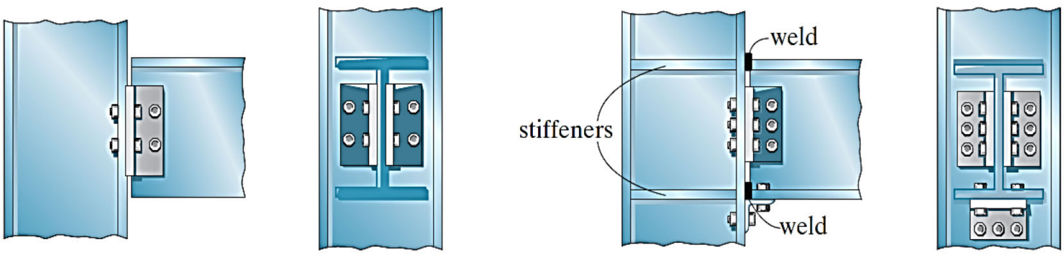
2 DETERMINACY AND STABILITY

2.1 Support Connections:

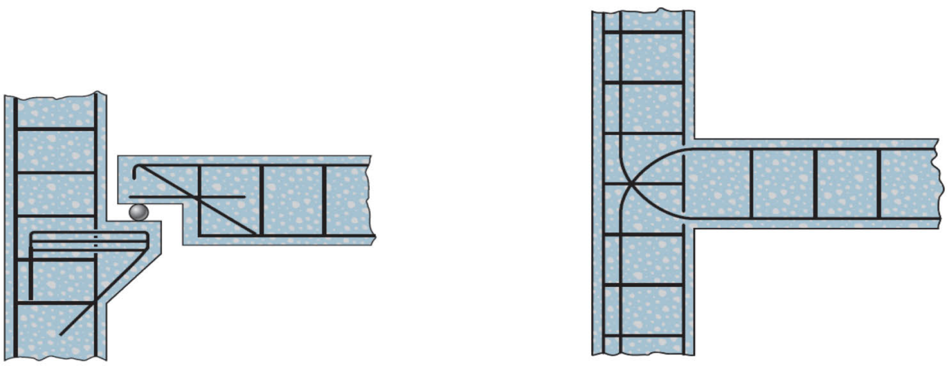
- ✓ Structural members are joined together in various ways depending on the intent of the designer.
- ✓ The three types of joints most often specified are the *pin connection, the roller support, and the fixed joint*.
- ✓ A ***pin-connected*** joint and a ***roller support*** allow some freedom for **slight rotation**.
- ✓ ***Fixed joint*** allows no relative rotation between the connected members and is consequently more expensive to fabricate. Examples of these joints, fashioned in metal and concrete, are shown in Figs. 2–1 and 2–2, respectively.
- ✓ For most timber structures, the members are assumed to be pin connected, since bolting or nailing them will not sufficiently restrain them from rotating with respect to each other.
- ✓ In reality, however, all connections exhibit some stiffness toward joint rotations, owing to friction and material behavior. In this case a more appropriate model for a support or joint might be that shown in Fig. 2–3c. If the torsional spring constant $\boldsymbol{k} = \mathbf{0}$, the joint is a pin, and if $\boldsymbol{k} = \boldsymbol{\infty}$, the joint is fixed.



typical "pin-supported" connection (metal)
(a)

typical "fixed-supported" connection (metal)
(b)

Fig. 2–1



typical "roller-supported" connection (concrete)
(a)

typical "fixed-supported" connection (concrete)
(b)

Fig. 2–2

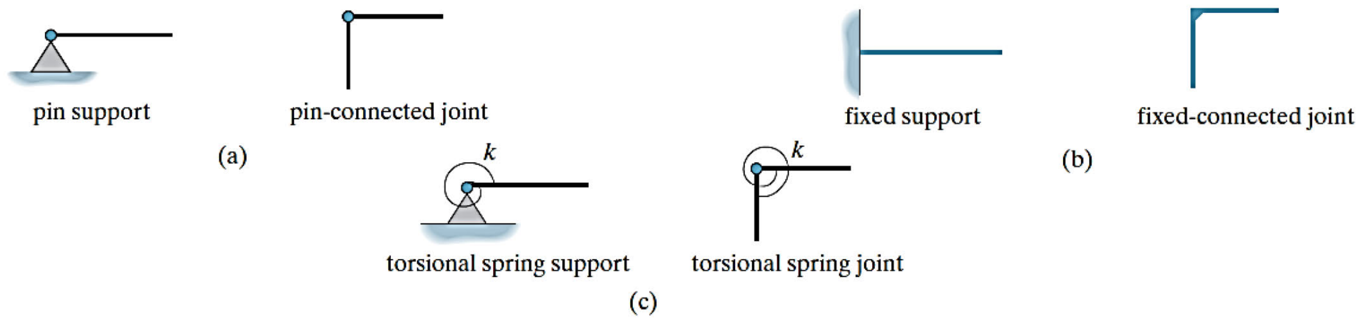
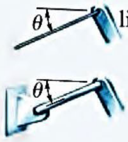
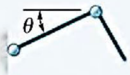
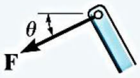
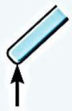
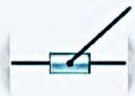
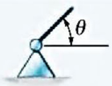
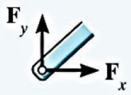
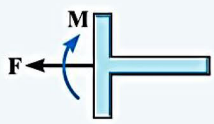
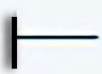
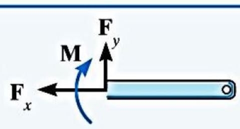


Fig. 2–3

✓ In reality, all supports actually exert *distributed surface loads* on their contacting members. The concentrated forces and moments shown in Table 2–1 represent the *resultants* of these load distributions. This representation is, of course, an idealization; however, it is used here since the surface area over which the distributed load acts is considerably *smaller* than the *total* surface area of the connecting members.

TABLE 2–1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable, weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers, rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4)  smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)  smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider  fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

2.2 Determinacy and Stability

Determinate Structure

The structure is said to be determinate if,

Number of Unknowns = Total Number of Equilibrium Equations.

Indeterminate Structure

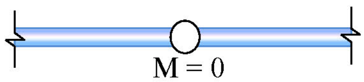
The structure is said to be indeterminate if,

Number of Unknowns > Total Number of Equilibrium Equations.

Equations of Condition (C)

1. Interior hinge connecting two members

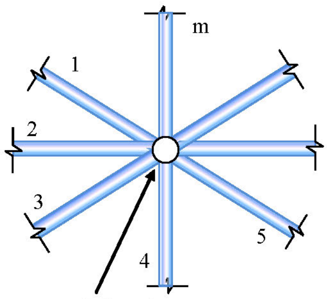
C = 1



M = 0
Equations of Condition = 1

2. Interior hinge connecting (m) members

C = m – 1

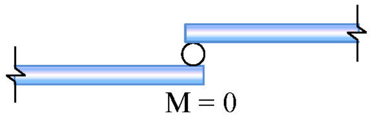


M = 0
Equations of Condition = m - 1

3. Interior roller

`

C = 2



M = 0
$F_x = 0$
Equations of Condition = 2

Basic Equations of Equilibrium

In plan structures, the basic equations of equilibrium are three, which are:-

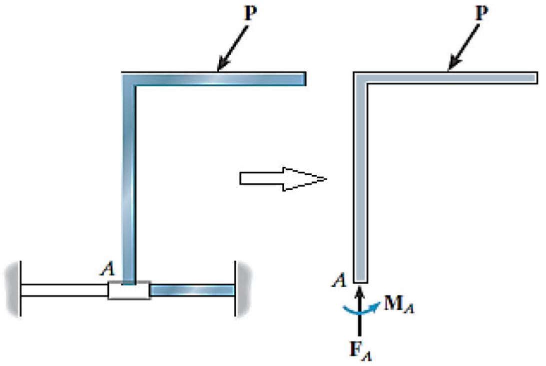
$$\sum F_x = 0 \ ; \ \sum F_y = 0 \ ; \ \sum M = 0$$

✓ It must be emphasized her that for one structural element, only three unknowns can be evaluated by using these equations. (no matter how several times the equation $\sum M = 0$)

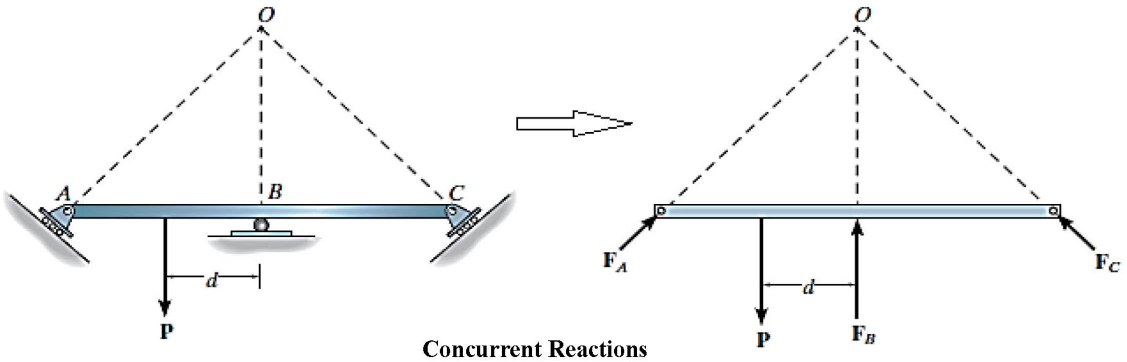
Unstable Structure

The structure is said to be unstable if any one of the following four conditions available

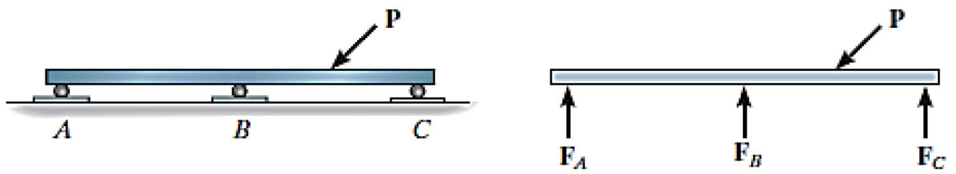
1. When number of unknowns < Total Number of equilibrium equations.
2. When all reaction are concurrent (meeting at one point).
3. When all reaction are parallel.
4. When the structure is geometrically unstable.
   (ex. Three hinges in one span makes a mechanism unstable)



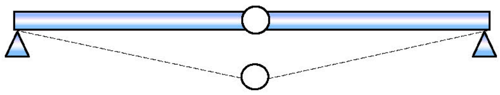
Partial Constraints
Number of Unknowns < Total Number of Equilibrium Equations



Concurrent Reactions



parallel reactions



Structure is Geometrically Unstable

2.2.1 Stability and Determinacy of Beams

Let **r = Number of Reactions (Unknowns).**

Total Number of Equilibriums Equations = 3 + C

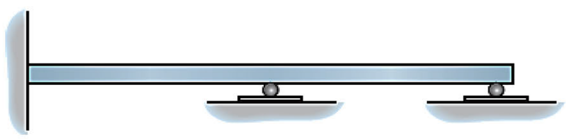
Therefore,

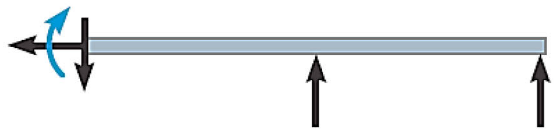
If	**$r < 3 + C$**	**The beam is unstable**
If	**$r = 3 + C$**	**The beam is determinate <u>if</u> stable**
If	**$r > 3 + C$**	**The beam is indeterminate <u>if</u> stable**

EXAMPLE 2.2.1.1
Classify each of the beams as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.

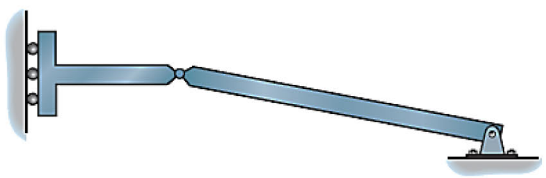
Solution

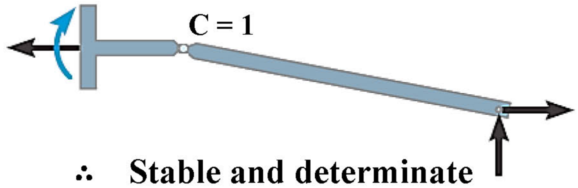
$r = 3$, $3 + C = 3 + 0 = 3$ ⇨ **$r = 3 + C$ $(3 = 3)$** ∴ **Stable and determinate**



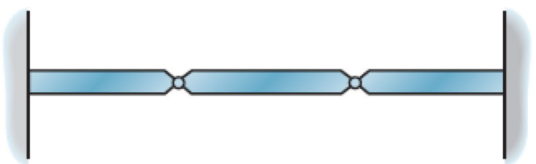


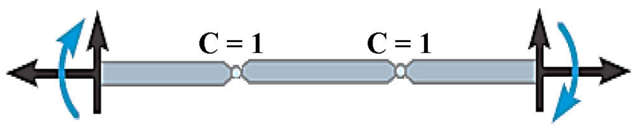
$r = 5$, $3 + C = 5$ ⇨ **$r > 3 + C$ $(5 > 3)$** ∴ **Stable and indeterminate to 2nd degree**





$r = 4$, $3 + C = 3 + 1 = 4$ ⇨ **$r = 3 + C$ $(4 = 4)$** ∴ **Stable and determinate**

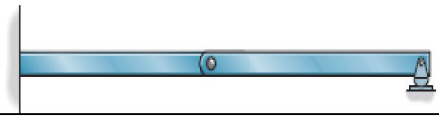
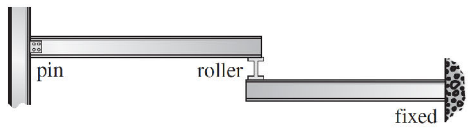
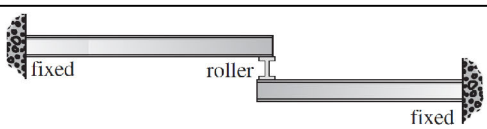




$r = 6$, $3 + C = 3 + 2 = 5$ ⇨ **$r > 3 + C$ $(6 = 5)$** ∴ **Stable and indeterminate to 1st degree**

EXAMPLE 2.2.1.2

Classify each of the beams as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.

Beam	r	C	3 + C	< r = 3 + C >	Stability and Determinacy
	3	0	3	3 = 3	Stable and determinate
	3	0	3	3 = 3	Unstable (parallel reactions)
	4	1	4	4 = 4	Stable and determinate
	5	0	3	5 > 3	Stable and indeterminate 2^{nd} degree
	5	1	4	5 > 4	Stable and indeterminate 1^{st} degree
	5	2	5	5 =5	Unstable (geometrically unstable)
	6	2	5	6 > 5	Stable and indeterminate 1^{st} degree
	5	2	5	5 = 5	Stable and determinate
	6	2	5	6 > 5	Stable and indeterminate 1^{st} degree
	5	0	3	5 > 3	Stable and indeterminate 2^{nd} degree

2.2.2 Stability and Determinacy of Trusses

Let **r = Number of Reactions.**
b = Number of Bars.
J = Number of Joints.

Number of Unknowns = b + r

Total Number of Equilibriums Equations = 2J

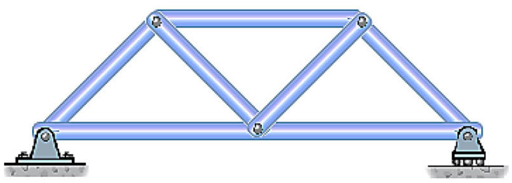
Therefore,

If	**b + r < 2J**	**The truss is unstable**
If	**b + r = 2J**	**The truss is determinate <u>if</u> stable**
If	**b + r > 2J**	**The truss is indeterminate <u>if</u> stable**

EXAMPLE 2.2.2.1

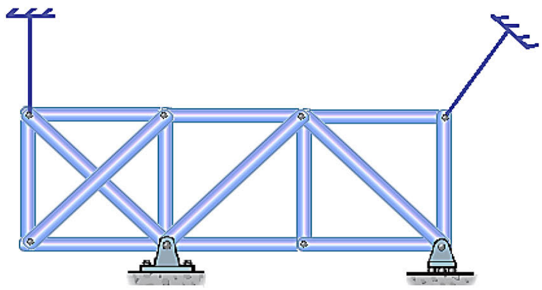
Classify each of the trusses as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.

Solution



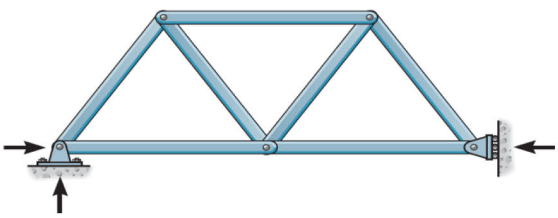
r = 3 , b = 7 , 2J = 2(5) ⇨ **r +b = 2J (10 = 10)**

∴ **Stable and determinate**



r = 5 , b = 14 , 2J = 2(8) ⇨ **r +b > 2J (19 > 16)**

∴ **Stable and indeterminate to 3rd degree**

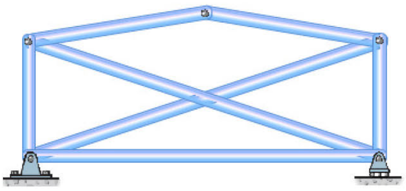
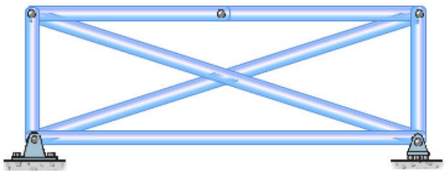
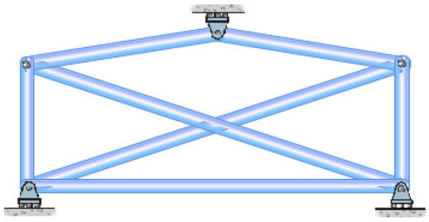
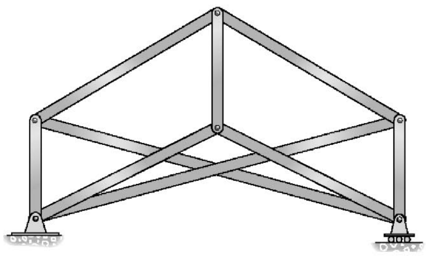
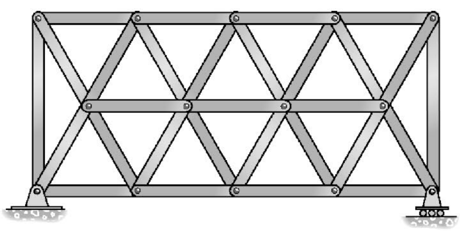
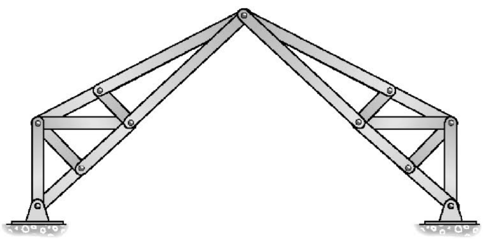


r = 3 , b = 7 , 2J = 2(5) ⇨ **r +b = 2J (10 = 10)**

∴ **Unstable (concurrent reactions)**

EXAMPLE 2.2.2.2

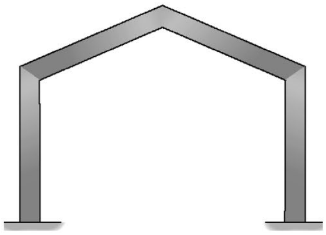
Classify each of the trusses as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.

Truss	r	b	J	< r + b = 2J >	Stability and Determinacy
	3	7	5	10 = 10	Stable and determinate
	3	7	5	10 = 10	Unstable (geometrically unstable)
	3	7	5	10 = 10	Unstable (parallel reactions)
	3	9	6	12 = 12	Stable and determinate
	3	29	14	32 > 28	Stable and indeterminate 4^{th} degree
	4	18	11	22 = 22	Stable and determinate

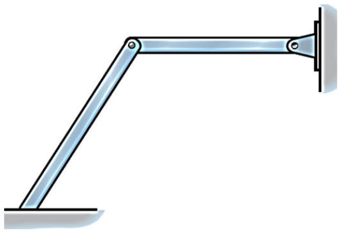
2.2.3 Stability and Determinacy of Frames and Arches

1. Open Frames and Arches

Can be treated similar to beams by apply $r \lesseqgtr 3 + C$



r = 6 , 3 + C = 3 + 0 = 3
r > 3 + C (6 > 3)
∴ Stable and indeterminate 3rd degree



r = 5 , 3 + C = 3 + 1 = 4
r > 3 + C (5 > 4)
∴ Stable and indeterminate 1st degree

2.Closed Frames and Arches

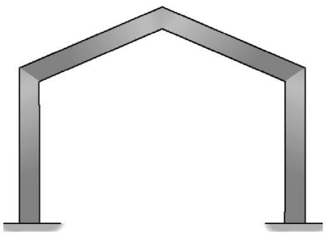
Let **r = Number of Reactions.**
b = Number of Members.
J = Number of Joints.

Number of Unknowns = 3b + r

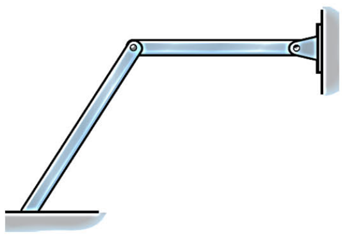
Total Number of Equilibriums Equations = 3J + C

Therefore,

If	**3b + r < 3J + C**	**The frame is unstable**
If	**3b + r = 3J + C**	**The frame is determinate <u>if</u> stable**
If	**3b + r > 3J + C**	**The frame is indeterminate <u>if</u> stable**



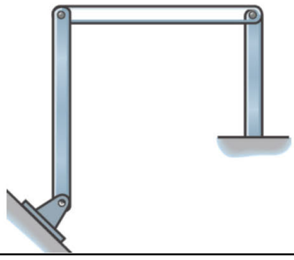
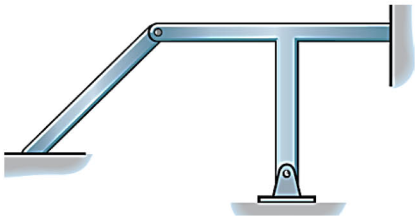
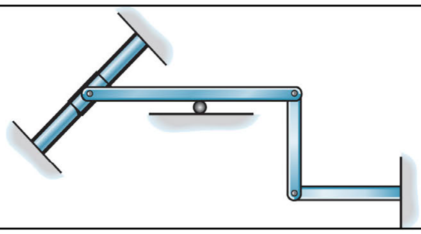
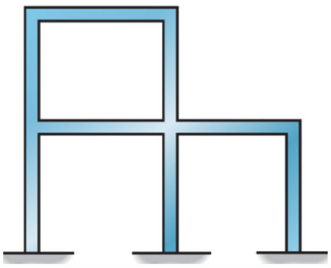
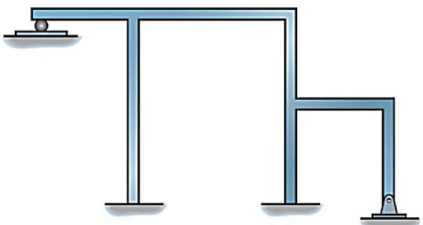
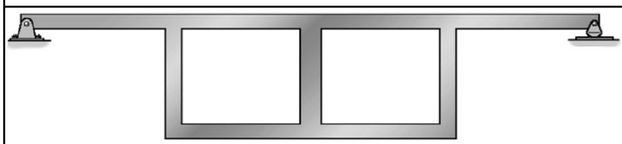
b = 4 , r = 6 , J = 5 , C = 0
3b + r = 3(4) + 6 = 18 , 3J = 15
3b + r > 3J + C (18 > 15)
∴ Stable and indeterminate 3rd degree



b = 2 , r = 5 , J = 3 , C = 1
3b + r = 3(2) + 5 = 11 , 3J = 9
3b + r > 3J + C (11 > 10)
∴ Stable and indeterminate 1st degree

EXAMPLE 2.2.3.1

Classify each of the Frames as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.

Frame	b	r	C	J	< 3b+r =3J+C >	Stability and Determinacy
	3	5	2	4	14 = 14	Stable and determinate
	4	8	1	5	20 = 16	Stable and indeterminate 4^{th} degree
	4	10	4	6	22 = 22	Stable and determinate
	8	9	0	8	33 > 24	Stable and indeterminate 9^{th} degree
	7	9	0	8	30 > 24	Stable and indeterminate 6^{th} degree
	9	3	0	8	30 > 24	Stable and indeterminate 6^{th} degree