4 ANALYSIS OF STATICALLY DETERMINATE TRUSSES

4.1 Common Types of Trusses

A ***truss*** is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts, metal bars, angles, or channels. The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate,* as shown in Fig. 4–1, or by simply passing a large bolt or pin through each of the members. Planar trusses lie in a single plane and are often used to support roofs and bridges.

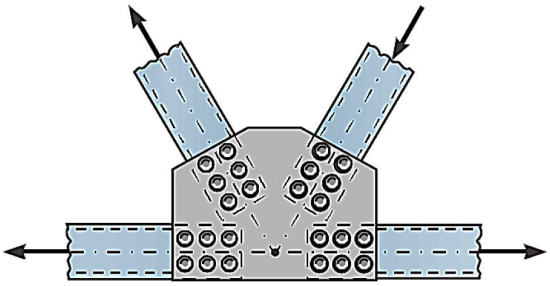


Fig. 4-1

Roof Trusses. Roof trusses are often used as part of an industrial building frame, such as the one shown in Fig. 4–2. Here, the roof load is transmitted to the truss at the joints by means of a series of *purlins*. The roof truss along with its supporting columns is termed a *bent*. Ordinarily, roof trusses are supported either by columns of wood, steel, or reinforced concrete, or by masonry walls.To keep the bent rigid, and thereby capable of resisting horizontal wind forces, knee braces are sometimes used at the supporting columns.

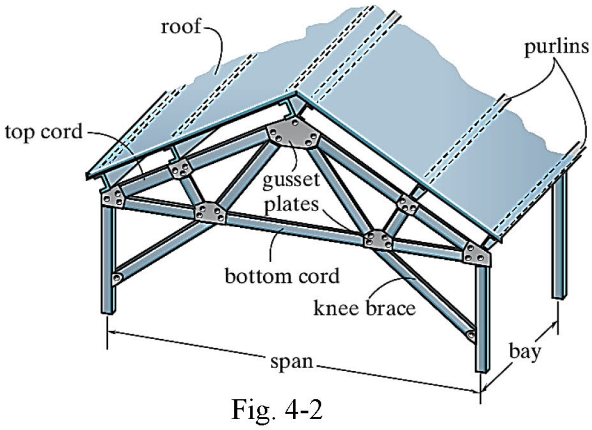


Fig. 4-2

Trusses used to support roofs are selected on the basis of the span, the slope, and the roof material. Some of the more common types of trusses used are shown in Fig. 4–3. In particular, the scissors truss, Fig. 4–3a, can be used for short spans that require overhead clearance.The Howe and Pratt trusses, Figs. 4–3b and 4–3c, are used for roofs of moderate span, about 60 ft (18 m) to 100 ft (30 m). If larger spans are required to support the roof, the fan truss or Fink truss may be used, Figs. 4–3d and 4–3e. These trusses may be built with a cambered bottom cord such as that shown in Fig. 4–3f. If a flat roof or nearly flat roof is to be selected, the Warren truss, Fig. 4–3g, is often used. Also, the Howe and Pratt trusses may be modified for flat roofs. Sawtooth trusses, Fig. 4–3h, are often used where column spacing is not objectionable and uniform lighting is important. A textile mill would be an example. The bowstring truss, Fig. 4–3i, is sometimes selected for garages and small airplane hangars; and the arched truss, Fig. 4–3j, although relatively expensive, can be used for high rises and long spans such as field houses, gymnasiums, and so on.

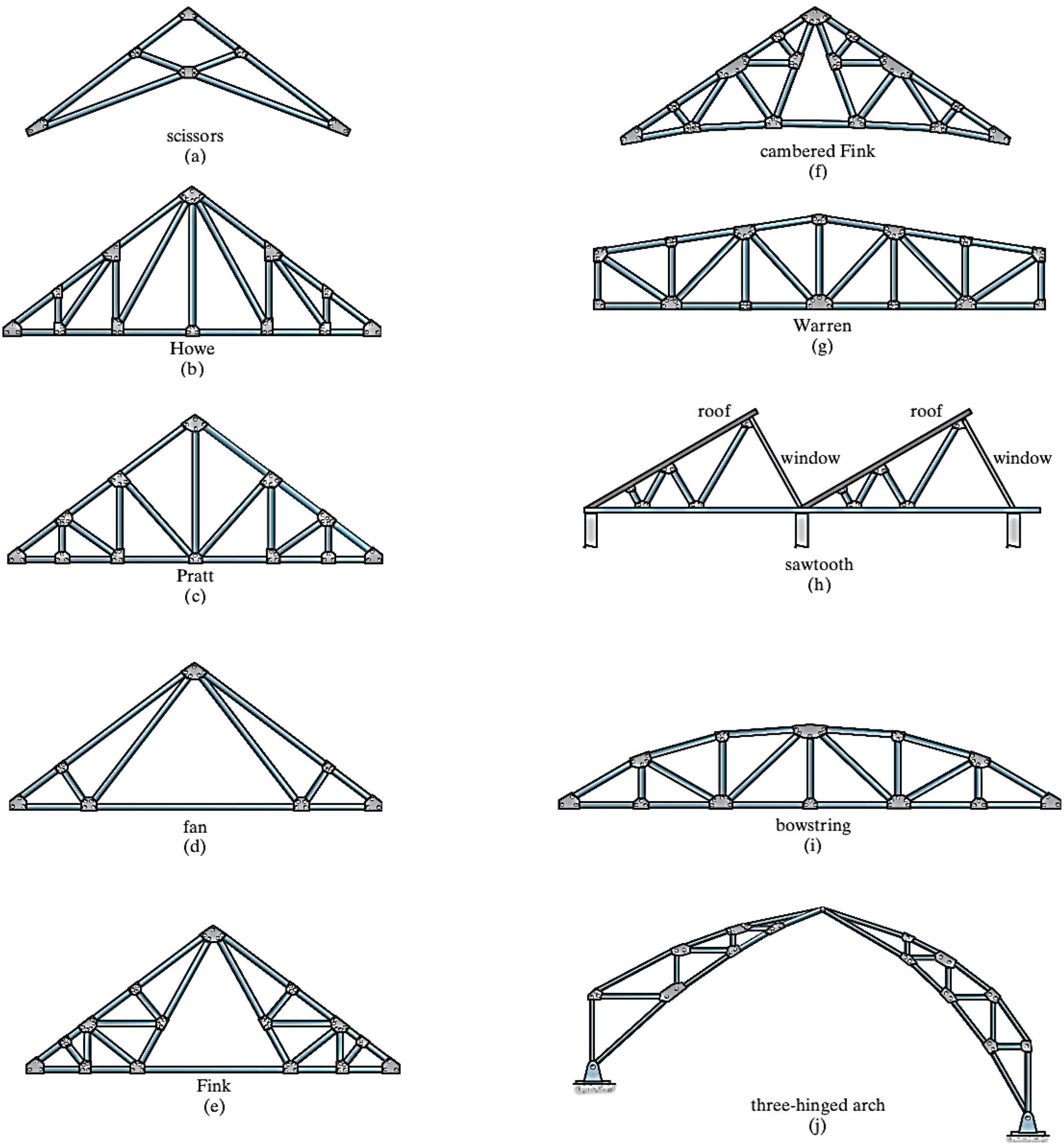

scissors
(a)
Howe
(b)
Pratt
(c)
fan
(d)
Fink
(e)
cambered Fink
(f)
Warren
(g)
roof
window
roof
window
sawtooth
(h)
bowstring
(i)
three-hinged arch
(j)

Fig. 4-3

Bridge Trusses. The main structural elements of a typical bridge truss are shown in Fig. 4–4. Here it is seen that a load on the *deck* is first transmitted to *stringers,* then to *floor beams,* and finally to the joints of the two supporting side trusses.The top and bottom cords of these side trusses are connected by top and bottom *lateral bracing,* which serves to resist the lateral forces caused by wind and the sidesway caused by moving vehicles on the bridge. Additional stability is provided by the *portal* and *sway bracing*. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.

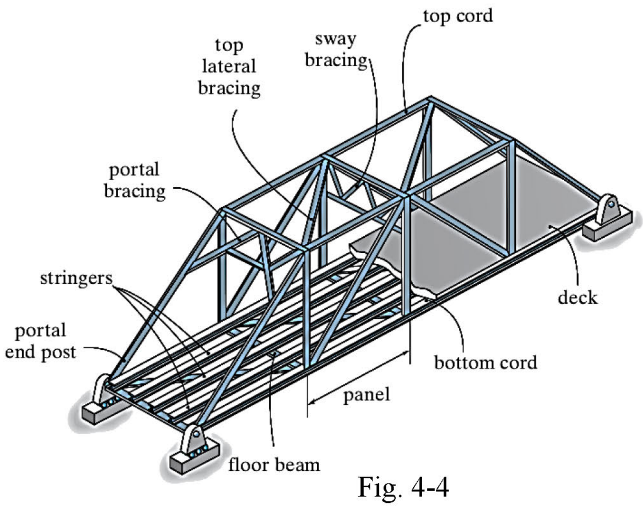


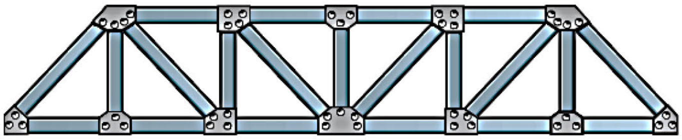
Fig. 4-4

A few of the typical forms of bridge trusses currently used for single spans are shown in Fig. 4–5. In particular, the Pratt, Howe, and Warren trusses are normally used for spans up to 200 ft (61 m) in length. The most common form is the Warren truss with verticals, Fig. 4–5*c*. For larger spans, a truss with a polygonal upper cord, such as the Parker truss, Fig. 4–5*d*, is used for some savings in material. The Warren truss with verticals can also be fabricated in this manner for spans up to 300 ft (91 m). The greatest economy of material is obtained if the diagonals have a slope between 45° and 60° with the horizontal. If this rule is maintained, then for spans greater than 300 ft (91 m), the depth of the truss must increase and consequently the panels will get longer. This results in a heavy deck system and, to keep the weight of the deck within tolerable limits, *subdivided* trusses have been developed.Typical examples include the Baltimore and subdivided Warren trusses, Figs. 4–5*e* and 4–5*f*. Finally, the K-truss shown in Fig. 4–5*g* can also be used in place of a subdivided truss, since it accomplishes the same purpose.

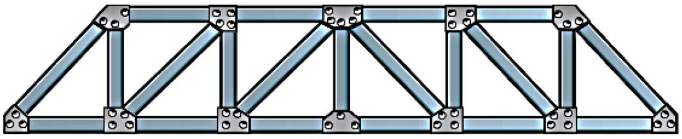
Assumptions for Design. To design both the members and the connections of a truss, it is first necessary to determine the *force* developed in each member when the truss is subjected to a given loading. In this regard, two important assumptions will be made in order to idealize the truss.

1 *The members are joined together by smooth pins*. In cases where bolted or welded joint connections are used, this assumption is generally satisfactory provided the center lines of the joining members are concurrent at a point, as in Fig. 4–1.

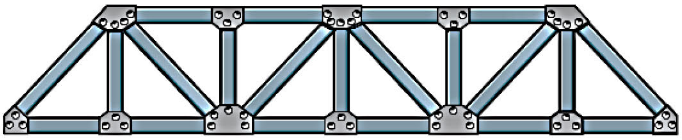
2 *All loadings are applied at the joints*. In most situations, such as for bridge and roof trusses, this assumption is true. Frequently in the force analysis, the weight of the members is neglected, since the force supported by the members is large in comparison with their weight.



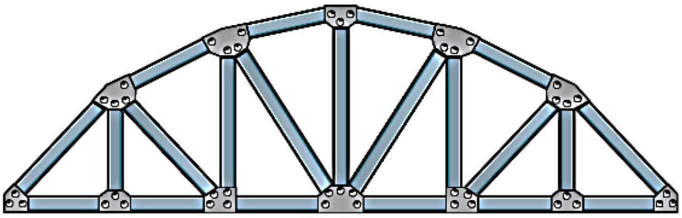
Pratt
(a)



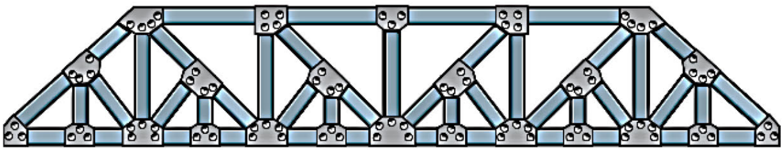
Howe
(b)



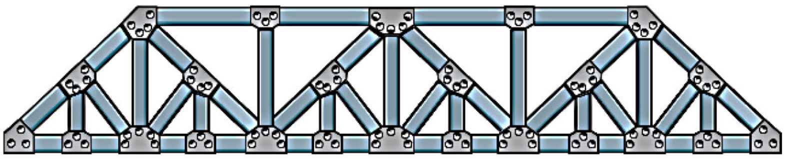
Warren (with verticals)
(c)



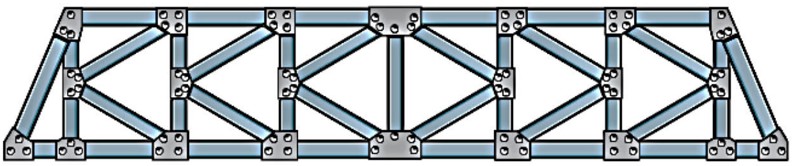
Parker
(d)



Baltimore
(e)



subdivided Warren
(f)



K-truss
(g)

Fig. 4-5

4.2 Classification of Coplanar Trusses

Before beginning the force analysis of a truss, it is important to classify the truss as simple, compound, or complex, and then to be able to specify its determinacy and stability.

Simple Truss.
To prevent collapse, the framework of a truss must be rigid. Obviously, the four-bar frame ***ABCD*** in Fig. 4–6 will collapse unless a diagonal, such as ***AC***, is added for support. The simplest framework that is rigid or stable is a triangle. Consequently, a simple truss is constructed by starting with a basic triangular element, such as ***ABC*** in Fig. 4–7, and connecting two members (***AD*** and ***BD***) to form an additional element. Thus it is seen that as each additional element of two members is placed on the truss, the number of joints is increased by one.

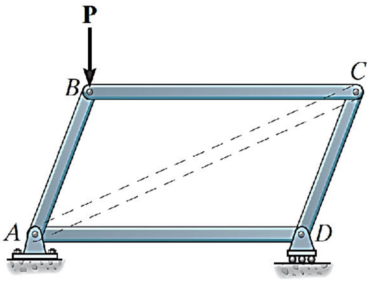


Fig. 4–6

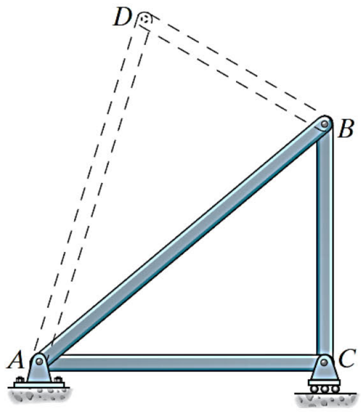
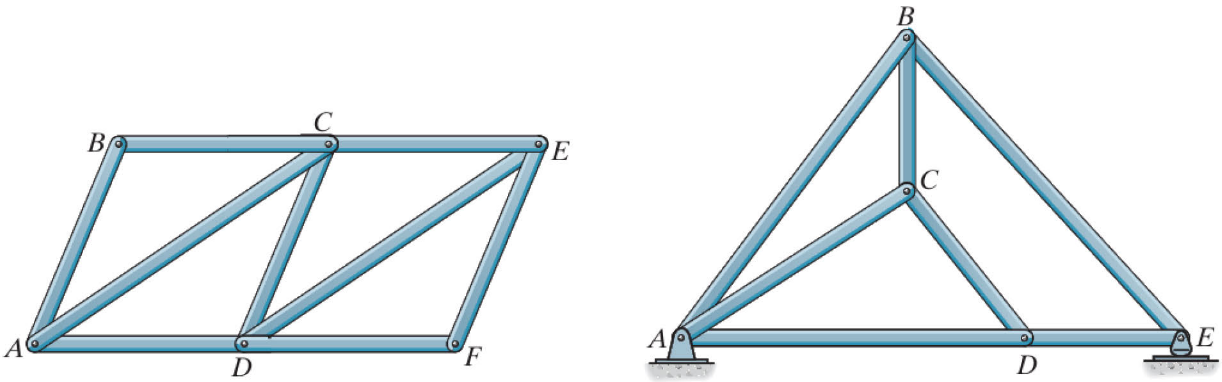


Fig. 4–7



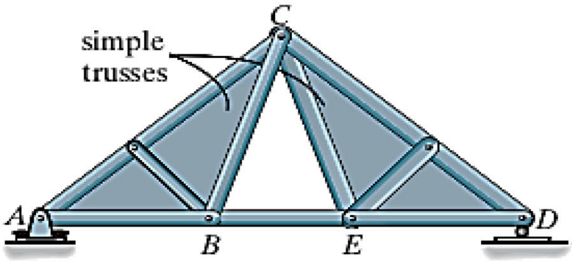
simple truss

simple truss

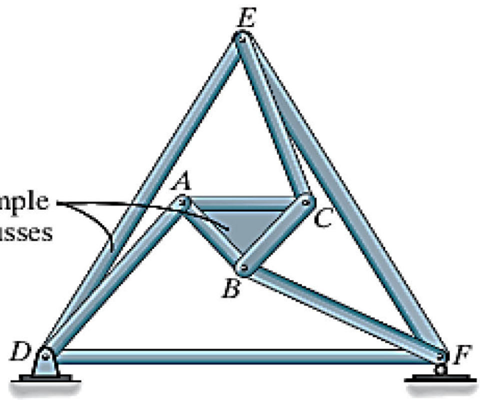
Fig. 4–8

Compound Truss.
A *compound truss* is formed by connecting two or more simple trusses together. Quite often this type of truss is used to support loads acting over a *large span,* since it is cheaper to construct a somewhat lighter compound truss than to use a heavier single simple truss.
The trusses may be connected by a common **joint** and **bar**. An example is given in Fig. 4–9*a*, where the shaded truss ***ABC*** is connected to the shaded truss ***CDE*** in this manner. The trusses

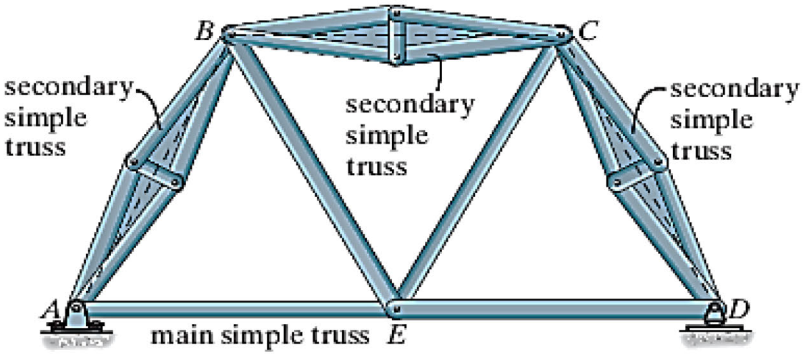
may be joined by three bars, as in the case of the shaded truss *ABC* connected to the larger truss ***DEF***, Fig. 4–9*b*.And finally, the trusses may be joined where bars of a large simple truss, called the *main truss,* have been *substituted* by simple trusses, called *secondary trusses*. An example is shown in Fig. 4–9*c*, where dashed members of the main truss ***ABCDE*** have been *replaced* by the secondary shaded trusses.



(a)



(b)



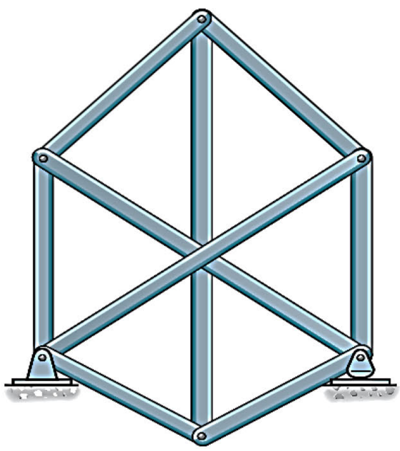
(c)

Various types of compound trusses

Fig. 4–9

Complex Truss.

A *complex truss* is one that cannot be classified as being either simple or compound. The truss in Fig. 4–10 is an example.



Complex truss

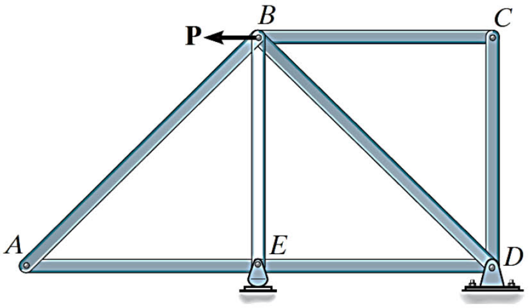
Fig. 4–10

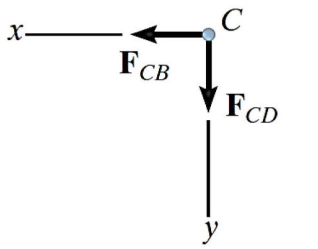
4.3 Analysis of Trusses

4.3.1 Zero-Force Members

Truss analysis using the method of joints is greatly simplified if one is able to first determine those members that support ***no loading***. These ***zero-force members*** may be necessary for the stability of the truss during construction and to provide support if the applied loading is changed. The zero-force members of a truss can generally be determined by inspection of the joints, and they occur in two cases.

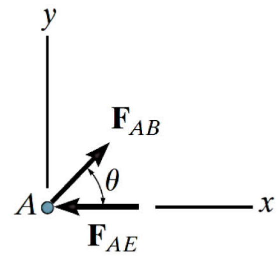
- ✓ If **only two non-collinear** members form a truss joint and no external load or support reaction is applied to the joint, the members must be zero-force members.





$\overset{+}{\leftarrow} \Sigma F_x = 0;\ F_{CB} = 0$

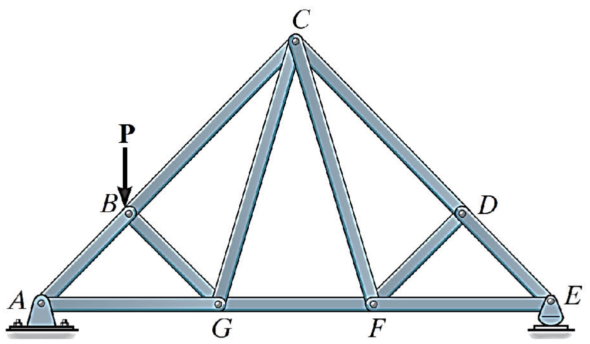
$+\downarrow \Sigma F_y = 0;\ F_{CD} = 0$

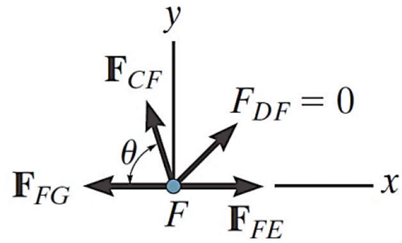


$+\uparrow \Sigma F_y = 0;\ F_{AB} \sin\theta = 0$
$F_{AB} = 0$ (since $\sin\theta \neq 0$)

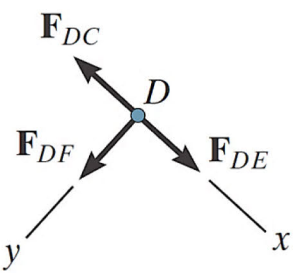
$\overset{+}{\rightarrow} \Sigma F_x = 0;\ -F_{AE} + 0 = 0$
$F_{AE} = 0$

- ✓ If **three members** form a truss joint for which **two of the members are collinear**, the **third member is a zero-force member**, provided no external force or support reaction is applied to the joint.





$+\uparrow \Sigma F_y = 0;\ F_{CF} \sin\theta + 0 = 0$
$F_{CF} = 0$ (since $\sin\theta \neq 0$)

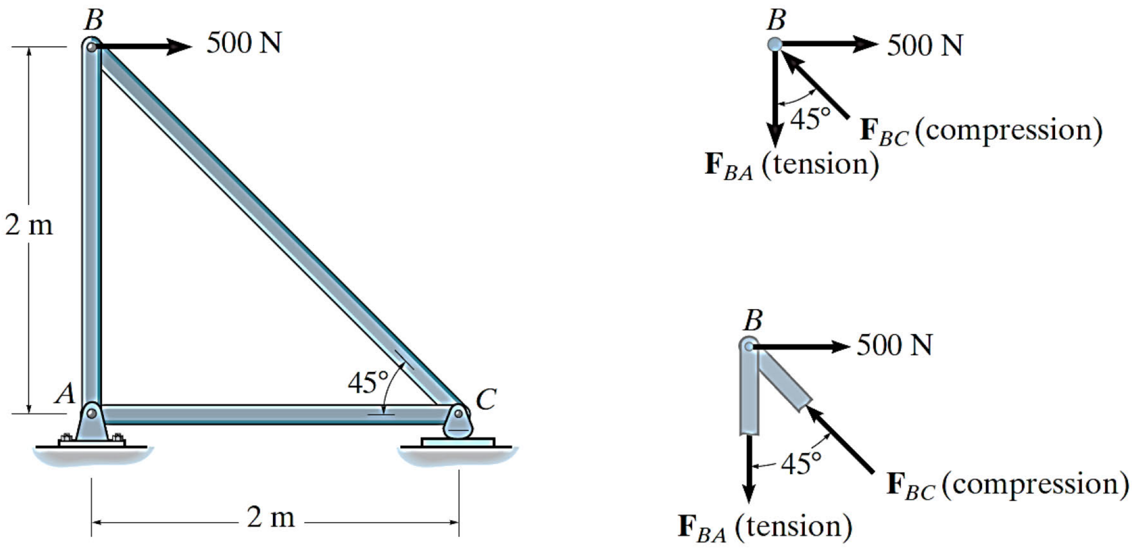


$+\swarrow \Sigma F_y = 0;\ F_{DF} = 0$

4.3.2 The Method of Joints

If a truss is in equilibrium, then each of its joints must also be in equilibrium. Hence, the method of joints consists of satisfying the equilibrium conditions $\sum F_x = 0$ and $\sum F_y = 0$ for the forces exerted *on the pin* at each joint of the truss.

- ✓ The joint analysis should start at a joint having at **least *one known*** force and at **most two unknown** forces, as in Figure.



- ✓ Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. ***(If this joint is at one of the supports, it may be necessary to calculate the external reactions at the supports by drawing a free-body diagram of the entire truss.)***

- ✓ ***Always assume the unknown member forces acting on the joint's free-body diagram to be in tension, i.e., "pulling" on the pin.*** **this is done**, then numerical solution of the equilibrium equations will yield ***positive scalars for members in tension and negative scalars for members in compression.***

- ✓ Once an unknown member force is found, use its *correct* magnitude and sense (**T** or **C**) on subsequent joint free-body diagrams.

- ✓ Continue to analyze each of the other joints, where again it is necessary to choose a joint having at most two unknowns and at least one known force.

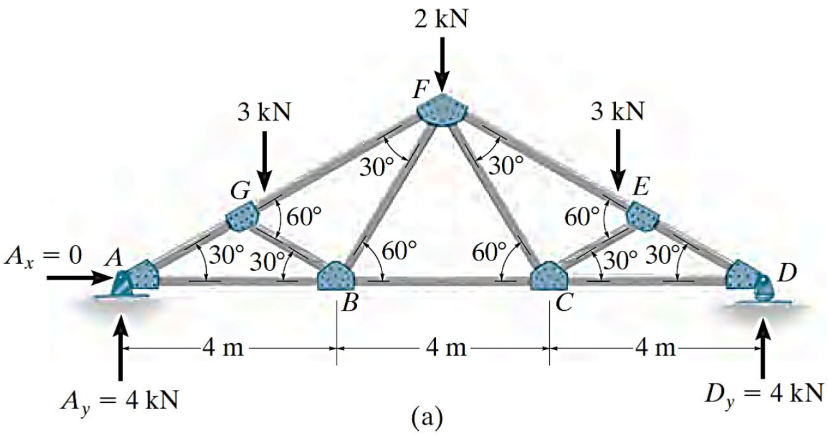
4.3.3 The Method of Sections

If the forces in only a few members of a truss are to be found, the method of sections generally provides the most direct means of obtaining these forces.

- ✓ It may first be necessary to determine the truss's ***external*** **reactions**,
- ✓ Try to select a section that, in general, passes through *not more than* ***three*** *members in which the forces are unknown.*
- ✓ Draw the free-body diagram of that part of the sectioned truss which has the least number of forces on it.
- ✓ Moments ($\sum M = 0$) should be summed about a point that lies ***at the intersection of the lines*** of action of ***two unknown*** forces; in this way, ***the third unknown force is determined*** directly from the equation.
- ✓ If two of the unknown forces are ***parallel***, forces may be summed ***perpendicular*** to the direction of these unknowns to determine ***directly*** **the third unknown** force.

EXAMPLE 4.3.1

Determine the force in each member of the roof truss shown in the photo. The dimensions and loadings are shown in **Fig. *a***. State whether the members are in tension or compression.



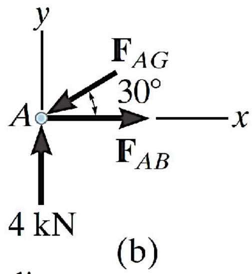
(a)

Solution

Only the forces in half the members have to be determined, since the truss is symmetric with respect to *both* loading and geometry**.**

Joint A, Fig. b.

The free-body diagram is shown in **Fig. *b***.



(b)

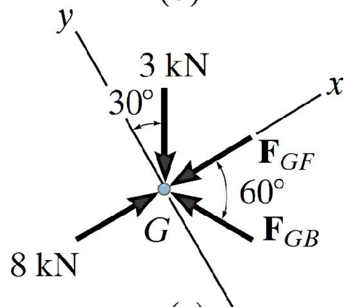
$$+\uparrow \sum F_y = 0;\quad 4 - F_{AG}\sin 30^\circ = 0 \quad \Rightarrow \quad F_{AG} = 8 \text{ kN (C)}$$

$$+\rightarrow \sum F_x = 0;\quad F_{AB} - 8\cos 30^\circ = 0 \quad \Rightarrow \quad F_{AB} = 6.928 \text{ kN (T)}$$

Joint G, Fig. c.

$$+\nwarrow \sum F_y = 0;\quad F_{GB}\sin 60^\circ - 3\cos 30^\circ = 0 \quad \Rightarrow \quad F_{GB} = 3 \text{ kN (C)}$$

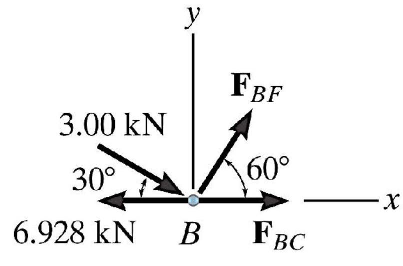
$$+\nearrow \sum F_x = 0;\quad 8 - 3\sin 30^\circ - 3\cos 60^\circ - F_{GF} = 0 \quad \Rightarrow \quad F_{GF} = 5 \text{ kN (C)}$$



(c)

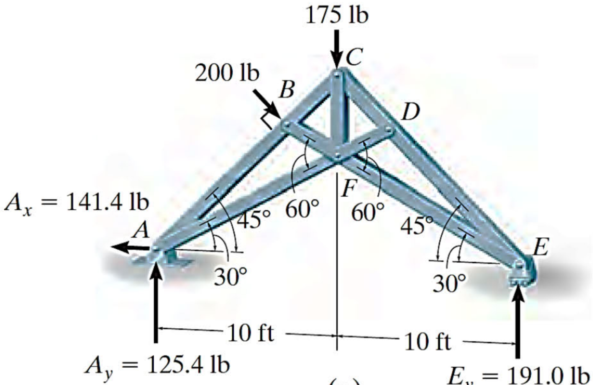
Joint B, Fig. d.

$$+\uparrow \sum F_y = 0; \quad F_{BF} \sin 60^\circ - 3\sin 30^\circ = 0$$
$$\Rightarrow \quad F_{BF} = 1.73 \text{ kN (T)}$$
$$+\rightarrow \sum F_x = 0; \quad F_{BC} + 1.73\cos 60^\circ + 3\cos 30^\circ - 6.928 = 0$$
$$\Rightarrow \quad F_{BC} = 3.46 \text{ kN (T)}$$



(d)

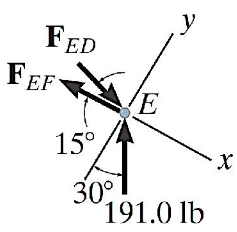
EXAMPLE 4.3.2
Determine the force in each member of the scissors truss shown in **Fig. *a***. State whether the members are in tension or compression. The reactions at the supports are given.



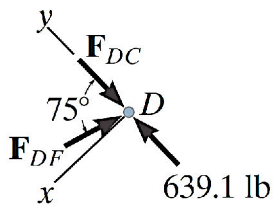
(a)

Solution
Joint E, Fig. b.

$$+\nearrow \sum F_y = 0; \quad 191.0\cos 30^\circ - F_{ED} \sin 15^\circ = 0$$
$$\Rightarrow \quad F_{ED} = 639.1 \text{ lb (C)}$$
$$+\searrow \sum F_x = 0; \quad 639.1\cos 15^\circ - F_{EF} - 191.0\sin 30^\circ = 0$$
$$\Rightarrow \quad F_{EF} = 521.8 \text{ lb (T)}$$



(b)



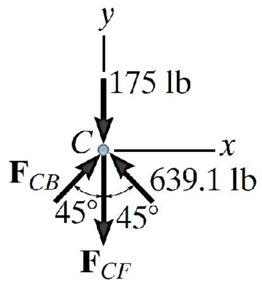
(c)

Joint D, Fig. c.

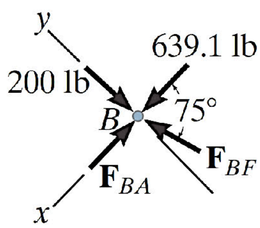
$$+\swarrow \sum F_x = 0; \quad -F_{DF} \sin 75 = 0 \quad \Rightarrow \quad F_{DF} = 0$$
$$+\nwarrow \sum F_y = 0; \quad -F_{DC} + 639.1 = 0 \quad \Rightarrow \quad F_{DC} = 639.1 \text{ lb (C)}$$

Joint C, Fig. d.

$$+\rightarrow \sum F_x = 0; \quad F_{CB} \sin 45^\circ - 639.1\sin 45^\circ = 0$$
$$\Rightarrow \quad F_{CB} = 639.1 \text{ lb (C)}$$
$$+\uparrow \sum F_y = 0; \quad -F_{CF} - 175 + 2(639.1)\cos 45^\circ = 0$$
$$\Rightarrow \quad F_{DC} = 728.8 \text{ lb (T)}$$



(d)



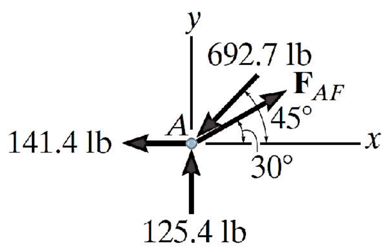
(e)

Joint B, Fig. e.

$$+\nwarrow \sum F_y = 0; \quad F_{BF} \sin 75^\circ - 200 = 0 \quad \Rightarrow \quad F_{BF} = 207.1 \text{ lb (C)}$$
$$+\swarrow \sum F_x = 0; \quad 639.1 + 207.1\cos 75^\circ - F_{BA} = 0 \Rightarrow \quad F_{BA} = 692.7 \text{ lb (C)}$$

Joint A, Fig. f.

$$+\rightarrow \sum F_x = 0; \quad F_{AF} \cos 30^\circ - 692.7\cos 45^\circ - 141.4 = 0$$
$$\Rightarrow F_{AF} = 728.9 \text{ lb (T)}$$
$$+\uparrow \sum F_y = 0; \quad 125.4 - 692.7\sin 45^\circ + 728.9\sin 30^\circ = 0 \quad \text{Check}$$



(f)