Electric Potential Energy and Electric Potential

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Energy Considerations

- When a force, F, acts on a particle, work is done on the particle in moving from point *a* to point *b*

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l}$$

If the force is a conservative, then the work done can be expressed in terms of a change in potential energy

$$W_{a\to b} = -\left(U_b - U_a\right) = -\Delta U$$

Also if the force is conservative, the total energy of the particle remains *constant*

$$KE_a + PE_a = KE_b + PE_b$$

Electric Potential Energy

The work done by the force is the same as the change in the particle's potential energy

$$\boldsymbol{W_{a\to b} = -(U_b - U_a) = -\Delta U}$$

$$U_b - U_a = -\int_a^b \vec{F} \cdot dl = -qE_{uniform}(y_b - y_a)$$

The work done only depends upon the *change* in position

Electric Potential Energy

General Points

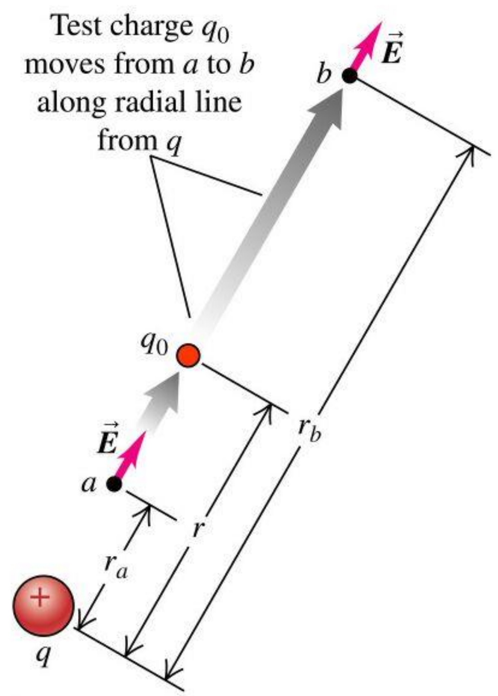
1) Potential Energy *increases* if the particle moves in the direction *opposite* to the force on it

Work will have to be done by an external agent for this to occur

and

2) Potential Energy *decreases* if the particle moves in the *same* direction as the force on it

Potential Energy of Two Point Charges



Suppose we have two charges q and q_0 separated by a distance r

The force between the two charges is given by Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

We now displace charge q_0 along a radial line from point a to point b

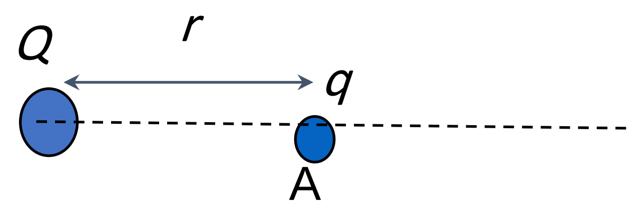
The force is not constant during this displacement

$$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2} dr = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

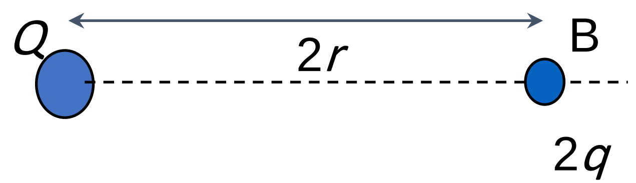
Example 1

Two test charges are brought separately to the vicinity of a positive charge Q

Charge $+q$ is brought to pt A, a distance r from Q



Charge $+2q$ is brought to pt B, a distance $2r$ from Q



I) Compare the potential energy of q (U_A) to that of $2q$ (U_B)

(a) $U_A < U_B$ $\boxed{\text{(b) } U_A = U_B}$ (c) $U_A > U_B$

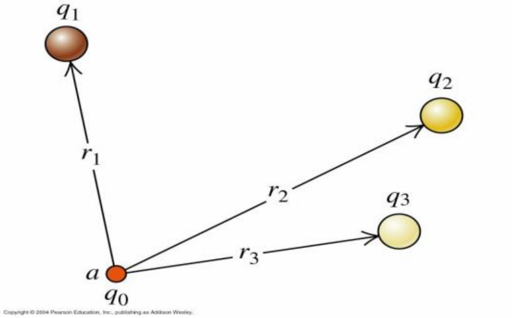
The potential energy of q is proportional to Qq/r

The potential energy of $2q$ is proportional to $Q(2q)/(2r) = Qq/r$

Therefore, the potential energies U_A and U_B are EQUAL!!!

Potential Energy

Recall Case 1 from before



The potential energy of the test charge, q_0, was given by

$$PE_{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_0 q_i}{r_i}$$

Notice that there is a part of this equation that would remain the same regardless of the test charge, q_0, placed at point *a*

The value of the test charge can be pulled out from the summation $PE_{q_0} = q_0 \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$

Electric Potential (V)

The potential at a given point

Represents the potential energy that a positive unit charge would have, if it were placed at that point

It has units of

$$\mathbf{Volts} = \frac{\mathbf{Energy}}{\mathbf{charge}} = \frac{\boldsymbol{joules}}{\boldsymbol{coulomb}}$$

Electric Potential (V)

We define the term to the right of the summation as the electric potential at point *a*

$$Electric\_Potential_a = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

Like energy, potential is a ***SCALAR***

<u>We define the potential of a given point charge as being</u>

$$\boldsymbol{Potential} = \boldsymbol{V} = \frac{1}{4\boldsymbol{\pi\varepsilon}_0}\frac{\boldsymbol{q}}{\boldsymbol{r}}$$

This equation has the convention that the potential is zero at infinite distance

Electric Potential

General Points for either positive or negative charges

The Potential *increases* if you move in the direction *opposite* to the electric field

and

The Potential *decreases* if you move in the same *same* direction as the electric field

Example 2

Points A, B, and C lie in a uniform electric field.

What is the potential difference between points A and B?

$\Delta V_{AB} = V_B - V_A$

a) $\Delta V_{AB} > 0$ b) $\Delta V_{AB} = 0$ c) $\Delta V_{AB} < 0$

The electric field, *E*, points in the direction of decreasing potential

Since points A and B are in the same relative horizontal location in the electric field there is on potential difference between them

Example 3

Points A, B, and C lie in a uniform electric field.

Point C is at a higher potential than point A.

True False

As stated previously the electric field points in the direction of *decreasing* potential

Since point C is further to the right in the electric field and the electric field is pointing to the right, point C is at a lower potential

The statement is therefore **FALSE**

Example 4

Points A, B, and C lie in a uniform electric field.

If a negative charge is moved from point A to point B, its electric potential energy

a) Increases. b) decreases. c) doesn't change.

The potential energy of a charge at a location in an electric field is given by the product of the charge and the potential at the location

As shown in Example 4, the potential at points A and B are the same

Therefore the electric potential energy also doesn't change

Work and Potential (V)

The work done by the electric force in moving a test charge from point *a* to point *b* is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

Dividing through by the test charge q_0 we have

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

Rearranging so the order of the subscripts is the same on both sides

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

Electric Potential

From this last result $$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

We get $$dV = -\vec{E} \cdot d\vec{l} \text{ or } \frac{dV}{dx} = -E$$

We see that the electric field points in the direction of *decreasing* potential

We are often more interested in potential differences as this relates directly to the work done in moving a charge from one point to another

Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of *e* (1.6×10^{-19} C) that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$W = q\Delta V = 1.6\times10^{-19}\ joules = 1\,eV$$

Question: A particle of charge $q_1 = +6.0\,\mu\text{C}$ is located on the x-axis at the point $x_1 = 5.1$ cm. A second particle of charge $q_2 = -5.0\,\mu\text{C}$ is placed on the x-axis at $x_2 = -3.4$ cm. What is the absolute electric potential at the origin ($x = 0$)? How much work must we perform in order to slowly move a charge of $q_3 = -7.0\,\mu\text{C}$ from infinity to the origin, whilst keeping the other two charges fixed?

$$V_1 = k_e \frac{q_1}{x_1} = (8.988 \times 10^9) \frac{(6 \times 10^{-6})}{(5.1 \times 10^{-2})} = 1.06 \times 10^6\,\text{V}.$$

$$V_2 = k_e \frac{q_2}{|x_2|} = (8.988 \times 10^9) \frac{(-5 \times 10^{-6})}{(3.4 \times 10^{-2})} = -1.32 \times 10^6\,\text{V}.$$

The net potential V at the origin is simply the **algebraic** sum of the potentials due to each charge taken in isolation. Thus,

$$V = V_1 + V_2 = -2.64 \times 10^5\,\text{V}.$$

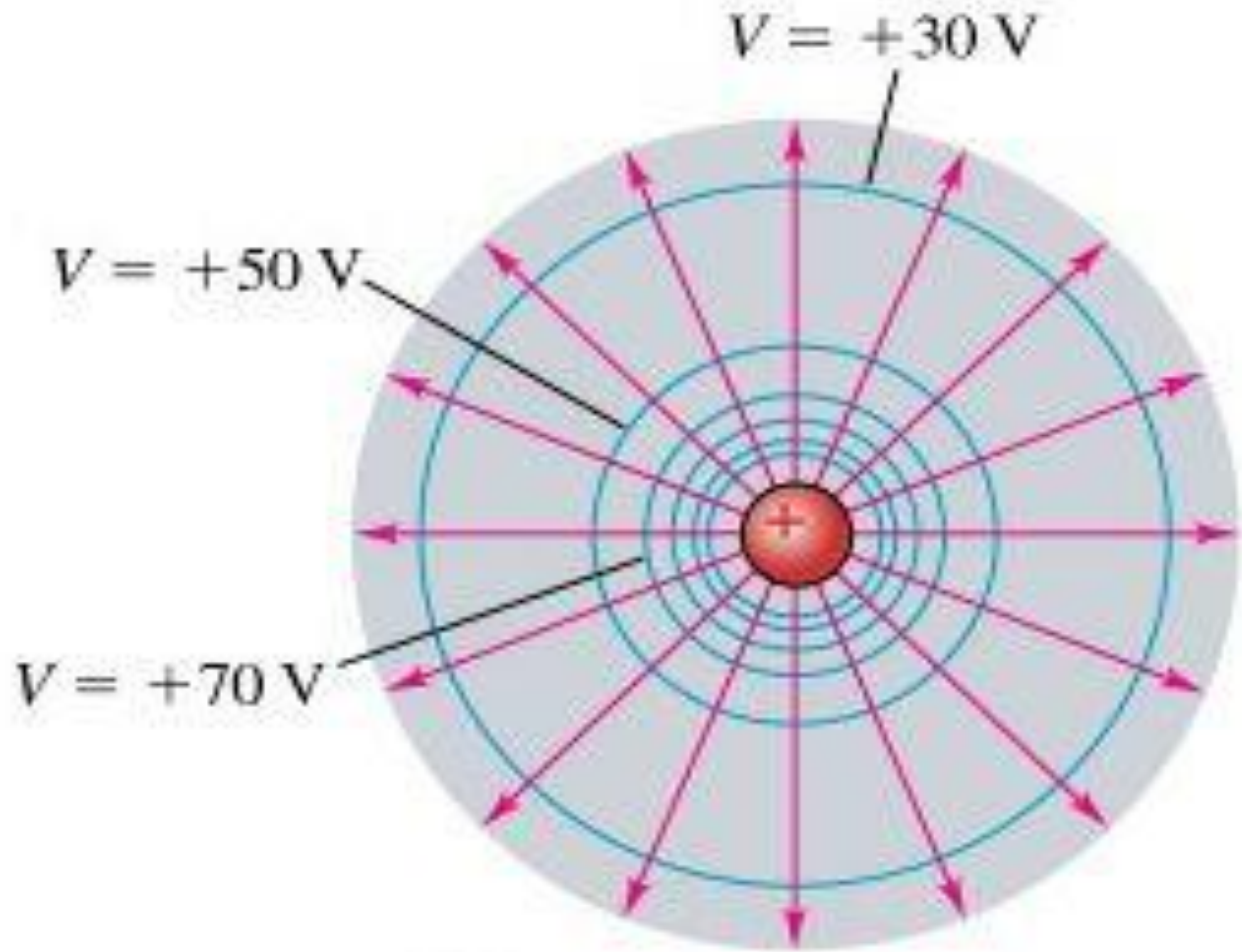
The work W which we must perform in order to slowly moving a charge q3 from infinity to the origin is simply the product of the charge and the potential difference. Thus,

$$W = q_3\,V = (-7 \times 10^{-6})\,(-2.64 \times 10^5) = 1.85\,\text{J}.$$

Equipotential Surfaces

- It is possible to move a test charge from one point to another without having any net work done on the charge.
- This occurs when the beginning and end points have the same potential
- It is possible to map out such points and a given set of points at the same potential form an *equipotential surface*

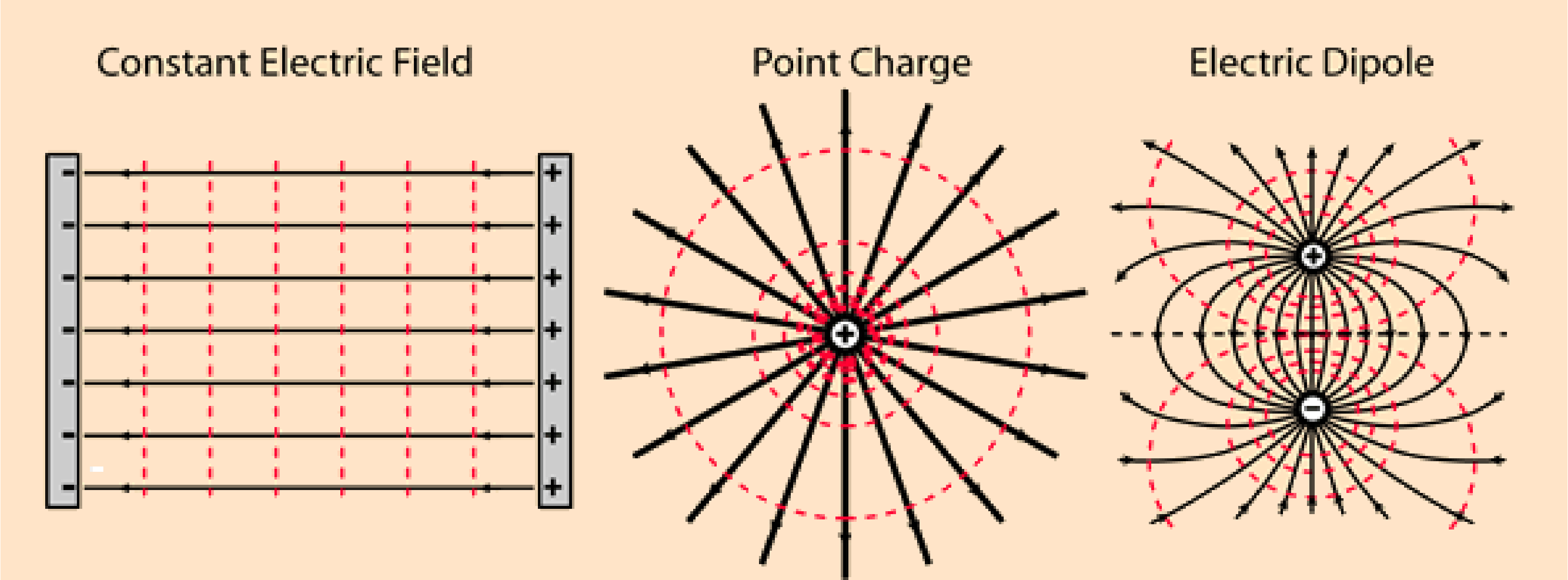
Equipotential Surfaces



Point Charge

Equipotential Surfaces

- The electric field does no work as a charge is moved along an equipotential surface
- Since no work is done, there is no force, qE, along the direction of motion
- The electric field is *perpendicular* to the equipotential surface



Example 5

Points A, B, and C lie in a uniform electric field.

Compare the potential differences between points A and C and points B and C.

a) $V_{AC} > V_{BC}$ b) $V_{AC} = V_{BC}$ c) $V_{AC} < V_{BC}$

In Example 2 we showed that the potential at points A and B were the same

Therefore the potential difference between A and C and the potential difference between points B and C are the same

Also remember that potential and potential energy are scalars and directions do not come into play

Example 6

If you want to move in a region of electric field without changing your electric potential energy. You would move

a) Parallel to the electric field

b) $\boxed{\text{Perpendicular to the electric field}}$

The work done by the electric field when a charge moves from one point to another is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

The way no work is done by the electric field is if the integration path is perpendicular to the electric field giving a zero for the dot product

Potential Gradient

The equation that relates the derivative of the potential to the electric field that we had before

$$\frac{dV}{dx} = -E$$

can be expanded into three dimensions

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{E} = -\left(\hat{i}\frac{dV}{dx} + \hat{j}\frac{dV}{dy} + \hat{k}\frac{dV}{dz} \right)$$

What about Conductors

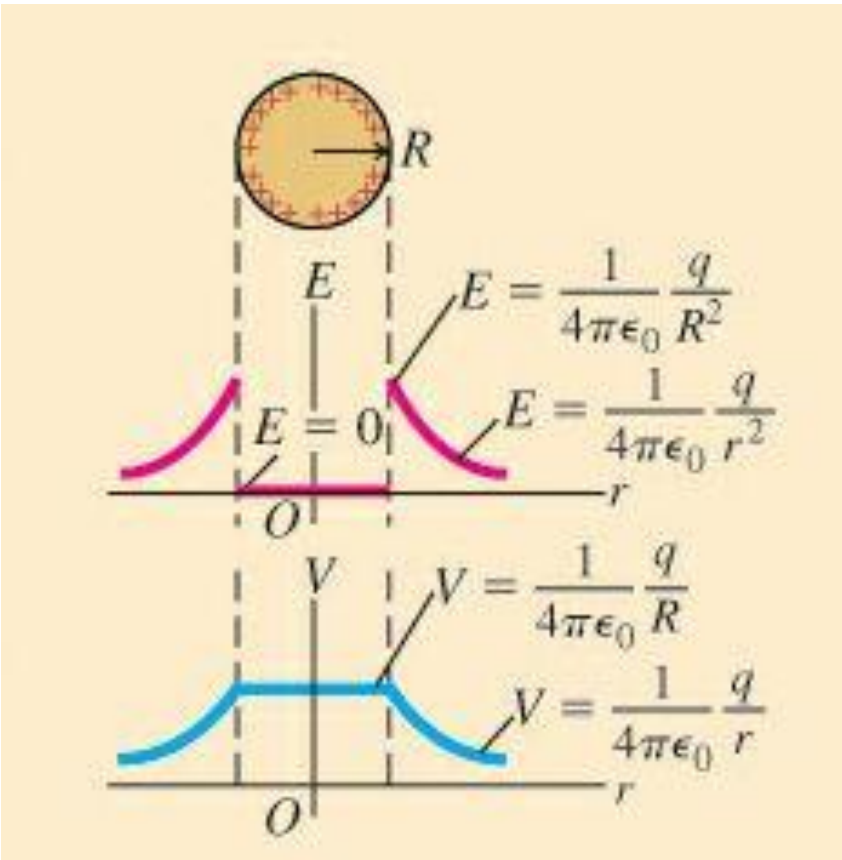
- In a static situation, the surface of a conductor is an equipotential surface
- But what is the potential inside the conductor if there is a surface charge?
- In electrostatics free charges in a good **conductor** reside only on the surface. Thus, the free charge **inside the conductor** is zero. Therefore, field in it is caused by charges on the surface. Since charges are of the same nature and distribution is UNIFORM, the **electric** fields cancel each other

Thus E = 0 inside the conductor

This leads to

$$\frac{dV}{dx} = 0 \text{ or } V = \text{constant}$$

What about Conductors



The value of the potential inside the conductor is chosen to match that at the surface

Example 7

The electric potential in a region of space is given by

$$V(x) = 3x^2 - x^3$$

The x-component of the electric field E_x at $x = 2$ is

$\boxed{\text{(a)} \quad E_x = 0}$ (b) $E_x > 0$ (c) $E_x < 0$

We know $V(x)$ "everywhere"

To obtain E_x "everywhere", use

$$\vec{E} = -\vec{\nabla} V \quad \Rightarrow \quad E_x = -\frac{dV}{dx} \quad \Rightarrow \quad E_x = -6x + 3x^2$$

$$E_x(2) = -6(2) + 3(2)^2 = 0$$

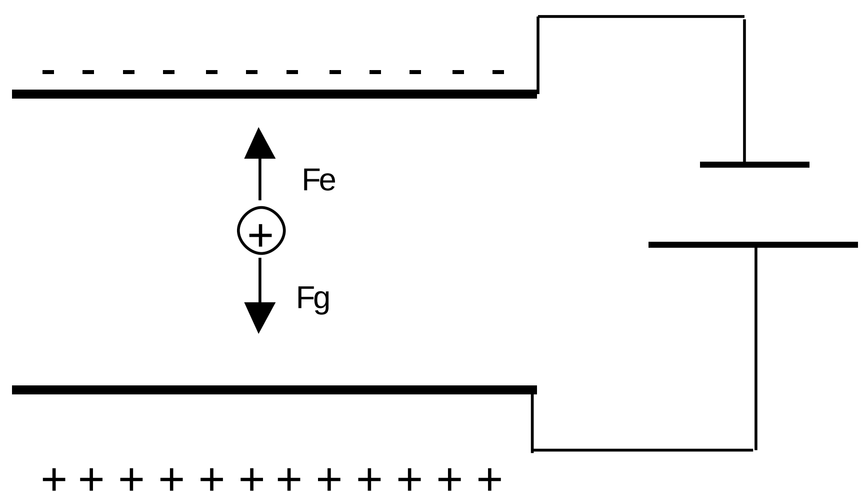
Sample Problem

- A charged particle of mass 4.0×10^{-15} kg is balanced between charged plates. The battery is rated at 5000 V and the plates are 4.0 cm apart.

m

V

r = .04 m



Find?

1. Electric field strength
2. Electric force on the particle
3. Amount of charge on the object
4. Number of elementary charges on the particle?