DESIGN OF STEEL STRUCTURES

Syllabus

- Introduction
- Tension Members
- Connections
- Compression Members
- Flexural members (Beams)
- Members under Biaxial Bending
- Beam-column

References

1- Steel Design by Segui, Fourth Edition, 2007.
2- Structural Steel Design by Mc Cormac and Csernak, Fifth Edition, 2012.
3- AISC-LRFD Manual. Handbook and Specifications

CHAPTER ONE

INTROUDACTION

1.1 General

Structural steel is one of the basic materials used by structural engineers. Steel, as a structural material has exceptional strength, stiffness, and ductility properties. As a result of these properties, steel is readily produced in a extensive variety of structural shapes to satisfy a wide range of application needs. The wide spread use of structural steel makes it necessary for structural engineers to be well versed in its properties and uses. Following some of the required concepts that need to be understood:

- **Static's**
  - ✓ The ability to compute reactions on basic structures under given loading.
  - ✓ The ability to determine stability and determinacy
  - ✓ The ability to determine internal forces in statically determinate structures.
    - Develop shear and moment diagrams
  - ✓ The ability to solve truss problems (both 2D and 3D) by using
    - Method of joints
    - Method of sections
  - ✓ The ability to solve "machine" problems
  - ✓ The ability to compute of section properties including
    - Cross sectional area
    - Moments of Inertia for section of homogenous materials
    - Moments of Inertia for composite sections

- **Mechanics**
  - ✓ An understanding of stress and strain concepts
  - ✓ The ability to compute stress including
    - Axial stress
    - Bending stress
    - Shear stress (due to both bending and torsion)
    - Principle stress
    - Stress on arbitrary planes
  - ✓ The ability to compute the buckling capacity of columns
  - ✓ The ability to compute deflection in beams
  - ✓ The ability to compute reactions and internal forces for statically indeterminate structures.

- **Properties of Materials**
  - ✓ The ability to read stress-strain diagrams to obtain critical material properties including:
    - Yield stress
    - Ultimate stress
    - Modulus of Elasticity
    - Ductility
  - ✓ An understanding of the statistical variation of material properties.

- **Structural Analysis**
    - An understanding of the nature of loads on structures
    - The ability to compute and use influence diagrams.
    - The ability to solve truss problems (forces and deflections)
    - The ability to solve frame problems (forces and deflections)
    - The ability to use at structural analysis software
- **Structural Engineering**
  - ✓ Design of different structures (Buildings, bridges, dams, etc.):
    - Satisfy needs or functions
    - Support its own loads
    - Support external loads

- **Steel Design**
  - ✓ Selection of structural form .
  - ✓ Determination of external loads.
  - ✓ Calculation of stresses and deformations.
  - ✓ Determination of size of individual members.

1.2 Advantages & Disadvantages of Steel as a Construction Material

✓ ***Advantages:***

1. High load resisting (High resistance)
2. High ductility and toughness
3. Easy control for steel structure
4. Elastic properties
5. Uniformity of properties
6. Additions to existing structure

✓ ***Disadvantages:***

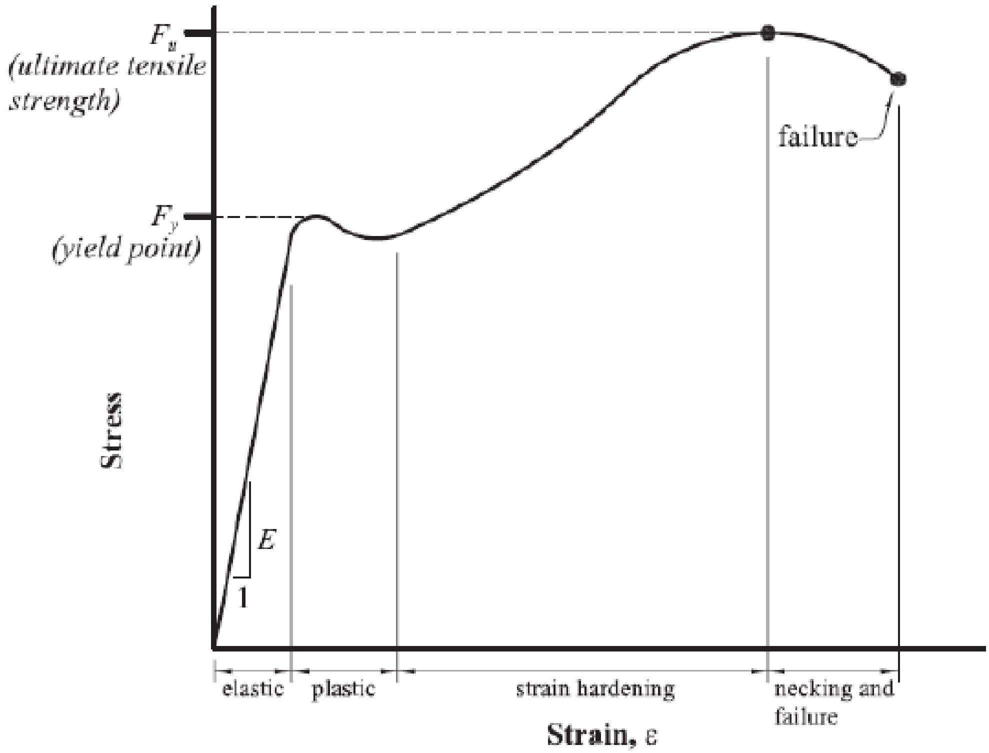
1. No ability to resist the fire (Fireproofing cost)
2. No ability to resist the corrosion (Maintenance cost)
3. High cost
4. Susceptibility to buckling, fatigue and brittle fracture

1.3 Materials

✓ ***Structural Steels***

For the purposes of the Specification for Structural Steel Buildings, four quantities are particularly important for a given steel type:

- The minimum yield stress (f_y).
- The specified minimum tensile strength ($\mathbf{F_u}$).
- The modulus of elasticity ($\mathbf{E_s}$).
- The shear modulus (**G**).



Stress–Strain Diagram for structural steel

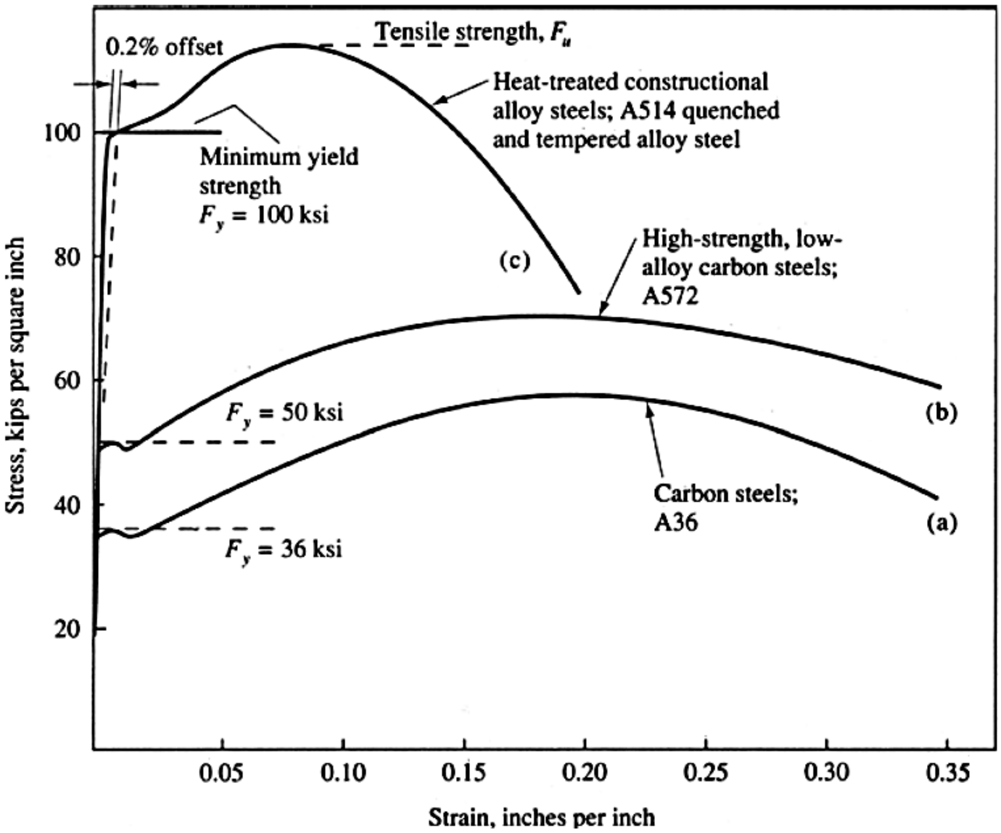
There are several types of steel as following :

- **Carbon Steels:**
  1. Low carbon **[C < (0.15%)]**.
  2. Mild carbon **[0.15% < C< 0.29%]** such as **A-36, A-53**.
  3. Medium carbon **[0.3% C < 0.59%] A-500, A-529**.
  4. High carbon **[0.6% < C < 1.7%] A-570**.

- **High-Strength Low-Alloy Steels:**
  Having f_y **40 ksi** to **70 ksi,** may include chromium, copper, manganese, nickel in addition to carbon. e.g. **A572, A618, A913,** and **A992**.

- **Alloy Steels:**

These alloy steels which are quenched and tampered to obtain f_y > **80 ksi**. They do not have a well-defined yield point, and are specified a yield point by the "offset method", example is **A852**.



Typical stress-strain curve for different types structural steel

✓ ***Bolts***

Bolting is a very common method of fastening steel members. Bolting is particularly cost effective in the field. The precursor to bolting was riveting. Riveting was a very dangerous and time consuming process. It involved heating the rivets to make them malleable then inserting them in hole and flattening the heads on both sides of the connection. The process required an intense heat source and a crew of three or more workers. In the mid 1900s, high strength bolts were introduced and quickly replaced rivets as the preferred method for connecting members together in the field because of their ease of installation and more consistent strengths. High strength is necessary since most bolts are highly tensioned in order to create large clamping forces between the connected elements. They also need lots of bearing and shear strength so as to reduce the number of fasteners needed. The types of bolts are:

- **Carbon Steel Bolts (A-307):**
  These are common non-structural fasteners with minimum tensile strength **(F_u)** of **60 ksi**.
- **High Strength Bolts (A-325):**
  These are structural fasteners (bolts) with low carbon, their ultimate tensile strength could reach **120 ksi**.

- **Quenched and Tempered Bolts (A-449):**
  These are similar to **A-307** in strength but can be produced to large diameters exceeding **1.5 inch**.

- **Heat Treated Structural Steel Bolts (A-490):**
  These are in carbon content (**up to 0.5%**) and has other alloys. They are quenched and re-heated (tempered) to **900^{o}F**. The minimum yield strength **(f_y)** for these bolts ranges from **115 ksi** upto **130 ksi**. The ultimate tensile strengths for **A490** bolts are **150 ksi**.

ASTM A325

ASTM A307

ASTM A490


Solid rivets

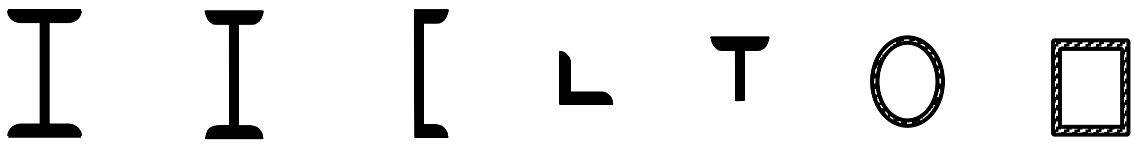
✓ ***Welding Materials:***

Welding is the process of uniting two metal parts by melting the materials at their interface so that they will bond together. A filler material is typically used to join the two parts together. The parts being joined are referred to as base metal and the filler is referred to as weld metal. Since structural welding is typically done by an electrical arc process, the weld metal is typically supplied via weld electrodes, sometimes known as welding rods.

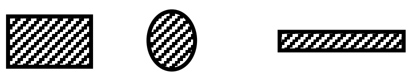
The Shielded Metal Arc Welding (SMAW) Process Electrodes

1.4 Type of Structural Steel Sections

✓ ***Hot-Rolled Sections:*** The Standard rolled shapes are shown in the figure.



(a) Wide-flange Shape
(b) American Standard Beam
(c) American Standard Channel
(d) Angle
(e) Structural T
(f) Pipe Section
(g) Structural Tubing

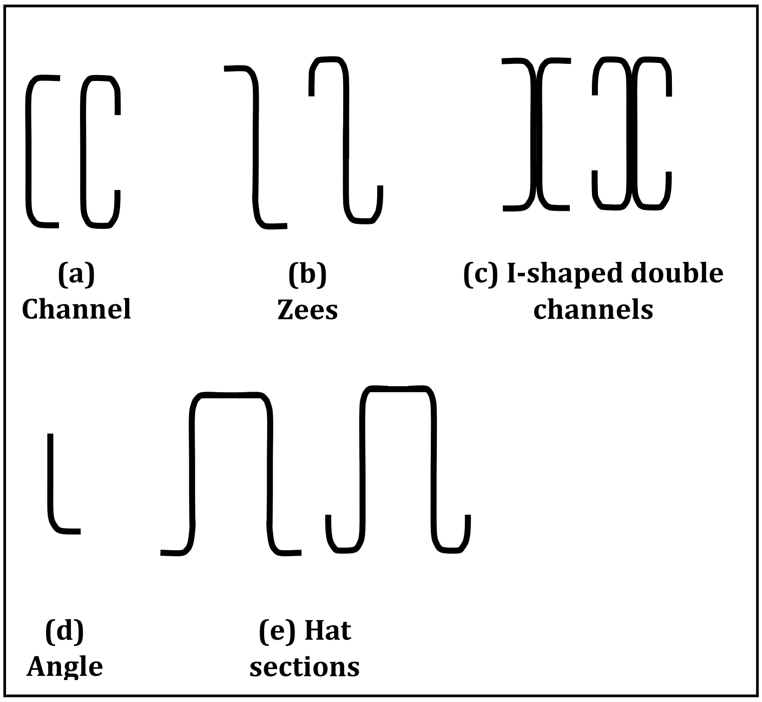


(h) Bars
(i) Plates

a – Wide-flange : W 18 × 97
b – Standard (I) : S 12 × 35
c – Channel : C 9 × 20
d – Angles : L 6 × 4 × ½
e – Structural Tee : WT, MT or ST e.g. ST 8 × 76
f & g Hollow Structural Sections
HSS: 9 or 8 × 8

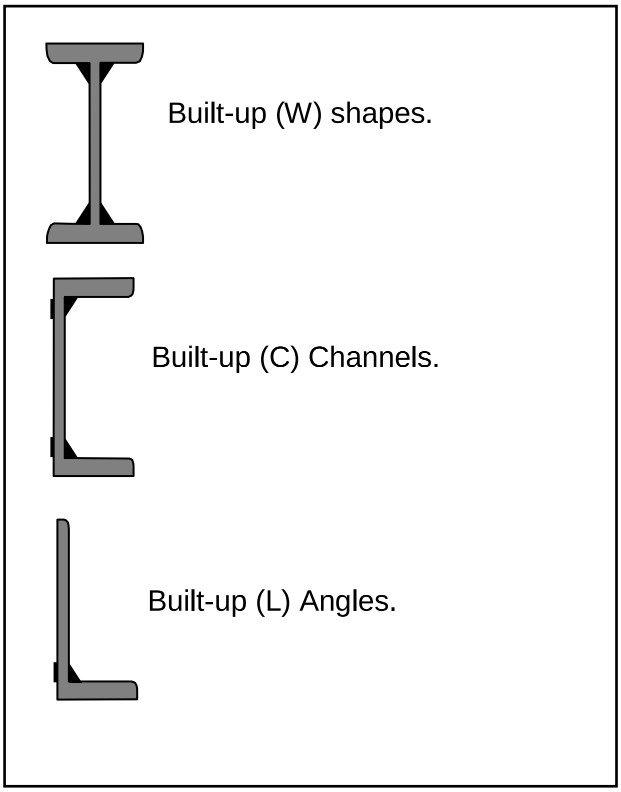
Standard rolled shapes

- ***Cold Formed Sections:*** as shown in the figure.



Cold Formed Sections

- ***Built-Up Sections:*** as shown in the figure.



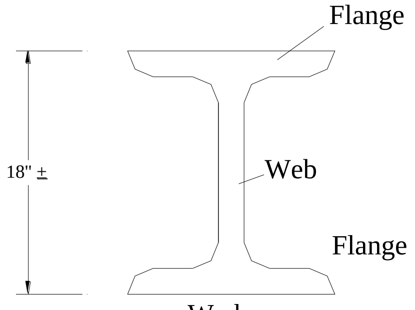
Built-up sections

1.5 Cross-Sections of Some of the more Commonly Used Hot-Rolled Shapes

- ***W- shape or Wide –flange Shape:*** For example :(W 18×50)
  W-type shape.

  18: section depth in inches .

  50: section weight in pounds per foot .

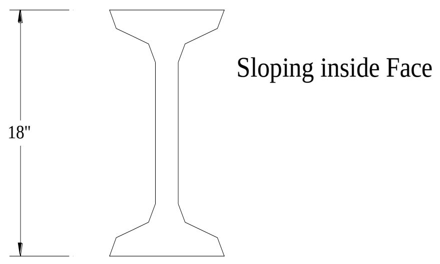


W-shape
W18×50

- ***S- shape or American standard S:*** For example :(S 18×70)
  S-type of shape

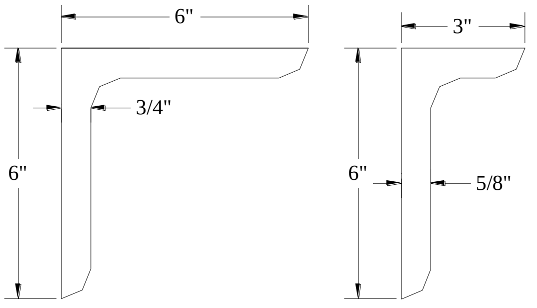
  18: section depth in inches .

  70: section weight in pounds per foot .



American Standard. S
S18 × 70

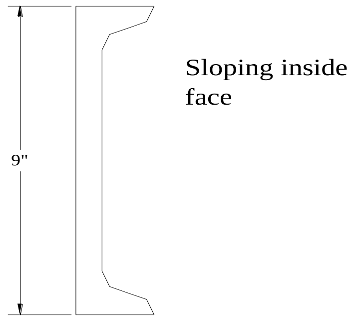
- ***L- shape or Angle shape:*** For example :
  - (L6 ×L6 ×¾’’)
  - (L6 ×L3 ×5/8’’)



Equal-Leg angle, L
L6 × 6 × 3/4

Unequal-Leg angle
L6 × 3 × 5/8

C- shape: For example (C18 ×70)



American Standard.
Channel, C
C18 × 70

1.6 Loads

1. Dead Loads: Also known as gravity loads, includes the weight of the structure and all fixed and permanent attachments.
2. Live Loads: Also belong to gravity loads, but their intensity and location may vary (non-permanent loads).
3. Highways / Rail Live Loads – Impact Loads
4. Snow Loads
5. Wind Loads
6. Earthquake Load
7. Thermal Loads
8. Other Loads: e.g.
   - ✓ Rain Loads
   - ✓ Hydrostatic Loads
   - ✓ Blast Loads.

**** Loads can be also classified to:***

1. Static Loads: applied slowly that the structure remains at rest during loading.
2. Dynamic Loads: applied rapidly to cause the structure to accelerate as a consequence of inertia forces.

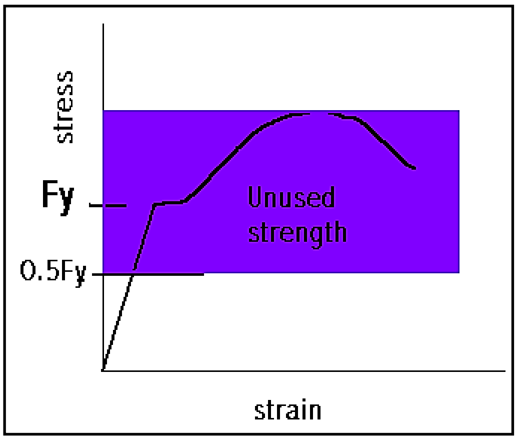
1.7 Philosophies of Design

Any design procedure require the confidence of engineer on the analysis of load effects and strength of the materials. The two distinct procedures employed by designers are **Allowable Stress Design (ASD) & Load & Resistance Factor Design (LRFD).**

- ✓ **Allowable Stress Design (ASD):**
  Safety in the design is obtained by specifying, that the effect of the loads should produce stresses that is a fraction of the yield stress f_y, say one half. This is equivalent to:

  FOS = Resistance, R/ Effect of load, Q $= f_y/0.5f_y = 2$

Since the specifications set limit on the stresses, it became allowable stress design (ASD). It is mostly reasonable where stresses are uniformly distributed over X-section (such on determinate trusses, arches, cables etc.).



Mathematical Description of ASD:

$$\frac{\phi R_n}{\gamma} \geq \sum Q_i$$

R_n = Resistance or Strength of the component being designed

Φ = Resistance Factor or Strength Reduction Factor

γ = Overload or Load Factors

Φ/γ = Factor of Safety FS

Q_i = Effect of applied loads

- ✓ **Load and Resistance Factor Design (LRFD):** To overcome the deficiencies of ASD, the LRFD method is based on **Strength of Materials.** It consider the variability not only in resistance but also in the effects of load and it provides measure of safety related to probability of failure. Safety in the design is obtained by specifying that the reduced Nominal Strength of a designed structure is less than the effect of factored loads acting on the structure

$$\phi R_n \geq n \sum \gamma Q_i$$

R_n = Resistance or Strength of the component being designed

Q_i = Effect of Applied Loads

n = Takes into account ductility, redundancy and operational imp .

Φ = Resistance Factor or Strength Reduction Factor

γ = Overload or Load Factors

Φ/γ = Factor of Safety FS

LRFD accounts for both variability in resistance and load and it achieves fairly uniform levels of safety for different limit states.

1.8 Building Codes

Buildings must be designed and constructed according to the provisions of a building code, which is a legal document containing requirements related to such things as structural safety, fire safety, plumbing, ventilation, and accessibility to the physically disabled. A building code has the force of law and is administered by a governmental entity such as a city, a county.

Building codes do not give design procedures, but they do specify the design requirements and constraints that must be satisfied. Of particular importance to the structural engineer is the prescription of minimum live loads for buildings. Although the engineer is encouraged to investigate the actual loading conditions and attempt to determine realistic values, the structure must be able to support these specified minimum loads.

1.9 Design Specifications

The specifications of most interest to the structural steel designer are those..-; published by the following organizations.

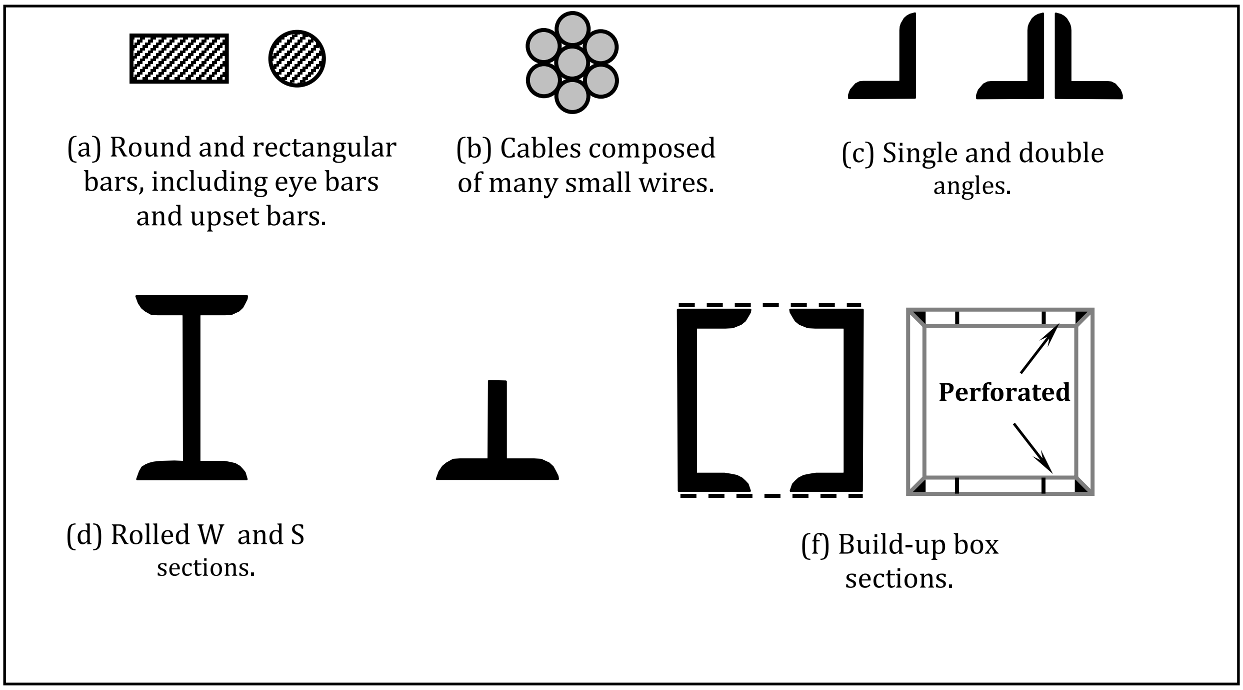
1. **American Institute of Steel Construction (AISC)**: This specification provides for the design of structural steel buildings and their connections.
2. **American Association of State Highway and Transportation Officials (AASHTO):** This specification covers the design of highway bridges and related structures. It provides for all structural materials normally used in bridges, including steel, reinforced concrete and timber.
3. **American Railway Engineering and Maintenance-of-Way Association (AREMA):** The AREMA Manual of Railway Engineering covers the design of railway bridges and related Structures.
4. **American Railway Engineering Association (AREA)**.
5. **American Iron and Steel Institute (AISI):** This specification deals with cold-formed steel.

CHAPTER TWO

TENSION MEMBERS

1.1 Overview

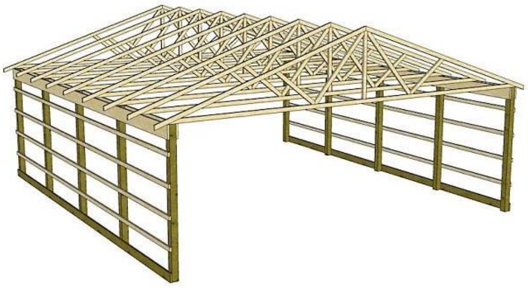
Tension member: is a structural elements which subjected to axial tensile forces. Steel shapes, which are used as tension members, are shown in the figure below.



Steel shapes use as tension members

Generally the used in:

1- ***Trusses in Frames & Bridges***



2- bracing for building & bridge

3- cables such as: suspended roof systems, suspension & bridges

The stress in an axially loaded tension member is given by:

$$f = P/A$$

Where,

P is the magnitude of load, and
A is the cross-sectional area normal to the load.

The stress in tension member is uniform throughout the cross-section except

- ✓ Near the point of application of load, and
- ✓ At the cross-section with holes for bolts.

➢ The cross-sectional area will be reduced by amount equal to the area removed by holes.

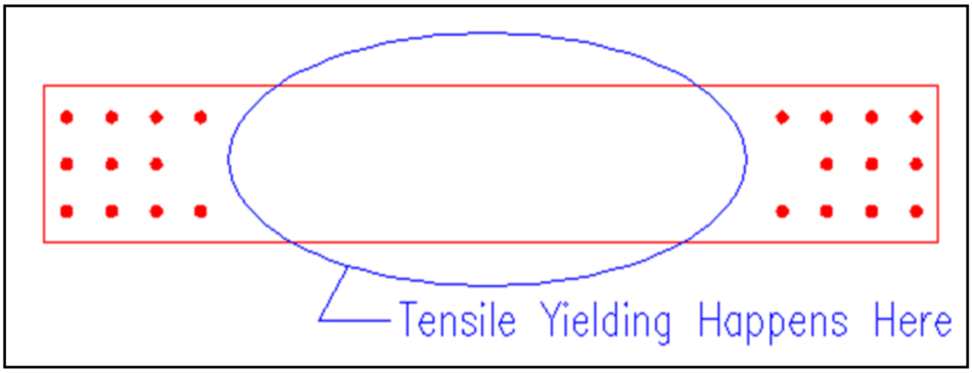
1.2 Controlling Limit States

There are three limit states that relate to the member itself. These limit states that will be considered are:

- ✓ Tensile yielding
- ✓ Tensile rupture
- ✓ Slenderness

1.2.1 Tension yielding:

Tensile yielding is considered away from the connections in the mid part of the member & excessive deformation can occur due to the yielding of the gross section. The figure shows the general region of concern for a flat plate member.



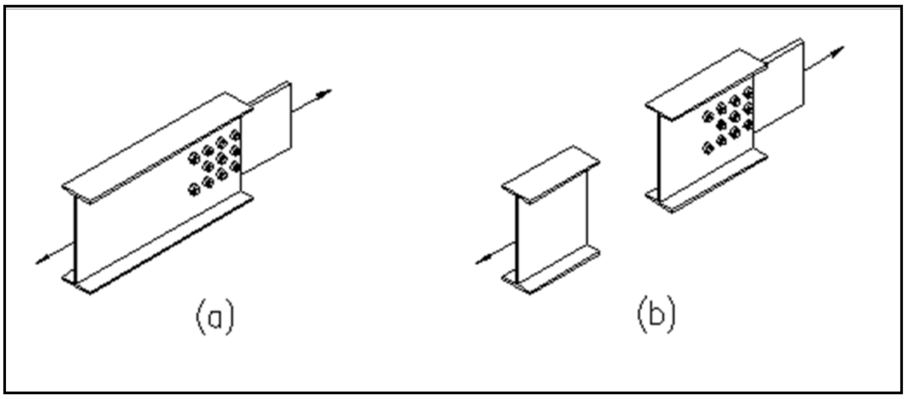
Tensile Yielding Region

Tensile yielding is illustrated in Figure (b). This failure mode looks at yielding on the gross cross sectional area, $\mathbf{A_g}$, of the member under consideration. Consequently, the critical area is located away from the connection as shown in Figure a.

To prevent excessive deformation, the stress at gross sectional area must be smaller than yielding strength:

$$f < \mathbf{F_y} \qquad \text{i.e.} \qquad \mathbf{P/A < F_y}$$

The nominal strength in yielding is: $\mathbf{P_{n1} = F_y * A_g}$



Tensile Strength Limit States

The statement of the limit states and the associated reduction factor and factor of safety are given here:

$$P_{u1} \le \varphi_t P_{n1}$$
$$\varphi_t = 0.90$$

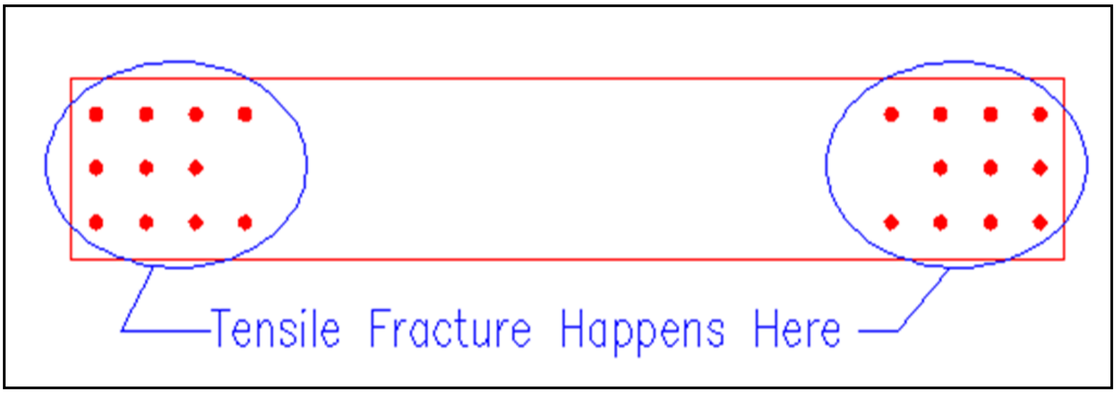
The values of $\mathbf{P_{u1}}$ and $\mathbf{P_{n1}}$ are the **LRFD** factored load and nominal tensile yielding strength of the member, respectively, applied to the member.

1.2.2 Tensile Rupture of a Member:

Tensile rupture is a strength based limit state similar to the tensile yielding limit state that we just considered. When the cross section is reduced by holes or if not all the cross sectional elements of a particular section (such as the flanges on a W section) are transferring force to a connection, then less of the section is effective in supporting the applied forces. Stress concentrations will also cause localized yielding. Local yielding to relieve stress concentrations is not a major problem for ductile materials so the yielding limit state is not considered where the connections are made. The concern at these locations is actual rupture so the applied forces are compared against the rupture strength in the region of reduced effective section. The figure illustrations where the concern is for sample flat bar member with bolted end connections. To prevent fracture, the stress at the net sectional area must be smaller than ultimate strength:

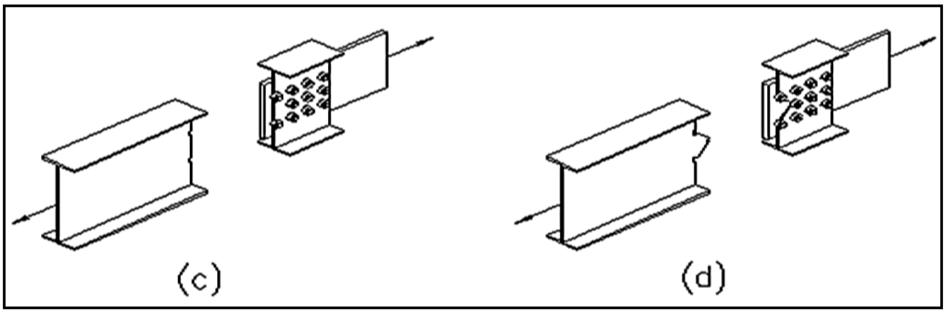
$$f < F_u \qquad \text{i.e.} \qquad P/A < F_u$$

The nominal strength in yielding is: $\mathbf{P_{n2} = F_u * A_e}$



Tensile Yielding Region

In this case we have two potential failure paths that see the full force of the member. These are shown in Figures (c) and (d). Tensile rupture is complicated by the need to get the forces out of the flanges, through the web, and into the bolts. This means that we need to account for the stress concentrated in and around the bolts.



Tensile Strength Limit States

The statement of the limit states and the associated reduction factor and factor of safety are given here:

$$P_{u2} \leq \varphi_t \, P_{n2}$$

$$\varphi_t = 0.75$$

The values of $\mathbf{P_{u2}}$ and $\mathbf{P_{n2}}$ are the **LRFD** factored load and nominal tensile rupture strength of the member, respectively, applied to the member.

1.2.3 Block Shear Rupture:

Block shear is, in some ways, similar to tensile rupture in that the main part of the member tears away from the connection i.e. the tension member can fail due to 'tear-out' of material at the connected end. The difference is that there is now a combination of tension and shear on the failure path. Like tensile rupture, there frequently is more than one failure path. The figure shows three possible block shear failure paths for a **WT** section. Block shear strength is determined as the **sum** of the **shear strength** on a **failure path** and the **tensile strength** on a **perpendicular segment**:

Block shear strength = gross yielding strength of the shear path
+ gross yielding strength of the tension path

Or

Block shear strength= gross yielding strength of the shear path
+ net section fracture strength of the tension path

When:

- $F_u A_{nt} \geq 0.6 F_u A_{nv}$:

$$\varphi_t R_{n3} = \varphi_t (0.6 F_y A_{gv} + F_u A_{nt}) \leq \varphi_t (0.6 F_u A_{nv} + F_u A_{nt}) \quad P_{u3} \leq \varphi_t R_{n3}$$

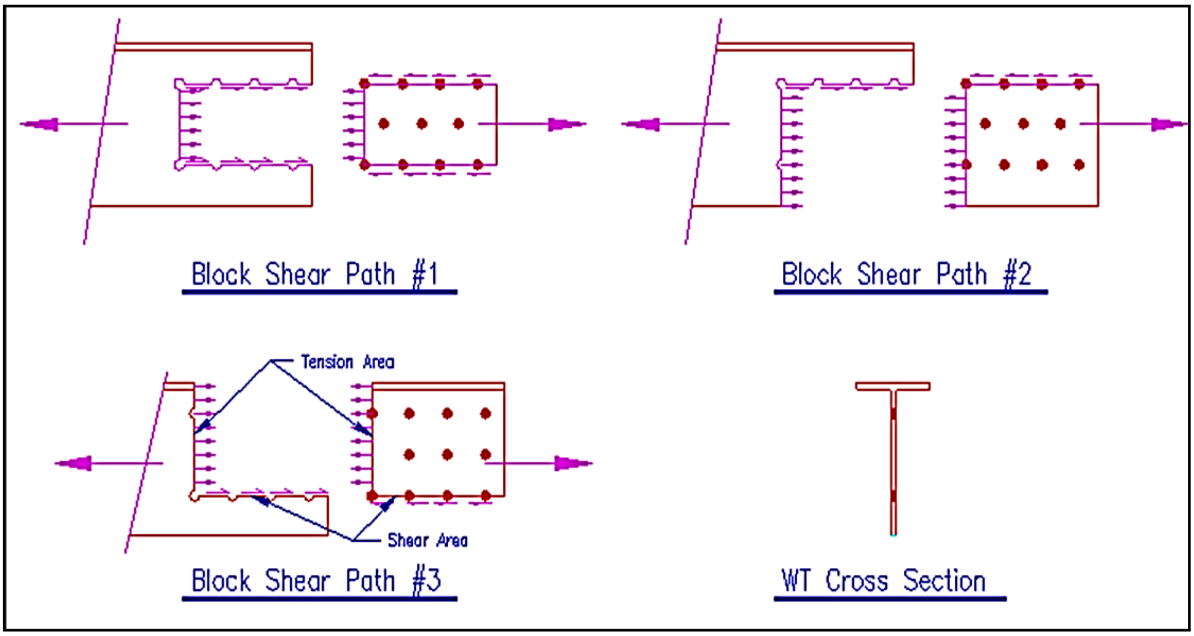
- $F_u A_{nt} < 0.6 F_u A_{nv}$:

$$\varphi_t R_{n3} = \varphi_t (0.6 F_u A_{nv} + F_y A_{gt}) \leq \varphi_t (0.6 F_u A_{nv} + F_u A_{nt})$$

$$P_{u3} \leq \varphi_t R_{n3}$$

Where: $\varphi_t = 0.75$
A_{gv} = gross area subjected to shear
A_{gt} = gross area subjected to tension
A_{nv} = net area subjected to shear
A_{nt} = net area subjected to tension

and values of P_{u3} and R_{n3} are the LRFD factored load and nominal resistance or strength associated with block shear of the member, respectively.



Block Shear Failure Paths

1.2.4 Slenderness Limits:

Slenderness is a *serviceability* limit state, not a strength limit state, so failure to adhere to the suggestion is unlikely to cause an unsafe condition.
The limit state is written as:

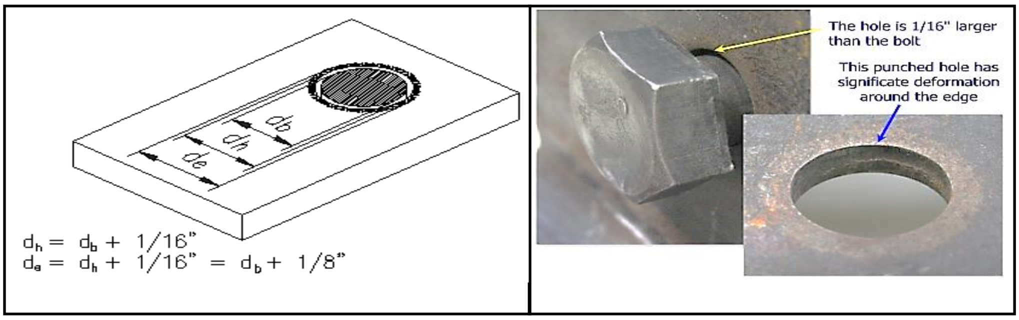
$$\mathbf{L/r_{min.} \leq 300}$$

Where "$\mathbf{r_{min}}$" is the least radius of gyration. "**r**" is a *section property that equals the square root of the moment of inertia divided by the cross section area*. Every member has an "**r**" for each of the principle axes.

1.3 Area Determination

1.3.1 Net Area (An):

The net area computation requires computation of a reduced section due to *holes* made in the member as well a *failure path* for the rupture surface. The figure shows a typical standard hole and the dimensions that are related to it.



Bolt Holes

For A_n calculations, is to be taken as **1/8"** larger than the bolt (i.e. **1/8" = 1/16"** *for the actual hole diameter plus an additional* **1/16"** *for damage related to punching or drilling.)* So, if you specify **3/4"** bolts in standard holes, the effective width of the holes is **7/8"** (i.e. **3/4"** for the bolt diameter + **1/16"** for the hole diameter **+1/16"** damage allowance.).

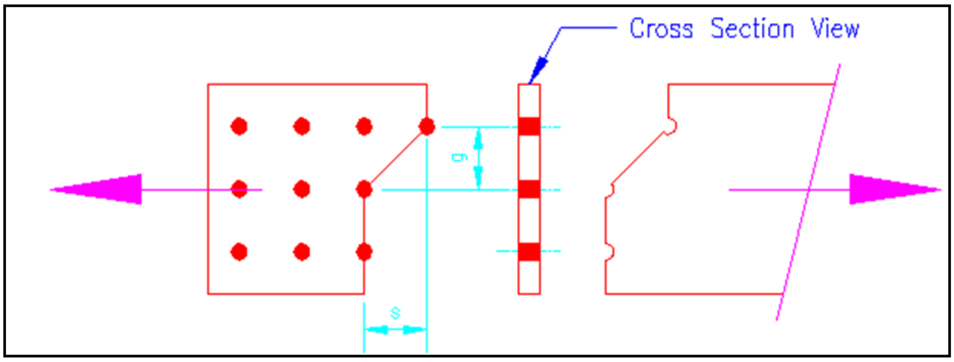
The next concept that needs discussing is the concept of failure paths. Failure paths are the approximate locations where a fracture may occur. For bolted tension member, maximum net area can be achieved if the bolts are

Placed in a single line. The connecting bolts can be staggered for several reasons:

1- To get more capacity by increasing the effective net area
2- To achieve a smaller connection length
3- To fit the geometry of the tension connection itself

The figure shows a failure path that has a component that is not perpendicular to the line of action for the force. The stagger is characterized by a "**pitch**" of **s** and a "**gage**" of **g** as shown.

$$\mathbf{A_n = A_g - (\sum d + \sum s^2/4g) * t}$$



Failure Path with Staggered Bolts

1.3.2 Effective Net Area, A_e:

In cases where *SOME BUT NOT ALL* of the cross sectional elements are used to transfer force to/from the member at the connection, then not all the net area is really effective for tensile rupture. This is the result of a phenomena called shear lag. Shear lag affects both bolted and welded connections. Therefore, the effective net area concept applied to both type of connections.

- ✓ For bolted connection, the effective net area is $\mathbf{A_e = UA_n}$
- ✓ For welded connection, the effective net area is $\mathbf{A_e = UA_g}$

Where, the reduction factors U is given by: U = 1 – x/L

Where, **x** is the distance from the centriod of the connected area to the plane of the connection, and **L** is the length of the connection.

1.3.3 Reduction Coefficient "U":

The AISC manual also gives values of **U** that can be used instead of calculating **x/L** as follow:

1.3.3.1 Bolted Members

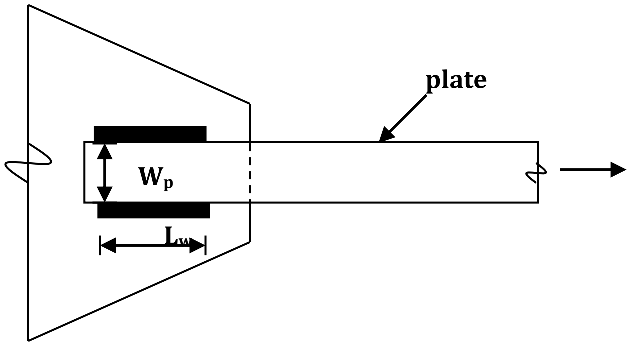
- For W, M, I, and S shapes with $b_f/d \geq 2/3$ with at least three fasteners per line in the direction of applied load …….U=0.9
- For T – shape with $b_f/d \geq 4/3$ with at least three fasteners per line in the direction of applied load …….U= 0.9
- For I- & T- shapes not meeting the above conditions & all other shapes including build up section ...U=0.85
- For all other shapes section with only two fasteners per line ...U=0.75
- When the load is transmitted through all of the cross section, U=1

1.3.3.2 Welded Members

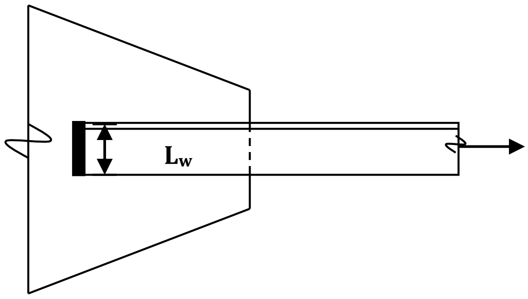
- When a plate is connected by only longitudinal weld to all
  - U = 0.75 when $1.0 \leq (L_w/W_p) < 1.5$
  - U = 0.87 when $1.5 \leq (L_w/W_p) < 2.0$
  - U = 1.00 when $(L_w/W_p) \geq 2.0$

Where L_w = length of longitudinal weld, in

W_p = plate width, in



- When tensile load is transmitted by transverse welds only $A_n=A_e$ & U=1.0



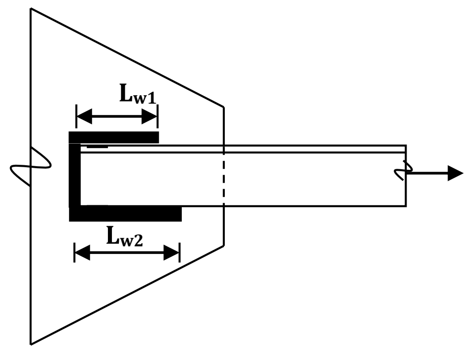
– When tensile load is transmitted only by longitudinal weld to a member other than plate, or longitudinal welds in combination with transverse welds:

$$A_n = A_g \quad \& \quad U = \min[(1 - x_{con}/L_{con}), 0.9]$$

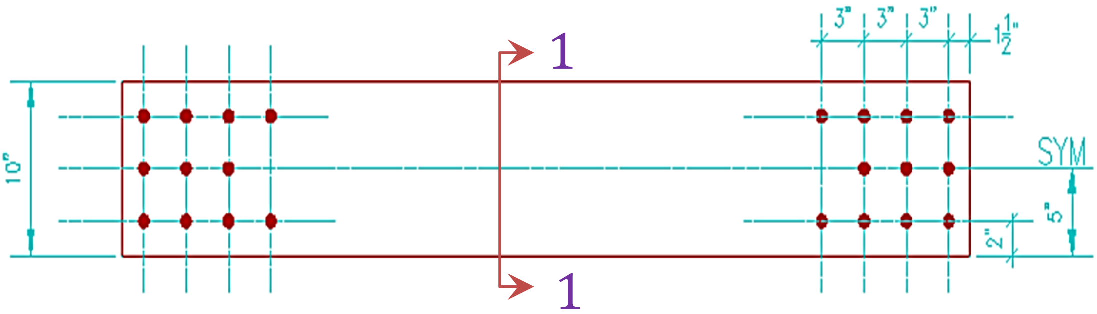
Where A_g = gross area of the members, in^2

L_{con}= connection length, taken as the length of longer longitudinal weld, in

= max. [L_{w1}, L_{w2}]



<u>Example 2-1</u>: **A 3/4" x 10"** plate of **Gr. 36** steel have span of **5 ft** long and has standard holes for **3/4"** bolts at each end for attachment to other structural members. The figure shows a face view of the plate. The service level loads that the member will be subject to are **140 kips** of dead load and **30 kips** of live load. Determine the axial tension capacity of the member.



Solution:

The problem solution is pursued in the following steps:

Determine the demand on the member.

P_u = 1.2D + 1.6L = 1.2(140 k) + 1.6(30 k) = 216 kips

Check size based on the slenderness limit state.

Our member is 5 feet long and the least value of r is computed as:

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{(10")(0.75")^3/12}{(10")(0.75")}} = 0.217in$$

The correct computation of L/r = (5 ft)(12 in/ft) / (0.217 in)= 277 < 300 ... The limit state is satisfied

Determine the capacity of the member based on the

- ✓ tensile yielding limit state

$$P_{n1} = F_y * A_g = (36\ ksi)(7.500\ in^2) = 270\ kips$$

$$\phi_t\ P_{n1} = 0.9*270 = 243\ k > P_u \quad ....Ok.$$

- ✓ tensile rupture limit state

First let's compute the net area A_n for each of the two failure paths identified in Figure 2-5-1.

Path #2

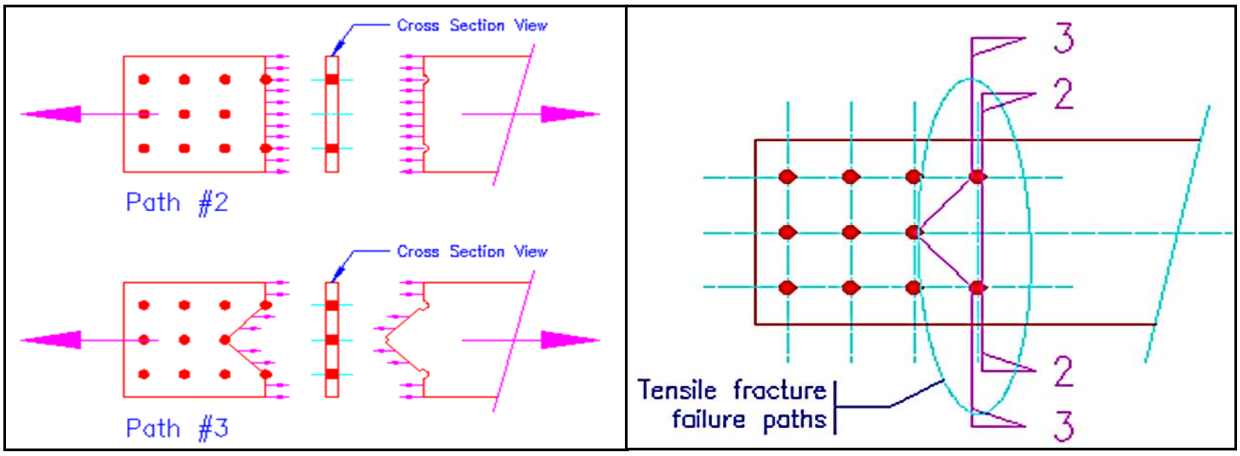
A_{n2} = A_g - hole area + gage area
= A_g - (num holes)
*(d_b+1/16"+1/16")(t_{pl})
= 7.50 in^2 - (2 holes)
*(0.75 in +1/8")(0.75 in)

$A_{n2} = 6.19\ in^2$

Path #3

A_{n3} = A_g - hole area + gage area
= A_g - (num holes)(d_b+1/16"+1/16")(t_{pl})
+ (t_{pl})($s^2/4g)_1$ + (t_{pl})($s^2/4g)_2$
= 7.50 in^2 - (3 holes)(0.75 in +1/8")(0.75 in)
+ (0.75 in)(3 in)^2/(4*(3 in))
+ (0.75 in)(3 in)^2/(4*(3 in))

$A_{n3} = 6.66\ in^2$



Tensile Rupture Failure Paths

The controlling net area is $\mathbf{A_{n2}}$ as it has the smaller value. This means that, if tensile rupture were to actually occur, this is the path that it would take. Therefore, for this problem:

$$\mathbf{A_n = 6.19\ in^2}$$

In this problem we have only one cross sectional element (i.e. one plate element in the cross section) and it is attached to the bolts leading us to **U = 1.0.** This means that there is no shear lag for this problem.

$$A_e = UA_n = (1)(6.19\ in^2) = 6.19\ in^2$$

$$P_{n2} = F_u * A_e = (58\ ksi)(6.19\ in^2) = 359\ kips$$

$$\phi_t\ P_{n2} = 0.75*359 = 269\ k > P_u \quad \ldots\ldots Ok.$$

2.4 Design of Tension Members

In design problems, the required tensile strength of member, $\mathbf{P_u}$, is known. The design task then consist of selecting a section and end connection such that the design tensile strength of member, $\boldsymbol{\phi P_n}$**,** is greater than or equal to the required strength $\mathbf{P_u}$. thus for design:

$$\boldsymbol{\phi P_n = \min [\phi P_{n1}, \phi P_{n2}] \geq P_u \quad or \quad \phi P_{n1} \geq P_u \quad \& \quad \phi P_{n2} \geq P_u}$$

$\mathbf{P_u}$**,** $\mathbf{P_{n1}}$ and $\mathbf{P_{n2}}$ are the LRFD factored load (or required tensile strength of member), nominal tensile yielding strength of the member, and nominal tensile rupture strength of the member, respectively. To satisfy the limit state of yielding in the gross section, the gross area must satisfy the relation:

$$\mathbf{A_{g1} \geq P_u /(0.9*F_y)}$$

While to satisfy the limit state of fracture in the net section, the net area must satisfy the relation:

$$\mathbf{A_n \geq P_u /(0.75*F_y*U)}$$

Then $\qquad \mathbf{A_{g2} \geq P_u /(0.75*F_y*U)}$ **+ estimated loss in area due to bolt holes**

$$\mathbf{A_g \geq max.\ [A_{g1} , A_{g2}]}$$

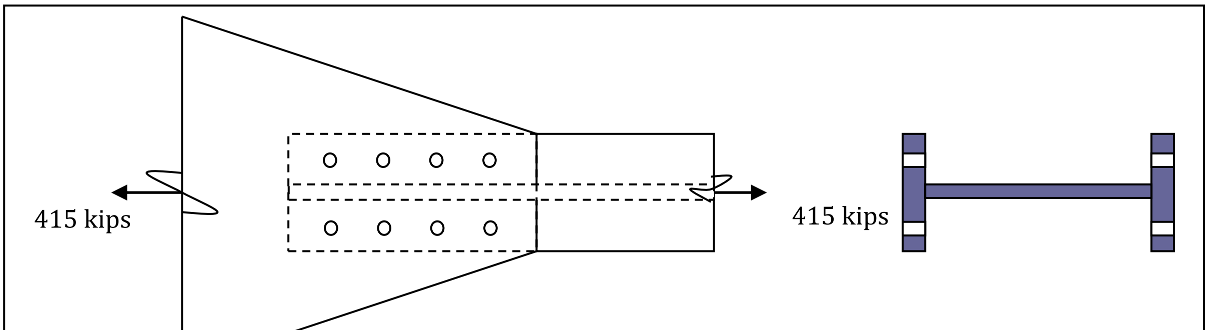
So, only section that satisfy the these relation are retained for further consideration in design.

$$\mathbf{L/r_{min} \leq 300}$$

Example 2-2: Select the lightest **W16*?** Shown in the figure, as a member of truss to transmit a factored tensile load of **415 kips**, the member is 30 ft long. **A588 Grade 50** shapes are available. Use ⅞-in bolt in two line in each flange.

Solution:

A588 Grade 50 steel; $\mathbf{F_y = 50}$ **ksi** and $\mathbf{F_u = 70}$ **ksi** . (**Table 2-3, page 2-39**)
Required member strength $\mathbf{P_u = 415}$ **kips**



Example 2-2

$$P_{n1} = F_y * A_g \geq P_u$$

$$A_{g1} \geq P_u /(0.9*F_y) = 415/(0.9*50) = 9.22 \text{ in}^2$$

$$r_{min} \geq L/300 = (3*12)/300 = 1.2 \text{ in}$$

from the LRFD Manual, W16×36 **(Page 1-20)** satisfy the two requirements

$$A_g = 10.6 \text{ in}^2 > 9.22 \text{ in}^2 \quad \& \quad r_y = 1.52 > 1.2 \text{ in} \quad \text{O.K.}$$

$b_f = 6.99$ in, $d = 15.9$ in., $b_f /d = 0.439 < ⅔$ ……. $U = 0.85$

$$d_e = d_b + 1/8 = 7/8 + 1/8 = 8/8 = 1 \text{ in}$$

$$A_n = A_g - 4d_e t_f = 10.6 - 4(1)(0.43) = 8.88 \text{ in}^2$$

$$\phi_t P_{n2} = 0.75 F_u * A_e = (0.75)(70 \text{ ksi})(0.85*8.88) = 396 \text{ kips} < P_u = 415 \quad \text{…….N.G.}$$

Select the next heavier section, a W16×40:

$$A_g = 11.8 \text{ in}^2 > 9.22 \text{ in}^2 \quad \& \quad r_y = 1.57 > 1.2 \text{ in} \quad \text{O.K.}$$

$$b_f = 7.00 \text{ in}, d = 16 \text{ in.}, b_f /d = 0.4375 < ⅔ \text{ ……. } U = 0.85$$

$$d_e = d_b + 1/8 = 7/8 + 1/8 = 8/8 = 1 \text{ in}$$

$$A_n = A_g - 4d_e t_f = 11.8 - 4(1)(0.505) = 9.78 \text{ in}^2$$

$\phi_t P_{n2} = 0.75\, F_u * A_e = (0.75)(70 \text{ ksi})(0.85*9.78) = 436 \text{ kips} > P_u = 415$ …….O.K.

CHAPTER THREE

CONNECTORS

3-1 Overview

The primary structural fasteners used in steel construction have typically been rivets, bolts and pins. These fasteners can be field installed, cheaper and with less problems than welding.

Bolts are generally installed so that they are either perpendicular to the force (i.e. the force causes shear in the fastener) or parallel to the force (i.e. the force causes tension in the fastener) that they are transferring between members. In some cases they have both shear and tension.

Rivets have essentially disappeared from modern steel construction, One thing to note is that rivets provide a very inconsistent clamping force so determining friction capacity for shear transfer is problematic. The capacity of rivet connections is best done considering only the bearing capacity.

Pins are generally smooth large diameter fasteners that are not threaded. These fasteners are not very common. Pins are always placed perpendicular to the load direction and are in shear. Since pins are not threaded, they do not clamp the connected members together and, consequently, do not enable friction based force transfer between the connected members.

Welding is the process of joining two steel pieces (the base metal) together by heating them to the point that molten filler material mixes with the base metal to form one continuous piece.

This chapter will focus principally on the capacity of bolts and welding as they are the preferred structural steel fastener.

3-2 Bolted Connections

Where the load direction is perpendicular to the bolt axis as shown in Figure 3-1-1. In this situation the principle force in the bolt is shear. Less frequently, the bolts are placed such that their axis is parallel to the direction of force as shown in Figure 3-1-2. Here the principle force in the bolts is tension. Then the failure of the connection results either from exceeding the shear capacity of the bolt or one of the bearing limit states discussed with tension members.

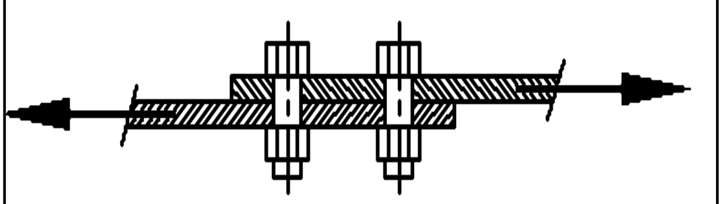


Figure 3-1-1 Bolts in Shear

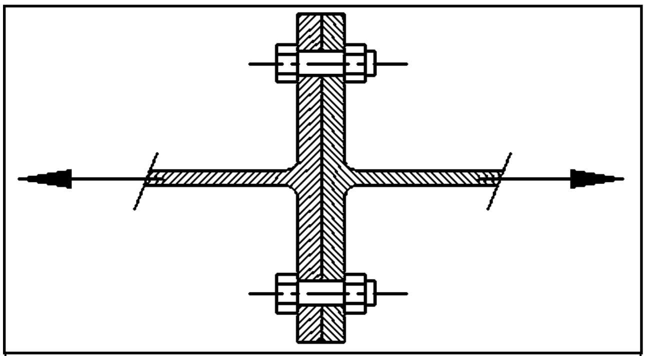


Figure 3-1-2 Bolts in Tension

3-2-1 Design Strength of Bolts in Shear

If the connections are to place in a tension test, shown in Figure 3-1-1, the force vs deformation curve would look something like what is shown in Figure 3-2.

As the load is progressively applied to the connection, the major force transfer between the connected plates would be by friction. The friction capacity is the result of the normal force (N) between the plates created by the bolt tension and the roughness of the contact surfaces (quantified by the friction coefficient, μ). Once the applied force exceeds the friction capacity (i.e. the nominal slip capacity), the connected members slip relative to each other until they bear on the bolts. After slip occurs the force is then transferred by bearing between the edge of the hole and the bolt to the bolt. The bolt carries the force by shear to the adjacent connected plate where it is transferred to the plate by bearing between the bolt and the edge of the hole.

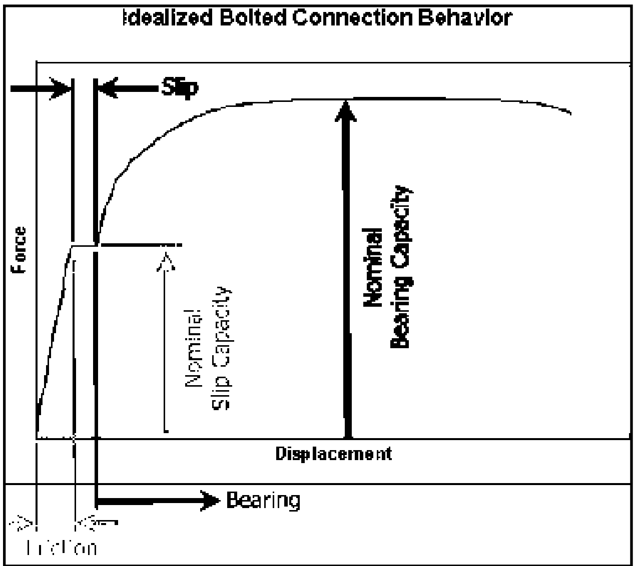


Figure 3-2 Bolt Load/Deformation Curve

As can be seen in Figure 3-2, every connection will have two shear capacities:

- The capacity to carry load without slip and
- The capacity to carry load without shear failure of the bolts

The first is called the **NOMINAL SLIP CRITICAL** capacity.

The second is called the **NOMINAL BEARING** capacity.

In a snug tight connection slip occurs at much smaller loads so the nominal slip capacity is negligible. The only capacity available for a snug tight connection is the nominal bearing capacity.

The location of maximum shear in the bolt is commonly referred to as a **SHEAR PLANE**. The bolt depicted in Figure 3-3-1 is referred to as a "single shear bolt" since it has only one critical shear plane. It is possible to have more than one critical shear plane. Figure 3-3-2 shows a bolt that has two critical shear planes. These bolts are said to be in "double shear" and can transfer twice as much force as a bolt in single shear. It is possible to have even more planes of shear.

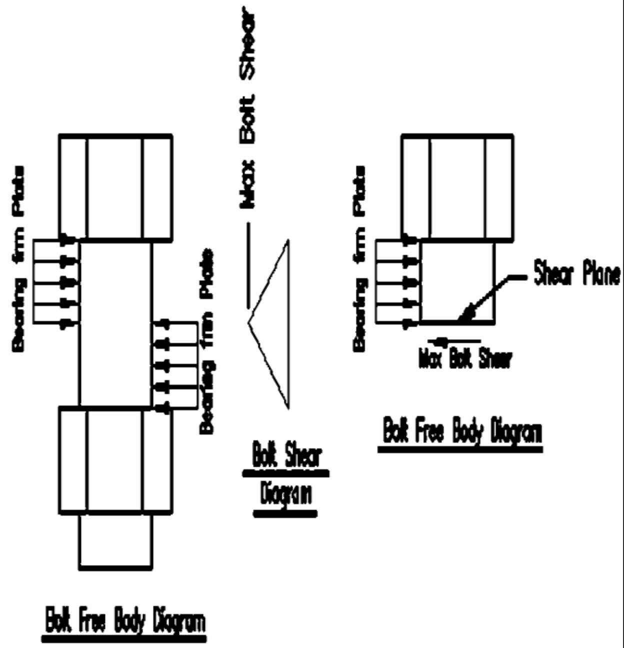


Figure ٣-٣-1Single Shear Bolt

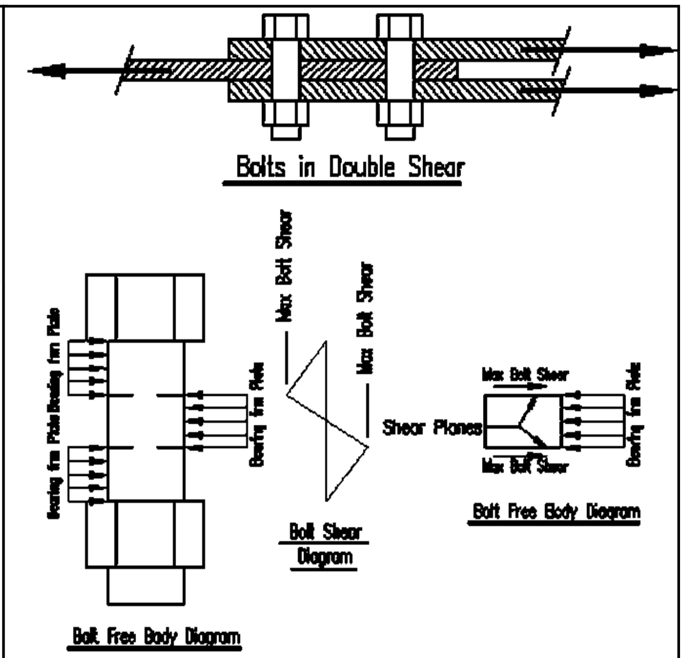


Figure 3-3-2 Double Shear Bolt

In this case R_{nv} is the nominal shear strength *<u>of a shear plane</u>* is computed using the equation:

$$R_{nv} = F_{nv}A_bN_s$$

Where: F_{nv} is obtained from LRFD Table J3.2

A_b is the nominal cross sectional area of the bolt ($\pi d_b^2/4$)

- N_s is the No. of shear planes

$$\varphi R_{nv} = \varphi F_{nv}A_bN_s$$

$$R_{dv} = \varphi R_{nv}\, N_b$$

Where: φ =0.75

- R_{dv} is design shear strength of connector in joint
- φR_{nv} is design shear strength per bolt

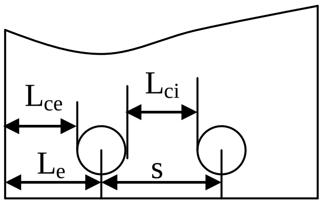
- N_b is the No. of bolts

While R_{nb} is the nominal bearing strength *of a shear plane* is computed using the equation:

$R_{nb} = 1.2\ F_u t\ L_c \leq 2.4\ F_u t\ d_b$

Where: L_c = clear distance $L_{ce} = L_e - 0.5d_h$ (at edge)

$L_{ci} = s - d_h$ (internal)



- F_u is ultimate tensile stress of member material, ksi
- t is the thickness of member, in
- d_b is the nominal diameter of bolt, in
- d_h is the diameter of hole, in …. $d_h = d_b + 1/16$
- L_e = end distance (Table J3.4, pp107 in LRFDM)

- Min spacing of bolt $s \geq 2\ ⅔d$ a distance of 3*d* is preferred.

$$R_{db} = \varphi R_{nb} * N_b = (\varphi R_{nbe} * N_{be}) + (\varphi R_{nbi} * N_{bi})$$

$$R_d = \min[R_{dv}, R_{db}]$$

Where: $\varphi = 0.75$

- R_{nb} is nominal bearing strength per bolt
- φR_{db} is design bearing strength per bolt
- N_{be} is the No. of external bolts
- N_{bi} is the No. of internal bolts
- R_d is design strength of connector in joint

3-2-2 Design Strength of A Bolt in Tension

The mechanics of a bolt in tension are less complicated than for a bolt in shear. In this case there is no slip to consider. Also there are no shear planes. The capacity of a bolt is the same regardless of the number of plates being connected together. The tensile force is parallel to the bolt axis and is considered to be concentric with the bolt's cross sectional area, resulting in uniform stress across the section as depicted in Figure 3-4.

As tensile load is applied to a connection it will reduce the contact pressure between connected members. The bolts see no tensile force beyond the pretension force until the contact stress between the connected members is overcome.

In this case R_{nt} the nominal tensile strength *of a bolt* is computed using the equation:
$R_{nt} = F_{nt}\,A_b$

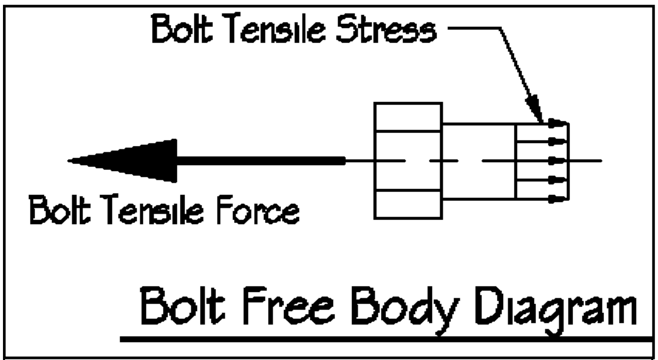


Figure 3-4 Bolt in Tension

Where: F_{nt} = nominal tensile strength per unit area, obtained from ASIC-LRFD manual, Table J3.2 (p. 107), as:

- F_{nt} = 90 ksi for A325 bolts
  113 ksi for A490 bolts

- A_b is the nominal cross sectional area of the bolt ($\pi d_b^2/4$)

$$\varphi R_{nt} = \varphi F_{nt} \,.A_b$$

$$R_{dt} = \varphi R_{nt} * \text{No. of bolt}$$

Where: φ =0.75 and R_{dt} is design tensile strength of a connector

3-2-3 Design Strength of A Bolt in Combined Shear and Tension

The bolts in wind bracing connections are often subjected to both shear and tension under applied loads. The interaction of applied shear and tension creates a situation where the principle stress is neither perpendicular nor parallel to the axis of the fastener. Figure 3-5 shows a connection where the bolts see both shear and tension.

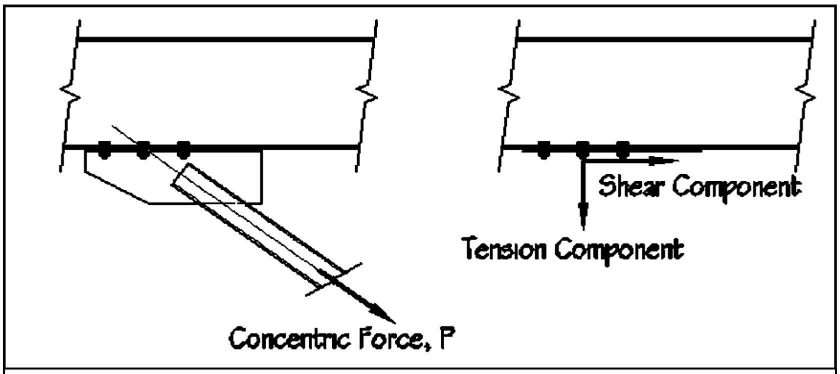


Figure 3-5 Connection with Combined Shear and Tension

The elliptic interaction formula approach can be used:

$$\left(\frac{T_u}{\phi R_{nt}}\right)^2 + \left(\frac{V_u}{\phi R_{nv}}\right)^2 \leq 1.0$$

Where: T_u is the factored tensile load in bolt

- φR_{nt} is design tensile strength of a high-strength bolt
- V_u is the factored shear load in bolt
- φR_{nv} is design shear strength per bolt
- T_u is the factored tensile load in bolt

For combined shear and tension, equations for tension stress limit are given in the ASIC-LRFD manual, Table J3.5 (p. 6-84), as:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\varphi F_{nv}} f_v \leq F_{nt}$$

***Example Problem* 3-1:** Determine the max. axial tensile load P (30% dead load & 70% live load) that can be transmitted by the bolts in the butt splice shown in Figure 3–6. . The main plates are ½-in. thick, and the cover plates are ⅜-in. thick. Assume 1-in. dia. A 490 bolts in standards holes with threads eXcluded from shear planes . The plates are of A572 Gr 55 steel.

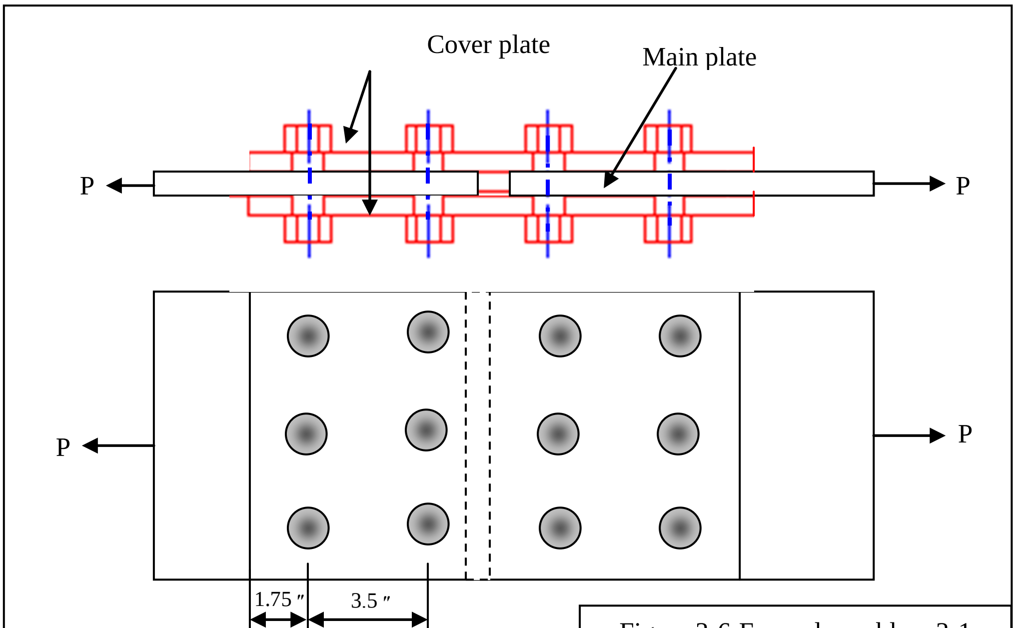


Figure 3-6 Example problem 3-1

Solution: - Design shear strength

From table **J 3.2** the value of F_{nv} for A490 bolt with eXcluded threads is 75.0 ksi

$\varphi R_{nv} = \varphi F_v A_b N_s = 0.75 * 75 * \pi(1)^2/4 * 2 = 88.4$ kips

$R_{dv} = \varphi R_{nv} N_b = 88.4 * 6 = 530.4$ kips

- Design bearing strength

@ edge bolt: $L_{ce} = L_e - 0.5\, d_h = 1.75 - 0.5(1 + 1/16) = 1.22$ in.

$\varphi R_{nbe} = 0.75(1.2\, F_u t\, L_c) = 0.75*1.2*70*1.22*1/2 = 38.4$ kips

@ interior bolt: $L_{ci} = s - d_h = 3.5 - (1 + 1/16) = 2.44$ in.

$\varphi R_{nbi} = 0.75(1.2\, F_u t\, L_c) = 0.75*1.2*70*2.44*1/2 = 76.9$ kips $> 0.75(2.4\, F_u t\, d_b)$ $= 0.75*2.4*70* ½ * 1 = 63$ kips

Take $\varphi R_{nbi} = 63$ kips

$R_{db} = (N_{be} * \varphi R_{nbe}) + (N_{bi} * \varphi R_{nbi}) = (3*38.4) + (3*63) = 304.2$ kips

$R_d = \min(R_{dv}, R_{db})$ .... $R_d = 304.2$ kips $\geq P_u$

$P_u = 1.2DL + 1.6LL = 1.2(0.3\, P_s) + 1.6(0.7\, P_s) \leq 478$ kips

$P_{s,\,max} = 205.5$ kips

***Example Problem* 3-2:** A lap joint connecting two ½-in. plates transmits axial service tensile loads P_D = 60 kips and P_L=60 kips using 1-in. dia. A325 high-strength bolts in standard holes with threads iNcluded in the shear plane. Assume A572 Gr 50 steel. Determine the No. of bolt required for a bearing type joint.

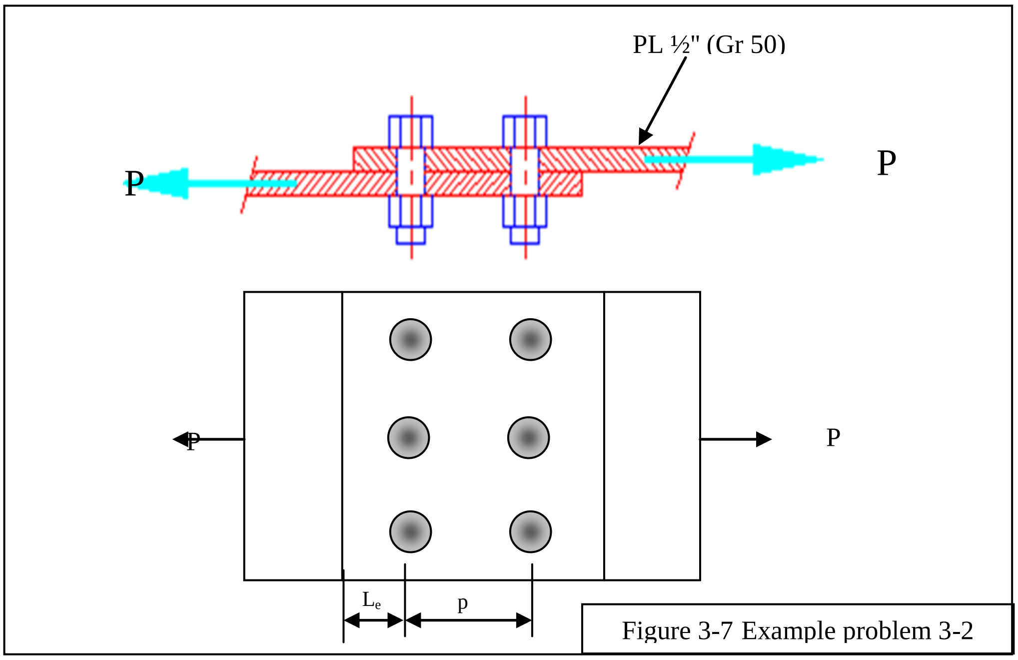


Figure 3-7 Example problem 3-2

Solution: -

P_u= 1.2DL + 1.6LL = 1.2(60) + 1.6(60) = 168 kips

$\varphi R_{nv} = \varphi F_{nv} A_b N_s = 0.75*48*\pi(1)^2/4*1 = 28.3$ kips

$\varphi R_{nbi} = \varphi R_{nbe} = 0.75(2.4\ F_u t\ d) = 0.75*2.4*65*1*½ = 58.5$ kips

R_d = min. (R_{dv}, R_{db}) = min. ($N_b\ \varphi R_{nv}$, $N_b \varphi R_{nb}$) .... 28.3 N_b kips $\geq P_u$

No. of bolts (N_b) = 168/28.3 = 5.9

Provide 6 bolts. That is 3 bolts in each vertical row.

Example Problem 3-3: Determine the required number of ¾ in. diameter A490 bolts for the connection below. It subjected to axial service tensile loads P_D = 14 kips and P_L=126 kips.

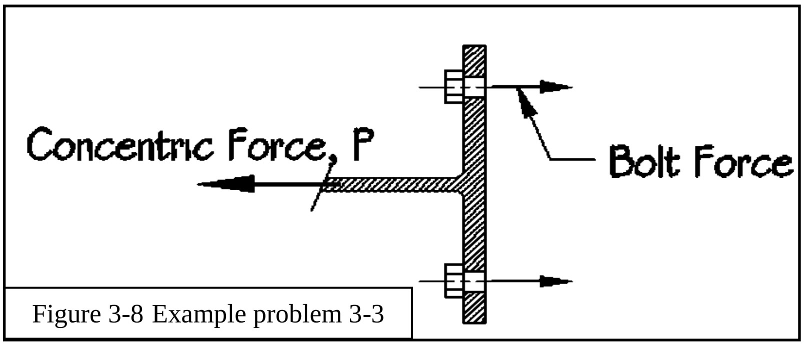


Figure 3-8 Example problem 3-3

Solution: -

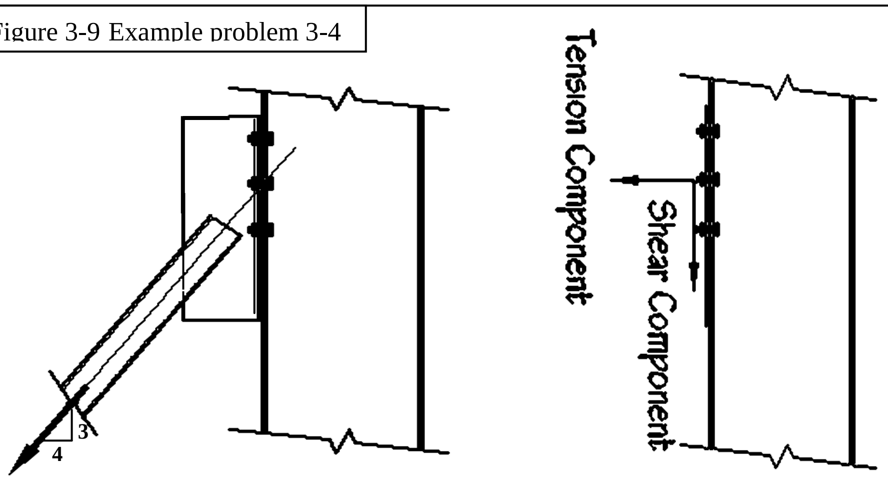
P_u= 1.2DL + 1.6LL = 1.2(14) + 1.6(126) = 218.4 kips

$\varphi R_{nt} = \varphi F_{nt} A_b = 0.75 * 113 * \pi(¾)^2/4 = 37.5$ kips

No. of bolts = $P_u / \varphi R_{nt}$ = 5.8 say 6 bolts

***Example Problem* 3-4:** A WT10.5×31 A36 Gr.36 is used as a bracket to transmit axial service tensile loads P_D = 15 kips and P_L=45 kips. Determine the adequacy of the ⅞ in. diameter A325bolts with threads in shear plan for the connection below.

Figure 3-9 Example problem 3-4



Solution: -

P_u= 1.2DL + 1.6LL = 1.2(15) + 1.6(45) = 90 kips

- The bolts in shear:

$V_{u,\ total}$ = 3/5 * 90 = 54 kips

- Shear strength:

$\varphi R_{nv} = \varphi F_{nv} A_b N_s = 0.75 * 48 * \pi(7/8)^2/4 = 21.7$ kips

$R_{dv} = \varphi R_{nv}\ N_b = 21.7 * 3 = 64.95$kips

- Bearing strength: the flange of TW-section controls

t_f = 0.615 "

$\varphi R_{nb} = 0.75(2.4\ F_u\ d_b t) = 0.75*2.4*58*7/8*0.615 = 56.18$ kips

$R_{db} = N_b * \varphi R_{nb} = 3 * 56.18 = 168.54$ kips

$R_d = \min(R_{dv}, R_{db})$ .... $R_d = 64.95$ kips > V_u = 54 kips O.K.

- The bolts in tension:

$T_{u,\ total}$ = 4/5 * 90 = 72 kips,

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\varphi F_{nv}} f_v \le F_{nt}$$

$f_v = V_u/(A_b$* No. of bolts) = 54/(0.6016*3) = 22.45 ksi

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)} 22.45 = 67.11\text{ksi} < 90 \text{ ksi} \quad \text{O.K.}$$

$\varphi R_{nt} = \varphi F'_{nt}\ A_b = 0.75 * 67.11 * \pi(7/8)^2/4 = 30.3$ kips

$R_{dt} = \varphi R_{nt}$ * No. of bolts = 3*30.3 = 90.9 kips >T_u = 72 kips O.K.

3-3 Welded Connections

Welding is the process of joining two steel pieces (the base metal) together by heating them to the point that molten filler material mixes with the base metal to form one continuous piece.

There are many welding processes, however the two most common processes used in structural steel fabrication:

- **Shielded Metal Arc Welding (SMAW):** A manual process that is typically used when welding in the field. It is also used frequently when welding in a fabrication shop.
- **Submerged Arc Welding (SAW):** An automated welding process that frequently used when welding in a fabrication shop.

There are five basic types of welded joints, as depicted in Figure 3-10: Butt Joints, Lap Joints, Tee Joints, Corner Joints, and Edge joints

The basic weld types are groove welds, fillet welds, slot & plug welds.

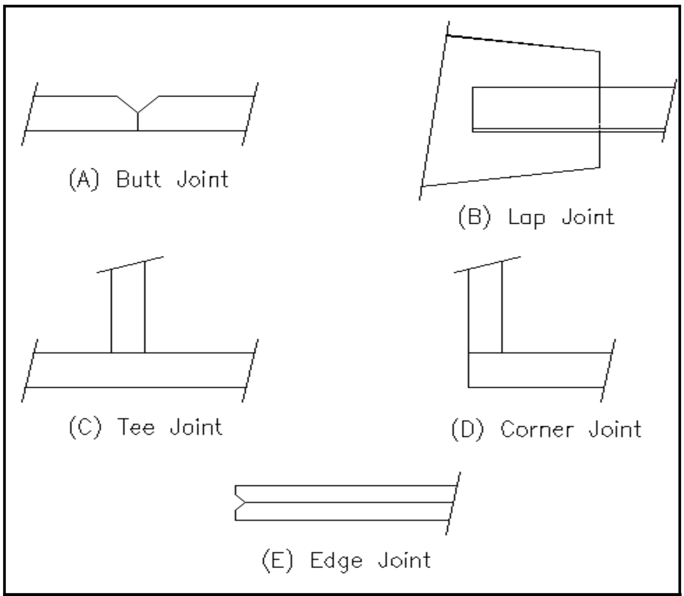


Figure 3-10 Welded Joint Types

Groove Welds: Groove welds are generally used to fill the gap between the two pieces being connected. Groove welds are considered to be either "complete joint penetration" (CJP) or "partial joint penetration" (PJP).

A CJP weld completely fills the gap between the two pieces as shown in Figure 3-11 parts A, B, and C. A PJP weld only fills a portion of the gap as seen in Figure 3-11 parts D, E, F, and G.

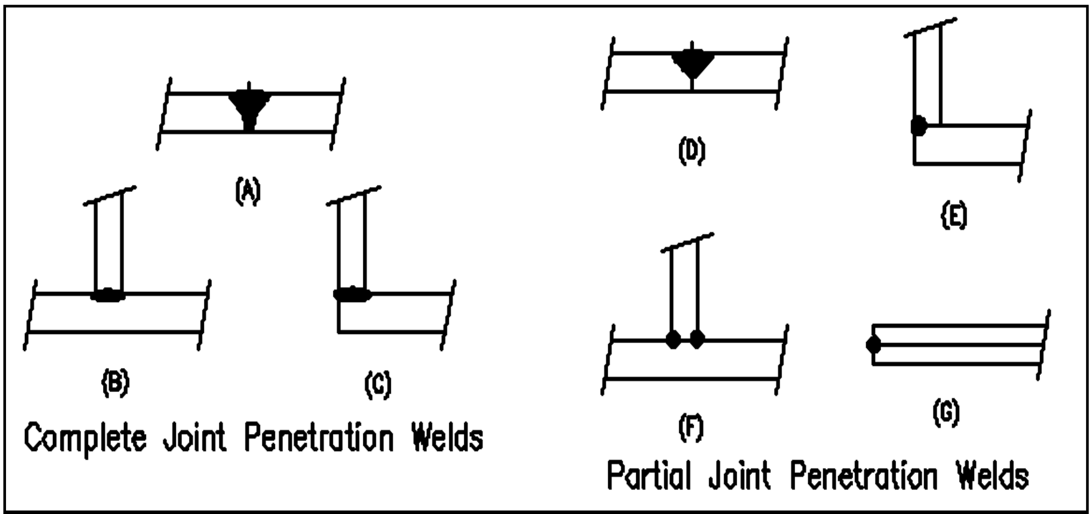


Figure 3-11 Groove Weld Examples

Fillet Welds: Fillet welds do not penetrate the gap between the parts being connected. A fillet weld generally has a triangular cross section with one leg of the triangle being attached to each piece being connected. Fillet welds are very common and are used for a variety of connections. A typical fillet weld is shown in Figure 3-12

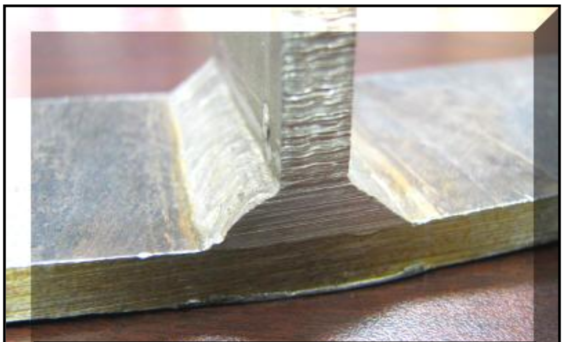


Figure 3- 12 Typical Fillet Welded T

Slot & Plug Welds: Slot & Plug welds are similar to fillet welds in that they do not penetrate the gap between the parts being connected. These welds fill a slot or hole in one of the pieces being connected with the connection being between the edge of the slot or hole on the one piece and the surface of the other piece.

3-4 Size and Effective Area of Fillet Welds

- **Minimum allowed size of fillet welds:** the minimum size of fillet welds shall not be less than size shown in Table J2.4 (LRFDM, pp. 96). This means that the weld needs to be big enough to heat the base material sufficient to create a good bond between the base metal and the weld metal.

- **Maximum allowed size of fillet welds:** The specification limits the weld size (LRFDM, J2.2b, pp. 96):

$$\omega_{, \max} \leq t \;\ldots\ldots\ldots \quad \text{if } t < ¼"$$
$$\omega_{, \max} \leq t - 1/16" \;\ldots\ldots\; \text{if } t \geq ¼"$$

Where t is the thickness of thickest connected member.

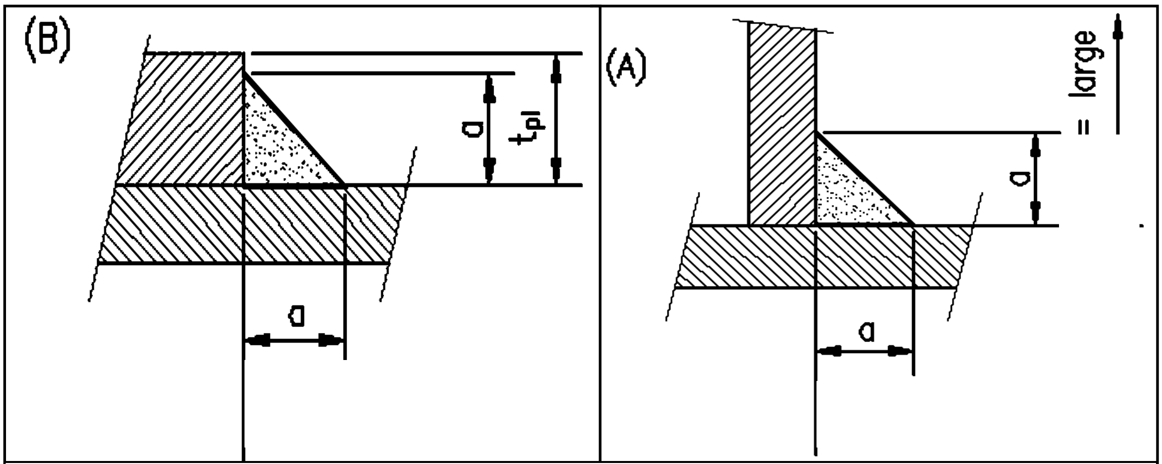


Figure 3-13-1 Maximum Fillet Weld Size

- ***Throat size of fillet weld:*** The effective thickness of throat, $\mathbf{t_e}$, for a fillet weld is taken as the least distance from the root of the weld (i.e. where the two connected pieces meet) to the outer surface of the weld as shown in Figure 3-13-2.

$$t_e = a \sin 45° = 0.707\,\omega$$

Where: t_e = effective throat or effective length of a fillet weld, in.
ω = leg size of a fillet weld, in.

- **Effective Areas:** The effective area of your typical fillet weld equals the effective throat times the length of the weld as shown in Figure 3-13-3.

$$A_w = t_e * L_w$$

Where L_w = gross length of a fillet weld, in.; $L_w \geq L_{w,\min} = 4\,\omega$
t_e = effective length of a fillet weld, in.
A_w = effective area of a fillet weld, in.^{2}

- The actual weld length should be: $L_w/\,\omega \leq 100$
- If $L_w/\,\omega > 100$ multiply by β
- $\beta = 1.2 - 0.002(L_w/a) \leq 1.0$
- If $L_w/\,\omega > 300$ Take $\beta = 0.6$

(LRFDM, J2.2b, pp. 96)

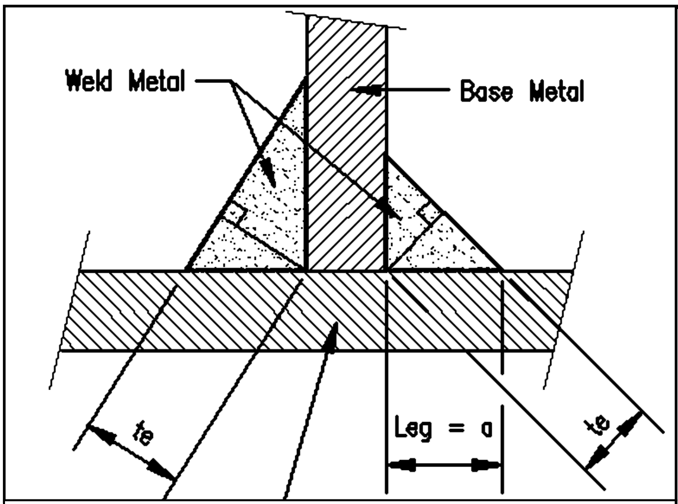


Figure 3-13-2 Fillet Weld Sectional Dimensions

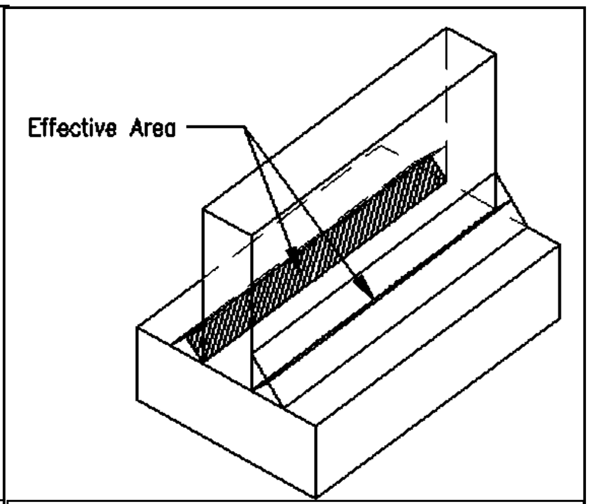


Figure 3-13-3 Effective Area of Fillet Welds

3-5 <u>*Design Strength of Fillet Weld*</u>

In this case two limit state are to be considered; weld metal strength and base metal strength:

- Weld metal design strength

  – Strength based on LRFDS Table J2.5
  $R_{dW} = \varphi F_w A_w = 0.75(0.6F_{EXX})t_e\,L_w = 0.45\,F_{EXX}\,t_e\,L_w$

  Where: A_w = effective area of weld, $in^2 = t_e\,L_w$
  F_w = nominal strength of weld metal, ksi = $0.6F_{EXX}$ (Table J2.5, pp. 100)

  Or $F_w = 0.6F_{EXX}\,[1.0+0.5(\sin\theta)^{1.5}]$ …for linear group loaded in plane through the center of gravity (J2.4, pp. 100)
  θ= angle of loading measured from the weld longitudinal axis, degree

  – Check base metal shear yielding strength
  $\varphi\,R_{BM1} = \varphi(0.6F_y)t_p\,L_w$ $\varphi=1$

  – Check base metal shear rupture strength
  $\varphi\,R_{BM2} = \varphi(0.6F_u)\,t_p\,L_w$ $\varphi=0.75$

  $R_d = \min.\,[R_{dW},\ \varphi\,R_{BM1},\ \varphi\,R_{BM2}]$

Example Problem 3-5: Determine the design shear strength of a 4-in long 5/16 in. fillet weld. Assume SMAW process and E70 electrodes. Assume that the applied load passes through the center of gravity of the weld. The weld is: (a) a longitudinal weld, (b) a transverse load, (c) an oblique weld, with the load inclined at 30^o with axis of the weld. Use: (1) LRFDS Table J2.5; (2) LRFDS Appendix J2.4.

Solution: - Weld size, a = 5/16 in. , Effective length, $L_w = 4.0$ in.
SMAW process: $t_e = a \sin 45° = 0.707\ \omega = 0.707\ (5/16) = 0.221$ in.
E70 electrodes. So, $F_{EXX} = 70.0$ ksi
As no details are given, assume that the base material does not control the design of weld.

1. Strength based on LRFDS Tables J2.5: In this approach, the design strength of the weld is independent of the orientation of the applied load.
   $$R_{dw} = 0.45F_{EXX}\ t_e\ L_w = 0.45*70*0.221*4 = 27.85 \text{ kips}$$

2. Strength based on LRFDS Appendix J2.4

a. Longitudinal weld: θ = 0.0, sin θ = 0.0
$$R_{dw\,(\theta=0.0)} = 0.45F_{EXX}\ t_e\ L_w\ [1.0 + 0.5\ (\sin\theta)^{1.5}]$$
$$= 0.45*70*\ 0.221*4\ [1.0 + 0.0] = 27.85 \text{ kips}$$

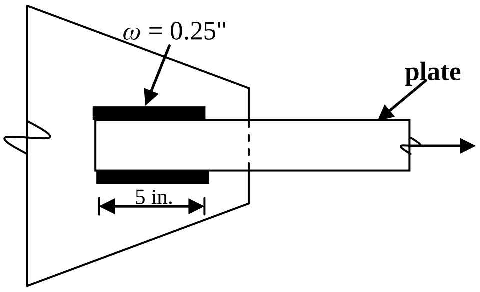
b. Transverse weld: θ = 90.0, sin θ = 1
$$R_{dw\,(\theta=90.0)} = 0.45F_{EXX}\ t_e\ L_w\ [1.0 + 0.5\ (\sin\theta)^{1.5}]$$
$$= 0.45*70 * 0.221*4\ [1.0 +0.5\ (1)^{1.5}] = 41.8 \text{ kips}$$

c. Transverse weld: θ = 30.0, sin θ = 0.5
$$R_{dw\,(\theta=30.0)} = 0.45F_{EXX}\ t_e\ L_w\ [1.0 + 0.5\ (\sin\theta)^{1.5}]$$

$$= 0.45*70*\ 0.221*4\ [1.0 +0.5\ (0.5)^{1.5}] = 32.8 \text{ kips}$$

Observe that the transverse weld is 50% stronger than the longitudinal one and the oblique weld 17.7%. the LRFDS Table J5.2 ignores this additional strength.

Example Problem 3-6: Determine the design strength of the tension member and connection system shown below. The tension member is a 4 in. × ⅜ in. thick rectangular bar. It is welded to a ½ in. thick gusset plate of A572 Gr 50 steel, using E70XX electrode. Consider the shear strength of the weld metal and the surrounding base metal.



Solution: -

– Check size limitation of weld

t_{bar}= ⅜ " & t_{plate}= ½ "

$\omega_{, max}$ = t - 1/16" ……. t > ¼"

= 1/2 - 1/16 = 7/16"

$\omega_{, min}$ = 3/16 (Table J2.4)

$\omega_{, min}$ = 0.1875" < ω = 0.25" < $\omega_{, max}$ = 0.4375" ……. OK

L_w = 5" > $L_{w, min}$ = 4 ω= 4*0.25= 1" ………OK

- L_w/ω = 5/.25= 20< 100 ……… β=1.0

– Design strength of the weld

$R_{dW} = \varphi F_w A_w = 0.45 F_{EXX}\, t_e\, L_w = 0.45 F_{EXX}\, (0.707\,\omega)\, L_w$

= 0.45*70 * (0.707*0.25) * 10 = 55.68 kips

– Check base metal shear yielding strength

$\varphi\, R_{BM1} = \varphi(0.6F_y)t_p\, L_w$ = (1) (0.6*36 ksi) (10*3/8 in^2) = 80 kips

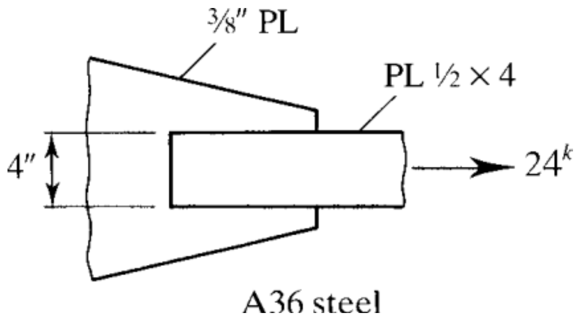
– Check base metal shear rupture strength

$\varphi\, R_{BM2} = \varphi(0.6F_u)t_p\, L_w$ = (0.75) (0.6*58 ksi) (10*3/8 in^2) = 97.9 kips

– Design strength of the system

R_d=55.68 kips

Example 3-7: A plate 1 ⁄2* 4 " of A36 steel is used as a tension member to carry a service dead load of 6 kips and a service live load of 18 kips. It is to be attached to a 3⁄8-inch gusset plate, as shown in Figure. Design a welded connection.



Solution:

The base metal is A36 steel, so E70XX electrodes will be used.
t_{min} = 3/ 8" (gusset plate)
t_{max}=0.5" (Member)
Thus from Table J2.4 of AISC with t_{min} = 3/ 8" thus

$$w_{min} = 3/16 \text{in}$$

For t_{max} = 0.5

$$w_{max} = t - 1/16 = 0.5 - 1/16 = 7/16 \text{in}$$

design w= 3/16 in
Shear strength of the weld

$$F_w = 0.6 F_{EXX} = 0.6*70 = 42 \text{ksi}$$

$$\phi R_n = 0.75 * F_w * 0.707 * w * L$$
$$= 0.75*42*0.707*3/16*L = 4.176\ L$$

Base Metal Strength

$$\phi R_n = \min\{1.0\,(0.6\,F_y\,t\,L), 0.75\,(0.6\,F_u\,t\,L)\}$$

$$\phi R_n = \min\begin{Bmatrix} 1.0\,(0.6*36*3/8*L), \\ 0.75\,(0.6*58*3/8*L) \end{Bmatrix}$$

$$\phi R_n = \min\{8.1L, 9.79L\} = 8.1\ L$$

Thus weld strength control and the connection strength = 4.176L

$$P_u = 1.2P_D + 1.6P_L = 1.2*6 + 1.6*18 = 36 \text{kips}$$

$$\phi R_n = 4.176L = P_u$$

$$\text{Required length}, L = \frac{36}{4.176} = 8.62 \text{in}$$

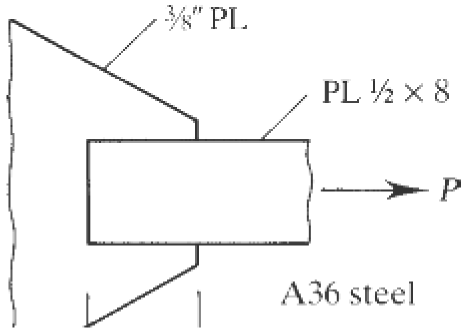
L=8.62>L_{min}=4w=4*3/16=0.75 in OK

Use two 4.5 in side weld

Total length=2*4.5=9 > 8.62

Length of side weld = 4.5 > transverse distance between welds = 4 " OK

Example 3-8: A plate 1⁄2*8 of A36 steel is used as a tension member and is to be connected to a 3⁄8- inch-thick gusset plate, as shown in Figure.. Design a weld to develop the full tensile capacity of the member. Use U=1.0.



Solution

Find design tensile strength of member.

- Yielding of gross area

$$\phi P_n = 0.9\, F_y\, A_g = 0.9*36*(0.5*8) = 129.6\,\text{kips}$$

Fracture of net area

$$A_e = UA_g = 1.0*0.5*8 = 4\ \text{in}^2$$

$$\phi P_n = 0.75\, F_u\, A_e = 0.75*58*(0.5*8) = 174.0\,\text{kips}$$

Thus design tensile strength = P_u = 129.6 kips

The base metal is A36 steel, so E70XX electrodes will be used.

t_{min} = 3/ 8" (gusset plate)
t_{max}=0.5" (Member)
Thus from Table J2.4 of AISC with t_{min} = 3/ 8" thus

$$w_{min} = 3/16\,\text{in}$$

For t_{max} = 0.5

$$w_{max} = t - 1/16 = 0.5 - 1/16 = 7/16\,\text{in}$$

design w= 3/16 in

Shear strength of the weld

$$F_w = 0.6 F_{EXX} = 0.6*70 = 42\,\text{ksi}$$

$$\phi R_n = 0.75*F_w*0.707*w*L$$
$$= 0.75*42*0.707*3/16*L = 4.176\ L$$

Base Metal Strength

$$\phi R_n = \min\left\{1.0\left(0.6\, F_y\, t\,L\right), 0.75\left(0.6\, F_u\, t\,L\right)\right\}$$

$$\phi R_n = \min\left\{\begin{matrix} 1.0\left(0.6*36*3/8*L\right), \\ 0.75\left(0.6*58*3/8*L\right) \end{matrix}\right\}$$

$$\phi R_n = \min\left\{8.1L, 9.79L\right\} = 8.1\ L$$

Thus weld strength control and the connection strength = 4.176L

$$P_u = 1.2P_D + 1.6P_L = 1.2*6 + 1.6*18 = 36\,kips$$

$$\phi R_n = 4.176L = P_u$$

$$\text{Required length}, L = \frac{129.6}{4.176} = 31.03\,in$$

L=31.03>L_{min}=4w=4*3/16=0.75 in OK

Use two 16 in side weld

Total length=2*16=32 > 31.03

Length of side weld = 16 > transverse distance between welds = 8 " OK

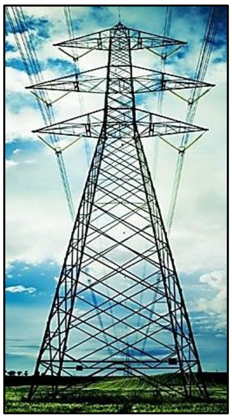
CHAPTER FOUR

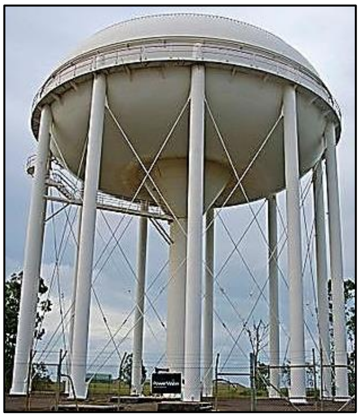
COMPRESSION MEMBERS

4.1 Overview

There are several types of compression members, the column being the best known. Among the other types are the top chords of trusses and various bracing members. In addition, many other members have compression in some of their parts. These include the compression flanges of rolled beams and built-up beam sections, and members that are subjected simultaneously to bending and compressive loads. Columns are usually thought of as being straight vertical members whose lengths are considerably greater than their thicknesses. Compression member: is a structural member which carries pure axial compression loads like compression members in:
Generally the used in:

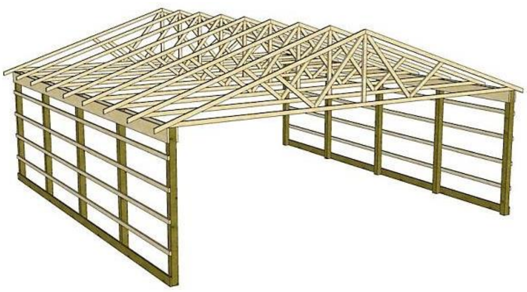
1- ***Column Supports & Towers***





Columns as supports & compressive member in towers

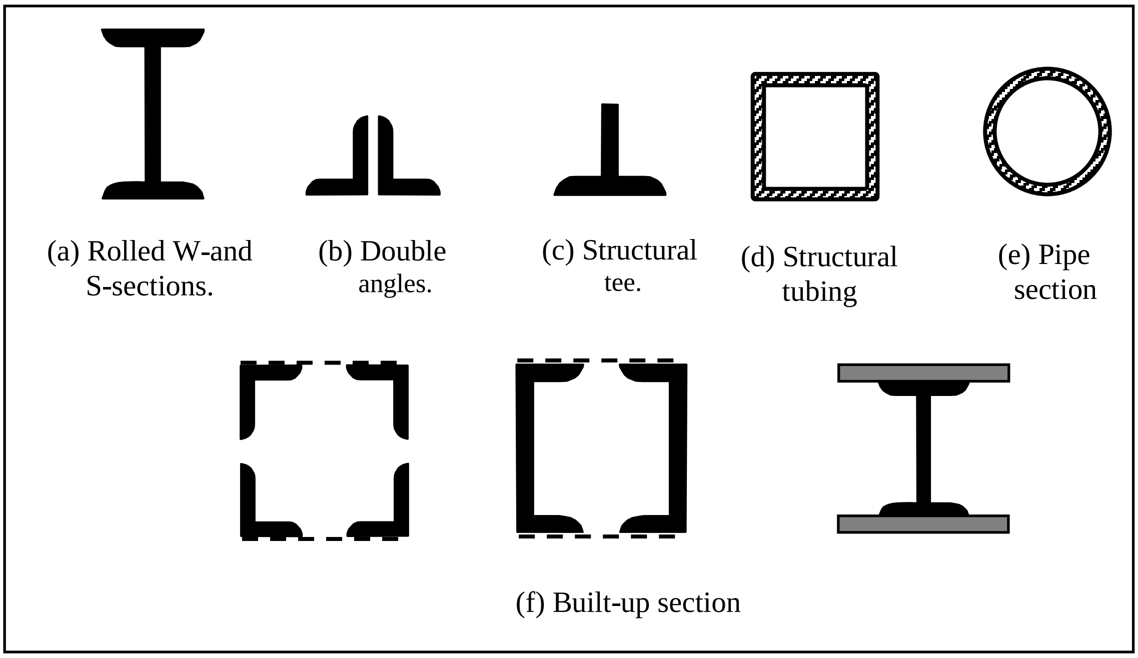
2- ***Trusses & Bridges***




Compressive member in truss

3- ***Columns in building frames***

Steel shapes, which are used as compression members, are shown in the figure below.



Steel shapes used as compression members

The stress in the column cross-section is given by:

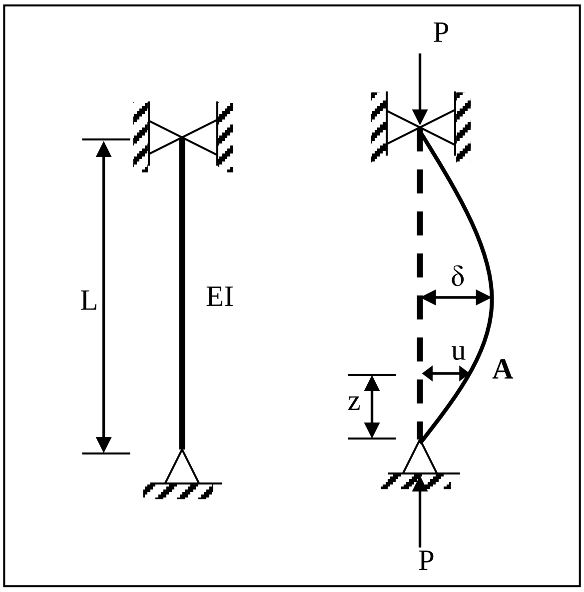
$$\boldsymbol{f} = \mathbf{P/A}$$

Where,
f is compressive stress which is assumed to be uniform over the entire cross-section,
P is the magnitude of load,
A is the cross-sectional area normal to the load.
If the applied load increased slowly, it will ultimately reach a value $\mathbf{P_{cr}}$ that will cause buckling of the column, $\mathbf{P_{cr}}$ is called the critical buckling load of the column, i.e. $\mathbf{P > P_{cr}}$ lead to buckling.

4.2 Elastic Flexural Buckling of a Pin-Ended Column

The deflection at distance **z** is denoted by **u**
Moment equilibrium about **A** in the buckling state gives:

$$\mathbf{M - P.u = 0.0 \;\ldots.\; M = P.u}$$
$$\mathbf{M = EI\Phi = -\,EI\,(d^2u/d^2z) = P.u}$$
$$\mathbf{EI\,(d^2u/d^2z) + P.u = 0.0}$$
$$\mathbf{d^2u/d^2z + (P/EI).\,u = 0.0}$$
$$\mathbf{d^2u/d^2z + \alpha^2.\,u = 0.0 \quad \ldots\ldots\; \alpha^2 = P/EI}$$
$$\mathbf{u = A\sin\alpha z + B\cos\alpha z}$$
$$\mathbf{u = 0.0 \quad @\; z = 0.0 \quad \ldots\ldots\ldots\ldots\; B = 0.0}$$
$$\mathbf{u = 0.0 \quad @\; z = L \;\;\ldots.\; A \neq 0.0}$$
$$\mathbf{then\;\; \sin\alpha L = 0.0 \quad \alpha L = n\pi \quad \ldots\alpha = n\pi/L}$$
Where n = 1, 2, 3 ….
$$\mathbf{Then\;\; P = P_{crn} = (n\pi/L)^2\, EI}$$
Thus, the Euler load of a pin – ended column is: $\mathbf{P_E = P_{cr1} = (\pi/L)^2\, EI}$



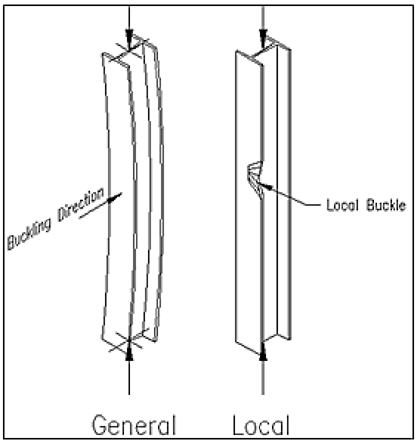
Pin – ended column under axial load

4.3 Buckling Basics

There are two main modes of buckling failure that may be experienced by
steel members: Overall (or general) buckling and local buckling. The Swiss mathematician Leonhard Euler developed an equation that predicts the critical buckling load $\mathbf{P_{cr}}$, for a straight pinned end column. The equation is:

$$\mathbf{P_{cr} = \text{Л}^2 EI/L^2}$$

Where, I = moment of inertia about axis of buckling.
This equation to be valid:



- The member must be elastic
- Its ends must be free to rotate but translate laterally

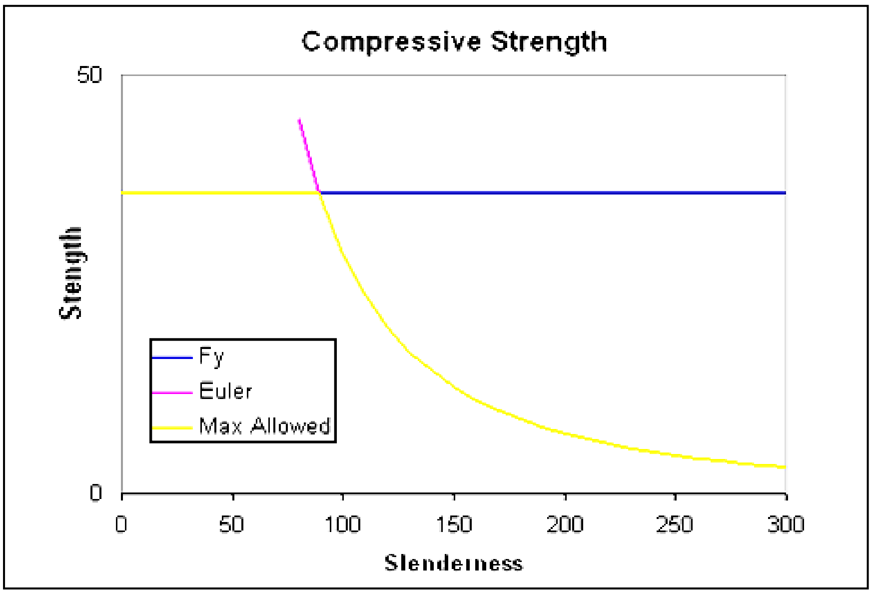
Dividing by the area of the element, we get an equation for the critical buckling stress:

$$\sigma_{cr} = \pi^2 E/(L/r)^2$$

Where the member cross sectional dependent term **(L/r)** is referred to as the "**slenderness**" of the member.

$$\sigma_{max} = \text{minimum}[\pi^2 E/(L/r)^2, F_y]$$

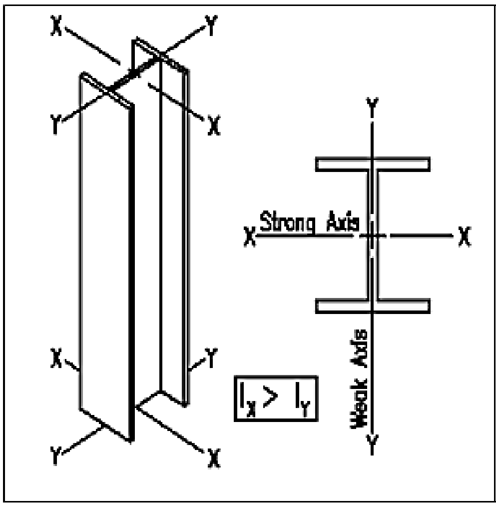
This relationship is graphed in the figure below



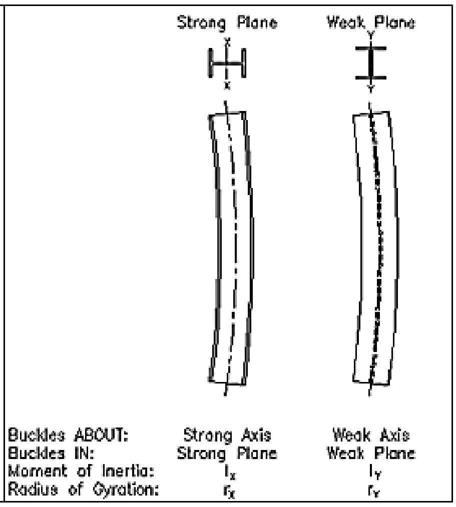
Theoretical Maximum Compressive Stress

4.4 General Member Buckling Concepts

The figure below illustrates the principle axes of a typical wide flange compression member. Other members shapes can be similarly drawn. With the exception of circular (pipe) sections, all the available shapes have a readily identifiable set of principle axes. Buckling is a two dimensional (planar) event. In other words it happens **IN** a **PLANE** that is perpendicular to the **AXIS** that it happens **ABOUT**.



Compression Member Principle Axis



Compression Member Principle Planes

3.4.1 Effective Length Coefficients and End Support Conditions

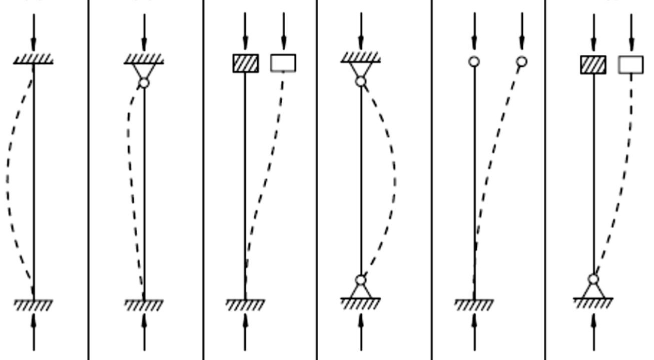
Theoretically, end supports are either pinned or fixed. In reality they can be designed to be pinned or rigid and may actually fall somewhere in between truly pinned or fixed. The support conditions will have an impact on the effective length, $\mathbf{L_e}$. Effective length, $\mathbf{L_e}$, of a compression member is the distance between where inflection points (Inflection point is a location of zero moment) are on a compression member. Effective length can be expressed as:

$$L_e = K\,L \quad \ldots\ldots \quad P_e = \pi^2 EI/(KL)^2$$

Where K is an effective length coefficient, **L** is the actual length of the compression member in the plane of buckling & $\mathbf{P_e}$ is elastic flexural buckling load in column. Different end conditions give different lengths for equivalent half-sine wave as shown in the figure below.

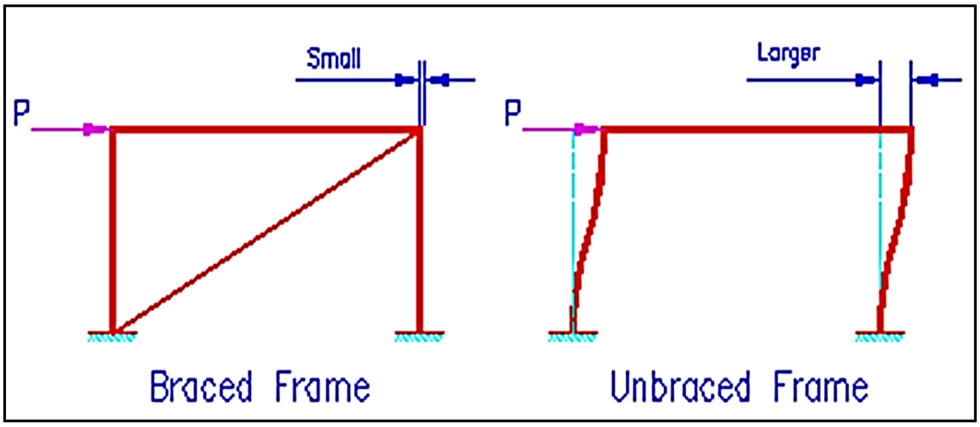
- The theoretical values of effective length coefficients assume that joints are completely fixed against rotation or totally free to rotate. Reality is usually somewhere in between. This affects the value of K.
- Table C-C2.2 is presented in LRFDM p. 240 to predicted the both theoritical and recommended design value of K of isolated column and its depended on support condition.

Table C-2.
Effective Length Factors (*K*) for Columns

Buckled shape of column is shown by dashed line	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*
						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code		Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free				

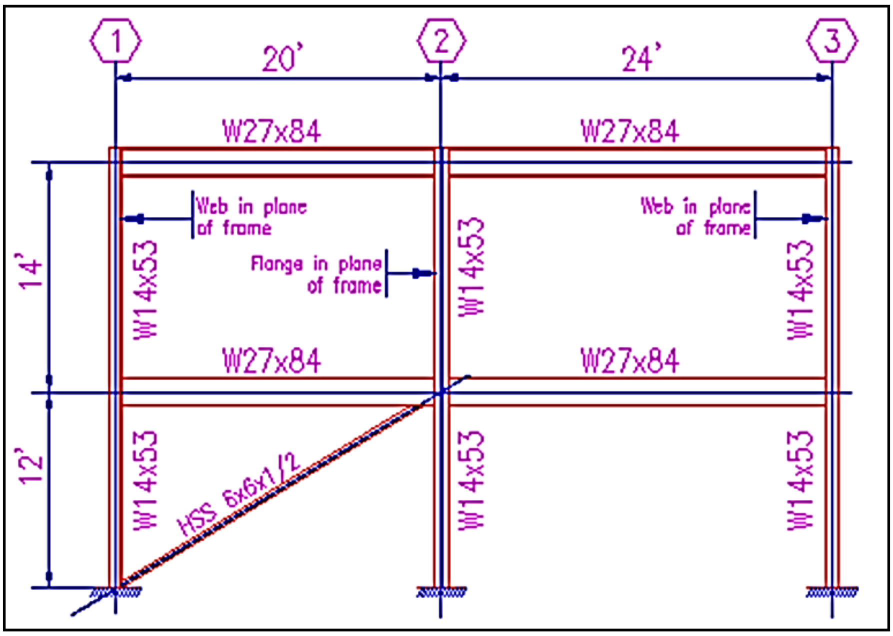
An theoretical effective length coefficient (K) values for different supports conditions for isolated column

- In the building, the cases with no joint translation are considered to be "braced frames" since some kind of bracing between the two levels is necessary to prevent lateral movement under shearing loads. The other cases are referred to as being "unbraced frames".



Braced vs. Unbraced Frame

- The figure below shows that different lengths of the same column can have different effective length coefficients in the same plane of buckling. Consider everything in the plane of buckling. Upper portion of this frame is **UNBRACED**. Lower portion of this frame is **BRACED**.



- Note that all the columns shown have an out-of-plane direction that must also be considered as well. Each direction will have totally independent lateral support and end conditions. It is highly recommended that you to draw both elevations of the column so that you can clearly see the conditions that apply to each column. Also note that the columns may have different laterally unsupported lengths in each direction as well.

- Two Charts are presented in LRFDM p. 241 to predicted the value of K of column in frames. One for braced frames (sidesway inhibited) and one for unbraced frames (sidesway uninhibited). To use these charts you must determine the rotational stiffness, **G**, of each joint in the plane of buckling being considered.

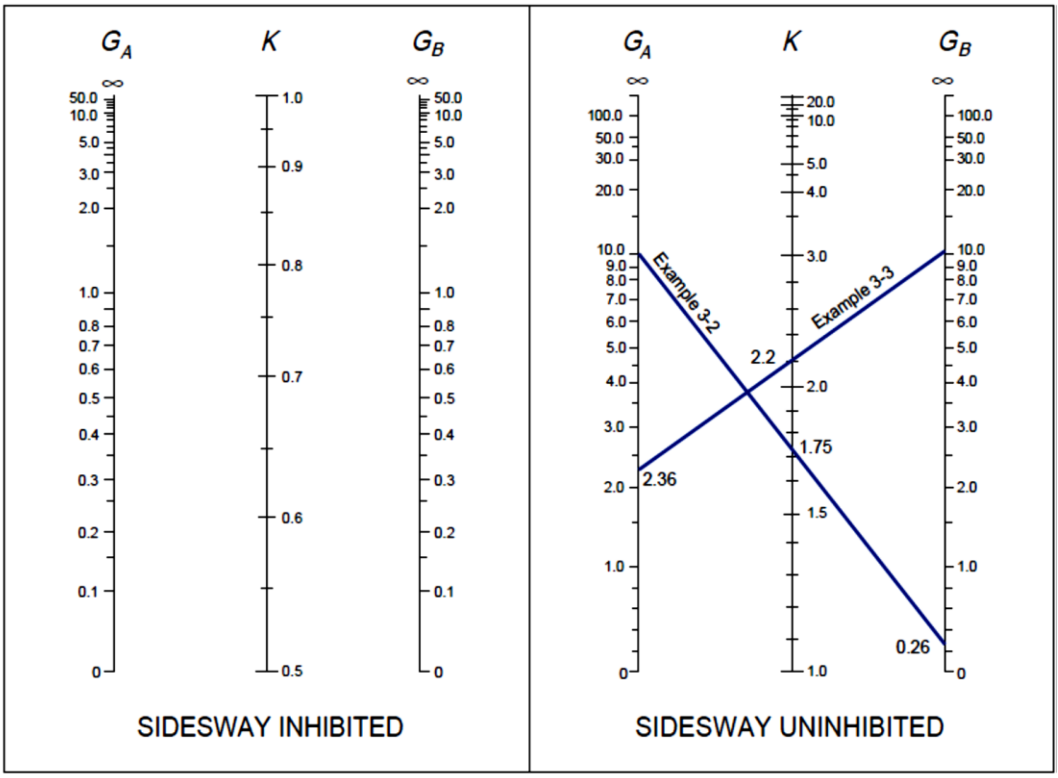
$$G = \frac{\sum \left(I_c / L_c \right)}{\sum \left(I_g / L_g \right)}$$

Where:

Σ indicates a summation of all member rigidity connected to that joint and lying on the plane in which buckling of column is considered.

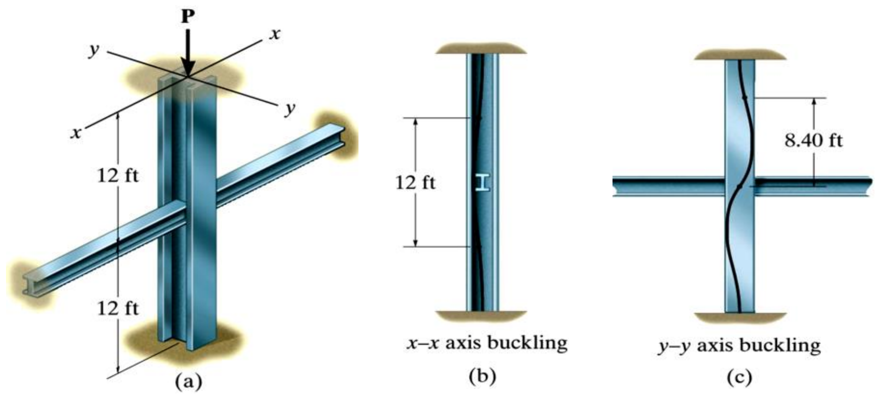
I_c & I_g moment of inertia of column & girder section, respectively.

L_c & L_g unsupported length of column & girder section, respectively.



Alignment charts for effective length of columns in continuous frames. The subscripts A and B refer to the joints at the two ends of the column section being considered.

The figure shows a typical framed joint, and Effective lengths in different directions



Column Ic = Ix
Beam Ig = Ix

Column Ic = Iy
Beam Ig = Iy

Example 3-1*:* Determine the buckling strength of **W12*50** column. Its length **20'**, the minor (weak) axis of buckling pinned at both ends, while major (strong) axis of buckling pinned at one end and fixed at the other end. **E = 29 ksi**.

Solution:

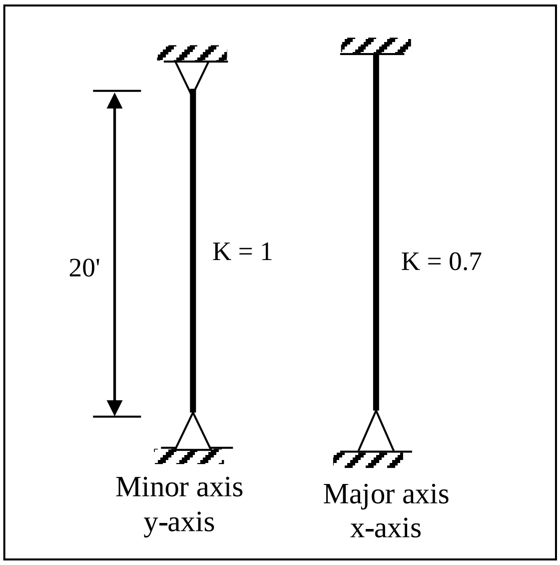
Note: for W-section x-axis is the strong Axis, while y-axis is the weak one.

P 1-39 $I_x = 394\ in^4$, $I_y = 56.3\ in^4$

The buckling Euler strength

$$P_e = Л^2 EI/(KL^2)$$

The value of K of isolated column for different end condition can be predicted from table C-2.2 in LRFDM p. 240.



$P_{e\ (x\text{-}x)} = Л^2(29000)*394/(0.8*20*12)^2$
$= 3035.8$ kips

$P_{e\ (y\text{-}y)} = Л^2(29000)*56.3/(1*20*12)^2$
$= 279.8$ kips

Example 3-2

(a) A W10 × 22 is used as a 15-ft long pin-connected column. Using the Euler expression, determine the column's critical or buckling load. Assume that the steel has a proportional limit of 36 ksi.

(b) Repeat part (a) if the length is changed to 8 ft.

Solution

(a) Using a 15-ft long W10 × 22 ($A = 6.49\text{ in}^2$, $r_x = 4.27$ in, $r_y = 1.33$ in)

Minimum $r = r_y = 1.33$ in

$$\frac{L}{r} = \frac{(12\text{ in/ft})(15\text{ ft})}{1.33\text{ in}} = 135.34$$

$$\text{Elastic or buckling stress } F_e = \frac{(\pi^2)(29 \times 10^3\text{ ksi})}{(135.34)^2}$$

$$= 15.63\text{ ksi} < \text{the proportional limit of 36 ksi}$$

OK column is in elastic range

$$\text{Elastic or buckling load} = (15.63\text{ ksi})(6.49\text{ in}^2) = 101.4\text{ k}$$

(b) Using an 8-ft long W10 × 22,

$$\frac{L}{r} = \frac{(12\text{ in/ft})(8\text{ ft})}{1.33\text{ in}} = 72.18$$

$$\text{Elastic or buckling stress } F_e = \frac{(\pi^2)(29 \times 10^3\text{ ksi})}{(72.18)^2} = 54.94\text{ ksi} > 36\text{ ksi}$$

∴ column is in inelastic range and Euler equation is not applicable.

3.4.2 Long Columns

The Euler formula predicts very well the strength of long columns where the axial buckling stress remains below the proportional limit. Such columns will buckle *elastically*.

3.4.3 Short Columns

For very short columns, the failure stress will equal the yield stress and no buckling will occur.

3.4.4 Intermediate Columns

For intermediate columns, some of the fibers will reach the yield stress and some will not. The members will fail by both yielding and buckling, and their behavior is said to be *inelastic*. Most columns fall into this range. (For the Euler formula to be applicable to such columns, it would have to be modified according to the reduced modulus concept or the tangent modulus concept to account for the presence of residual stresses.)

3.5 Column Formulas

The AISC Specification provides one equation (the Euler equation) for long columns with elastic buckling and an empirical parabolic equation for short and intermediate columns. With these equations, a flexural buckling stress, F_{cr}, is determined for a compression member. Once this stress is computed for a particular member, it is multiplied by the cross-sectional area of the member to obtain its nominal strength P_n.

P_n is the nominal compressive strength of the member is computed by the following equation :

$$nP_n = F_{cr}\,A_g$$

$$P_d = \emptyset_c P_n = \emptyset_c F_{cr}\,A_g \;\; = \text{LRFD compression strength } \boldsymbol{(\emptyset_c = 0.9)}$$

Where:

- F_{cr} is the critical flexural buckling stress.
- A_g is the gross cross sectional area of the member.

The criteria for selecting which formula to use is based on either the slenderness ratio for the member or the relationship between the Euler buckling stress and the yield stress of the material. The selection can be stated as:

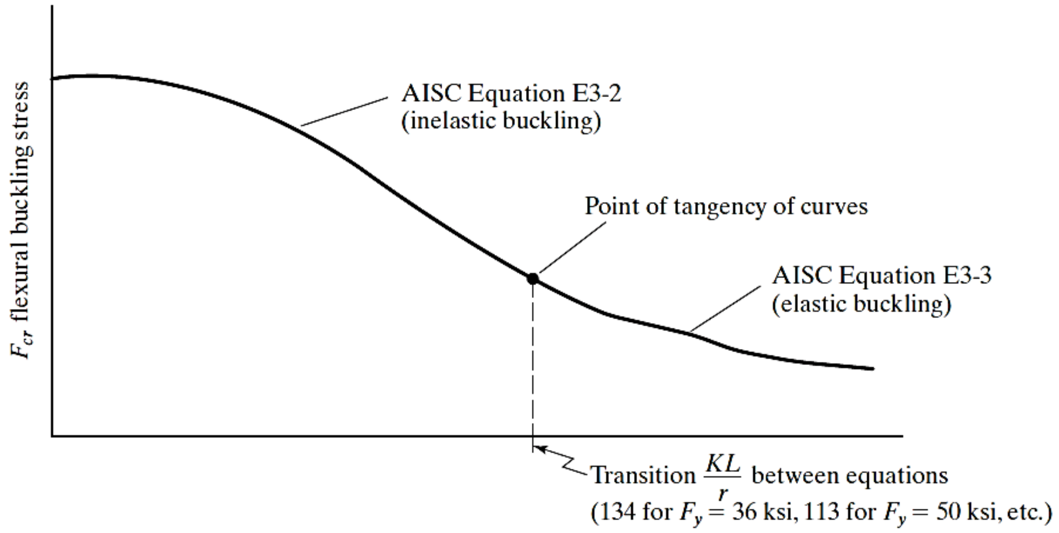
- If $KL/r \leq 4.71^*\sqrt{\dfrac{E}{F_y}}$ then $F_{cr} = [0.658^{\,Fy/Fe}]\,F_y$
- If $KL/r > 4.71^*\sqrt{\dfrac{E}{F_y}}$ then $F_{cr} = 0.877\,F_e$

In these expressions, $\mathbf{F_e}$ is the elastic critical buckling stress—that is, the Euler stress—calculated with the effective length of the column **KL**.

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

Note: The **AISC** Manual provides computed values of critical stresses $\boldsymbol{\emptyset_c F_{cr}}$ in their **Table 4-22 PP (4-318)**. The values are given for practical **KL/r** values (**0** to **200**) and for steels with **Fy = 36, 42, 46,** and **50 ksi.**

These equations are represented graphically in the figure below



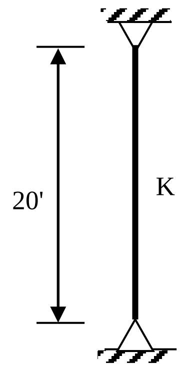
AISC column curve.

Example 3-3:

Determine the design strength of W14*74 column. Its length 20', it's pinned at both ends. E = 29 ksi.

Solution:

Ag = 21.8 in^2, r_x = 6.04 in^4, r_y = 2.48 in^4 , f_y = 36 ksi



$$\frac{K_y L}{r_y} = \frac{1*20*12}{2.48} = 96.77.......(control)$$

$$\frac{K_x L}{r_x} = \frac{1*20*12}{6.04} = 39.73$$

$$\max.\frac{KL}{r} = 96.77 < 200......ok$$

$$4.71\sqrt{\frac{F_y}{E}} = 4.71\sqrt{\frac{50}{29000}} = 113$$

$$\frac{KL}{r} = 96.77 < 113.....\text{use AISC Equation E3 - 2. p33}$$

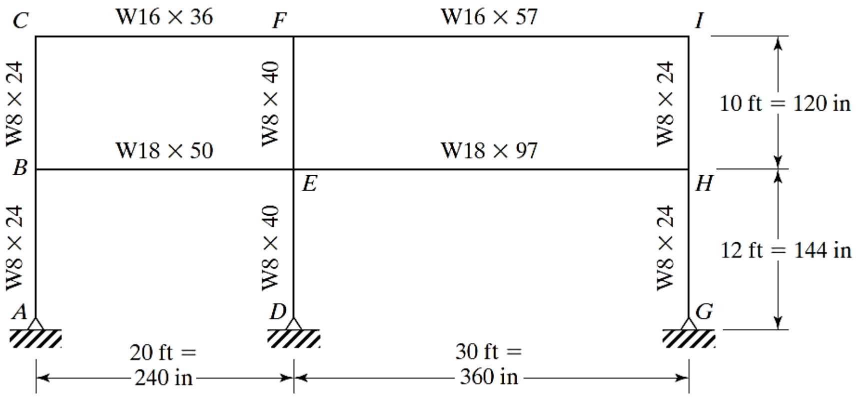
F_e = Л^{2}E/(KL/r)^2= 30.56 ksi

F_{cr} = [0.658 $^{Fy/Fe}$] F_y= 25.21 ksi

P_d = ϕ_c F_{cr} A_g = 0.9 (25.21)(21.8)= 495 kips

Example 3-4:

Determine the effective length factor for each of the columns of the frame shown in the figure, if the frame is not braced against sidesway.



Solution.

Stiffness factors: *E* is assumed to be 29,000 ksi for all members and is therefore neglected in the equation to calculate *G*.

	Member	Shape	I	L	I/L
Columns	AB	W8 × 24	82.7	144	0.574
	BC	W8 × 24	82.7	120	0.689
	DE	W8 × 40	146	144	1.014
	EF	W8 × 40	146	120	1.217
	GH	W8 × 24	82.7	144	0.574
	HI	W8 × 24	82.7	120	0.689
Girders	BE	W18 × 50	800	240	3.333
	CF	W16 × 36	448	240	1.867
	EH	W18 × 97	1750	360	4.861
	FI	W16 × 57	758	360	2.106

G factors for each joint:

Joint	$\Sigma(I_c/L_c)/\Sigma(I_g/L_g)$	G
A	Pinned Column, $G = 10$	10.0
B	$\dfrac{0.574 + 0.689}{3.333}$	0.379
C	$\dfrac{0.689}{1.867}$	0.369
D	Pinned Column, $G = 10$	10.0
E	$\dfrac{1.014 + 1.217}{(3.333 + 4.861)}$	0.272
F	$\dfrac{1.217}{(1.867 + 2.106)}$	0.306
G	Pinned Column, $G = 10$	10.0
H	$\dfrac{0.574 + 0.689}{4.861}$	0.260
I	$\dfrac{0.689}{2.106}$	0.327

Column **K** factors from the chart

Column	G_A	G_B	K
AB	10.0	0.379	1.76
BC	0.379	0.369	1.12
DE	10.0	0.272	1.74
EF	0.272	0.306	1.10
GH	10.0	0.260	1.73
HI	0.260	0.327	1.10

3.4 Local Member Buckling Concepts

The cross sections of steel shapes tend to consist of an assembly of thin plates. When the cross section of a steel shape is subjected to large compressive stresses, the thin plates that make up the cross section may buckle before the full strength of the member is attained if the thin plates are too slender. When a cross sectional element fails in buckling, then the member capacity is reached. Consequently, local buckling becomes a limit state for the strength of steel shapes subjected to compressive stress. The figure below shows an example of flange local buckling. This member failed before the full strength of the member was realized because the slender flange plate buckled first.

- In the Euler equation the parameter **(L/r)** is known as the slenderness of the member. For a plate, the slenderness parameter is a function of the width/thickness **(b/t)** ratio, **λ**, of a slender plate cross sectional element.

- There are two different types of plate elements in a cross section: ***Stiffened*** and ***Unstiffened***.

Flange Local Buckling Example

- If a plate's edges are restrained against buckling, then the force required to buckle the plate increases. If one edge is restrained (i.e. "unstiffened" plate element) the force to cause out-of-plane buckling is less than that required to buckle a plate with two edges restrained against out-of-plane buckling (i.e. "stiffened" plate element). An intersecting plate at a plate edge adds a significant moment of inertia out of plane to the edge which prevents deflection at the attached edge. The figure below illustrates the modes of buckling for a stiffened and unstiffened plate elements.

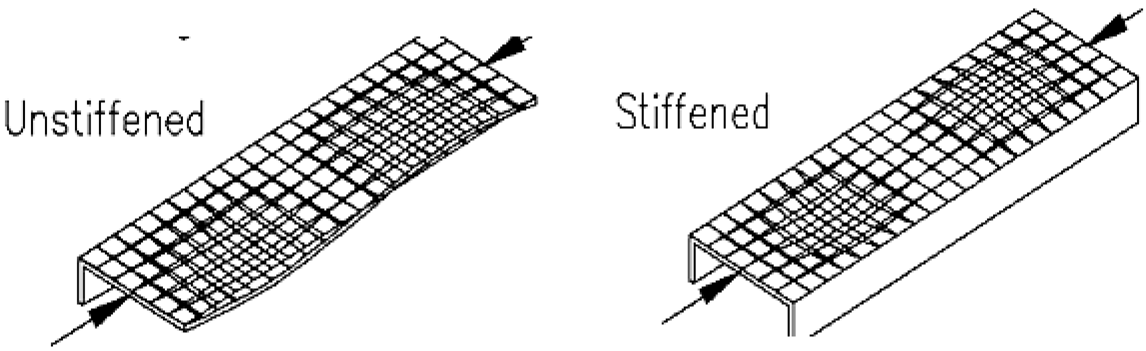
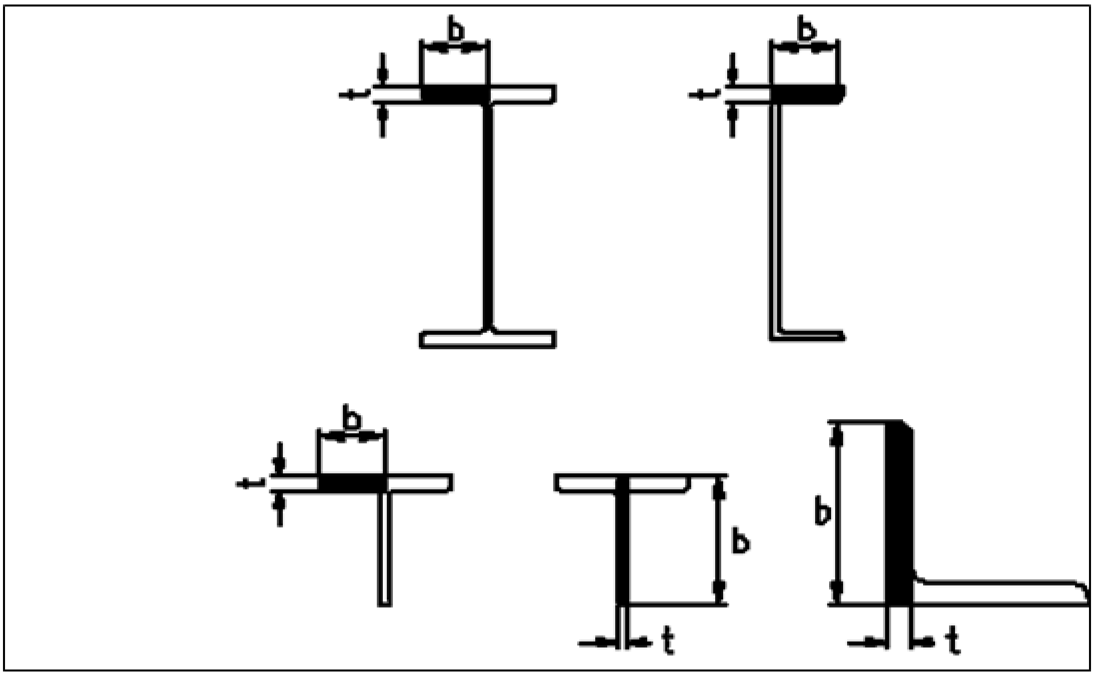


Plate Buckling Modes

- The figure below shows the *unstiffened elements* on some typical steel sections and the measurement of the element width, **b**, and thickness, **t**.



Unstiffened Elements

For example to prevent local buckling the plate slenderness, λ, should be less than limiting width-to-thickness ratio, λr, as following:

– Flange of I-, W- or T- shape: $\lambda_f = b_f/2t_f \leq \lambda_{rf} = 0.56\sqrt{E/F_y}$

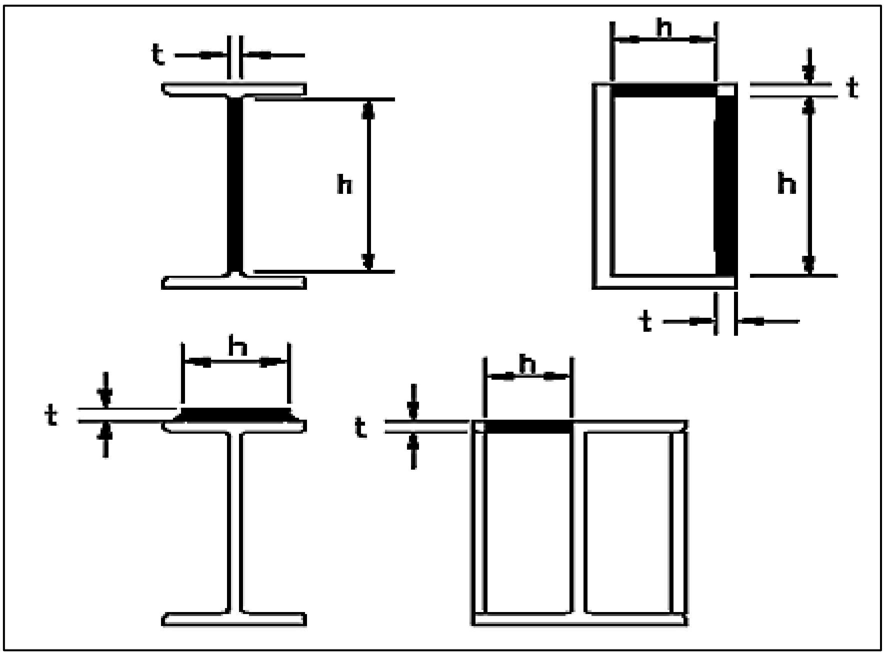
– Flange of C- shape: $\lambda_f = b_f/\, t_f \leq \lambda_{rf} = 0.56\sqrt{E/F_y}$

– Web of W- or C-shape: $\lambda_w = h/t_w \leq \lambda_{rw} = 0.75\sqrt{E/F_y}$

– Web of T- shape: $\lambda_w = d/t_w \leq \lambda_{rw} = 0.75\sqrt{E/F_y}$

– For single angle: $\lambda_a = b_{larger\ leg}/t \leq \lambda_{rw} = 0.45\sqrt{E/F_y}$

- The figure below shows the stiffened elements on some typical steel sections and the measurement of the element width, **h**, and thickness, **t**.



Stiffened Elements

For example to prevent local buckling for stiffness element:

– Web of W- or C-shape: $\lambda_w = h/t_w \leq \lambda_{rw} = 1.49\sqrt{E/F_y}$

– Side of tube: $\lambda_{tube} = h/t \leq \lambda_{r,\,tube} = 1.40\sqrt{E/F_y}$

- See table B4.1 p.16
- If $\lambda \leq \lambda_r$, the shape is non-slender. Otherwise, the shape is slender.
- If the width-to-thickness ratio λ is greater than λ_r, ($\lambda > \lambda_r$) use the provisions of AISC E7 and compute a reduction factor Q. • Compute KL/r and F_e as usual.

- If $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{QF_y}}$ or $\frac{QF_y}{F_e} \leq 2.25$,

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y \qquad \text{(AISC Equation E7-2)}$$

- If $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$ or $\frac{QF_y}{F_e} > 2.25$,

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E7-3)}$$

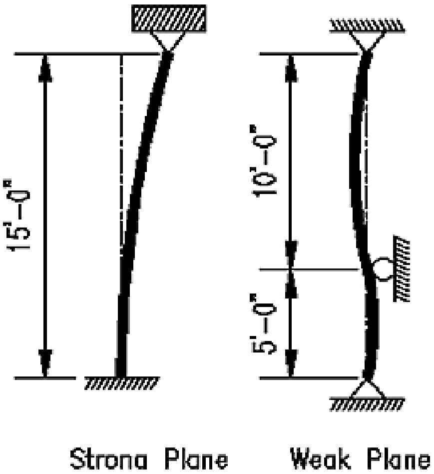
- The nominal strength is $P_n = F_{cr}A_g$ (AISC Equation E7-1)

The reduction factor Q is the product of two factorsQ_s for unstiffened elements and Q_a for stiffened elements.

- If the shape has no slender unstiffened elements, $Q_s = 1.0$.
- If the shape has no slender stiffened elements, $Q_a = 1.0$.

To calculate fQ_s for unstiffened elements and Q_a for stiffened elements see AISC E7-4 to E7-19. P40 to p.43

Example 3-6: A W8×35 Gr. 36 column is to be 15 ft long. In the strong plane, the column is part of an unbraced frame, one end is to be considered fixed and the other pinned. In the weak plane, the column is part of a braced frame, both ends are to be considered pinned and there is a lateral support provided 5 ft from one end.



Solution: - For W8×35

$A = 10.3$ in.^{2}, $r_x = 3.51$ in., $r_y = 2.03$ in.,

$L = 15$ ft; $L_x = 15$ ft; $L_{y1} = 5$ ft; $L_{y2} = 10$ ft
$K_x L_x = 0.8*15 = 12$ ft
$K_y L_{y1} = 1*5 = 5$ ft;
$K_y L_{y2} = 1*10 = 10$ ft (control)

$$\frac{K_x L}{r_x} = \frac{12*12}{3.51} = 50.9$$

$$\frac{K_y L}{r_y} = \frac{10*12}{2.03} = 59.11 \text{.....Controls (largest KL/r)}$$

$$< 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 ksi}{65 ksi}} = 99.49$$

$$F_e = \frac{\pi^2(29{,}000 \text{ ksi})}{59.11^2} = 82 \text{ ksi}$$

$$F_{cr} = 0.658^{\left(36/82\right)}(36\text{ksi}) = 29.96\text{ksi}$$

$$P_n = (29.96 \text{ ksi})(10.3 \text{ in}^2) = 308.57 \text{ kips}$$

$P_d = \phi_c P_n = 0.9*556.89 = 277.7$ kips

Or one can find ϕ_c F_{cr} from table 4-22 p. 4-319

- Local buckling checking

$\lambda_f = b_f/2t_f = 8.10 < \lambda_{rf} = 0.56\sqrt{E/F_y} = 11.8$ (unstiffener) O.K.

$\lambda_w = h/t_w = 20.5 < \lambda_{rw} = 1.49\sqrt{E/F_y} = 31.5$ (stiffener) O.K.

Neither flange local buckling nor web local buckling will precede member buckling. So, the design axial compressive strength of the column is 501.2.

Width-to-Thickness Ratios: Compression Elements in Members Subject to Axial Compression

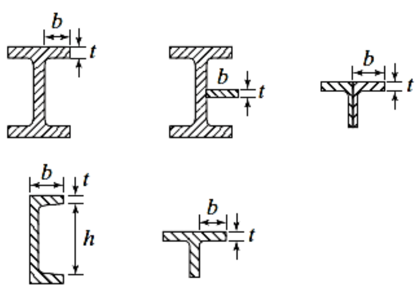
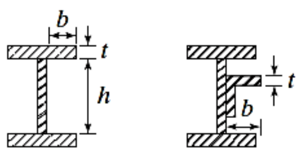
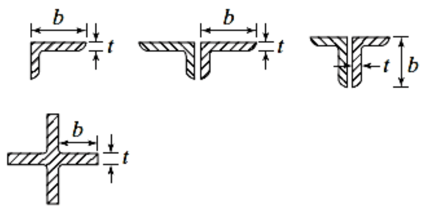
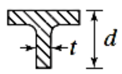
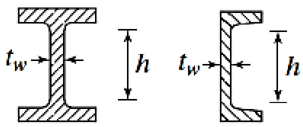
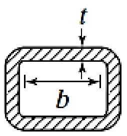
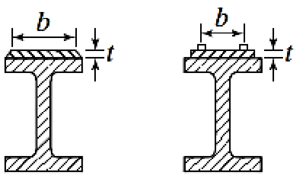
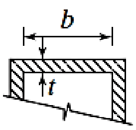
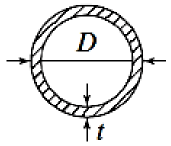
	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender / slender)	Examples
Unstiffened Elements	1	Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections, out-standing legs of pairs of angles connected with continuous con-tact, flanges of channels, and flanges of tees	b/t	$0.56\sqrt{\frac{E}{F_y}}$	
	2	Flanges of built-up I-shaped sec-tions and plates or angle legs projecting from built-up I-shaped sections	b/t	$0.64\sqrt{\frac{k_c E}{F_y}}$[a]	
	3	Legs of single angles, legs of double angles with separators, and all other unstiffened elements	b/t	$0.45\sqrt{\frac{E}{F_y}}$	
	4	Stems of tees	d/t	$0.75\sqrt{\frac{E}{F_y}}$	

TABLE 5.2 Continued

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_r (nonslender / slender)	Examples
Stiffened Elements	5	Webs of doubly symmetric I-shaped sections and channels	h/t_w	$1.49\sqrt{\dfrac{E}{F_y}}$	
	6	Walls of rectangular HSS and boxes of uniform thickness	b/t	$1.40\sqrt{\dfrac{E}{F_y}}$	
	7	Flange cover plates and diaphragm plates between lines of fasteners or welds	b/t	$1.40\sqrt{\dfrac{E}{F_y}}$	
	8	All other stiffened elements	b/t	$1.49\sqrt{\dfrac{E}{F_y}}$	
	9	Round HSS	D/t	$0.11\dfrac{E}{F_y}$	

Example 3-7: An **HSS 16 x 16 x ½** with $\mathbf{F_y}$ is used for an **18-ft**-long column with simple end supports.
(a) Determine $\boldsymbol{\phi_c P_n}$ with the appropriate AISC equations.
(b) Repeat part (a), using Table 4-4 in the AISC Manual.

Solution:

(a) Using an HSS

$$16 \times 16 \times \frac{1}{2}(A = 28.3 \text{ in}^2, t_{\text{wall}} = 0.465 \text{ in}, r_x = r_y = 6.31 \text{ in})$$

Calculate $\frac{b}{t}$ (AISC Table B4.1a, Case 6)

b is approximated as the tube size $-2 \times t_{\text{wall}}$

$$\frac{b}{t} = \frac{16 - 2(0.465)}{0.465} = 32.41 < 1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000}{46}}$$

$$= 35.15 \quad \therefore \text{ Section has no slender elements}$$

$\frac{b}{t}$ ratio also available from Table 1-12 of Manual

Calculate $\frac{KL}{r}$ and F_{cr}

$K = 1.0$

$$\left(\frac{KL}{r}\right)_x = \left(\frac{KL}{r}\right)_y = \frac{(1.0)(12 \times 18) \text{ in}}{6.31 \text{ in}} = 34.23$$

$$< 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.26$$

$\therefore$ Use AISC Equation E3-2 for F_{cr}

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{(\pi^2)(29{,}000)}{(34.23)^2} = 244.28 \text{ ksi}$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y = \left[0.658^{\frac{46}{244.28}}\right]46$$

$$= 42.51 \text{ ksi}$$

LRFD $\phi_c = 0.90$

$\phi_c F_{cr} = (0.90)(42.51) = 38.26$ ksi

$\phi_c P_n = \phi_c F_{cr} A = (38.26)(28.3)$

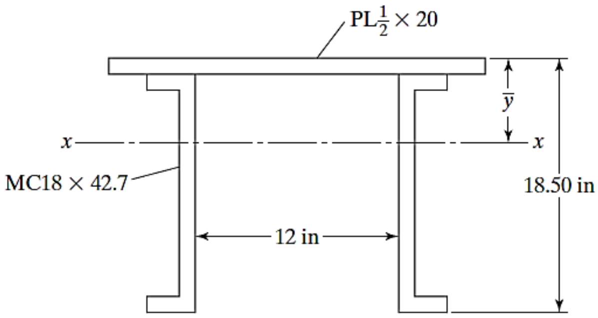
$= 1082$ k

(b) From the Manual, **Table 4-4**

$\emptyset_c P_n = 1080$ k

Example 3-8:

Determine the LRFD design strength $\boldsymbol{\emptyset_c P_n}$ for the axially loaded column shown in the figure. If **KL = 19 ft** and **50-ksi** steel is used.



Solution

A =12.6 in^2, d=18.00 in,
I_x = 554 in^4, I_y =14.3 in^4, P.P(1-36)
$\bar{x}$ = 0.877 in from back of C)

$$A_g = (20)\left(\tfrac{1}{2}\right) + (2)(12.6) = 35.2 \text{ in}^2$$

$$\bar{y} \text{ from top} = \frac{(10)(0.25) + (2)(12.6)(9.50)}{35.2} = 6.87 \text{ in}$$

$$I_x = (2)(554) + (2)(12.6)(9.50 - 6.87)^2 + \left(\frac{1}{12}\right)(20)\left(\frac{1}{2}\right)^3 + (10)(6.87 - 0.25)^2$$
$$= 1721 \text{ in}^4$$

$$I_y = (2)(14.3) + (2)(12.6)(6.877)^2 + \left(\tfrac{1}{12}\right)\left(\tfrac{1}{2}\right)(20)^3 = 1554 \text{ in}^4$$

$$r_x = \sqrt{\frac{1721}{35.2}} = 6.99 \text{ in}$$

$$r_y = \sqrt{\frac{1554}{35.2}} = 6.64 \text{ in}$$

$$\left(\frac{KL}{r}\right)_x = \frac{(12)(19)}{6.99} = 32.62$$

$$\left(\frac{KL}{r}\right)_y = \frac{(12)(19)}{6.64} = 34.34 \leftarrow$$

From the Manual, Table 4-22, we read for $\dfrac{KL}{r} = 34.34$ that $\phi_c F_{cr} = 41.33$ ksi for 50 ksi steel.

$$\phi_c P_n = \phi_c F_{cr} A_g = (41.33)(35.2) = 1455 \text{ k}$$

Example 3-9:
Using F_y= 50 ksi select the lightest W14 available for the service column loads P_D = 130 k and P_L = 210 k. KL = 10 ft.
Solution

$$P_u = (1.2)(130\text{ k}) + (1.6)(210\text{ k}) = 492\text{ k}$$

$$\text{Assume } \frac{KL}{r} = 50$$

Using $F_y = 50$ ksi steel

$\phi_c F_{cr}$ from AISC Table 4-22 $= 37.5$ ksi

$$\text{A Reqd} = \frac{P_u}{\phi_c F_{cr}} = \frac{492\text{ k}}{37.5\text{ ksi}} = 13.12\text{ in}^2$$

Try W14 × 48 ($A = 14.1\text{ in}^2$, $r_x = 5.85$ in,
$r_y = 1.91$ in)

$$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(10\text{ ft})}{1.91\text{ in}} = 62.83$$

$\phi_c F_{cr} = 33.75$ ksi from AISC Table 4-22

$$\phi_c P_n = (33.75\text{ ksi})(14.1\text{ in}^2)$$
$$= 476\text{ k} < 492\text{ k N.G.}$$

Try next larger section W14 × 53 ($A = 15.6\text{ in}^2$,
$r_y = 1.92$ in)

$$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(10\text{ ft})}{1.92\text{ in}} = 62.5$$

$$\phi_c F_{cr} = 33.85\text{ ksi}$$

$$\phi_c P_n = (33.85\text{ ksi})(15.6\text{ in}^2)$$

$$= 528\text{ k} > 492\text{ k} \quad \textbf{OK}$$

Use W14 × 53.

Example 3-10:

Select the lightest available W12 section, using the LRFD for the following conditions: F_y = 50 ksi, P_D = 250 k, P_L = 400 k, K_xL_x = 26 ft and K_yL_y = 13 ft.

(a) By trial and error

(b) Using AISC tables

Solution

(a) Using trial and error to select a section, using the LRFD expressions, and then checking the section with the LRFD method.

$$P_u = (1.2)(250\text{ k}) + (1.6)(400\text{ k}) = 940\text{ k}$$

$$\text{Assume } \frac{KL}{r} = 50$$

$$\text{Using } F_y = 50\text{ ksi steel}$$
$$\phi_c F_{cr} = 37.5\text{ ksi (AISC Table 4-22)}$$

$$A\text{ Reqd} = \frac{940\text{ k}}{37.5\text{ ksi}} = 25.07\text{ in}^2$$

$$\text{Try W}12 \times 87\ (A = 25.6\text{ in}^2, r_x = 5.38\text{ in}, r_y = 3.07\text{ in})$$

$$\left(\frac{KL}{r}\right)_x = \frac{(12\text{ in/ft})(26\text{ ft})}{5.38\text{ in}} = 57.99 \leftarrow \ \therefore \left(\frac{KL}{r}\right)_x \text{ controls}$$

$$\left(\frac{KL}{r}\right)_y = \frac{(12\text{ in/ft})(13\text{ ft})}{3.07\text{ in}} = 50.81$$

$$\phi_c F_{cr} = 35.2\text{ ksi (Table 4-22)}$$

$$\phi_c P_n = (35.2\text{ ksi})(25.6\text{ in}^2)$$

$$= 901\text{ k} < 940\text{ k } \mathbf{N.G.}$$

A subsequent check of the next-larger W12 section, a W12 x 96, shows that it will work for the LRFD procedure.

Inelastic Effective Length Factors

The discussion in Section 4-4-1 concerning the evaluation of effective length factors in rectangular frames was restricted to the buckling of perfectly elastic frames. However, in reality, instability of steel frames is more likely to take place after the stresses at some parts of frame have reached the yield stress.

For elastic behavior the values of coefficients G_A and G_B given in the two charts of LRFDM (p. 241 and p.242) can be used. If the elastic E still applies for the girder members, but inelastic for the columns, this can be accounted for by adjusting the G values as follows:

$$G_i = \frac{\sum E_i\left(I_c/L_c\right)}{\sum E_e\left(I_g/L_g\right)} = \frac{E_i}{E_e} G_e = \tau G_e$$

Where: G_e = elastic G factor assuming that both columns and girders behave elastically

G_i = inelastic G factor assuming that girders behave elastically while the columns behave inelastically

τ = stiffness reduction factor

Then $\tau = 1.0$ For $P_u/P_y \leq 0.39$

$\tau = 4(\alpha P_u/P_y)\,[1-(\alpha P_u/P_y)]$ For $P_u/P_y > 0.39$

Where: $P_y = F_y A_g$ and α=1 for LRFD

Vales for stiffness reduction factor τ, for different values of P_u/A_g are presented in LRFDM for steel with F_y= 35, 36, 42, 46 and 50 ksi (p. 4-317). For values of P_u/A_g smaller than those with entries in this table, the columns behaves elastically, and the reduction factor τ=1.0. Note that G = 10.0 for pin end, and G = 1.0 for fixed end the value of G at that end should not multiply by the stiffness reduction factor τ.

Note: LRFDM Tables p.(4-10) to p.(4-21) can be used for calculating design strength of column for W sections, and these values are tabulated with respect to the effective length about the minor axis K_yL_y. For buckling about major axis calculate $(KL)_{eq.}$:

$$(KL)_{eq} = \frac{K_x L_x}{r_x / r_y}$$

Example Problem 4-6: Calculate the effective length for W10×60 A992 Gr.50 steel column AB in the unbraced frame shown below, which subjected to an axial factored compressive load of 450 kips.

The columns are oriented such that major axis bending occurs in the plane of frame. The columns are braced continuously along the length for out –of-plane buckling. The same column section is used for the story above. Check the column adequacy. All girders are W14×74 sections.

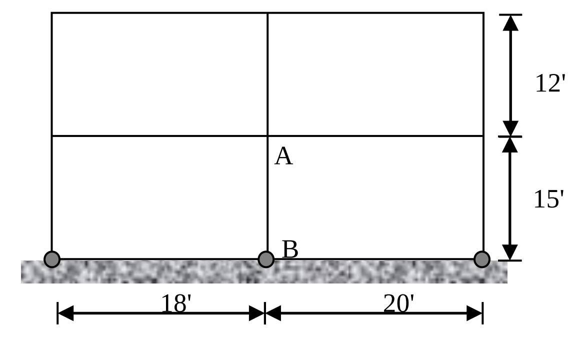


Figure 4-13: Example problem 4-6

Solution: -

- Since the columns are braced continuously along the length for out –of-plane buckling (minor axis), then L_y = 0.0 (No buckling occur about y-axis)
- Need to calculate K_x using alignment charts for unbraced frame:

  $I_x = 795\ in^4$ for W14×74 & $I_x = 341\ in^4$ for W10×60

$$G_A = \frac{341/12 + 341/15}{795/18 + 795/20} = 0.61 \qquad \& \qquad G_B = 10$$

- $P_y = F_y A_g = 50*17.6 = 880$ kips Then $P_u/P_y = 0.511 > 0.39$ the column partially plastifies
- Calculate $K_{x,\ inealstic}$: $P_u/A_g = 450/17.6 = 25.57$ ksi & $F_y = 50$ ksi

  Then τ=0.875

  $G_A = 0.61*0.875 = 0.53$ & $G_B = 10$ … $K_{x,\ inelastic} = 1.8$ (alignment chart)
- Design strength of the W10×60 column: $K_xL_x = 1.8*15 = 27$

  $r_x/r_y = 1.71$ Then $(KL)_{eq} = 27/1.71 = 15.79'$

  $P_{dc} = 533.67$ kips [LRFDM Table p.(4-19) - using interpolation]

  $P_{dc} = 533.67$ kips > $P_u = 450$ kips ……. OK

***Example Problem* 4-7:** Select the lightest W12 A992 Gr. 50 for the column AB in the unbraced frame shown below, which subjected to an axial factored compressive load of 500 kips.

The columns are oriented such that major axis bending occurs in the plane of frame.
The columns are braced at each story level for out –of-plane buckling.
A same section is used for columns of the stories above and below.
All girders are W14×68 sections.

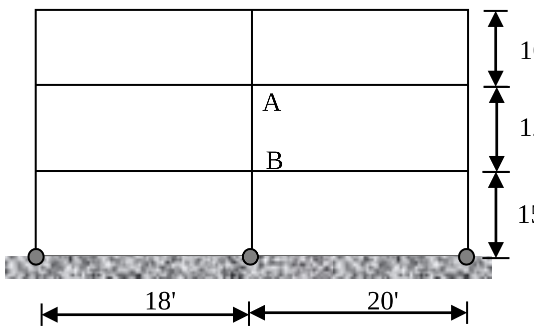


Figure 4-14: Example problem 4-7

Solution: -

- Since the columns are braced at each story level for out –of-plane buckling, then K_y = 1.0 …… K_yL_y = 1.0*12=12'
- Assume minor axis buckling governs, and F_y=50 ksi (A992 steel)

P_{dc} = 547 kips for W12× 53 [LRFDM Table p.(4-18)]

- $P_y=F_y A_g$= 50*15.6= 780 kips Then P_u/P_y=0.641> 0.5 the column partially plastifies
- Calculate $K_{x,\ inealstic}$:
- P_u/A_g = 500/15.6 = 32.05 ksi & F_y = 50 ksi

Then τ=0.662

$$G_A = \frac{0.662*\left(\frac{425}{10}+\frac{425}{12}\right)}{\frac{722}{18}+\frac{722}{20}} = 0.68$$

$$G_B = \frac{0.662*\left(\frac{425}{15}+\frac{425}{12}\right)}{\frac{722}{18}+\frac{722}{20}} = 0.55$$

From the chart : $K_{x\ inelastic} \approx 1.18$

- Check selected W12× 53 section for x-axis buckling:

K_xL_x= 1.18*12 = 14.16; r_x/r_y = 2.11…Then $(KL)_{eq}$ = 14.16/2.11= 6.71'

P_{dc} = 648.35 kips [LRFDM Table - using interpolation]

P_{dc} = 648.35 kips > P_u= 450 kips ……. OK

- Check for local buckling:

$$\lambda_f = b_f/2t_f = 8.69 < \lambda_{rf} = 0.56\sqrt{E/F_y} = 13.5 \qquad \text{O.K.}$$

$$\lambda_w = h/t_w = 28.1 < \lambda_{rw} = 1.49\sqrt{E/F_y} = 35.9 \qquad \text{O.K.}$$

So, select a W12×53 of A992 Grade 50 steel.

CHAPTER FIVE

BENDING MEMBERS

5.1 Overview

Beams are a structural members which support transverse loads and primary subjected to bending as shown in Figure 5-1-1. The principle limit states for selecting beams are related to flexure, shear, and deflection. These an appropriate beam size. Steel shapes, which are used as beams, are shown in Figure (5-1-2) below.

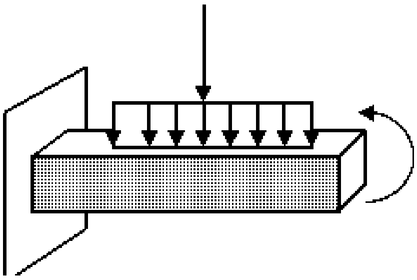


Figure 5-1-1 Cantilever Beam

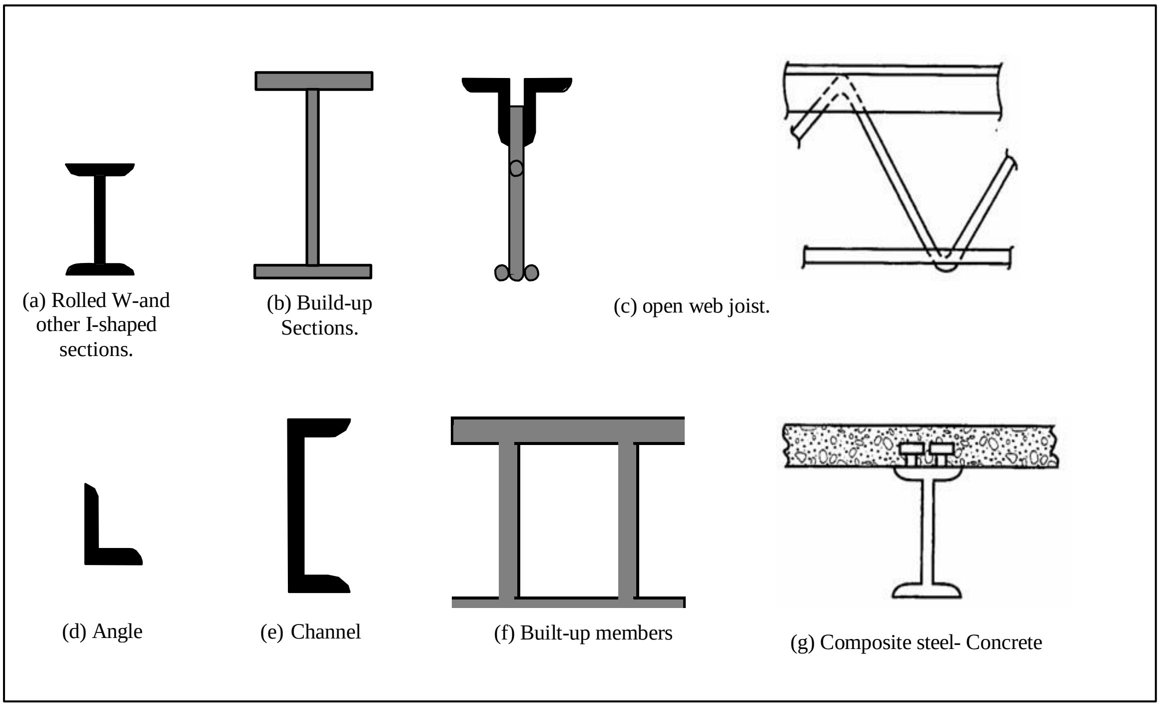


Figure 5-1-2 Steel shapes used as beams

5.2 Types of Beams

Based on the function and/or location in the building, beams may be classified as one of the following several types:

- *Girders*: Usually the most important beams. (see Figure 5-2-1).
- Stringers: Longitudinal bridge beams spanning between floor beams. (see Figure 5-2-1).
- Floor Beams: In buildings, a major beam usually supporting joists; a transverse beam in bridge floors. (see Figure 5-2-1).

- Joists: A beam supporting floor construction but not major beams. (see Figure 5-2-2).
- Purlins: Roof beam spanning between trusses. (see Figure 5-2-3).
- Girts: Horizontal wall beams serving principally to resist bending due to wind on the side of an industrial building. (see Figure 5-2-4).
- Lintels: Member supporting a wall over a window or door opening. (see Figure 5-2-5).

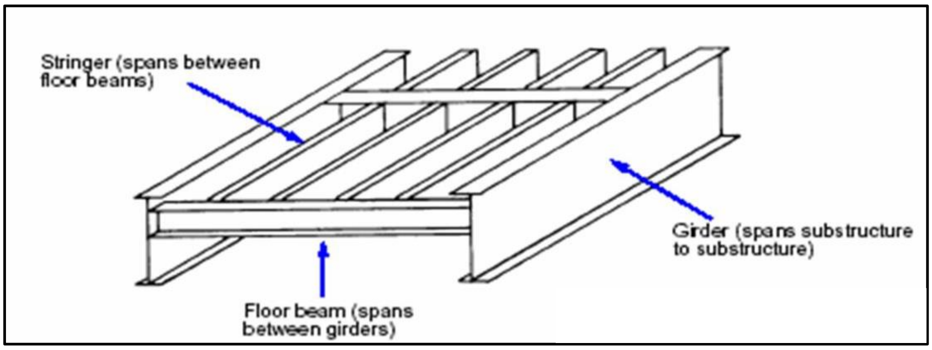


Figure 4-2-1 types of Beams

Figure 5-2-2 Joists

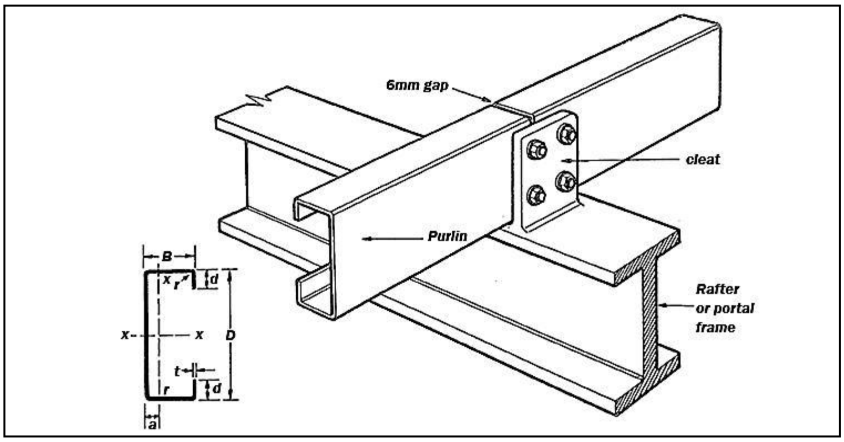


Figure 5-2-3 Purlin

Figure 5-2-4 Girts

Figure 5-2-5 Lintels

5.3 Bending Stresses

For an introduction to bending stresses, the rectangular beam and stress diagrams of Fig. 5-3-1 are considered. (For this initial discussion, the beam's compression flange is assumed to be fully braced against lateral buckling). If the beam is subjected to some bending moment, the stress at any point may be computed with the usual flexure formula,

$$f_b = \frac{Mc}{I}$$

It is to be remembered, however, that this expression is applicable only when the maximum computed stress in the beam is below the elastic limit. The formula is based on the usual elastic assumptions:

- ✓ Stress is proportional to strain,
- ✓ A plane section before bending remains a plane section after bending, etc.

The value of ***I/c*** is a constant for a particular section and is known as the ***section modulus*** **(*S*).**The flexure formula may then be written as follows:

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

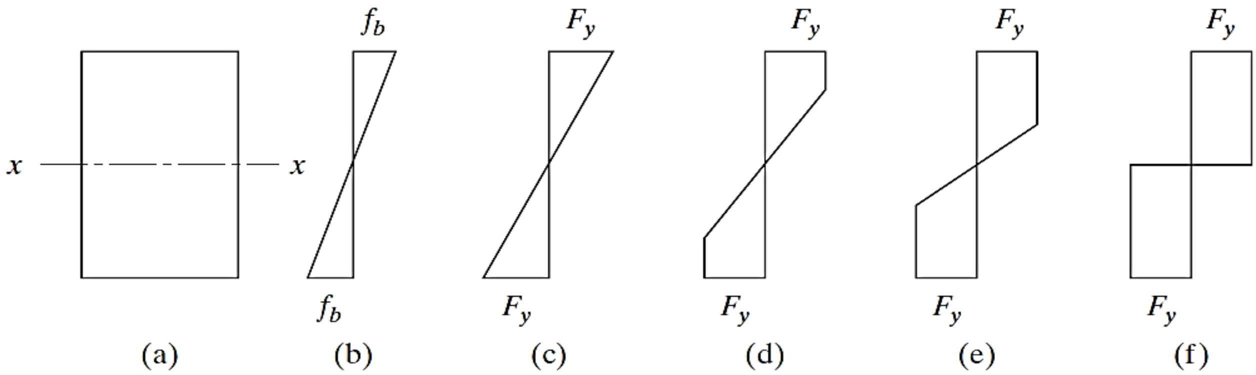


Figure 5-3-1 Variations in bending stresses due to increasing moment about *x* axis.

When the moment is applied to the beam, the stress will vary linearly from the neutral axis to the extreme fibers. This situation is shown in part (b) of Fig. 5-3-1.If the moment is increased, there will continue to be a linear variation of stress until the yield stress is reached in the outermost fibers, as shown in part (c) of the figure. The

yield moment of a cross section is defined as the moment that will just produce the yield stress in the outermost fiber of the section. If the moment in a ductile steel beam is increased beyond the yield moment, the outermost fibers that had previously been stressed to their yield stress will continue to have the same stress, but will yield, and the duty of providing the necessary additional resisting moment will fall on the fibers nearer to the neutral axis. This process will continue, with more and more parts of the beam cross section stressed to the yield stress (as shown by the stress diagrams of parts (d) and (e) of the figure), until finally a full plastic distribution is approached, as shown in part (f). **Note** that the variation of **strain** from the neutral axis to the outer fibers remains linear for **all** of these cases. When the stress distribution has reached this stage, **a *plastic hinge*** is said to have formed, because no additional moment can be resisted at the section. **Any additional moment** applied at the section will cause the beam to **rotate**, with little increase in stress. The ***plastic moment*** $\boldsymbol{M_P}$ is the moment that will produce full plasticity in a member cross section and create a plastic hinge. The ratio of the plastic moment to the yield moment $\boldsymbol{M_y}$ is called the ***shape factor***.The shape factor equals 1.50 for rectangular sections and varies from about 1.10 to 1.20 for standard rolled-beam sections.

5.4 Plastic Hinges

In Figure. 5-4-1.The load shown is applied to the beam and increased in magnitude until the yield moment is reached and the outermost fiber is stressed to the yield stress. The magnitude of the load is further increased, with the result that the outer fibers begin to yield. The yielding spreads out to the other fibers, away from the section of maximum moment, as indicated in the figure. The distance in which this yielding occurs away from the section in question is dependent on the loading conditions and the member cross section.

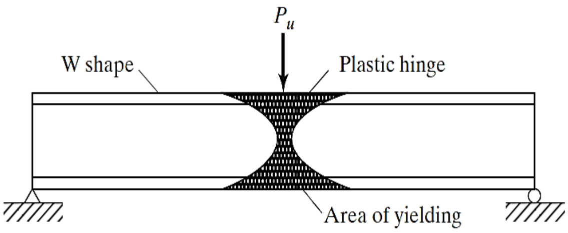


Figure. 5-4-1 a plastic hinge.

5.5 The Plastic Modulus

The yield moment $\boldsymbol{M_y}$ equals the yield stress times the elastic modulus. The elastic modulus equals $\boldsymbol{I/c}$ or $\boldsymbol{bd^2/6}$ for a rectangular section, and the yield moment equals $\boldsymbol{F_y bd^2/6}$. This same value can be obtained by considering the resisting internal couple shown in Fig. 5-5-1. The resisting moment equals T or C times the lever arm between them, as follows:

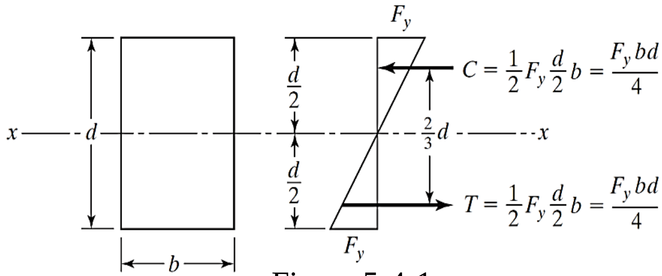


Figure 5-4-1

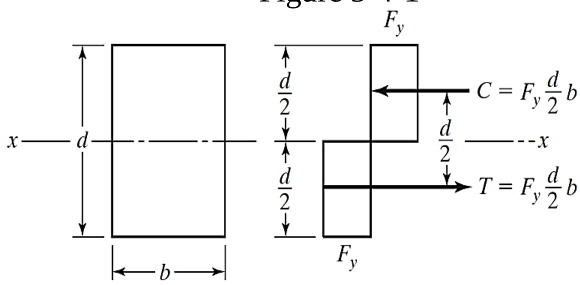


Figure 5-4-2

$$M_y = \left(\frac{F_y bd}{4}\right)\left(\frac{2}{3}d\right) = \frac{F_y bd^2}{6}$$

The resisting moment at full plasticity can be determined in a similar manner. The result is the so-called plastic moment, $\boldsymbol{M_P}$. *It is also the* ***nominal moment of the section***, $\boldsymbol{M_n}$. This plastic, or nominal, moment equals T or C times the lever arm between them. For the rectangular beam of Fig. 5-5-2, we have

$$M_P = M_n = T\frac{d}{2} = C\frac{d}{2} = \left(\frac{F_y bd}{2}\right)\left(\frac{d}{2}\right)$$

$$= \frac{F_y bd^2}{4}$$

The plastic moment is said to equal the yield stress times the plastic section modulus. From the foregoing expression for a rectangular section, the plastic section modulus $\boldsymbol{Z}$ can be seen to equal $\boldsymbol{bd^2/4}$ The shape factor, which equals $\boldsymbol{M_P/M_y}$,

$$\frac{M_P}{M_y} = \frac{F_y Z}{F_y S} \text{ or } \frac{Z}{S} \text{is } \left(bd^2/4\right)/\left(bd^2/6\right) = 1.5 \quad \text{for a rectangular section}$$

A study of the plastic section modulus determined here shows that

- ✓ *It equals the statical moment of the tension and compression areas about the plastic neutral axis*.
- ✓ Unless the section is symmetrical, the neutral axis for the plastic condition will not be in the same location as for the elastic condition.
- ✓ The total internal compression must equal the total internal tension.
- ✓ As all fibers are considered to have the same stress $\boldsymbol{F_y}$ in the plastic condition,
- ✓ The areas above and below the plastic neutral axis must be equal.
- ✓ This situation does not hold for unsymmetrical sections in the elastic condition

Example

Determine $\boldsymbol{M_y, M_n}$, and $\boldsymbol{Z}$ for the steel tee beam shown in Fig. 5-1. Also, calculate the shape factor and the nominal load ($\boldsymbol{w_n}$) that can be placed on the beam for a **12-ft** simple span. $\mathbf{F_y}$ **= 50 ksi**.

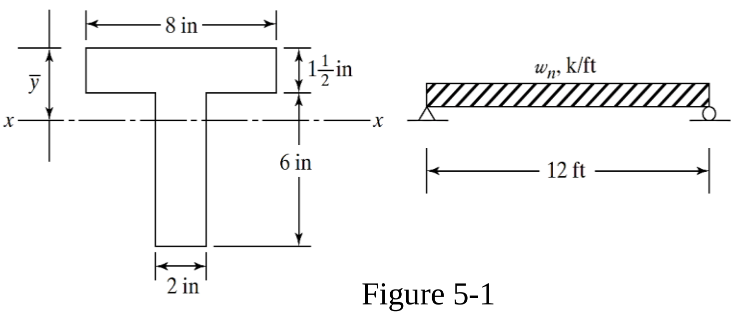


Figure 5-1

Solution.

Elastic calculations:

$$A = (8\text{ in})\left(1\frac{1}{2}\text{ in}\right) + (6\text{ in})(2\text{ in}) = 24\text{ in}^2$$

$$\bar{y} = \frac{(12\text{ in})(0.75\text{ in}) + (12\text{ in})(4.5\text{ in})}{24\text{ in}^2} = 2.625\text{ in from top of flange}$$

$$I = \frac{1}{12}(8\text{ in})(1.5\text{ in})^3 + (8\text{ in})(1.5\text{ in})(1.875\text{ in})^2 + \frac{1}{12}(2\text{ in})(6\text{ in})^3 + (2\text{ in})(6\text{ in})(1.875\text{ in})^2 = 122.6\text{ in}^4$$

$$S = \frac{I}{c} = \frac{122.6\text{ in}^4}{4.875\text{ in}} = 25.1\text{ in}^3$$

$$M_y = F_y S = \frac{(50\text{ ksi})(25.1\text{ in}^3)}{12\text{ in/ft}} = 104.6\text{ ft-k}$$

Plastic calculations (plastic neutral axis is at base of flange):

$$Z = (12\text{ in}^2)(0.75\text{ in}) + (12\text{ in}^2)(3\text{ in}) = 45\text{ in}^3$$

$$M_n = M_p = F_y Z = \frac{(50\text{ ksi})(45\text{ in}^3)}{12\text{ in/ft}} = 187.5\text{ ft-k}$$

$$\text{Shape factor} = \frac{M_p}{M_y} \quad \text{or} \quad \frac{Z}{S} = \frac{45\text{ in}^3}{25.1\text{ in}^3} = 1.79$$

$$M_n = \frac{w_n L^2}{8}$$

$$\therefore \quad w_n = \frac{(8)(187.5\text{ ft-k})}{(12\text{ ft})^2} = 10.4\text{ k/ft}$$

Table 3-23 P.P. (3-211)

Note: The values of the plastic section moduli for the standard steel beam sections are tabulated in **Table 3-2** (P.P 3-11) of the AISC Manual, entitled "W Shapes Selection by $\mathbf{Z_x}$.

5.6 Theory of Plastic Analysis

The basic plastic theory has been shown to be a major change in the distribution of stresses after the stresses at certain points in a structure reach the yield stress. The theory is that those parts of the structure that have been stressed to the yield stress cannot resist additional stresses. They instead will yield the amount required to permit the extra load or stresses to be transferred to other parts of the structure where the stresses are below the yield stress, and thus in the elastic range and able to resist increased stress. Plasticity can be said to serve the purpose of equalizing stresses in cases of overload.

For this discussion, the stress–strain diagram is assumed to have the idealized shape shown in Fig. 4-6-1. The yield stress and the proportional limit are assumed to occur at the same point for this steel, and the stress–strain diagram is assumed to be a perfectly straight line in the plastic range. Beyond the plastic range there is a range of strain hardening. This latter range could theoretically permit steel members to withstand additional stress, but from a practical standpoint the strains which arise are so large that they cannot be considered. Furthermore, inelastic buckling will limit the ability of a section to develop a moment greater than M_p, even if strain hardening is significant.

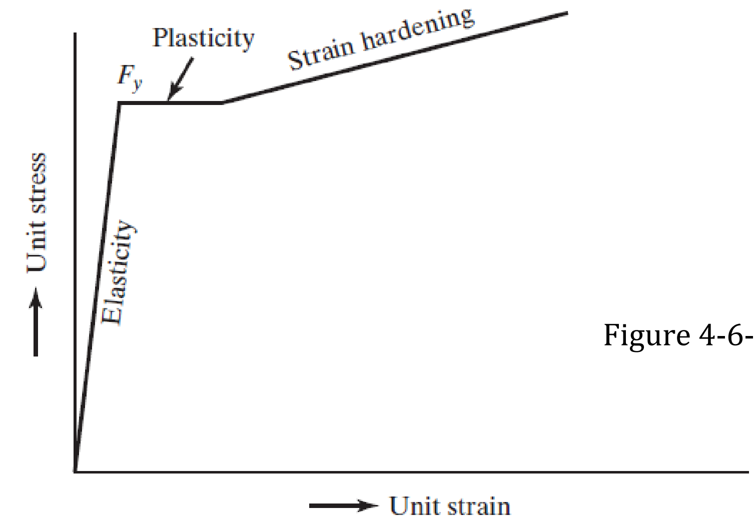


Figure 4-6-1

5.7 The Collapse Mechanism

- ✓ A statically determinate beam will fail if one plastic hinge develops.To illustrate this fact, the simple beam of constant cross section loaded with a concentrated load at midspan, shown in Fig. 4-7-1 (a), is considered. Should the load be increased until a plastic hinge is developed at the point of maximum moment (underneath the load in this case), an unstable structure will have been created, as shown in part (b) of the figure. Any further increase in load will cause collapse. P_n represents the nominal, or theoretical, maximum load that the beam can support.

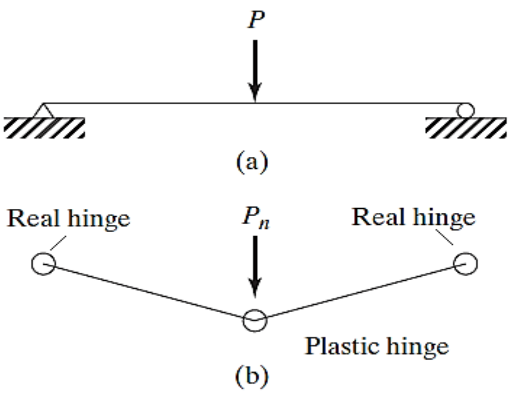


Figure 4-7-1

- ✓ For a statically indeterminate structure to fail, it is necessary for more than one plastic hinge to form. The number of plastic hinges required for failure of statically indeterminate structures will be shown to vary from structure to structure, but may never be less than two. The fixed-end beam of Fig. 4-7-2, part (a), cannot fail unless the three plastic hinges shown in part (b) of the figure are developed.

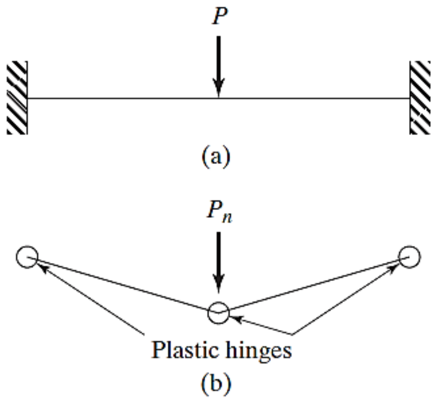


Figure 4-7-2

- ✓ Although a plastic hinge may have formed in a statically indeterminate structure, the load can still be increased without causing failure if the geometry of the structure permits.
- ✓ The plastic hinge will act like a real hinge insofar as increased loading is concerned.
- ✓ As the load is increased, there is a redistribution of moment, because the plastic hinge can resist no more moment.
- ✓ As more plastic hinges are formed in the structure, there will eventually be a sufficient number of them to cause collapse.
- ✓ Actually، some additional load can be carried after this time, before collapse occurs, as the stresses go into the strain hardening range, but the deflections that would occur are too large to be permissible.

The propped beam of Fig. 4-7-3, part (a), is an example of a structure that will fail after two plastic hinges develop. Three hinges are required for collapse, but there is a real hinge on the right end. In this beam, the largest elastic moment caused by the design concentrated load is at the fixed end. As the magnitude of the load is increased, a plastic hinge will form at that point.

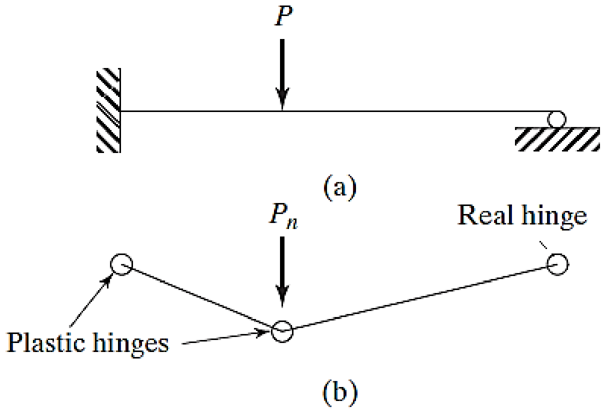


Figure 4-7-3

- ✓ The load may be further increased until the moment at some other point (here it will be at the concentrated load) reaches the plastic moment.
- ✓ Additional load will cause the beam to collapse. The arrangement of plastic hinges and perhaps real hinges that permit collapse in a structure is called the ***mechanism***. Parts (b) of Figs. 4-7-1, 4-7-2, and 4-7-3 show mechanisms for various beams.

5.8 The Virtual-Work Method

One very satisfactory method used for the plastic analysis of structures is the ***virtualwork method***.

- ✓ The structure in question is assumed to be loaded to its nominal capacity M_n, and is then assumed to deflect through a small additional displacement after the ultimate load is reached.
- ✓ The work performed by the external loads during this displacement is equated to the internal work absorbed by the hinges. For this discussion, the ***small-angle theory*** is used.
- ✓ By this theory, the sine of a small angle equals the tangent of that angle and also equals the same angle expressed in radians. In the pages to follow, the author uses these values interchangeably because the small displacements considered here produce extremely small rotations or angles.

The uniformly loaded fixed-ended beam Fig. 4-8-1.

This beam and its collapse mechanism are shown. Owing to symmetry, the rotations at the end plastic hinges are equal, and they are represented by in the figure؛ thus, the rotation at the middle plastic hinge will be 2θ.

The work performed by the total external load ($w_n L$) is equal to $w_n L$ times the ***average deflection*** of the mechanism. The average deflection equals one-half the deflection at the center plastic hinge **(1/2 x θ x L/2).**
The external work is equated to the internal work absorbed by the hinges, or to the sum of M_n at each plastic hinge times the angle through which it works. The resulting expression can be solved for M_n and w_n as follows:

$$M_n(\theta + 2\theta + \theta) = w_n L\left(\frac{1}{2} \times \theta \times \frac{L}{2}\right)$$

$$M_n = \frac{w_n L^2}{16}$$

$$w_n = \frac{16 M_n}{L^2}.$$

For the 18-ft span these values become

$$M_n = \frac{(w_n)(18)^2}{16} = 20.25\, w_n$$

$$w_n = \frac{M_n}{20.25}$$

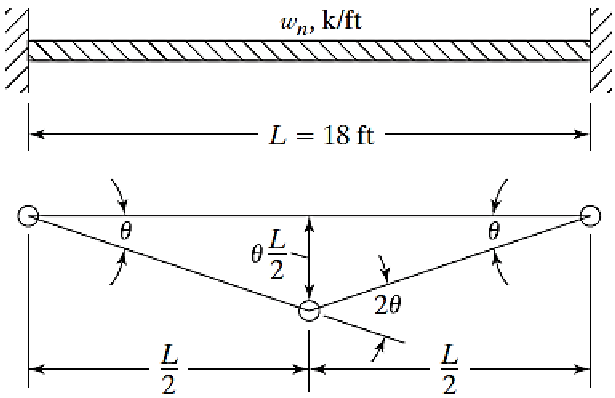


Figure 4-8-1

Plastic analysis can be handled in a similar manner for the propped beam of Fig. 4-8-2. There, the collapse mechanism is shown, and the end rotations (which are equal to each other) are assumed to equal θ.
The work performed by the external load P_n as it moves through the distance **(θ x L/2)** is equated to the internal work performed by the plastic moments

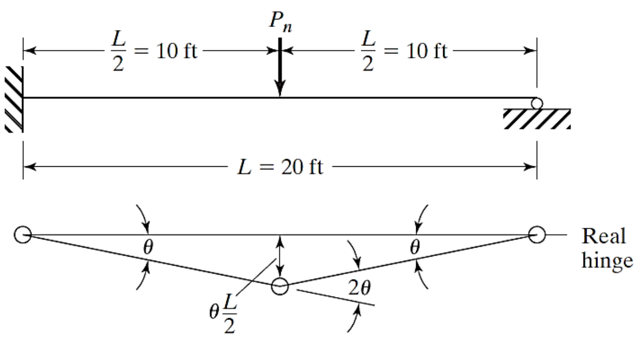


Figure 4-8-2

at the hinges; note that there is no moment at the real hinge on the right end of the beam.

$$M_n(\theta + 2\theta) = P_n\left(\theta\frac{L}{2}\right)$$
$$M_n = \frac{P_nL}{6} \quad \text{(or } 3.33P_n \text{ for the 20-ft beam shown)}$$
$$P_n = \frac{6M_n}{L} \text{ (or } 0.3M_n \text{ for the 20-ft beam shown)}$$

The fixed-end beam of Fig. 4-8-3, together with its collapse mechanism and assumed angle rotations, is considered next. From this figure, the values of $\boldsymbol{M_n}$ and $\boldsymbol{P_n}$ can be determined by virtual work as follows:

$$M_n(2\theta + 3\theta + \theta) = P_n\left(2\theta \times \frac{L}{3}\right)$$
$$M_n = \frac{P_nL}{9} \text{ (or } 3.33P_n \text{ for this beam)}$$
$$P_n = \frac{9M_n}{L} \text{ (or } 0.3M_n \text{ for this beam).}$$

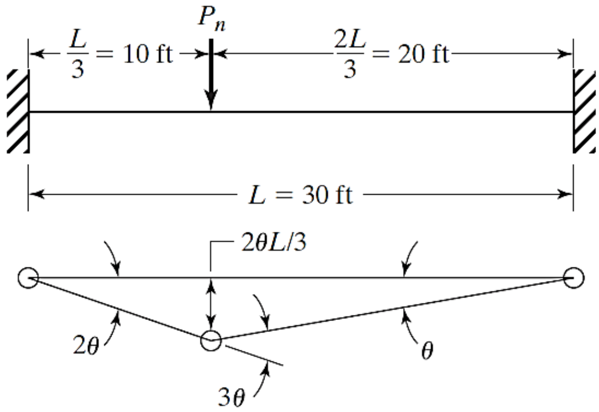


Figure 4-8-3

The plastic analysis of the propped beam of Fig. 4-8-4 is done by the virtual-work method. The beam with its two concentrated loads is shown, together with four possible collapse mechanisms and the necessary calculations. It is true that the mechanisms of parts (b), (d), and (e) of the figure do not control, but such a fact is not obvious to the average student until he or she makes the virtual-work calculations for each case. Actually, the mechanism of part (e) is based on the assumption that the plastic moment is reached at both of the concentrated loads simultaneously (a situation that might
very well occur).

Note: The value for which the collapse load $\boldsymbol{P_n}$ is the smallest in terms of $\boldsymbol{M_n}$ is the correct value (or the value where $\boldsymbol{M_n}$ is the greatest in terms of $\boldsymbol{P_n}$). For this beam, the second plastic hinge forms at the $\boldsymbol{P_n}$ concentrated load, and $\boldsymbol{P_n}$ equals **0.154** $\boldsymbol{M_n}$.

5.9

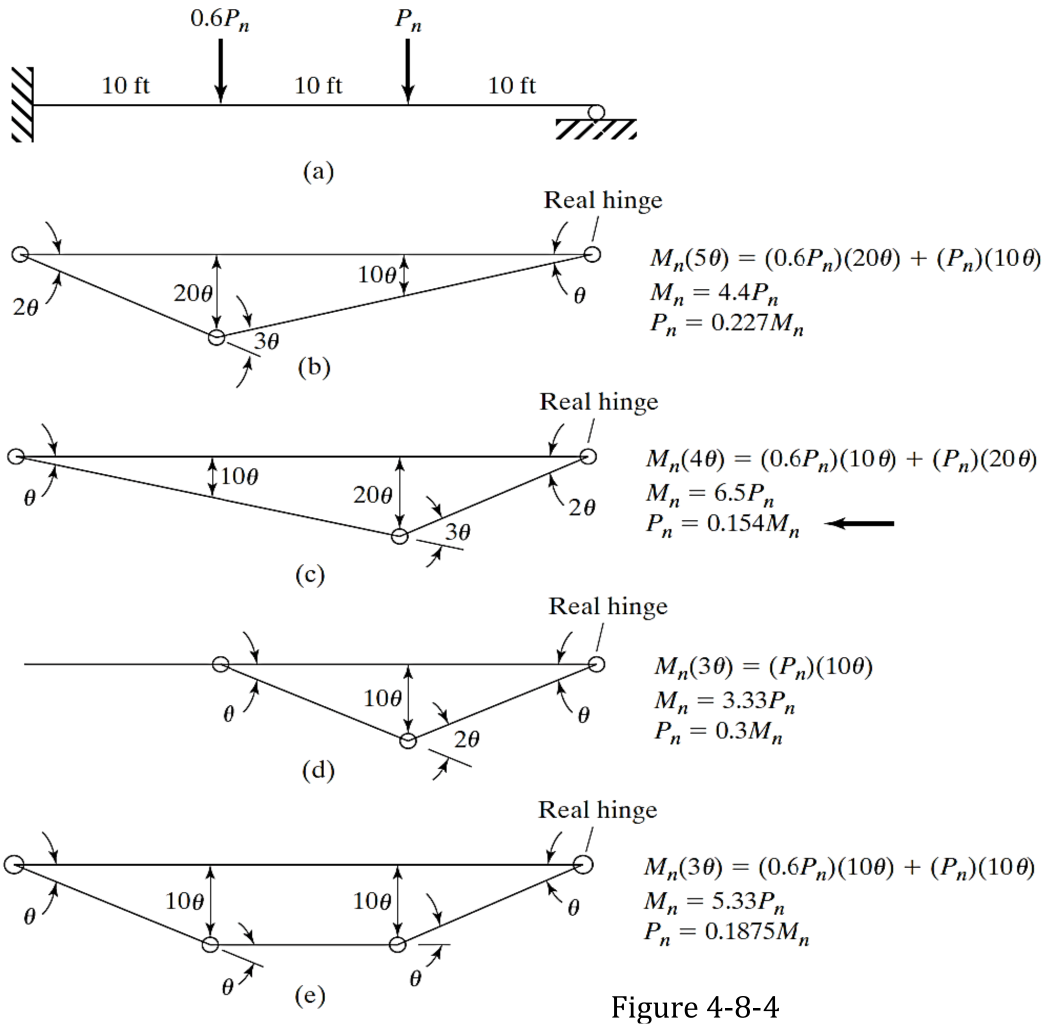


Figure 4-8-4

5.10 Location of Plastic Hinge for Uniform Loadings

There was no difficulty in locating the plastic hinge for the uniformly loaded fixed-end beam, but for other beams with uniform loads, such as propped or continuous beams, the problem may be rather difficult.

The elastic moment diagram for this beam is shown as the solid line in part (b) of the figure. As the uniform load is increased in magnitude, a plastic hinge will first form at the fixed end. At this time, the beam will, in effect, be a "simple" beam (so far as increased loads are concerned) with a plastic hinge on one end and a real hinge on the other. Subsequent increases in the load will cause the moment to change, as represented by the dashed line in part (b) of the figure. This process will continue until the moment at some other point (a distance *x* from the right support in the figure) reaches M_n and creates another plastic hinge.

The virtual-work expression for the collapse mechanism of the beam shown in part (c) of Fig. 4-9-1 is written as follows:

$$M_n\left(\theta+\theta+\frac{L-x}{x}\theta\right)=\left(w_nL\right)(\theta)(\mathrm{L}-\mathrm{x})\left(\frac{1}{2}\right)$$

Solving this equation for M_n, taking $dM_n/dx = 0$, the value of x can be calculated to equal **0.414*L***.This value is also applicable to uniformly loaded end spans of continuous beams with simple end supports.

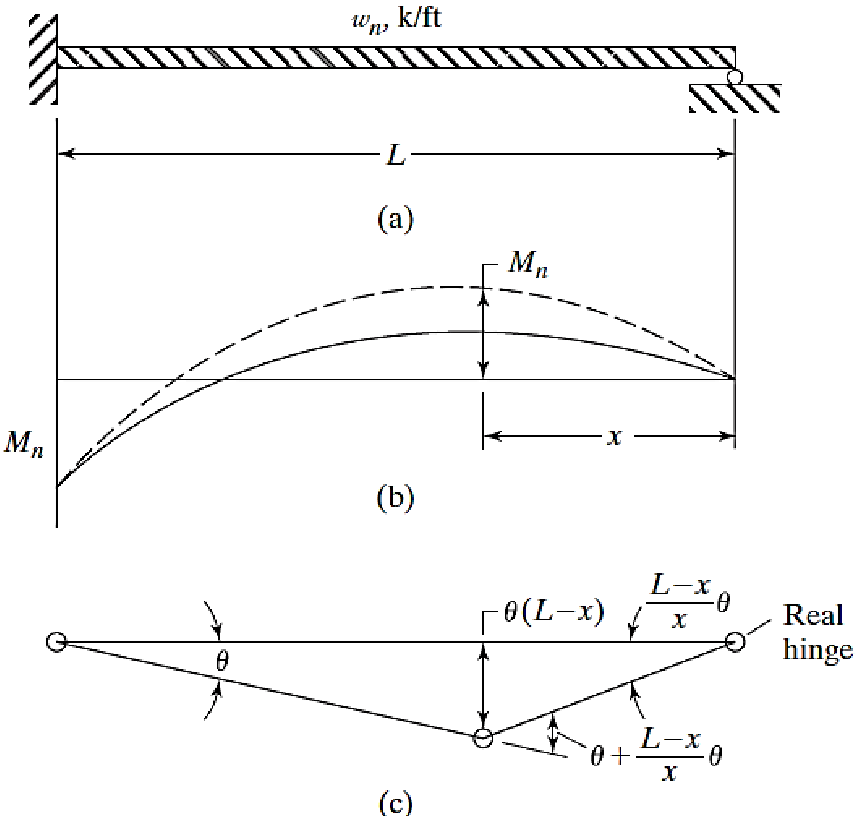


Figure 4-9-1

The beam and its collapse mechanism are redrawn in Fig. 4-9-2, and the following expression for the plastic moment and uniform load are written by the virtual-work procedure:

$$M_n(\theta + 2.414\theta) = (w_nL)(0.586\theta L)\left(\frac{1}{2}\right)$$

$$M_n = 0.0858 w_n L^2$$

$$w_n = 11.65\frac{M_n}{L^2}$$

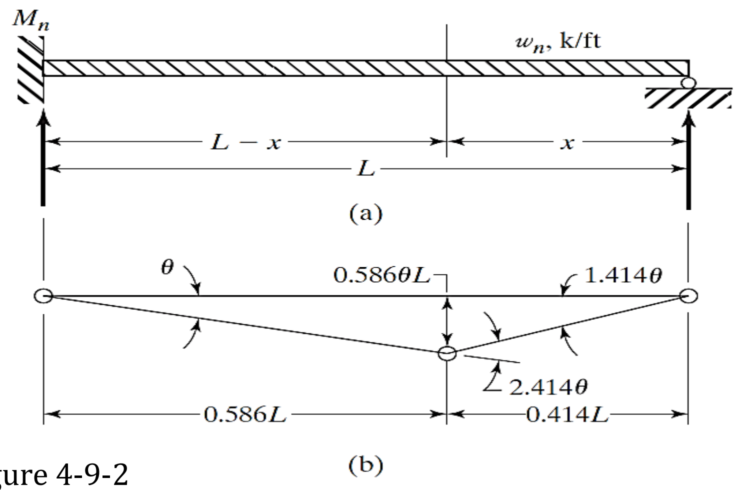


Figure 4-9-2

Example 4-2

A **W18 x 55** has been selected for the beam shown in Fig. 4-2.Using **50 ksi** steel and assuming full lateral support, determine the value of w_n.

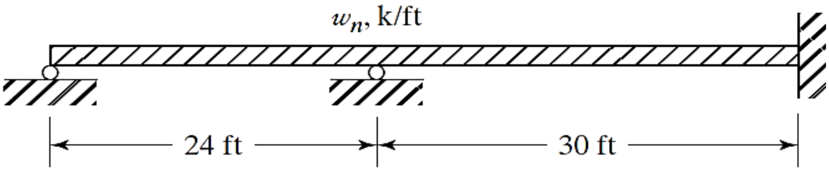


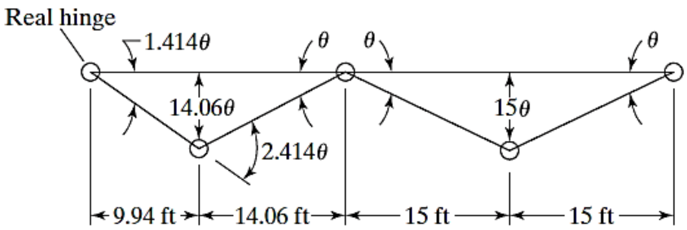
Figure 4-2

Solution

From the **Table 3-2** of the AISC Manual. Z_x **= 112 in³**

$$M_n = F_y Z = \frac{(50 \text{ ksi})(112 \text{ in}^3)}{12 \text{ in/ft}} = 466.7 \text{ ft-k}$$

Drawing the (collapse) mechanisms for the two spans:



Left-hand span:

$$(M_n)(3.414\theta) = (24w_n)\left(\frac{1}{2}\right)(14.06\theta)$$

$$w_n = 0.0202\ M_n = (0.0202)(466.7) = 9.43 \text{ k/ft}$$

Right-hand span:

$$(M_n)(4\theta) = (30w_n)\left(\frac{1}{2}\right)(15\theta)$$

$$w_n = 0.0178\ M_n = (0.0178)(466.7) = 8.31 \text{ k/ft} \leftarrow$$

Example 4-3

A **W12 x 72** is used for the beam and columns of the frame shown in Fig. 4-3. If **Fy = 50 ksi,** determine the value of P_n.

Solution

The virtual-work expressions are written for parts (b), (c), and (d) of Fig. 4-3 and shown with the respective parts of the figure. The combined beam and sidesway case is found to be the critical case, and from it, the value of P_n is determined as follows:

Z_x **= 108 in³**

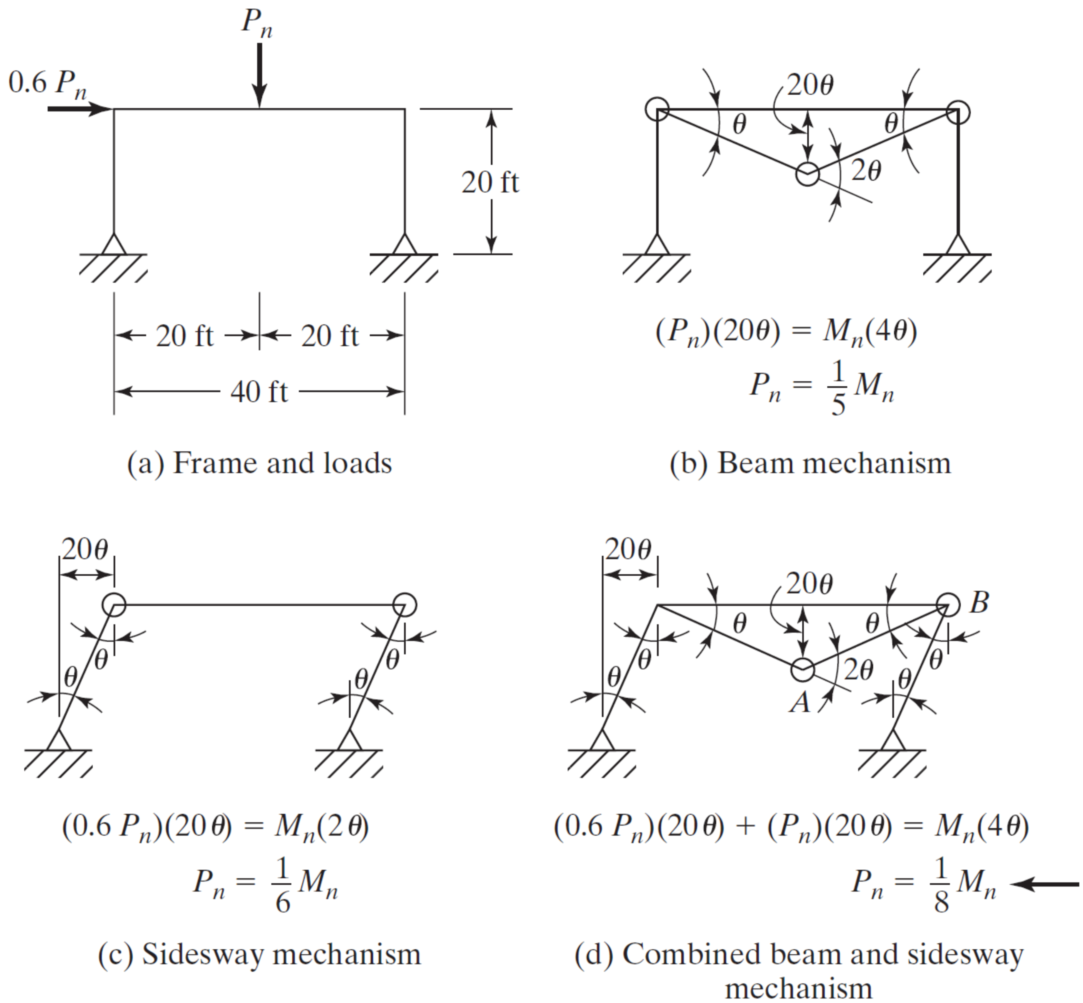


Figure 4-3

$$P_n = \frac{1}{8} M_n = \left(\frac{1}{8}\right)\left(F_y Z\right) = \left(\frac{1}{8}\right)\left(\frac{50x\,108}{12}\right) = 56.25\,\text{k}$$

5.4 Classification of Cross-Section

For the case of local buckling the slenderness is based on width/thickness ratios of the slender plate elements that make up the cross section of most steel members. The member cross sections are then classified by which of the three ranges their most slender element falls in as shown in Figure 5-4. If the most slender cross sectional element is not very slender (i.e. b/t is small) , then the cross section is said to be COMPACT. If the most slender element of the cross section falls in the transition range, then the cross section is said to be NON-COMPACT. Otherwise, when the most slender cross sectional element is very slender (i.e. b/t is large) then the cross section is said to be SLENDER. (pp. 3-5)

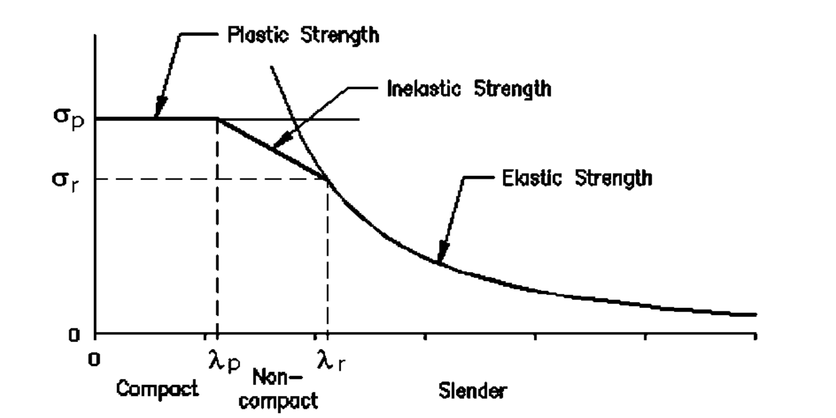


Figure 5-4 Theoretical Maximum Compressive Stress

It can be summarized as follows. Let : λ = width – thickness ratio
λ_p = upper limit for compact category
λ_r = upper limit for non-compact category
Then:

- If $\lambda \leq \lambda_p$ the shape is compact (an I-shape is compact if $\lambda_f \leq \lambda_{pf}$ and $\lambda_w \leq \lambda_{pw}$)
- If $\lambda_p < \lambda \leq \lambda_r$ the shape is noncompact
- If $\lambda > \lambda_r$ the shape is slender (an I-shape is slender if $\lambda_f \leq \lambda_{rf}$ and $\lambda_w \leq \lambda_{rw}$)

For rolled I-shape (Sec. B4, Table B 4-1, pp. 16 to18):

- $\lambda_f = b_f/2t_f$; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ and $\lambda_{rf} = 1.0\sqrt{E/F_y}$
- $\lambda_w = h/t_w$; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ and $\lambda_{rw} = 5.70\sqrt{E/F_y}$

These limits are also used for C-shape, except that λ for flange is: $\lambda_f = b_f/t_f$

5.5 *Design Strength of Beam*

5.5.1 Yielding Limit State

The specification computes the nominal moment capacity, M_n, as the maximum moment that a member can support. This maximum moment is considered to be when the cross section is fully yielded. Figure 5-5-1 Illustrates how the stress distribution changes as moment is increased on a section.

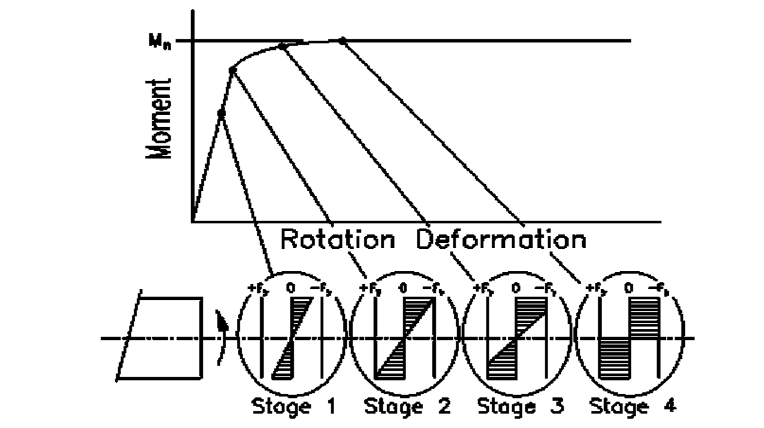


Figure 5-5-1 Flexural stress distribution

In this case M_n is the nominal flexural yielding strength of the member. For compact I-shaped members and channels bent about their major axis:

$$M_{nx} = M_{px} = F_y Z_x \qquad \text{for strong axis bending}$$

$$M_u \leq M_d = \phi_b M_{nx}$$

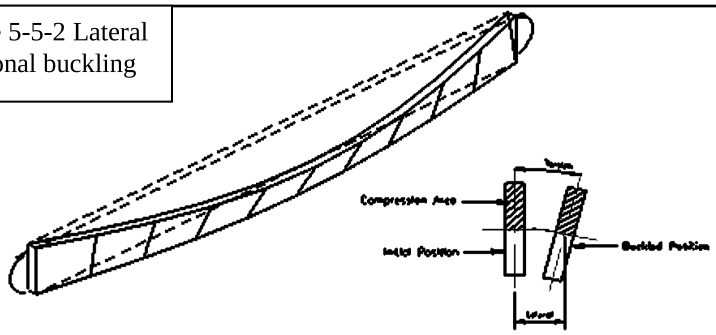
Where: $\phi_b = 0.9$

- M_p is the plastic flexural strength of the member.
- F_y is the material yield stress.
- Z is the plastic section modulus for the axis of bending being considered.

5.5.2 Lateral Torsional Buckling Limit State

5.5.2.1General: When a member is subjected to bending, one side of the member is in compression and wants to behave like a column. This means that it is subject to flexural buckling. Since the compression side is connected to the tension side (which is not prone to buckling), it cannot buckle in the plane of loading. This leaves the lateral direction as the direction of buckling. The tension side resists the buckling, resulting in the rotated cross section (i.e. the torsion). A simple experiment can be used to demonstrate this behavior, take a thin, flat bar (a typical "yard stick" works well) and apply end moments about the end with your hands. If you force bending about the strong axis, the member will buckle sideways and the section will rotate so that it is no longer vertical. This is lateral torsional buckling (LTB). The experiment is illustrated in Figure 5-5-2.

Figure 5-5-2 Lateral torsional buckling



If you bend the member about it's weak axis, this behavior is not observed. This is because the out-of-plane moment of inertia of the section is larger than in-plane moment of inertia. The out-of-plane inertia then creates a stiffness out-of-plane that is larger the in-plane, thus preventing the out-of-plane buckling. The result is that LTB is a strong axis phenomena. It need only be considered for strong axis bending. Like all buckling, the force that will cause LTB to happen (in this case, moment) is dependent on the length, or slenderness, of the "column". Figure 5-5-3 shows the general form of the curve used for LTB. For *LTB the length of the column* is length of laterally unsupported compression flange. If the length is short enough, then the member can develop it's full plastic strength. For longer lengths, there is inelastic buckling, and for long laterally unbraced lengths there is elastic buckling, following a typical buckling/plastic strength curve.

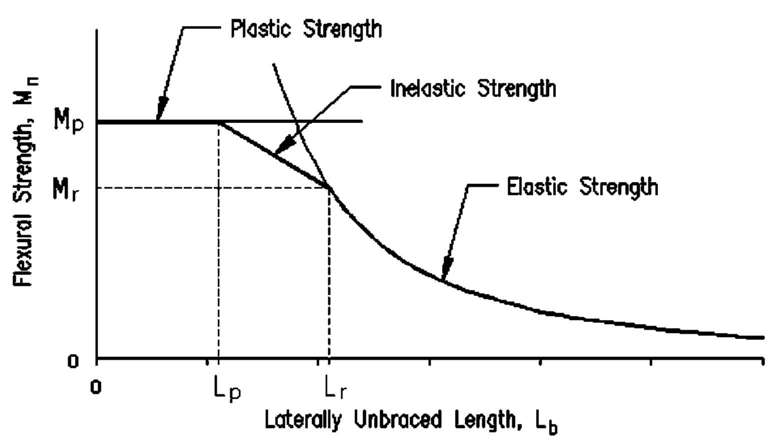


Figure 5-5-3 Strength (M_n) vs. Slenderness (L_b) for LTB

5.5.2.2 Laterally Unbraced Lengths(Classification of spans for flexure):

It is important to be able to identify laterally unbraced lengths in flexural members. The most important parameter in preventing the lateral buckling of the beam is the spacing, L_b, of the lateral bracing. There are a few criteria that must be considered.

1. The lateral support must be applied to the compression flange. Bracing at mid-height or at the tension flange is not sufficient.
2. The bracing must provide actual lateral support.

For the purlins to be effective as lateral supports (adequately braced beam), they must act to induce a point of inflection in the beam at the point of connection, as shown in Figure 5-5-4. In some cases, particularly cantilevered and continuous beams, the compression flange is on the bottom of the member so does not have any lateral support (.unbraced beam)

The general form of the LTB limit state follows the typical buckling curves. The slenderness parameter used is L_b, the *laterally unbraced length*. The limits of the buckling regions are specified by the terms L_p (the limit of the plastic region) and L_r (the limit of the inelastic buckling region) as shown in Figure 5-5-3. Hence:

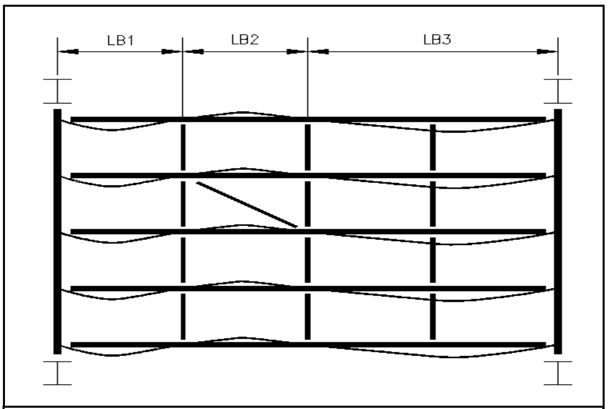


Figure 5-5-4 Lateral Bracing

- If $L_b \le L_p$ then the plastic strength, M_p, controls and LTB does not occur
- If $L_p < L_b \le L_r$ then inelastic LTB occurs
- If $L_b > L_r$ then elastic LTB occurs

Where: L_p = the limit of laterally unbraced length for plastic lateral buckling (Sec. F2, pp. 48) & (pp. 3-4 to 3-5)

$$= L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

L_r = the limit of laterally unbraced length for elastic lateral buckling (Sec. F2, pp. 48)

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.67\left(\frac{0.7F_y S_x h_o}{EJc}\right)^2}}$$

r_{ts} = effective radius of gyration, in (provided in AISC Table 1-1)
J = torsional constant, in^4 (AISC Table 1-1)
c = 1.0 for doubly symmetric I-shapes
h_o = distance between flange centroids, in (AISC Table 1-1)

5.5.2.3 Design moment:

- *Compact section*:

1. *Plastic Range (zone 1)*: As noted above, for a beam to be considered adequately braced, its compression flange should be either continuously braced, or the distance L_b between adjacent lateral braces should satisfy the relation (Sec. F2.1-pp. 47): $L_b \leq L_p$ (LTB does not happen) Consequently in the plastic range:

   $M_d = \phi_b M_p = \phi_b F_y Z_x$ (I-shape bent about the major axis)

2. *In-elastic Buckling Range (zone 2)*:A linear interpolating function is used to compute M_n in the in-elastic buckling range. The value resulting from the interpolation is then scaled by C_b. This value is compared with M_p to find the final M_n. Then the flexural design moment can be written as(Sec. F2.2-pp. 47):

$$M_d = \phi_b\, C_b\, (M_p - (M_p - 0.7 S_x F_y) * (L_b - L_p)/(L_r - L_p))$$

Or $$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

C_b = a coefficient which depends on variation in moments along the span (Sec. F1.)

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C}$$

Where:

M_{max} = largest moment in unbraced segment of a beam
M_A = moment at the ¼ point
M_B = moment at the ½ point
M_C = moment at the ¾ point

C_b = 1.0 for uniform distributed bending moment. Table 3-1(pp. 3-10) in LRDFM gives the value for C_b for simply supported beams.

3. *Elastic Buckling Range (zone 3)*: The nominal moment capacity, M_{nE}, in the elastic range is found by computing the elastic moment that creates the critical buckling stress, F_{cr}, in the compression flange(Sec. F3.2a-pp. 47).

$$M_n = F_{cr} S_x \le M_p$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

Where:

r_{ts} = effective radius of gyration, in (provided in AISC Table 1-1)
J = torsional constant, in^4 (AISC Table 1-1)
c = 1.0 for doubly symmetric I-shapes
h_o = distance between flange centroids, in (AISC Table 1-1)

- Non –compact section: if the section is non-compact because of flange or web ($\lambda_p < \lambda \le \lambda_r$)(Sec. F3.2, pp. 49):

$$M_n = \left[M_P - \left(M_p - 0.7F_y S_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right]$$

For built-up sections with slender flanges (that is, where $\lambda > \lambda_r$) (Sec. F3.2b, pp. 49):

$$M_n = \frac{0.9EK_c S_x}{\lambda^2}$$

$$K_c = \sqrt{\frac{h}{t_w}} \ge 0.35 \le 0.76$$

<u>Example Problem</u> 5-1: A compact W16×45 of A992 Gr. 50 steel is used as simply supported beam of 33-ft span, as shown in Figure. Determine the max. factored, uniform load that the beam can support if lateral supports are provide: (a) at 5.5 ft interval; (b) at 11 ft interval; (c) at 33 ft interval.

Solution: - From LRFDM, for W16×45:
A= 13.3 in^2; Z_x= 82.3 in^3; S_x= 72.7 in^3; I_y=32.8 in^4; r_y = 1.57 in and F_y= 50 ksi. J_c=1.11.

a) L_p = 5.55 Tables 3-2. p.(3-17)
L_b= 5.5' < L_p =5.55'
Then $M_d = \phi_b M_{px} = \phi_b F_y Z_x$ = 309ft-kips

$M_{max} = M_d = \frac{q_{u1}L^2}{8}$ ….. $q_{u1} = \frac{309*8}{33^2} = 2.27$ klf

Note: The max. factored, uniform load for F_y= 36 (For MC-Section) & F_y=50 ksi (For W-Section), are tabulate in LRFDM to Tables 3-6. p.(3-33) to p.(3-95) for fully braced beam or when $L_b < L_p$.
for our example enter Factored Uniform Loads
Q_u = 74.8 kips p. (3-61)
For W16×45, F_y= 50 ksi and L=33'
q_{u1} = 74.8/33 =2.27 klf

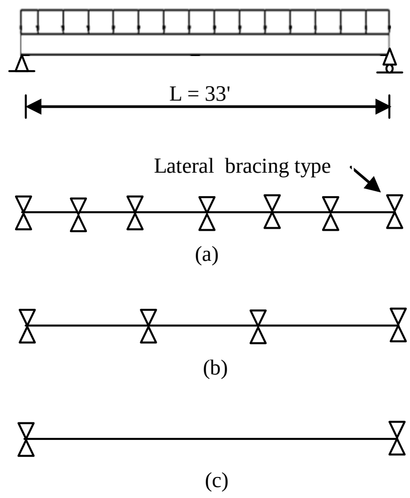


Figure: Example problem 5-1

b) $L_p = 5.55 < L_b$= 11' then calculate L_r

L_r = 15.2 > L_b= 11' Tables 3-2. p.(3-17)

$M_d = \phi_b C_b (M_{px} - (M_{px} - 0.7S_xF_y)*(L_b - L_p)/(L_r - L_p))$

Or $\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$

C_b = 1.01 …. (Table 3-1, p. 3-10) , BF= 10.8 ……Tables 3-2. p.(3-17)

M_d= 252.6 ft-kip

$M_{max} = M_d = \frac{q_{u1}L^2}{8}$ ….. $q_{u1} = \frac{252.6*8}{33^2} = 1.86$ klf

c) L_b= 33' > L_r

$$F_{cr} = \frac{C_b\pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$M_n = F_{cr}S_x \leq M_p$

C_b = 1.14 …. (Table 3-1, p. 3-10)

S_x= 72.7 in^3, h_o= 16.5 in, r_{ts}= 1.88 in, J= 1.11
M_d = 830.6 > $\phi_b M_{px}$

$M_{max} = M_d = 309 = \frac{q_{u1}L^2}{8}$ ….. $q_{u1} = \frac{309*8}{33^2} = 2.27$ klf

Example Problem 5-2: A W12×65 of A992 Gr. 50 steel has unbraced length of 11'. Determine the design bending moment.

Solution: - From LRFDM (Table 3-2, pp. 3-17) for W12×65; Z_x= 96.8 in^3; r_y=3.02" and F_y= 50 ksi.

$\lambda_f = b_f/2t_f = 9.92$; $\lambda_{pf} = 0.38\sqrt{E/F_y} = 9.15$ and $\lambda_{rf} = 1.0\sqrt{E/F_r} = 24.08$

$\lambda_w = h/t_w = 24.9$; $\lambda_{pw} = 3.76\sqrt{E/F_y} = 90.6$ and $\lambda_{rw} = 5.70\sqrt{E/F_y} = 137$

As $\lambda_{pf} < \lambda_f < \lambda_{rf}$ ……the flange is noncompact, but web is compact

$M_{px} = F_y Z_x = 50*87.9 = 4840$ in-kips = 403.33 ft-kips

$M_d = \phi_b M_n = \phi_b [M_{px} - (M_{px} - M_{rx})*(\lambda_b - \lambda_p)/(\lambda_r - \lambda_p)] = 395.7$ ft-kip.

5.6 Selecting Sections

The objective of the selection process is, generally, to select the least cost (this is also frequently the lightest) member that satisfies the design criteria. For beams, there are multiple limit states to consider. The selection criteria can be stated as: select the lightest section such that:

- Req'd $M_n \leq$ Actual M_n,
- Req'd $V_n \leq$ Actual V_n, and
- Actual $\delta \leq$ Allowed δ.

5.7 Shear Strength Limit State

Beam shear strength must be provided to resist the anticipated applied beam shears. In steel members, the elements of the cross section that resist shear may be very slender. As a result the shear elements may be subject to the normal ranges of the buckling curve, including plastic, inelastic buckling, and elastic buckling behaviors. The distribution of elastic beam shear stress on a given cross section is determined by the following equation:

$$\tau = VQ/(Ib)$$

Where: τ is the shear stress at some point on the cross section.

- V is the shear force acting on the cross section.
- Q is the first moment of area “above” the point where of interest is.
- I is the moment of inertia of the cross section.
- b is the breadth (i.e. width), parallel to the axis of bending, of the cross section at the point of interest.

The graph of this equation over the height of a rectangular section and an "I" shaped section is shown in Figure 5-7-1. Its appear that for I-shapes bent about their major axis, it is assumed that only the web resists the shear and that the intensity of shear stress is uniform throught the depth. The design shear strength.

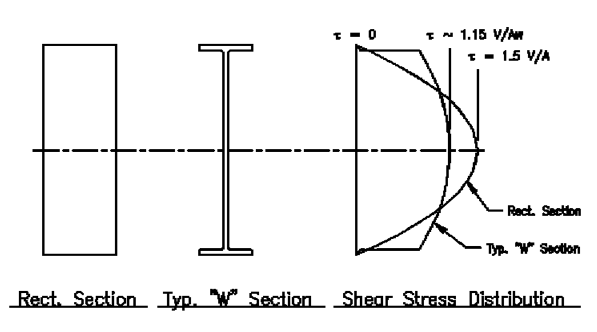


Figure 5-7-1 Shear stress distribution

For I-rolled for limit state of shear yielding of the web is (Sec. G2, pp. 64 to 65):

$$V_d = \varphi_v V_n = \varphi_v F_{yv} A_w = \varphi_v C_v (0.6F_y) A_w$$

Where: $\varphi_v = 0.90$

- F_{yv} is the shear yield strength of the steel=$0.6F_y$
- A_w is the shear are of a web. For I shaped members including channels, A_w equals the overall depth times the web thickness, d t_w.

$$V_d = \varphi_v V_n = 0.54 F_y A_w C_v$$

- C_v is a modifier that accounts for buckling behavior of the web.

$$\frac{h}{t_w} \le 1.10\sqrt{\frac{k_v E}{F_y}} \qquad C_v = 1.0$$

$$1.10\sqrt{\frac{k_v E}{F_y}} < \frac{h}{t_w} \le 1.37\sqrt{\frac{k_v E}{F_y}} \qquad C_v = \frac{1.10\sqrt{\frac{k_v E}{F_y}}}{\frac{h}{t_w}}$$

$$\frac{h}{t_w} > 1.37\sqrt{\frac{k_v E}{F_y}} \qquad C_v = \frac{1.51 E k_v}{\left(\frac{h}{t_w}\right)^2 F_y}$$

For webs without transverse stiffeners and with

$$\frac{h}{t_w} < 260 \qquad k_v = 5$$

5.8 Deflection Limit State

The calculations of deflection are done at service (i.e. actual) levels and for load combinations that make sense for the project and/or member under consideration. Typically, two different loadings are considered: Total load (dead plus transient loads such as live load and snow) and transient load only.
Total load deflections are important because these will have an impact on nonstructural elements that are near to or attached to the beam. The transient load deflections are important for maintaining the comfort of occupants. In the absence of more specific criteria, criteria for structures with brittle finishes (as found in code documents for years) is frequently used.
Standard American practice for buildings has been to limit service live-load deflections to approximately 1/360 of the span length.
This deflection is supposedly the largest value that ceiling joists can deflect without causing cracks in underlying plaster. The 1/360 deflection is only one of many maximum deflection values in use because of different loading situations, different engineers, and different specifications. (Table 3-23, pp. 3-211 to 3-226, for calculating moment, shear and deflection for different support conditions)

***Example Problem* 5-3:** Select s standard W-shape of A992 Gr. 50 steel for used as simply supported beam of 30-ft span, as shown in Figure 5-8. the beam has continuous lateral supports and support a uniform service live load of 4.5 kips/ft. Max allowable live load deflection is 1.5".

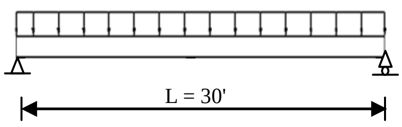


Figure 5-8: Example problem 5-3

Solution: - Ignore the beam weight initially then check
after a selection is made.

W_u = 1.6 L.L.= 1.6(4.5) = 7.2 kips/ft.

$M_u = \frac{1}{8} W_u L^2$ = 810 ft-kips

Assume the shape is compact with full lateral support

$M_d = \phi_b M_{px} = \phi_b F_y Z_x \geq M_u$ =810 ft-kips

$Z_x \geq M_u/\phi_b F_y = 216$ in^3

Try W 24×84 (LRFDM p.3-16), $Z_x = 224$ in^3

W_u = 1.2 D.L. + 1.6 L.L.= 1.2(0.084) + 1.6(4.5) = 7.3 kips/ft.

$M_u = \frac{1}{8} W_u L^2$ = 821.4 ft-kips

$Z_{x,\,req} = M_u/\phi_b F_y = 219$ in^3 < 224 in^3 OK

This shape is compact (noncompact shape are marked as such table)

$V_u = \frac{1}{2} W_u L$ = 110 kips

$\lambda_w = 21/0.47 = 44.68 < \lambda_{pv} = 2.45\sqrt{\dfrac{E}{F_y}} = 59$

$\phi_b V_n = \phi_b\, 0.6 F_y A_w = 0.9*0.6*50*(24.1*0.47) = 306$ kips $> V_u$ OK

$\Delta_{L.L} = \dfrac{5W_{LL}L^4}{384EI_x} = 1.19in < 1.5$ (p. 3-211) OK

***Example Problem* 5-4:** Select s standard W18×?-shape of A992 Gr. 50 steel for used as simply supported beam of 30-ft span, as shown in Figure 5-9. The beam supports a two equal concentrated service live and dead load of 24 and 10 kips/ft, respectively, at one-third and two-third. The beam is supported laterally at the points of load application. Max allowable live load deflection is 1.3".

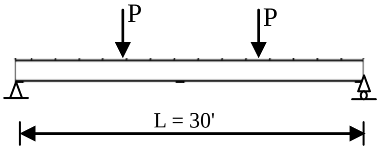


Figure: Example problem 5-4

Solution: - Ignore the beam weight initially and assume that $L_b < L_p$. Then one can use LRFDM Max. factored uniform loads Tables but first enter to the LRFD Table of Concentrated Load Equivalents on p.(3-208):

- Equivalent uniform load = 2.667 P_u (Table 3-22a, p. 3-208)

- Required factored uniform load:

$$P_u = 1.2(10) + 1.6(24) = 50.4$$
$$W_u = 2.667\, P_u = 135 \text{ kips}$$

- Enter factored uniform loads Table for F_y=50 ksi and $W_u \geq$ 135 kips

- 1st trial – W 18×71: W_u = 146 kips > 135 kips (p.3-58)
  L_b= 10' , L_p= 6' and L_r = 19.6' (p.3-16)
  Since $L_p < L_b < L_r$

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

C_b = 1…. (Table 3-1, p. 3-10) , $\phi_b M_{px}$= 548 kip-ft , BF= 15.7 …… p.(3-16)

M_d =485.2 kip-ft

$$M_{u,\,max} = (50.4*10) + \frac{0.071*35^2}{8} = 511.3 \text{ kip-ft} > M_d \quad \text{Not. Ok.}$$

- 2nd trial – W 18×76: W_u = 163 kips > 135 kips (p.4-87)
  L_b= 10' , L_p= 9.22' and L_r = 27.1'

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

C_b = 1…. (Table 3-1, p. 3-10) , $\phi_b M_{px}$= 611 kip-ft , BF= 12.8 …… p.(3-16)

M_d =601 kip-ft

$$M_{u,\,max} = (50.4*10) + \frac{0.076*35^2}{8} = 512 \text{ kip-ft} < M_d \text{ ….. OK}$$

Use W 18×76

- Check for shear requirement: $V_u = P_u = 1.2(10) + 1.6(24) = 50.4$ kips
  $\lambda_w = 15.5/0.425 = 36.47 < \lambda_{pv} = 2.45\sqrt{\frac{E}{F_y}} = 59$
  $\phi_b V_n = \phi_b\, 0.6 F_y A_w = 0.9*0.6*50*(15.5*0.425) = 177.86 \text{ kips} > V_u$ OK.

- Check for live load deflection: M_{LL} = 24*10 = 240 kip-ft
  The max. deflection is (See p. 3-7):

$$\Delta_{\text{max. L.L (@ mid span)}} = \frac{M_{LL}L^2}{C_1 I_x} = \frac{240(30)^2}{158(1330)} 1.03 in < 1.3 \quad \text{OK.}$$

***Example Problem* 5-5:** Select standard W-shape of A992 Gr. 50 steel for used as framed girder of 35-ft span, as shown in Figure 5-10, using LRFD Beam Design Moment Charts. it is supported a two equal concentrated service, which produce a required moment of 440 kip-ft in the center between the two loads. The beam is supported laterally at the points of load application.

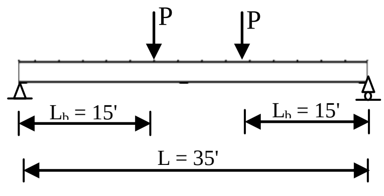


Figure : Example problem 5-5

Solution: - LRFD Beam Design Moment Charts (p.4-113 to p.4-166) can be used for $L_p < L_b \leq L_r$ and $C_b = 1.0$

For this load condition, $C_b = 1.0$ (the moment is uniform between the two loads. Since the 15' is longest unbraced length, one can expected that $L_p < L_b < L_r$.

With total span of 35' and M_u=440 kip-ft., assume weight of beam 70 ibs/ft

$$M_{u,\,total} = 440 + \left(1.2 * \frac{0.07 * 35^2}{8}\right) = 453 \text{ kip-ft.}$$

Enter the chart with L_b =15' and M_u=453 kip-ft, any beam listed above and to the right of intersecting point satisfies the design moment requirement. The solid portion of curves indicated the most economical section by weight, while the dashed portion of curves indicated ranges in which a lighter weight beam will satisfy the loading conditions.
For our example: Use W21×68 (p. 3-121)

$$M_d = 457 \text{ kip-ft} > M_u = 453 \text{ kip-ft}$$