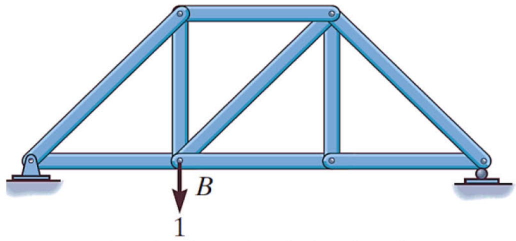
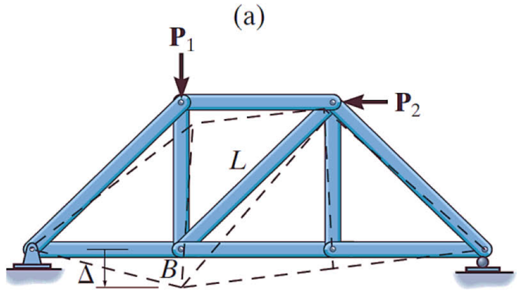
## 6.2 Method of Virtual Work: Trusses

The method of virtual work can be used to determine the displacement of a truss joint when the truss is subjected to an external loading, temperature change, or fabrication errors. Each of these situations will now be discussed.




Apply virtual unit load to *B*
(a)




Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$
(b)

***External Loading.***

For the purpose of explanation let us consider the vertical displacement Δ of joint ***B*** of the truss in **Fig.*a***. Here a typical element of the truss would be one of its ***members*** having a length *L*, **Fig.*b***. If the applied loadings $\mathbf{P_1}$ and $\mathbf{P_2}$ cause a ***linear elastic material response***, then this element deforms an amount,

$$\Delta L = \frac{NL}{AE}$$

where *N* is the normal or axial force in the member, caused by the loads. The virtual-work equation for the truss is therefore

$$1.\ \Delta = \sum \frac{nNL}{AE} \qquad ...(6-3)$$

where

- **1** = external virtual unit load acting on the truss joint in the stated direction of Δ.
- ***n*** = internal virtual normal force in a truss member caused by the external virtual unit load.
- Δ = external joint displacement caused by the real loads on the truss.
- ***N*** = internal normal force in a truss member caused by the real loads.
- ***E*** = modulus of elasticity of a member.
- ***A*** = cross-sectional area of a member.
- ***L*** = length of a member.

***Temperature.***

In some cases, truss members may change their length due to temperature. If $\alpha$ is the coefficient of thermal expansion for a member and $\Delta T$ is the change in its temperature, the change in length of a member is $\Delta L = \alpha\ \Delta T\ L$

Hence, we can determine the displacement of a selected truss joint due to this temperature change from.

$$1.\ \Delta = \sum n\ \alpha\ \Delta T\ L \qquad ...(6-4)$$

where

- **1** = external virtual unit load acting on the truss joint in the stated direction of Δ.
- ***n*** = internal virtual normal force in a truss member caused by the external virtual unit load.
- Δ = external joint displacement caused by the temperature change.
- $\alpha$ = coefficient of thermal expansion of member.
- $\Delta T$ = change in temperature of member.
- ***L*** = length of a member.

**Note:** If any of the members undergoes an ***increase in temperature***, $\Delta T$ will be ***positive***, whereas a ***decrease in temperature*** results in a ***negative*** value for $\Delta T$.

## *Fabrication Errors and Camber.*

Occasionally, errors in fabricating the lengths of the members of a truss may occur. Also, in some cases truss members must be made slightly longer or shorter in order to give the truss a camber. If a truss member is shorter or longer than intended, the displacement of a truss joint from its expected position can be determined from ,

$$1 \cdot \Delta = \sum n \ \Delta L \qquad ...(6-5)$$

where

- **1** = external virtual unit load acting on the truss joint in the stated direction of $\Delta$.
- $n$ = internal virtual normal force in a truss member caused by the external virtual unit load.
- $\Delta$ = external joint displacement caused by the fabrication errors.
- $\Delta L$ = difference in length of the member from its intended size as caused by a fabrication error.

**Note:** When a fabrication error ***increases the length*** of a member, $\Delta L$ **is** ***positive,*** whereas a ***decrease in length*** **is** ***negative***.

A combination of the right sides of **Eqs. 6–3** through **6–5** will be necessary if both external loads act on the truss and some of the members undergo a thermal change or have been fabricated with the wrong dimensions.

$$1 \cdot \Delta = \sum \frac{nNL}{AE} + \sum n \ \alpha \ \Delta T \ L + \sum n \ \Delta L$$

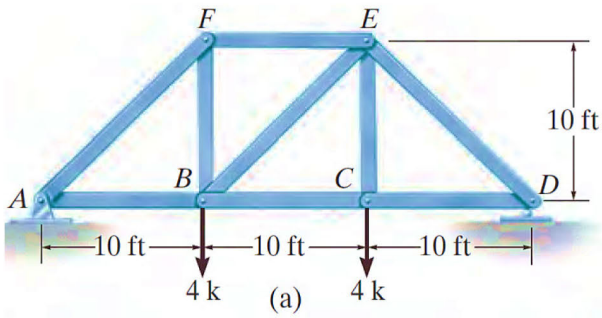
**EXAMPLE 6.2.1**

Determine the vertical displacement of joint $C$ of the steel truss shown in **Fig. *a***. The cross-sectional area of each member is $A = 0.5 \text{ in}^2$ and $E = 29(10^3)$ **ksi.**

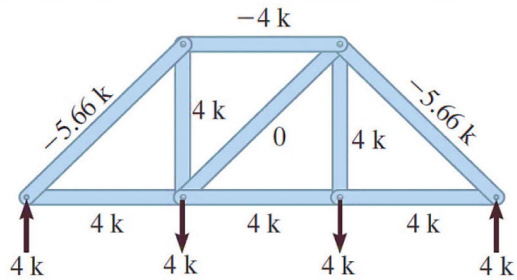
**Solution**

**Real Forces N.** The real forces in the members are calculated using the method of joints. The results are shown in **Fig. *b*.**

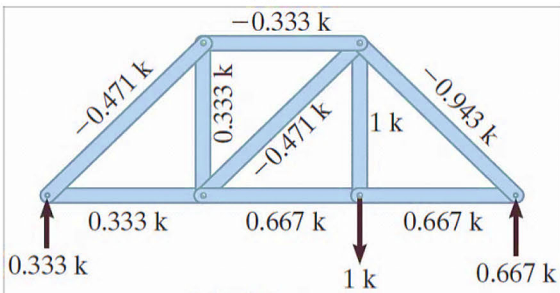
**Virtual Forces n.** Only a vertical **1-k** load is placed at joint $C$, and the force in each member is calculated using the method of joints. The results are shown in **Fig. *c*.** Positive numbers indicate tensile forces and negative numbers indicate compressive forces.



(a)



real forces **N**
(b)



virtual forces **n**
(c)

**Virtual-Work Equation.** Arranging the data in tabular form, we have

| Member | *n* (k) | *N* (k) | *L* ( ft) | *nNL* (k$^2$. ft) |
|---|---|---|---|---|
| *AB* | 0.333 | 4 | 10 | 13.320 |
| *BC* | 0.667 | 4 | 10 | 26.680 |
| *CD* | 0.667 | 4 | 10 | 26.680 |
| *DE* | -0.943 | -5.66 | 14.14 | 75.471 |
| *FE* | -0.333 | -4 | 10 | 13.320 |
| *EB* | -0.471 | 0 | 14.14 | 0.000 |
| *BF* | 0.333 | 4 | 10 | 13.320 |
| *AF* | -0.471 | -5.66 | 14.14 | 37.695 |
| *CE* | 1 | 4 | 10 | 40.000 |
| | | | | **∑ 246.486** |

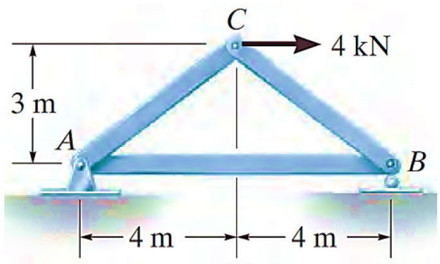
Thus

$$1\text{ k.}\ \Delta_{C_v} = \sum \frac{nNL}{AE} = \frac{246.486\ \text{k}^2.\text{ft}}{AE}$$

$$1\text{ k.}\ \Delta_{C_v} = \frac{\left(246.486\ \text{k}^2.\text{ft}\right)\left(12\ \text{in/ft}\right)}{\left(0.5\ \text{in}^2\right)\left(29\left(10^3\right)\ \text{k/in}^2\right)} = 0.204\ \text{in.}$$

**EXAMPLE 6.2.2**

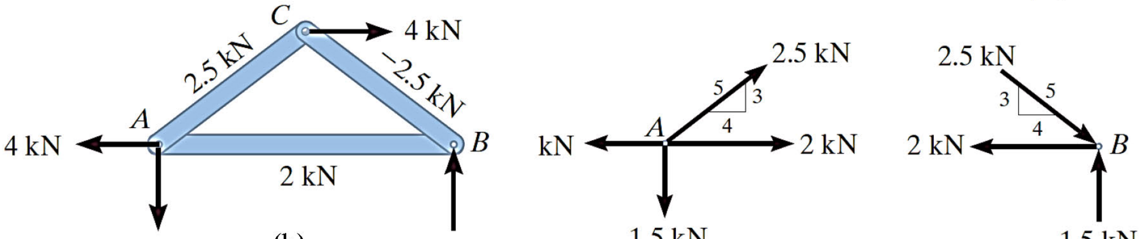
The cross-sectional area of each member of the truss shown in **Fig.*a*** is ***A*** **= 400 mm$^2$** and ***E*** **= 200 GPa. (a)** Determine the vertical displacement of joint ***C*** if a **4-kN** force is applied to the truss at ***C***. **(b)** If no loads act on the truss, what would be the vertical displacement of joint ***C*** if member ***AB*** were **5 mm** too short?



(a)

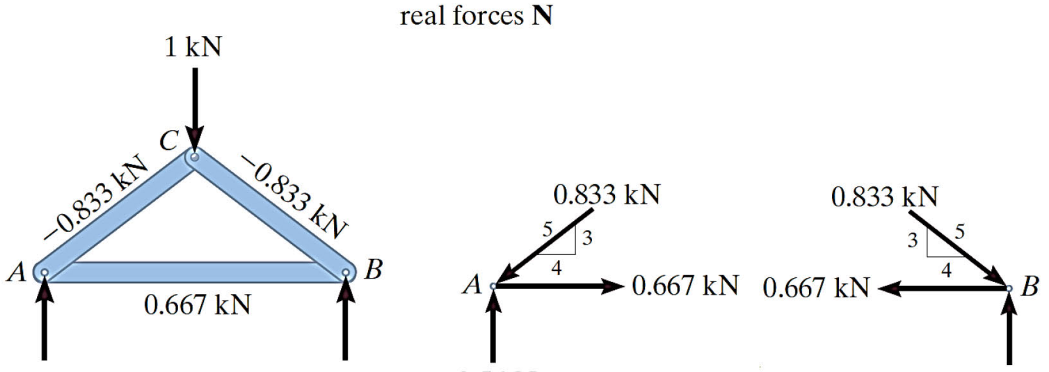
**Solution**

**(a)**



(b)

real forces **N**



(c)

virtual forces **n**

| Member | *n* (kN) | *N* (kN) | *L* ( m) | *nNL* ($kN^2$. m) |
|---|---|---|---|---|
| ***AB*** | 0.667 | 2 | 8 | 10.672 |
| ***AC*** | -0.833 | 2.5 | 5 | -10.413 |
| ***CB*** | -0.833 | -2.5 | 5 | 10.413 |
| | | | | ∑ **10.672** |

$$1\text{ kN. }\Delta_{C_v} = \sum \frac{nNL}{AE} = \frac{10.672\text{ kN}^2\text{.m}}{AE}$$

$$1\text{ kN. }\Delta_{C_v} = \frac{10.672\text{ kN}^2\text{.m}}{400(10^{-6})\text{ m}^2(200(10^{-6})\text{ kN/m}^2)}$$

$$\Delta_{C_v} = 0.000133\text{ m} = 0.133\text{ mm}$$

**(b)**
Since the vertical displacement of ***C*** is to be determined, we can use the results of **Fig. *c***. Only member ***AB*** undergoes a change in length, namely, of **Δ*L* = 0.005 m**.
Thus,

$$1.\ \Delta = \sum n\ \Delta L$$

$$1\text{ kN. }\Delta_{C_v} = (0.667\text{ kN})(-0.005\text{ m})$$

$$\Delta_{C_v} = -0.00333\text{ m} = -3.33\text{ mm}$$

The negative sign indicates joint ***C*** is displaced ***upward***, opposite to the **1-kN** vertical load.

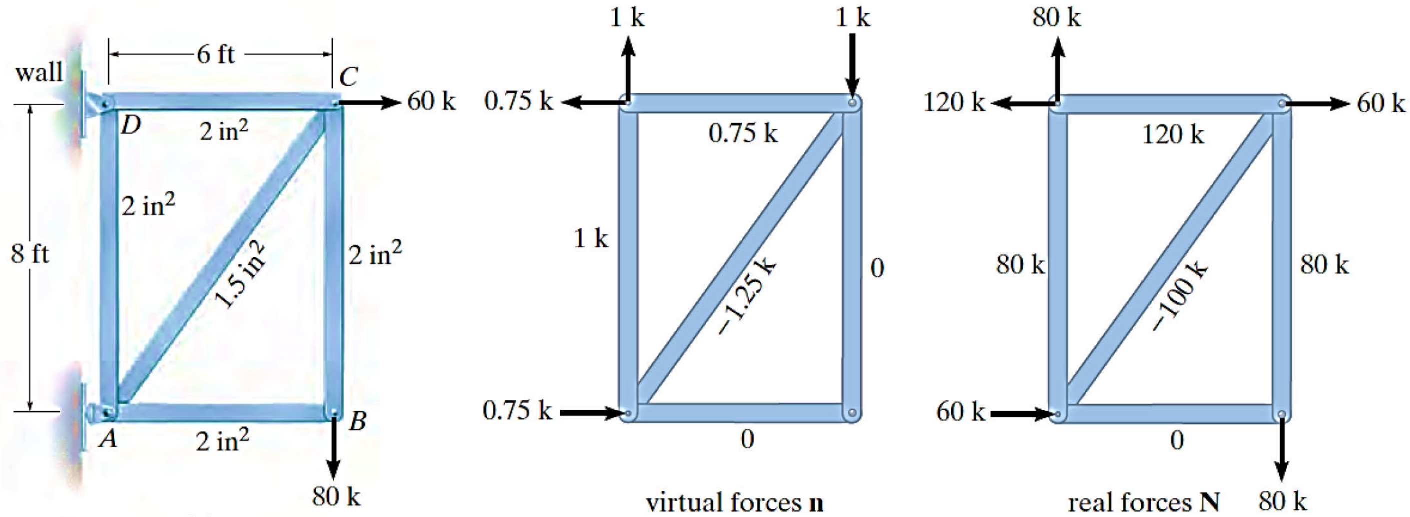
**Note**: If the **4-kN** load and fabrication error are both accounted for, the resultant displacement is then

$$\Delta_{C_v} = 0.133 - 3.33 = -3.20\text{ mm (upward)}.$$

**EXAMPLE 6.2.3**

Determine the vertical displacement of joint *C* of the steel truss shown in **Fig. *a***. Due to radiant heating from the wall, member ***AD*** is subjected to an ***increase*** in temperature of **$\Delta T$ = +120°F**.Take ***α* = 0.6(10$^{-5}$)/°F** and ***E* = 29(10$^{3}$) kis.** The cross-sectional area of each member is indicated in the figure.

**Solution**




(a) (b) (c)

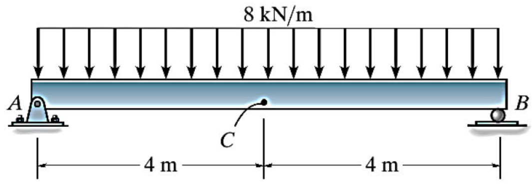
$$1.\ \Delta_{C_v} = \sum \frac{nNL}{AE} + \sum n\ \alpha\ \Delta T\ L$$

$$= \frac{(0.75)(120)(6)(12)}{2[29(10^3)]} + \frac{(1)(80)(8)(12)}{2[29(10^3)]}$$

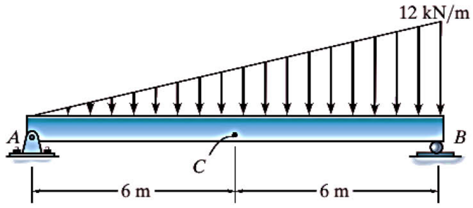
$$+ \frac{(-1.25)(-100)(10)(12)}{1.5[29(10^3)]} + (1)[0.6(10^{-5})](8)(12)$$

$$\Delta_{C_v} = 0.658\,\text{in}$$

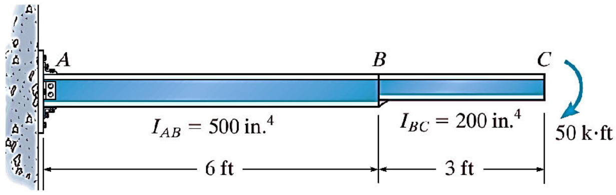
**Hw.15**
Determine the slope at *A* and displacement at point *C*. ***EI*** is constant. Use the principle of virtual work.



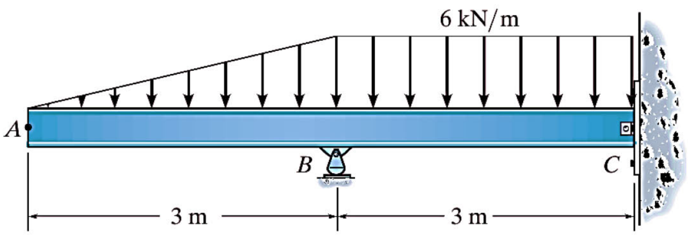

**Hw.16**
Determine the displacement at point *C*. ***EI*** is constant. Use the principle of virtual work.



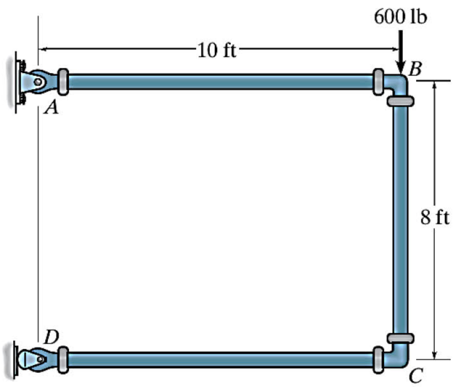

**Hw.17**
Determine the displacement and slope at point *C* of the cantilever beam.The moment of inertia of each segment is indicated in the figure. Take **E = 29(10$^3$) ksi**. Use the principle of virtual work.



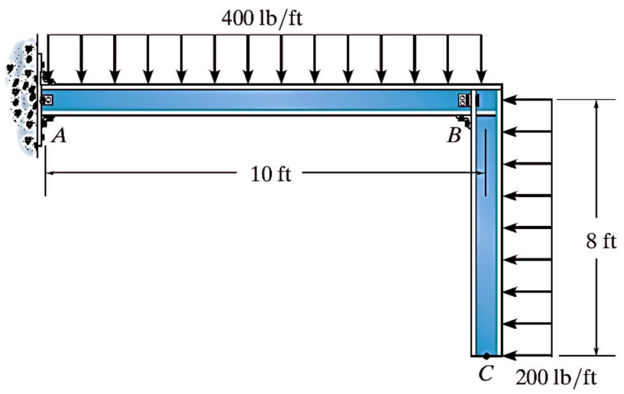

**Hw.18**
Determine the slope and displacement at point A. Assume *C* is pinned. Use the principle of virtual work. ***EI*** is constant.



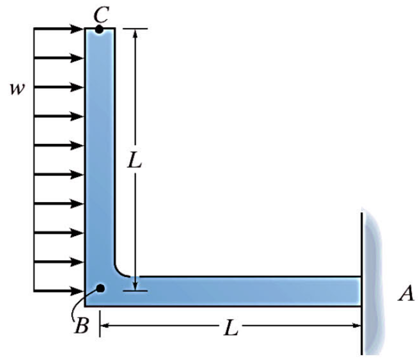

**Hw.19**
Use the method of virtual work and determine the vertical deflection at the rocker support *D. EI* is constant



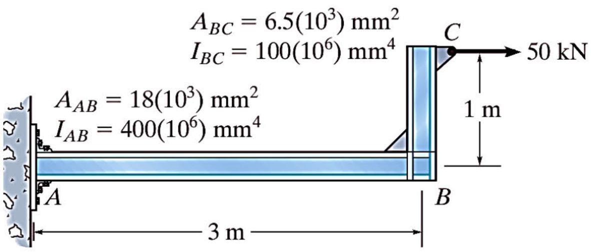

**Hw.20**
Determine the horizontal displacement of point *C. EI* is constant. Use the method of virtual work.



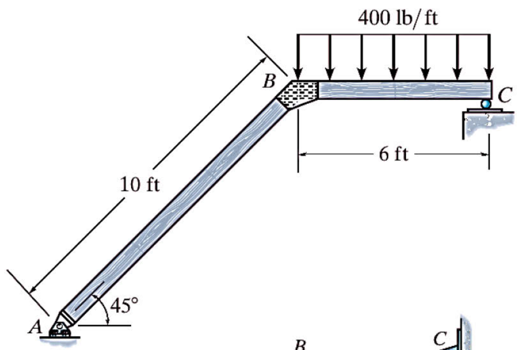
**Hw.21**
The **L-shaped** frame is made from two segments, each of length ***L*** and flexural stiffness ***EI***. If it is subjected to the uniform distributed load, determine the horizontal displacement of the end ***C,*** and the vertical displacement of point ***B***. Use the method of virtual work.



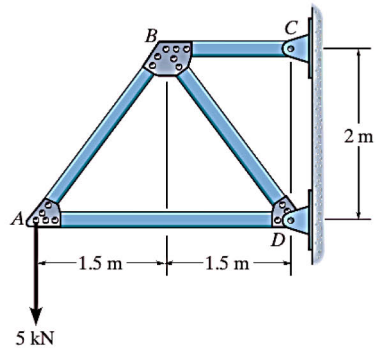

**Hw.22**
Determine the vertical deflection at *C*. The cross-sectional area and moment of inertia of each segment is shown in the figure. Take ***E*** **= 200 GPa** Assume ***A*** is a fixed support. Use the method of virtual work.



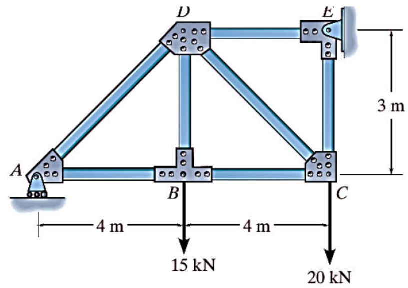

**Hw.23**
Use the method of virtual work and determine the horizontal deflection at *C*. ***EI*** is constant.There is a pin at ***A***. Assume ***C*** is a roller and ***B*** is a fixed joint.




**Hw.24**
Determine the vertical displacement of joint ***A***. Each bar is made of steel and has a cross-sectional area of **600 mm2**.Take ***E*** **= 200 GPa**. Use the method of virtual work.




**Hw.25**
Determine the vertical displacement of joint ***D***. Use the method of virtual work. ***AE*** is constant. Assume the members are pin connected at their ends.




**Hw.26**
- **(A)** Determine the vertical displacement of joint ***A*** if members ***AB*** and ***BC*** experience a temperature increase of $\Delta T$ = **200°F**. Take ***A*** **= 2 in$^2$** and ***E*** **= 29(10$^3$) ksi.** Also, **α= 6.60 (10-6)/°F.**
- **(B)** Determine the vertical displacement of joint ***A*** if member ***AE*** is fabricated **0.5 in**. too short.