 20%. The generator supply two motors over transmission line with transformers at both ends as shown in the one-line diagram of Fig. The motors have rated inputs of 15 and 7.5 MVA, both 10 kV with 25% sub transient reactance. The three-phase transformers are both rated 30 MVA, 10.8/121 kV, connection Δ-Y with leakage reactance of 10% of each. The series reactance of the line is 100 ohms. Draw the positive and negative sequence networks of the system with reactance marked in per unit.

Assume that the negative sequence reactance of each machine is equal to its sub transient reactance. Omit resistances. Select generator rating as base in the generator circuit.



Sol.:

$$\text{Transmission line voltage base} = 11 \times \frac{121}{10.8} = 11 \text{ kV}$$

$$\text{Motor voltage base} = 123.2 \times \frac{10.8}{121} = 11 \text{ kV}$$

The reactances of transformers line and motors are converted to pu values on appropriate bases as follows

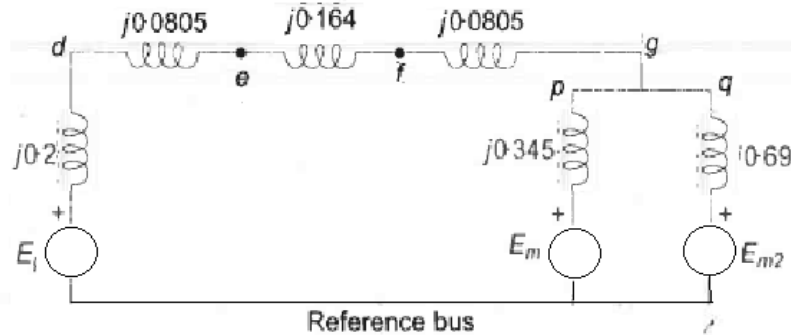
$$\text{Transformer reactance} = 0.1 \times \frac{25}{30} \times \left(\frac{10.8}{11}\right)^2 = 0.0805 \text{ pu}$$

$$\text{Line reactance} = \frac{100 \times 25}{(123.2)^2}$$

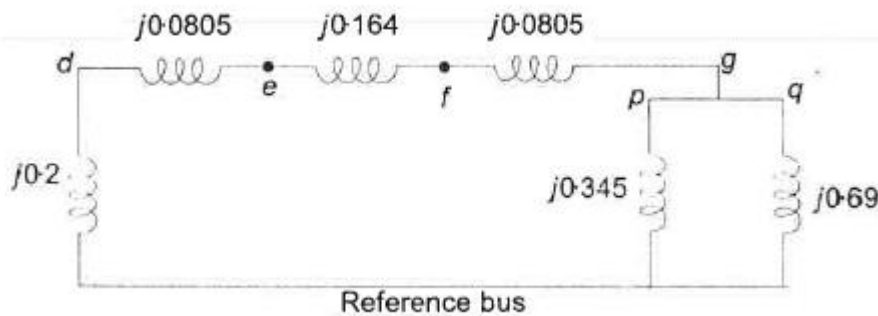
$$\text{Reactance of motor 1} = 0.25 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2 = 0.345 \text{ pu}$$

$$\text{Reactance of motor 2} = 0.25 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = 0.69 \text{ pu}$$

Positive sequence network is



Negative sequence network is



Ex3. Draw the zero-sequence network for the system described in Example 2. Assume zero sequence reactance for the generator and motors of 0.06 per unit. Current limiting reactors of 2.5 ohms each are connected in the neutral of the generator and motor No. 2. The zero-sequence reactance of the transmission line is 300 ohms.

Sol.:

Solution The zero sequence reactance of the transformer is equal to its positive sequence reactance. Hence

Transformer zero sequence reactance: 0.0805 pu

Generator zero sequence reactance : 0.06 pu

$$\text{Zero sequence reactance of motor 1} = 0.06 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2 = 0.082 \text{ pu}$$

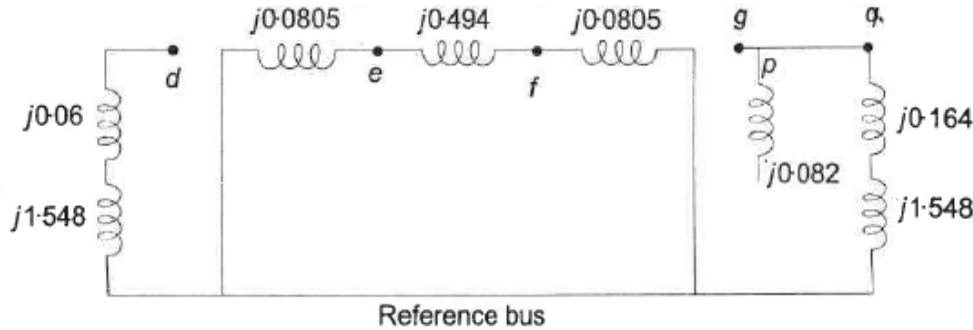
$$\text{Zero sequence reactance of motor 2} = 0.06 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = 0.164 \text{ pu}$$

$$\text{Reactance of current limiting reactors} = \frac{2.5 \times 25}{11^2} = 0.516$$

Reactance of current limiting reactor included in zero sequence network'
 = $3 \times 0.516 = 1.548 \text{ pu}$

Zero sequence reactance of transmission line = $\frac{300 \times 25}{123.2^2} = 0.494 \text{ pu}$

The zero sequence network is shown in Fig.



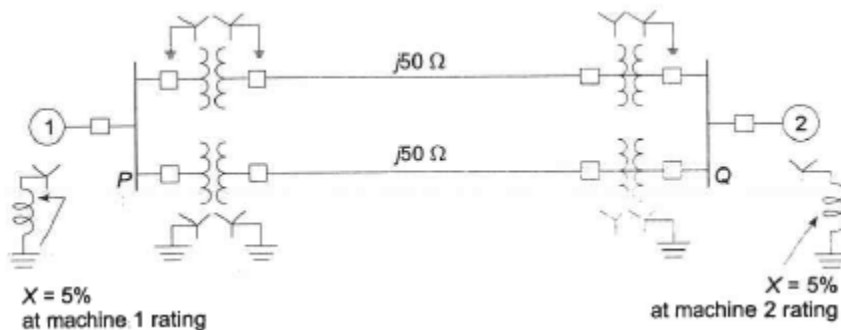
Q1/ Draw the positive, negative and zero sequence impedance networks for the power system of Fig. Choose a base of 50 MVA, 220 kV in the 50Ω transmission lines, and mark all reactance in pu. The ratings of the generators and transformers are:

G1: 25 MVA, 11 kV, $X'' = 20\%$

G2: 25 MVA, 11 kV, $X'' = 20\%$

Three-phase transformer (each): 20 MVA, 11Y /220Y kV, $X = 15\%$

The negative sequence reactance of each synchronous machine is equal to its sub transient reactance. The zero-sequence reactance of each machine is 8%. Assume that the zero-sequence reactance of lines are 250% of their positive sequence reactance.



Q2/ For the power system of Fig. draw the positive, negative and zero sequence networks. The generators and transformers are rated as follows:

Generator1 : 25 MVA, 11 kV, $X'' = 0.2$, $X_2 = 0.15$, $X_0 = 0.03 \text{ pu}$

Generator2 : 15 MVA, 11 kV, $X'' = 0.2$, $X_2 = 0.15$, $X_0 = 0.05 \text{ pu}$

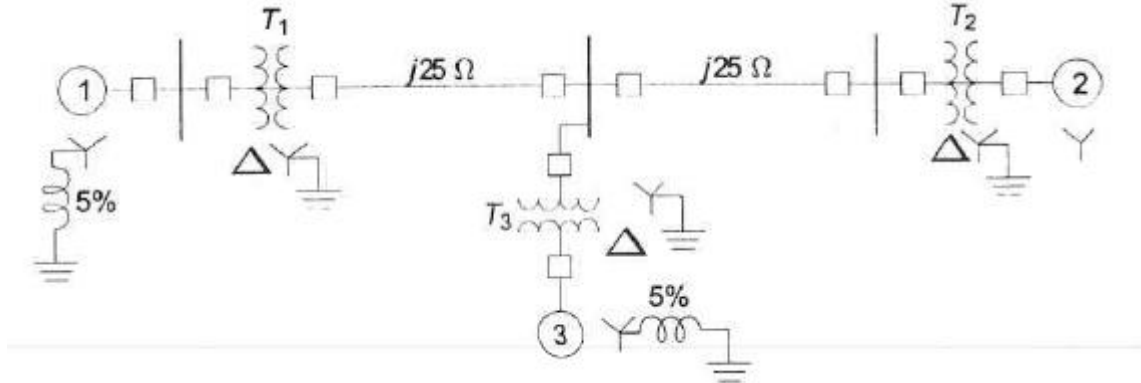
Synchronous Motor 3: 25 MVA, 11 kV, $X'' = 0.2$, $X_2 = 0.02$, $X_0 = 0.1 \text{ pu}$

Transformer 1: 25 MVA, 11Δ/120 Y kV, $X = 10\%$

Transformer 2: 12.5 MVA, 11Δ/120 Y kV, $X = 10\%$

Transformer 3: 10 MVA, I20Y/11 Y kV, X =10%

Choose a base of 50 MVA, 11 kV in the circuit of generator 1.



7- Unbalanced Faults

Most of the faults that occur on power systems are unsymmetrical faults, which may consist of unsymmetrical short circuits, unsymmetrical faults through impedances, or open conductors. Unsymmetrical faults occur as single line-to-ground faults, line-to-line faults, or double line-to-ground faults.

The path of the fault current from line to line or line to ground may or may not contain impedance.

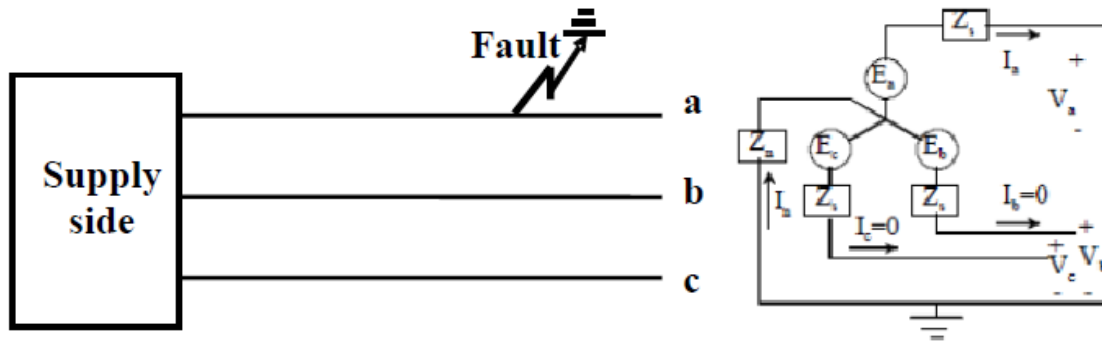
One or two open conductors result in unsymmetrical faults, through either the breaking of one or two conductors or the action of fuses and other devices that may not open The three phases simultaneously.

Since any unsymmetrical fault causes unbalanced currents to flow in the system, the method of symmetrical components is very used in the analysis to determine the currents and voltages in all parts of the system after the occurrence of the fault.

We will consider faults on a power system by applying Thévenin's theorem, which allows us to find the current in the fault by replacing the entire system by a single generator and series impedance, and we will show how the bus impedance matrix is applied to the analysis of unsymmetrical faults. The common types of asymmetrical faults occurring in a Power System are single line to ground faults and line to line faults, with and without fault impedance.

1- Single Line-To-Ground Fault (L-G fault)

The single line to ground fault can occur in any of the three phases. However, it is sufficient to analyze only one of the cases. Looking at the symmetry of the symmetrical component matrix, it is seen that the simplest to analyze would be the phase **a**. Consider an L-G fault with zero fault impedance as shown in figure. Since the fault impedance is 0, at the fault $V_a = 0, I_b = 0, I_c = 0$



since load currents are neglected. These can be converted to equivalent conditions in symmetrical components as follows.

$$\mathbf{V}_a = \mathbf{V}_{a^0} + \mathbf{V}_{a^1} + \mathbf{V}_{a^2} = \mathbf{0}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b = 0 \\ I_c = 0 \end{bmatrix},$$

giving $I_{a^0} = I_{a^1} = I_{a^2} = I_a/3$

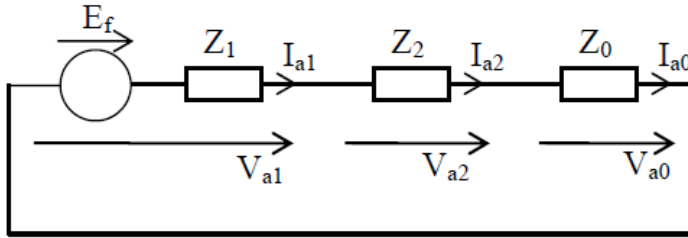
Mathematical analysis using the network equation in symmetrical components would yield the desired result for the fault current $I_f = I_a$.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} = I_a/3 \\ I_{a1} = I_a/3 \\ I_{a2} = I_a/3 \end{bmatrix}$$

Thus $\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = \mathbf{0} = -\mathbf{Z}_0 \cdot \mathbf{I}_a/3 + \mathbf{E}_f - \mathbf{Z}_1 \cdot \mathbf{I}_a/3 - \mathbf{Z}_2 \cdot \mathbf{I}_a/3$

Simplification, with $I_f = I_a$, gives

$$I_f = \frac{3E_f}{Z_1 + Z_2 + Z_0}$$

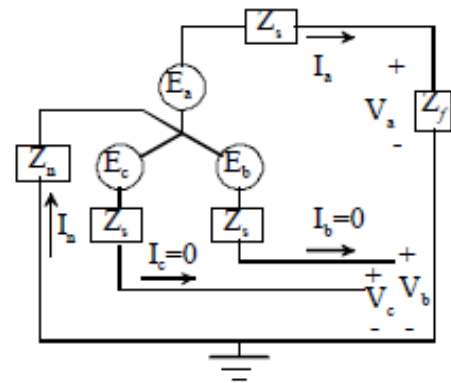
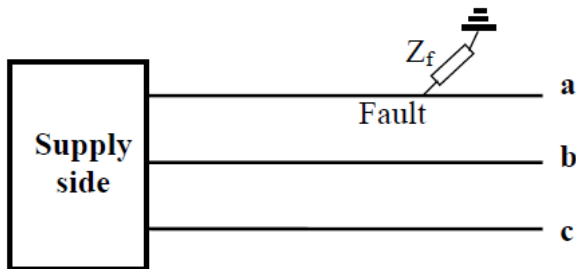


Also, considering the equations

$$V_{a0} + V_{a1} + V_{a2} = 0, \text{ and } I_{a0} = I_{a1} = I_{a2} .$$

Indicates that the three networks (zero, positive and negative) must be connected in series (same current, voltages add up) and short-circuited, giving the circuit shown in figure.

In this case, I_a corresponds to the fault current I_f , which in turn corresponds to 3 times any one of the components ($I_{a0} = I_{a1} = I_{a2} = I_a/3$). Thus the network would also yield the same fault current as in the mathematical analysis. In this example, the connection of sequence components is more convenient to apply than the mathematical analysis. Thus for a single line to ground fault (L-G fault) with no fault impedance, the sequence networks must be connected in series and short circuited. Consider now an L-G fault with fault impedance Z_f as shown in figure.



at the fault

$$V_a = I_a Z_f, \quad I_b = 0, \quad I_c = 0$$

These can be converted to equivalent conditions in symmetrical components as

follows.

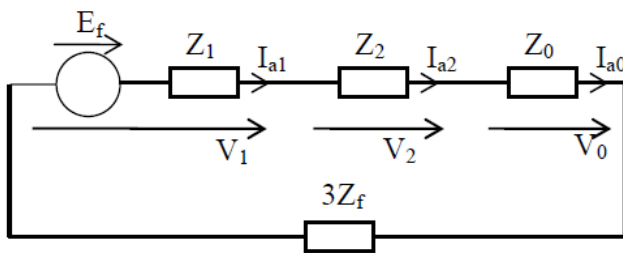
$$\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = (\mathbf{I}_{a0} + \mathbf{I}_{a1} + \mathbf{I}_{a2}) \cdot \mathbf{Z}_f$$

And
$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b = 0 \\ I_c = 0 \end{bmatrix},$$

giving $\mathbf{I}_{a0} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_a/3$

Mathematical analysis using the network equation in symmetrical components would yield the desired result for the fault current I_f as

$$I_f = \frac{3V_f}{Z_1 + Z_2 + Z_0 + 3Z_f}$$



Similarly, considering the basic equations, $\mathbf{I}_{a0} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_a/3$,

And $\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = 3\mathbf{I}_{a0} \cdot \mathbf{Z}_f$

or $\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = \mathbf{I}_{a0} \cdot 3\mathbf{Z}_f$,

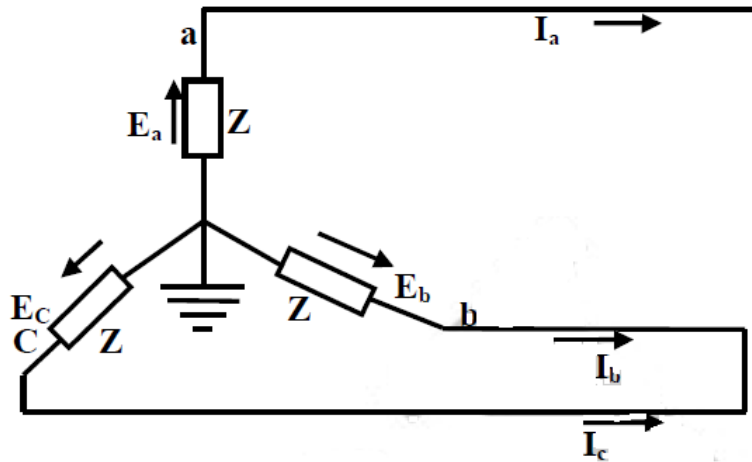
would yield a circuit connection of the 3 sequence networks in series and in series with an effective impedance of $3 Z_f$.

2- Line-To-Line Fault (L-L Fault):

The line to line fault takes place on phases ‘b’ and ‘c’

The boundary conditions are

$$\begin{array}{ll} V_b = Z_f I_b + V_c & \dots\dots\dots 1 \\ I_b = -I_c \text{ or } I_b + I_c = 0 & \dots\dots\dots 2 \\ V_b = V_c, \quad I_a = 0 & \dots\dots\dots 3 \end{array}$$



The sequence network equations are:

$$V_a^0 = 0 - Z^0 I_a^0$$

$$V_a^1 = E_a - Z^1 I_a^1$$

$$V_a^2 = 0 - Z^2 I_a^2$$

The solution of these six equations will give six unknowns V_{a0} , V_{a1} , V_{a2} , and I_{a0} , I_{a1} and I_{a2}

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

Expanding the matrix equation we have:

$$I_a^0 = (1/3) [0 + I_b - I_b]$$

$$I_a^0 = 0 \quad \dots\dots\dots 4$$

$$I_a^1 = (1/3) [0 + a I_b - a^2 I_b]$$

$$= (1/3) [a - a^2] I_b \quad \dots\dots\dots 5$$

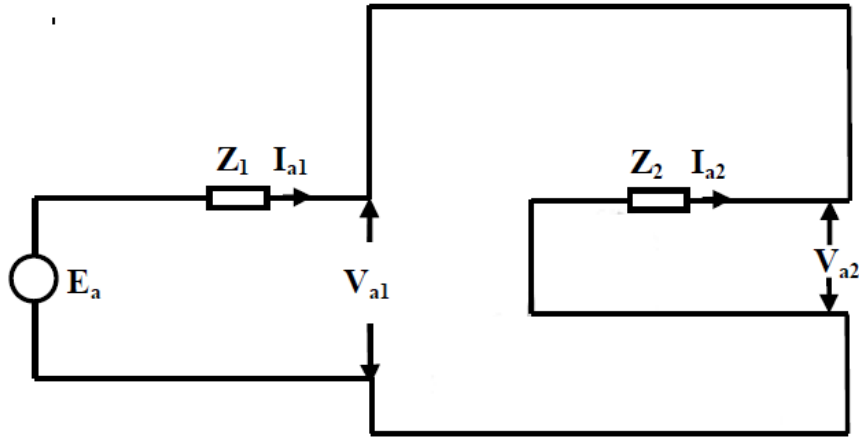
$$I_a^2 = (1/3) [0 + a^2 I_b - a I_b]$$

$$= (1/3) [a^2 - a] I_b \quad \dots\dots\dots 6$$

From 5&6 we have

$$I_a^1 = -I_a^2 \quad \dots\dots\dots 7$$

Which means for a line to line fault the zero sequence component of current is absent and positive sequence component of current is equal in magnitude but opposite in phase to negative sequence component of current, i.e., Transform equation 1 in terms of symmetrical components we have



$$V_b = a^2 V_a^1 + a V_a^2 + V_a^0 \quad \dots\dots 8$$

$$V_c = a V_a^1 + a^2 V_a^2 + V_a^0 \quad \dots\dots 9$$

Substituting the equations (8) and (9) equations (3)

$$a^2 V_a^1 + a V_a^2 + V_a^0 = a V_a^1 + a^2 V_a^2 + V_a^0$$

$$\therefore V_a^1 = V_a^2 \quad \dots\dots 10$$

i.e., positive sequence component of voltage equals the negative sequence component of voltage. This also means that the two sequence networks are connected in opposition. Now making use of the sequence network equation and the equation (11) $\therefore V_a^1 = V_a^2$.

$$E_a - I_a^1 Z_1 = -I_a^2 Z_2 = I_a^1 Z_2$$

$$I_a^1 = \frac{E_a}{Z_1 + Z_2}$$

$$I_f = I_b = -I_c = a^2 I_a^1 + a I_a^2 + I_a^0 \quad (I_a^2 = -I_a^1, I_a^0 = 0)$$

$$= (a^2 - a) I_a^1 = -J\sqrt{3} I_a^1$$

$$= \frac{-J\sqrt{3} E_a^1}{Z_1 + Z_2}$$

NOTE:

1. The connection of sequence currents are connected in parallel.
2. The phase difference between I_a^1 and I_a^2 for line – to – line fault should be 180° ($I_{a1} = I_{a2}$).

Line to Line fault with Z_f

$$I_f = \frac{-J\sqrt{3} E_a^1}{Z_1 + (Z_2 + Z_f)}$$

3- Double Line-To-Ground Fault (L-L-G fault)

The boundary conditions at the fault point are:

$$V_b = V_c = Z_f(I_b + I_c) \dots\dots\dots 1$$

$$I_a = 0 \dots\dots\dots 2$$

From equation 2

$$\boxed{I_a^0 + I_a^1 + I_a^2 = 0} \dots\dots\dots 3$$

V_b & V_c in terms of symmetrical components are

$$\left\{ \begin{aligned} V_b &= V_a^0 + a^2 V_a^1 + a V_a^2 \\ V_c &= V_a^0 + a V_a^1 + a^2 V_a^2 \end{aligned} \right\} \dots\dots\dots 4$$

$$\boxed{\therefore V_a^1 = V_a^2} \dots\dots\dots 5$$

Eqn. 1 in terms of symmetrical components

$$V_b = Z_f(I_a^0 + a^2 I_a^1 + a I_a^2 + I_a^0 + a I_a^1 + a^2 I_a^2)$$

$$= Z_f(2I_a^0 - I_a^1 - I_a^2)$$

$$V_b = 3Z_f I_a^0 \dots\dots\dots 6$$

Substitute 6&5 in 4

$$3Z_f I_a^0 = V_a^0 + (a^2 + a)V_a^1$$

$$\boxed{3Z_f I_a^0 = V_a^0 - V_a^1} \dots\dots\dots 7$$

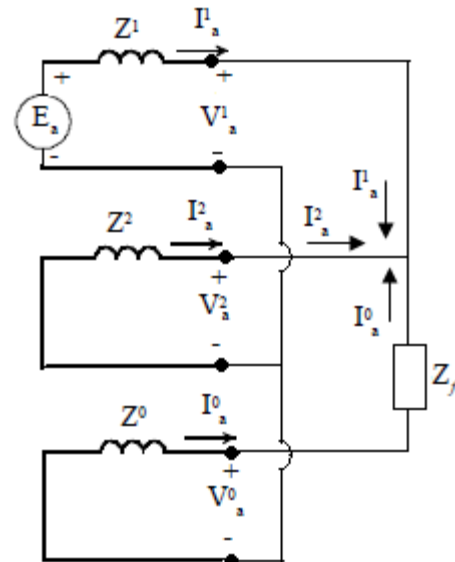
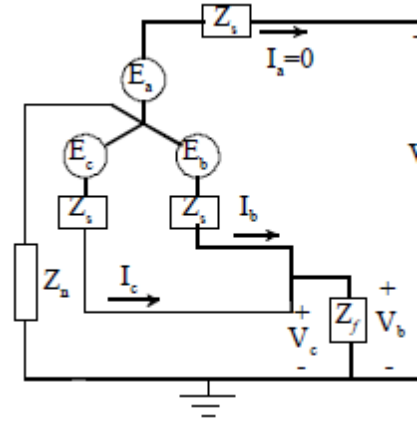
We have $\left\{ \begin{aligned} V_a^0 &= 0 - Z^0 I_a^0 \\ V_a^1 &= E_a - Z^1 I_a^1 \\ V_a^2 &= 0 - Z^2 I_a^2 \end{aligned} \right\} \dots\dots\dots 8$

Substitute 8 in 7

$$I_a^0 = -\frac{E_a - Z^1 I_a^1}{Z^0 + 3Z_f} \dots\dots\dots 9$$

Using 8 in 5

$$I_a^2 = -\frac{E_a - Z^1 I_a^1}{Z^2} \dots\dots\dots 10$$



Substitute 9&10 in 2&3

$$I_a^1 = \frac{E_a}{Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}} \dots\dots\dots 11$$

The terminal conditions of L-L-G fault are in the equations 3, 5 and 7.

Hence the impedance seen by E_a is: $Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}$ and from this we note that the impedance Z^1 in series with the parallel combination of Z^2 and $Z^0 + 3Z_f$.

Thus the sequence networks can be connected as shown in the figure above. Finally the fault current is found from:

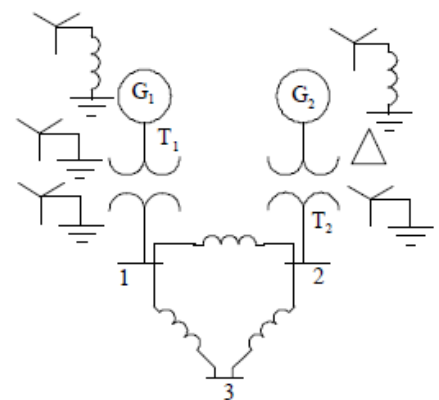
$$I_f = I_b + I_c = 3I_a^0 \dots\dots\dots 12$$

Example:

The one-line diagram of a simple power system is shown in the figure. The neutral of each generator is grounded through a current-limiting reactor of 0.25 per unit on a 100-MVA base. The system data is tabulated below. The generators are running on no-load at their rated voltage and rated frequency with their emfs in phase. Determine the fault current for the following faults:

- (a) A balanced three-phase fault at bus3 through fault impedance $Z_f = j0.1$ per unit.
- (b) A single line-to-ground fault at bus 3 through fault impedance $Z_f = j0.1$ per unit
- (c) A line-to-line fault at bus 3 through fault impedance $Z_f = j0.1$ per unit
- (d) A double line-to-ground fault at bus 3 through fault impedance $Z_f = j0.1$ per unit

Item	Base MVA	Voltage rating	X ¹	X ²	X ⁰
G ₁	100	20 KV	0.15	0.15	0.05
G ₂	100	20 KV	0.15	0.15	0.05
T ₁	100	20/220 KV	0.1	0.1	0.1
T ₂	100	20/220 KV	0.1	0.1	0.1
L ₁₂	100	220 KV	0.125	0.125	0.3
L ₁₃	100	220 KV	0.15	0.15	0.35
L ₂₃	100	220 KV	0.25	0.25	0.7125



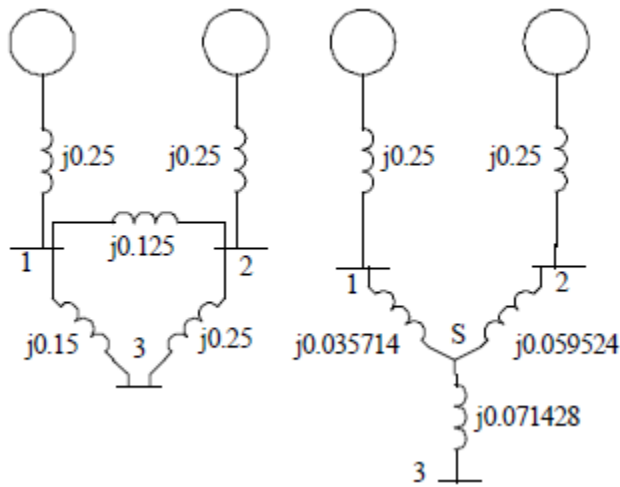
Solution:

Find the Thevenin impedance viewed from bus 3 (faulted bus). The delta transforms to a Y as in the figure below

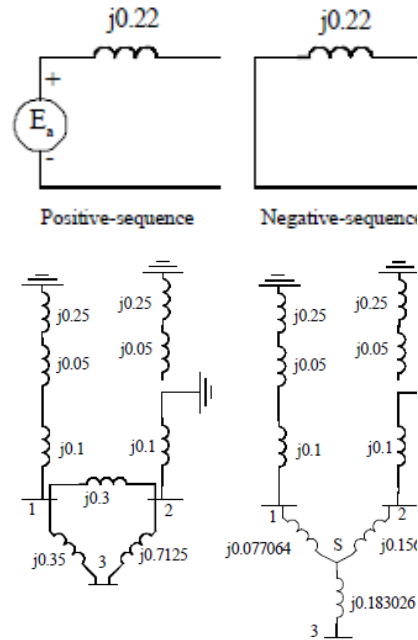
$$Z_{1S} = \frac{(j0.125)(j0.15)}{j0.525} = j0.0357143$$

$$Z_{2S} = \frac{(j0.125)(j0.25)}{j0.525} = j0.0595238$$

$$Z_{3S} = \frac{(j0.15)(j0.25)}{j0.525} = j0.0714286$$



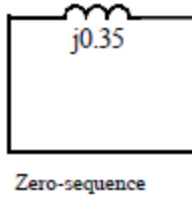
$$Z^1 = \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286 = j0.22$$



The positive- and negative-sequence networks are shown to the right. The only difference between them in this case is that the source is missing in the negative-sequence network. Hence we have:

$$Z^2 = Z^1 = j0.22$$

Now, the zero-sequence network is constructed based on the transformer connections and is shown in the figures to the right. Find the equivalent zero sequence circuit. The result is the simple circuit shown below.



(a) Balanced three-phase fault at bus 3.

In this case only a positive network will be used. All generators replaced by on voltage have a voltage of 1.0 per unit, hence the fault current is:

$$I_f = I_{3a} = \frac{V_{3a}(0)}{Z^1 + Z_f} = \frac{1}{j0.22 + j0.1} = -j3.125$$

(b) Single line-to-ground fault at bus 3.

$$I_{3a}^0 = I_{3a}^1 = I_{3a}^2 = \frac{V_{3a}(0)}{Z^1 + Z^2 + Z^0 + 3Z_f}$$

$$= \frac{1}{j0.22 + j0.22 + j0.35 + 3 \times j0.1}$$

$$= -j0.9174 \text{ p.u.}$$

$$\begin{bmatrix} I_{3a} \\ I_{3b} \\ I_{3c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{3a}^0 \\ I_{3a}^1 \\ I_{3a}^2 \end{bmatrix} = \begin{bmatrix} -j2.7523 \\ 0 \\ 0 \end{bmatrix} \text{ p.u.}$$

(c) Line-to-line fault at bus 3.

The zero-sequence component of current is zero, i.e. I_{3a}^0 .

$$I_{3a}^1 = -I_{3a}^2 = \frac{V_{3a}(0)}{Z^1 + Z^2 + Z_f}$$

$$= \frac{1}{j0.22 + j0.22 + j0.1}$$

$$= -j1.8519 \text{ p.u.}$$

The fault current is:

$$\begin{bmatrix} I_{3a} \\ I_{3b} \\ I_{3c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{3a}^0 \\ I_{3a}^1 \\ I_{3a}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2075 \\ 3.2075 \end{bmatrix} \text{ p.u.}$$

(d) Double line-to-ground fault at bus 3.

$$I_a^1 = \frac{V_{3a}(0)}{Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}}$$

$$= \frac{1}{j0.22 + \frac{j0.22(j0.35+0.3)}{j0.22+j0.35+0.3}}$$

$$= -j2.6017 \text{ p.u}$$

$$I_a^2 = -\frac{V_{3a}(0) - Z^1 I_a^1}{Z^2}$$

$$= \frac{1 - (j0.22)(-j2.6017)}{j0.22}$$

$$= j1.9438$$

$$I_a^0 = -\frac{V_{3a}(0) - Z^1 I_a^1}{Z^0 + 3Z_f} = \frac{1 - (j0.22)(-j2.6017)}{j0.35 + 0.3} = j0.6579$$

The fault current is:

$$\begin{bmatrix} I_{3a} \\ I_{3b} \\ I_{3c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{3a}^0 \\ I_{3a}^1 \\ I_{3a}^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.058 \angle 165.93 \\ 4.058 \angle 14.07 \end{bmatrix} \text{ p.u}$$

$$I_f = I_b + I_c = 1.9732 \angle 90$$

HW

Q1/ The reactance data for the power system shown in Figur in per unit on a common base is as follows:



Item	X^1	X^2	X^0
G_1	0.10	0.10	0.05
G_2	0.10	0.10	0.05
T_1	0.25	0.25	0.25
T_2	0.25	0.25	0.25
Line 1-2	0.30	0.30	0.50

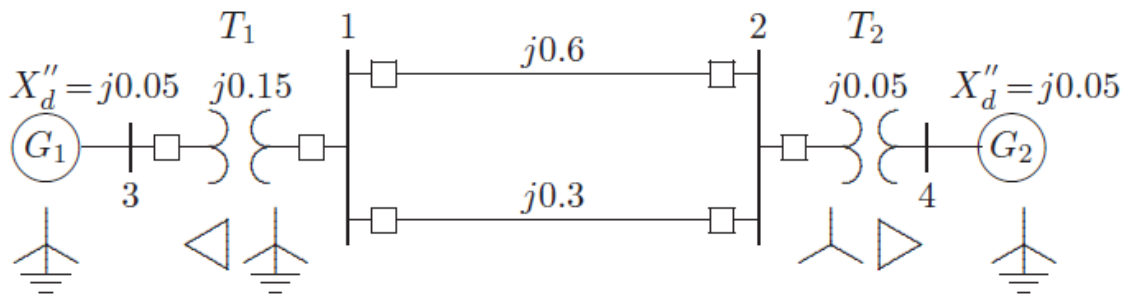
Obtain the Th´evenin sequence impedances for the fault at bus 1 and compute the fault current in per unit for the following faults:

- (a) A bolted three-phase fault at bus 1.
- (b) A bolted single line-to-ground fault at bus 1.

- (c) A bolted line-to-line fault at bus 1.
 (d) A bolted double line-to-ground fault at bus 1.

Q2/The positive-sequence reactances for the power system shown in Figure are in per unit on a common MVA base. Resistances are neglected and the negative-sequence impedances are assumed to be the same as the positive-sequence

impedances. A bolted line-to-line fault occurs between phases *b* and *c* at bus 2. Before the fault occurrence, all bus voltages are 1.0 per unit. Obtain the positive sequence bus impedance matrix. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase.



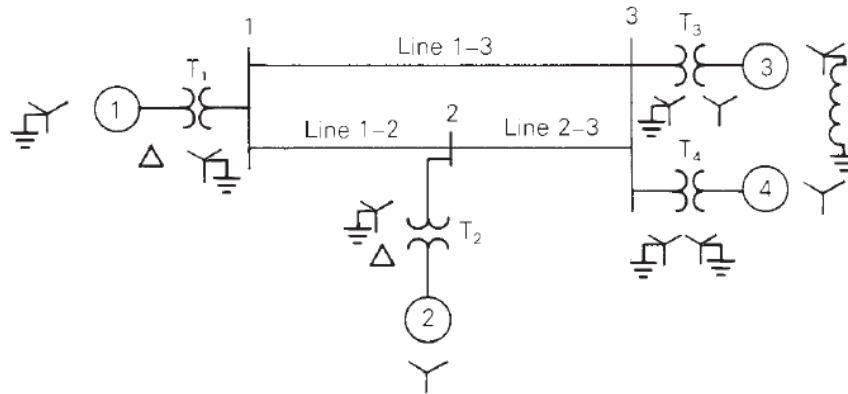
Q3/ The single-line diagram of a three-phase power system is shown in Figure. Equipment ratings are given as follows:

Synchronous generators:

G1	1000 MVA	15 kV	$X_d'' = X_2 = 0.18, X_0 = 0.07$ per unit
G2	1000 MVA	15 kV	$X_d'' = X_2 = 0.20, X_0 = 0.10$ per unit
G3	500 MVA	13.8 kV	$X_d'' = X_2 = 0.15, X_0 = 0.05$ per unit
G4	750 MVA	13.8 kV	$X_d'' = 0.30, X_2 = 0.40, X_0 = 0.10$ per unit

Transformers:

T1	1000 MVA	15 kV Δ /765 kV Y	$X = 0.10$ per unit
T2	1000 MVA	15 kV Δ /765 kV Y	$X = 0.10$ per unit
T3	500 MVA	15 kV Y/765 kV Y	$X = 0.12$ per unit
T4	750 MVA	15 kV Y/765 kV Y	$X = 0.11$ per unit

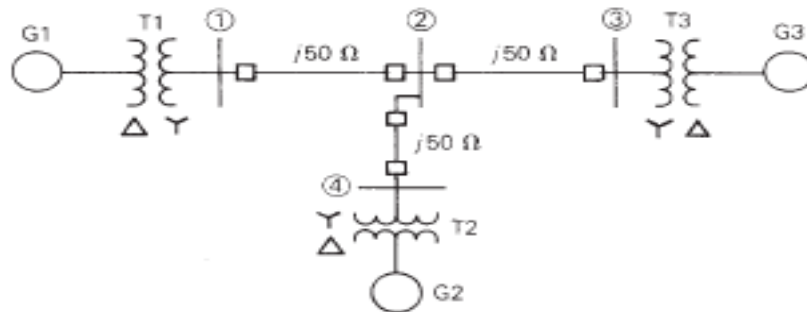


Transmission lines:

- 1-2 765 kV $X_1 = 50 \Omega$, $X_0 = 150 \Omega$
- 1-3 765 kV $X_1 = 40 \Omega$, $X_0 = 100 \Omega$
- 2-3 765 kV $X_1 = 40 \Omega$, $X_0 = 100 \Omega$

The inductor connected to Generator 3 neutral has a reactance of 0.05 per unit using generator 3 ratings as a base. Draw the zero-, positive-, and negative-sequence reactance diagrams using a 1000-MVA, 765-kV base in the zone of line 1-2.

Q4/ Equipment ratings for the four-bus power system shown in Figure are given as follows:



- Generator G1: 500 MVA, 13.8 kV, $X_d'' = X_2 = 0.20$, $X_0 = 0.10$ per unit
- Generator G2: 750 MVA, 18 kV, $X_d'' = X_2 = 0.18$, $X_0 = 0.09$ per unit
- Generator G3: 1000 MVA, 20 kV, $X_d'' = 0.17$, $X_2 = 0.20$, $X_0 = 0.09$ per unit
- Transformer T1: 500 MVA, 13.8 kV $\Delta/500$ kV Y, $X = 0.12$ per unit
- Transformer T2: 750 MVA, 18 kV $\Delta/500$ kV Y, $X = 0.10$ per unit
- Transformer T3: 1000 MVA, 20 kV $\Delta/500$ kV Y, $X = 0.10$ per unit
- Each line: $X_1 = 50$ ohms, $X_0 = 150$ ohms

The inductor connected to generator G3 neutral has a reactance of 0.028 pu. Draw the zero-, positive-, and negative-sequence reactance diagrams using a 1000-MVA, 20-kV base in the zone of generator G3

Q5/ A single-line diagram of a four-bus system is shown in Figure for which Z_{BUS} is given below:

$$Z_{BUS} = j \begin{bmatrix} 0.25 & 0.2 & 0.16 & 0.14 \\ 0.2 & 0.23 & 0.15 & 0.151 \\ 0.16 & 0.15 & 0.196 & 0.1 \\ 0.14 & 0.151 & 0.1 & 0.195 \end{bmatrix} \text{ per unit}$$

Let a three-phase fault occur at bus 2 of the network.

- Calculate the initial symmetrical rms current in the fault.
- Determine the voltages during the fault at buses 1, 3, and 4.
- Compute the fault currents contributed to bus 2 by the adjacent un faulted buses 1, 3, and 4.
- Find the current flow in the line from bus 3 to bus 1. Assume the pre fault voltage V_f at bus 2 to be $1 \angle 0$ pu, and neglect all pre fault currents.

