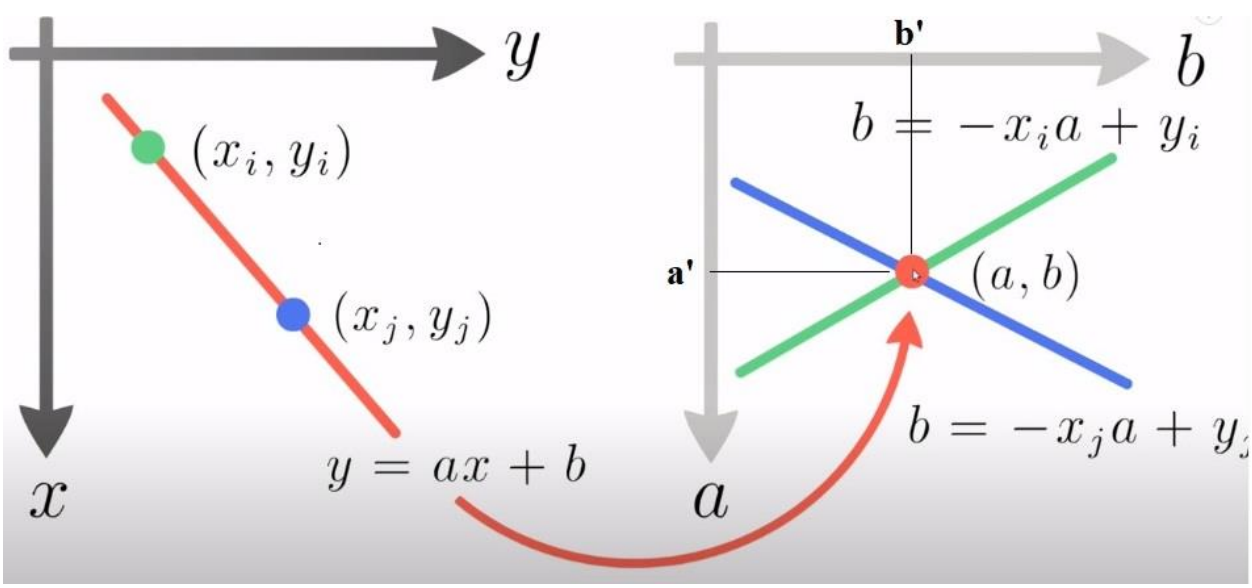
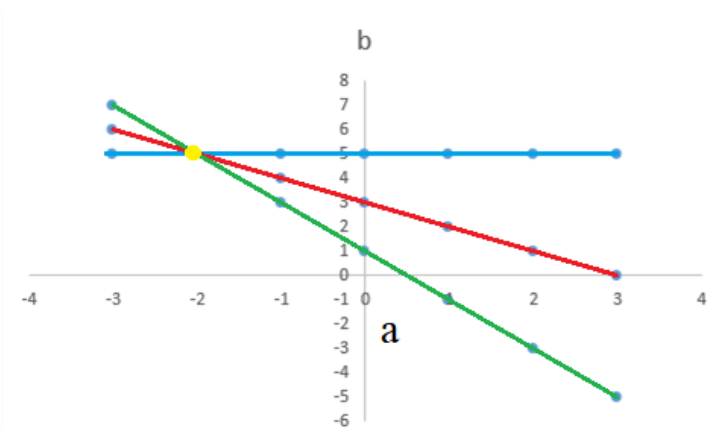
Image Segmentation

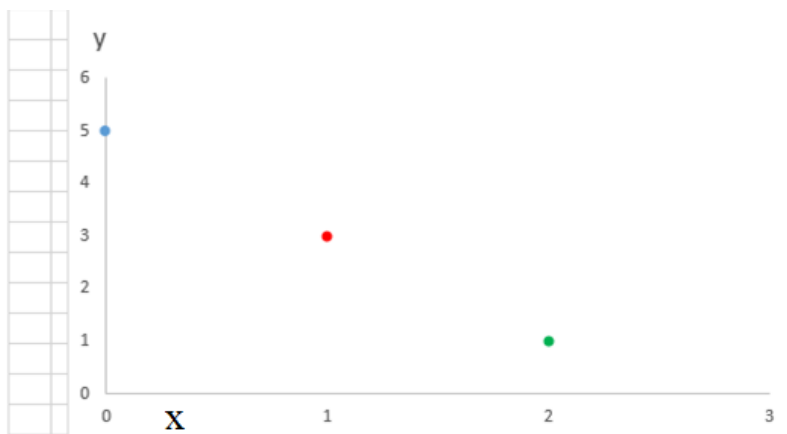
- **Line Detection using Hough Transform**
- Edge-detection algorithms are usually followed by linking procedure to solve the problem of discontinuities.
- Also, it suffers from noise problem.
- One approach that can be used to find and link line segments in an image is Hough Transform (HT).
- Whether set of pixels lie on curve of specified shape (e.g. line).
- HT considers a point $(xi\ ,\ yi)$ in the x-y plane and the general equation of a straight line in slope-intercept form $yi = axi + b$.
- Infinitely many lines pass through $(xi\ ,\ yi)$ but they are all satisfy equation $yi = axi + b$ for varying values of a and b.
- We can consider *ab-plane* (parameter space) and write the line equation as $b = -xia + yi$ which represent a single line for a fixed point $(xi\ ,\ yi)$ (green line in the figure).



- A second point $(xj\ ,\ yj)$ also has a line (blue line) in parameter space associated with it.
- This is true for all the points in the line in the xy-plane (left one).
- All these lines intersect at some point (a' , b') (red point in the parameter plane).
- In this way, we can found a principal line in the xy-plane by found the intersection points in the parameter space (where large numbers of parameter-space lines intersect).
- **Example**
- 3 points in x-y plane [(0,5), (1,3), (2,1)] shown in right figure (blue, red and green).
- In x-y plane the line is defined by $yi = axi + b$ while in a-b plane (left figure) the line is defined by $b = -xia + yi$.
- The details of mapping each point from x-y plane to a-b plane is shown in the table.

	b = -ax + y	a = -3	a = -2	a = -1	a = 0	a = 1	a = 2	a = 3
x = 0 , y = 5	b = 0 + 5	5	5	5	5	5	5	5
x = 1 , y = 3	b = -a + 3	6	5	4	3	2	1	0
x = 2 , y = 1	b = -2a + 1	7	5	3	1	-1	-3	-5

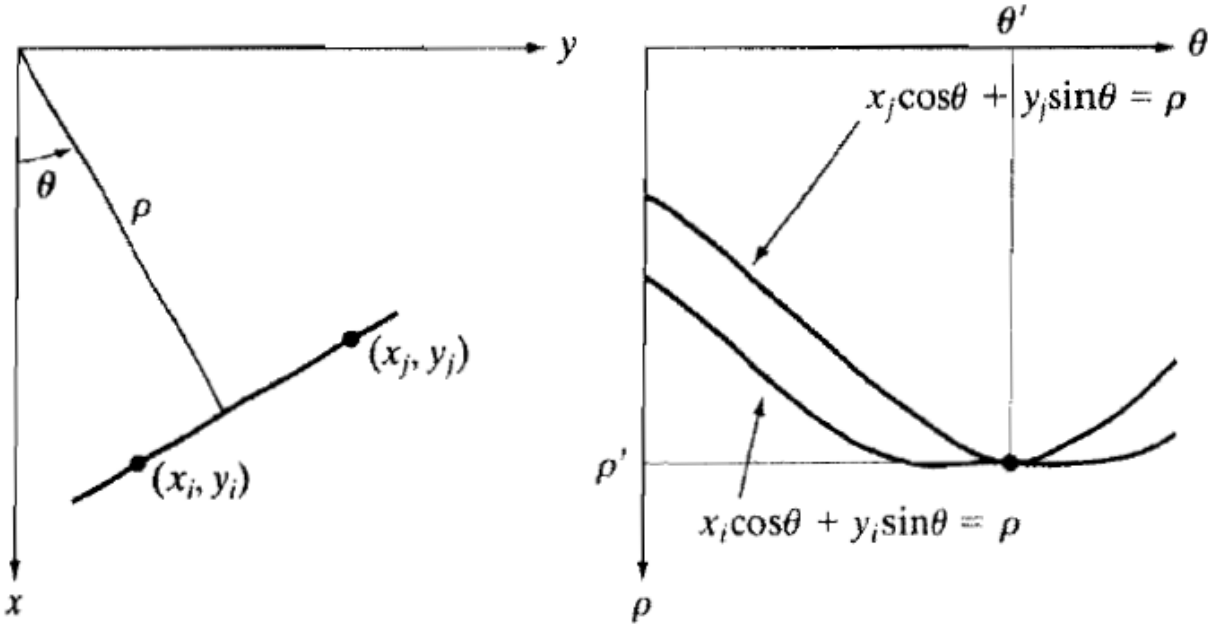




- After observing the figure (left) we can see that a = -2, b = 5, so the equation of the line that link the 3 points in x-y plane is:

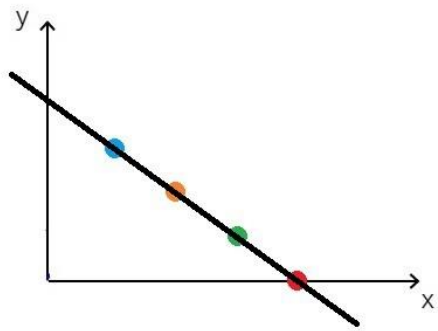
  $yi = -2xi + 5$

- A difficulty with this approach is that a (slope of the line (a)) approaches infinity as the line approaches the vertical direction.
- To avoid this difficulty with ab-plane, a normal representation of line was used. $x\cos\theta + y\sin\theta = \rho$. See the below right figure.
- Each sinusoidal curve in the right figure, represents the family of lines that pas through a particular point $(xk\ ,\ yk)$ in the x-y plane.
- The intersection point $(\rho'\ ,\theta')$ in the figure corresponds to the line that passes through both $(xi\ ,\ yi)$ and $(xj\ ,\ yj)$.

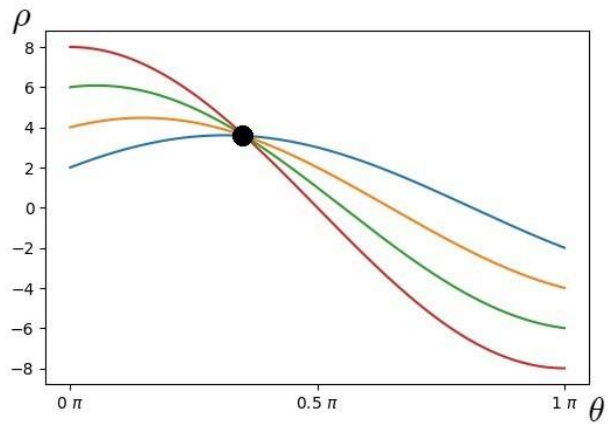


- **Example of four point in x-y plane and their sinusoidal.**
- The mapping from x-y plane to ***roh-theta*** plane is similar as previous example.

- The x and y values are known for every point and we just need to substitute for all θ values (e.g. from 90 to -90) in the ($x \cos\theta + y \sin\theta = \rho$) to draw the sinusoidal curve.

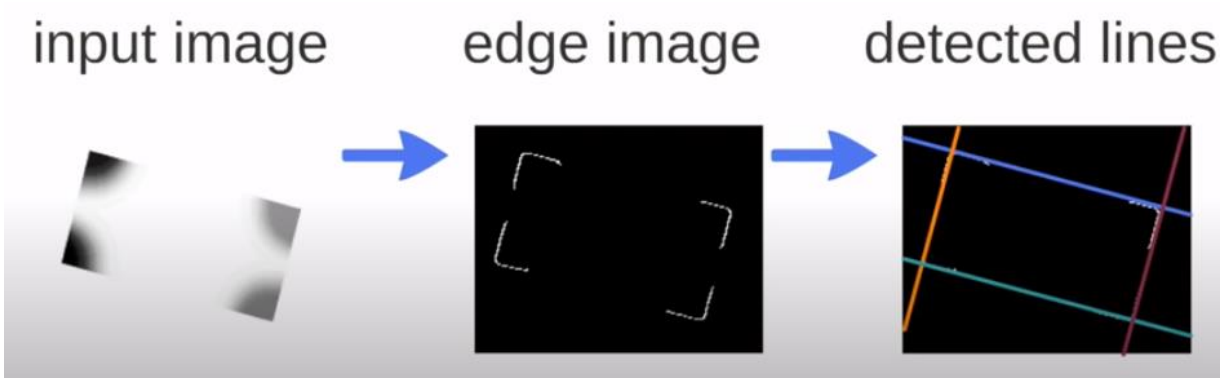


Points which form a line



Bunch of sinusoids intersecting at one point

- Example of line detection using Hough Transform.



- **References**

- Szeliski, Richard. *Computer vision: algorithms and applications*. Springer Science & Business Media, 2010.
- Gonzalez, Rafael C., and Richard E. Woods. "Digital image processing." (2002).