**Ministry of Higher Education**
**& Scientific Research**
**University of Anbar**
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**lectures**
**Subject: Vector analysis.**
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## Linear combination:

**Definition (3):** A vector $W$ is called **linear combination** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r$, if it can be expressed in the following form,

$$W = k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \dots + k_r\mathbf{v}_r \quad \dots (1)$$

Where, $k_1, k_2, \dots, k_r$, are scalars ($\in \mathbb{R}$).

***Note:*** If $r=1$, then the equation (1), reduced to $W = k_1\mathbf{v}_1$, $W$ is a linear combination of a single vector $\mathbf{v}_1$ if it a scalar multiple of $\mathbf{v}_1$.

(Vectors in $\mathbb{R}^3$ are a **linear combination** of **i**, **j**, and **k**.)

Every vector $\mathbf{v} = <a, b, c>$ in $\mathbb{R}^3$ is expressible as a linear combination of the standard basis vectors $\mathbf{i} = <1, 0, 0>$, $\mathbf{j} = <0, 1, 0>$, and $\mathbf{k} = <0, 0, 1>$

$$\mathbf{v} = <a, b, c> = a<1, 0, 0> + b<0, 1, 0> + c<0, 0, 1>$$

$$= a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

Also for the vectors in $\mathbb{R}^2$.

$$\mathbf{v} = <a, b> = a<1, 0> + b<0, 1>$$

$$= a\mathbf{i} + b\mathbf{j}$$

**Example (1):** Show that the vector $\mathbf{w}_1 = <9, 2, 7>$ is a linear combination of the vectors $\mathbf{u} = <1, 2, -1>$ and $\mathbf{v} = <6, 4, 2>$ in $\mathbb{R}^3$, and $\mathbf{w}_2 = <4, -1, 8>$ is not a linear combination of these vectors.

Solution:

$\mathbf{w}_1 = k_1\mathbf{u} + k_2\mathbf{v}$, $k_1$ and $k_2 \in \mathbb{R}$.

$<9, 2, 7> = k_1<1, 2, -1> + k_2<6, 4, 2>$

$= <k_1, 2k_1, -k_1> + <6k_2, 4k_2, 2k_2>$

$= <k_1 + 6k_2, 2k_1 + 4k_2, -k_1 + 2k_2>$

and, $k_1 + 6k_2 = 9$

$2k_1 + 4k_2 = 2$

$-k_1 + 2k_2 = 7$

We obtain a system of linear equations and we can solve this system using Gauss-Jordan elimination method.

$$\begin{bmatrix} 1 & 6 & 9 \\ 2 & 4 & 2 \\ -1 & 2 & 7 \end{bmatrix} -R_1 + R_2 \to R_2, \text{ and } R_1 + R_3 \to R_3 \sim \begin{bmatrix} 1 & 6 & 9 \\ 0 & -8 & -16 \\ 0 & 8 & 16 \end{bmatrix} -\tfrac{1}{8}R_2 \to R_2$$

$$\begin{bmatrix} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 8 & 16 \end{bmatrix} -6R_2 + R_1 \to R_1 \text{ and } -8R_2 + R_3 \to R_3 \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$k_1 = -3$, and $k_2 = 2$

Thus, $\mathbf{w}_1$ is a linear combination of the vectors **u** and **v**.

$\mathbf{w}_1 = k_1\mathbf{u} + k_2\mathbf{v} = -3\langle 1, 2, -1\rangle + 2\langle 6, 4, 2\rangle$

$= \langle -3, -6, 3\rangle + \langle 12, 8, 4\rangle$

$= \langle 9, 2, 7\rangle$

$= \mathbf{w}_1$

Similarly, for $\mathbf{w}_2$ and we obtain

$$\begin{bmatrix} 1 & 0 & \frac{-11}{4} \\ 0 & 1 & \frac{9}{8} \\ 0 & 0 & 3 \end{bmatrix}$$

We note this system of linear equation is inconsistent, because the third equation 0 + 0= 3, and this system has no solution. Therefore, both $k_1$ and $k_2$ do not exist.

Hence, $\mathbf{w}_2$ is not a linear combination of the vectors **u** and **v**.

**Example (2):** Show that the vector $\mathbf{x} = \langle 2, 1, 5, -5\rangle$ is a linear combination of the vectors $\mathbf{x}_1 = \langle 1, 2, 1, -1\rangle$, $\mathbf{x}_2 = \langle 1, 0, 2, -3\rangle$ and $\mathbf{x}_3 = \langle 1, 1, 0, -2\rangle$ in $\mathbb{R}^4$. (**Homework**).

**Example (3):** Show whether the vector $\mathbf{x} = \langle 1, 0, 2\rangle$ is a linear combination of the vectors $\mathbf{x}_1 = \langle 1, 2, -1\rangle$ and $\mathbf{x}_2 = \langle 1, 0, -1\rangle$ in $\mathbb{R}^3$.

**Theorem (16):** If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$, are vectors in a vector space $V$, then

a) The set $W$ of all linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$, is a subspace of $V$.
b) $W$ is the smallest subspace of $V$, that contains $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$, in the sense that every other subspace of $V$, that contains $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$, must contain $W$.

## The Span of a set of vectors:

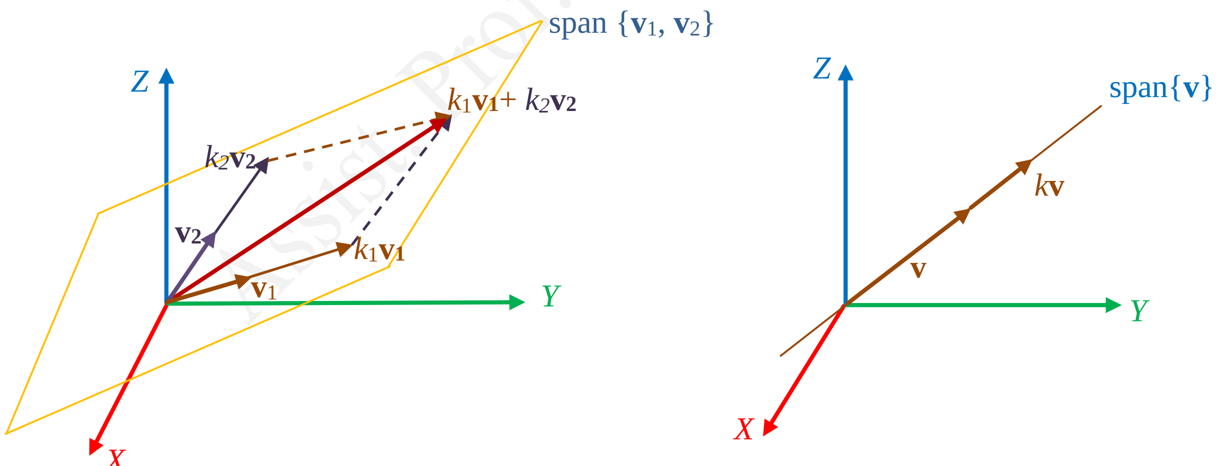
**Definition (4):** If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$, is a set of vectors in a vector space $V$, then the subspace $W$ of $V$, consisting of all linear combinations of the vectors in $S$ is called the **space spanned** by the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$, and we say that the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r$ **span** $W$. To indicate that $W$ is the space spanned by the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$, we write as the following,

$$W = \text{span}(S) \text{ or } W = \text{span}\,\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}.$$

We can say if $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$is a set of the vector space $V$, then $S$ is the span (span $V$), if each vector in $V$ is a linear combination of the vectors in $S$.

## Space spanned by one or two vectors:

If $\mathbf{v}_1$ and $\mathbf{v}_2$ are no collinear vectors in $\mathbb{R}^3$ with their initial points at the origin, then span($\mathbf{v}_1$, $\mathbf{v}_2$), which consists of all linear combinations $k_1\mathbf{v}_1 + k_2\mathbf{v}_2$, is the plane determined by $\mathbf{v}_1$ and $\mathbf{v}_2$, as shown in figure(4-a) Similarly, if **v is a nonzero vector in** $\mathbb{R}^2$ or $\mathbb{R}^3$, then the span{v}, which is the set of all scalar multiple $k\mathbf{v}$, is the line determined by $\mathbf{v}$ as shown in figure (4-b).




**(a)** Span $\{\mathbf{v}_1, \mathbf{v}_2\}$is the plane through origin determined by $\mathbf{v}_1$ and $\mathbf{v}_2$.

**(b)** Span $\{\mathbf{v}\}$is the line through origin determined by $\mathbf{v}$.

**Figure (4)**

**Example (4):** Let $V$ is a vector space of $\mathbb{R}^3$ and let $S = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$, such that $\mathbf{x}_1 = <1, 2, 1>$, $\mathbf{x}_2 = <1, 0, 2>$, and $\mathbf{x}_3 = <1, 1, 0>$, show whether $S$ span $\mathbb{R}^3$.

Solution:

To show that, we must determine whether every vector in $\mathbb{R}^3$ is a linear combination of the vectors in $S$.

Let $\mathbf{x}= <a, b, c>$ is any vector $\in \mathbb{R}^3$, where $a$, $b$, and $c \in \mathbb{R}$, then

$$\mathbf{x}= c_1\mathbf{x}_1 + c_2\mathbf{x}_2 + c_3\mathbf{x}_3, \text{ where } c_1, c_2, \text{ and } c_3 \text{ are scalars} \in \mathbb{R}$$

We must find $c_1$, $c_2$, and $c_3$.

$$<a, b, c>= c_1<1, 2, 1> + c_2<1, 0, 2> + c_3<1, 1, 0>$$

$$= < c_1, 2c_1, c_1> + < c_2, 0, 2c_2> + < c_3, c_3, 0>$$

$$= < c_1 + c_2 + c_3, 2c_1 + c_3, c_1 + 2c_2>$$

We obtain the following system of linear equations,

$$c_1 + c_2 + c_3, 2c_1= a$$

$$2c_1 + c_3= b$$

$$c_1 + 2c_2= c$$

Using Gauss-Jordan elimination method, we can solve this system, and if this system is consistent, then $S$ span $\mathbb{R}^3$.

$$\begin{bmatrix} 1 & 1 & 1 & a \\ 2 & 0 & 1 & b \\ 1 & 2 & 0 & c \end{bmatrix} -2R_1+R_2\to R_2 \text{ and } -R_1+R_3\to R_3 \sim \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & -2 & -1 & -2a+b \\ 0 & 1 & -1 & -a+c \end{bmatrix} -\tfrac{1}{2}R_2\to R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & \frac{1}{2} & \frac{2a-b}{2} \\ 0 & 1 & -1 & -a+c \end{bmatrix} -R_2+R_1\to R_1 \text{ and } -R_2+R_3\to R_3 \sim \begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{b}{2} \\ 0 & 1 & \frac{1}{2} & \frac{2a-b}{2} \\ 0 & 0 & \frac{-3}{2} & \frac{-4a+b+2c}{2} \end{bmatrix} -\tfrac{2}{3}R_3\to R_3$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{b}{2} \\ 0 & 1 & \frac{1}{2} & \frac{2a-b}{2} \\ 0 & 0 & 1 & \frac{4a-b-2c}{3} \end{bmatrix} -\tfrac{1}{2}R_3+R_2\to R_2 \text{ and } -\tfrac{1}{2}R_3+R_1\to R_1 \sim \begin{bmatrix} 1 & 0 & 0 & \frac{-2a+2b+c}{3} \\ 0 & 1 & 0 & \frac{a-b+c}{3} \\ 0 & 0 & 1 & \frac{4a-b-2c}{3} \end{bmatrix}$$

$c_1= \frac{-2a+2b+c}{3}$, $c_2= \frac{a-b+c}{3}$, $c_3= \frac{4a-b-2c}{3}$

Thus, $S$ span $\mathbb{R}^3$.

**Example (5):** Determine whether the vectors $\mathbf{v}_1= <1, 1, 2>$, $\mathbf{v}_2= <1, 0, 1>$, and $\mathbf{v}_3= <2, 1, 3>$ span the vector space $\mathbb{R}^3$.

Solution:

Let **b**= $<b_1, b_2, b_3>$ any arbitrary vector in $\mathbb{R}^3$, such that $b_1$, $b_2$, and $b_3 \in \mathbb{R}$.

Now, we must determine whether the vector **b** is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, such that

$$\mathbf{b}= k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k\mathbf{v}_3, \text{ where } k_1, k_2, \text{ and } k_3 \text{ are scalars} \in \mathbb{R}.$$

$< b_1, b_2, b_3>= k_1<1, 1, 2> + k_2<1, 0, 1> + k_3<2, 1, 3>$

$= < k_1, k_1, 2k_1> + < k_2, 0, k_2> + <2k_3, k_3, 3k_3>$

$= < k_1 + k_2 + 2k_3, k_1 + k_3, 2k_1 + k_2 + 3k_3>$

$$\begin{aligned} k_1 + k_2 + 2k_3 &= \mathbf{b}_1 \\ k_1 + \qquad k_3 &= \mathbf{b}_2 \\ 2k_1 + k_2 + 3k_3 &= \mathbf{b}_3 \end{aligned}$$

We obtain system of linear equations and we can solve it using Gauss-Jordan elimination method.

$$\begin{bmatrix} 1 & 1 & 2 & b_1 \\ 1 & 0 & 1 & b_2 \\ 2 & 1 & 3 & b_3 \end{bmatrix} -R_1+R_2\to R_2 \text{ and } -2R_1+R_3\to R_3 \sim \begin{bmatrix} 1 & 1 & 2 & b_1 \\ 0 & -1 & -1 & -b_1+b_2 \\ 0 & -1 & -1 & -2b_1+b_3 \end{bmatrix} -R_2\to R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & b_1 \\ 0 & 1 & 1 & b_1-b_2 \\ 0 & -1 & -1 & -2b_1+b_3 \end{bmatrix} -R_2+R_1\to R_1 \text{ and } R_2+R_3\to R_3 \sim \begin{bmatrix} 1 & 0 & 2 & b_2 \\ 0 & 1 & 1 & b_1-b_2 \\ 0 & 0 & 0 & -b_1-b_2+b_3 \end{bmatrix}$$

Now, if $-b_1 - b_2 + b_3 \neq 0$, then this system is inconsistent and $S$ do not span $\mathbb{R}^3$.

**Example (6):** Let $V$ is a vector space of all polynomials $P_2$ (degree $\leq 2$) and let $S= \{\boldsymbol{p}_1(t), \boldsymbol{p}_2(t)\}$, such that $\boldsymbol{p}_1(t)= t^2 + 2t + 1$ and $\boldsymbol{p}_2(t)= t^2 + 2$, show whether $S$ span $P_2$. (**Homework**)

**Example (7):** Show whether the set $S= \{\mathbf{e}_1, \mathbf{e}_2\}$ span $\mathbb{R}^2$, where $\mathbf{e}_1= \mathbf{i}= <1, 0>$ and $\mathbf{e}_2= \mathbf{j}= <0, 1>$.

Solution:

Let $\mathbf{v}= <v_1, v_2> \in \mathbb{R}^2$, such that $v_1$ and $v_2 \in \mathbb{R}$.

$\mathbf{v}= k_1\mathbf{e}_1 + k_2\mathbf{e}_2$

$<v_1, v_2>= k_1<1, 0> + k_2<0, 1>$

$= < k_1, 0> + <0, k_2>$

$= < k_1, k_2 > \to v_1= k_1, v_2= k_2$

$\mathbf{v}= v_1\mathbf{e}_1 + v_2\mathbf{e}_2$

$= v_1\mathbf{i} + v_2\mathbf{j}$

Therefore, every vector in $\mathbb{R}^2$, we can write it as a linear combination of the unit vectors of $\mathbb{R}^2$.

Thus, the set of the unit vectors in $\mathbb{R}^2$ formed span $\mathbb{R}^2$.

So, every vector $\mathbf{v}= <v_1, v_2, v_3> \in \mathbb{R}^3$, we can write it as a linear combination of the unit vectors in $\mathbb{R}^3$.

$\mathbf{v}= v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + v_3\mathbf{e}_3$, such that, $\mathbf{e}_1= \mathbf{i}= <1, 0, 0>$, $\mathbf{e}_2= \mathbf{j}= <0, 1, 0>$ and $\mathbf{e}_3= \mathbf{k}= <0, 0, 1>$

Therefore, the set of the unit vectors in $\mathbb{R}^3$ formed span $\mathbb{R}^3$.

So, the set of the vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$ in $\mathbb{R}^n$ formed span $\mathbb{R}^n$.

We can say that the set $S= \{1, t, t^2, \ldots, t^n\}$ formed span $P_n$, because every polynomial in $P_n$ be of the form $\boldsymbol{p}(t)= a_0 + a_1t + a_3t^2 + \ldots + a_nt^n$, which is a linear combination of $1, t, t^2, \ldots, t^n$.

Thus, the set $S$ span $P_n$ ($P_n$= span $\{1, t, t^2, \ldots, t^n\}$).

**Theorem (17):** If $S_1= \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$ and $S_2= \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ are two sets of vectors in a vector space $V$, then span $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$ = span $\{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_k\}$ if and only if each vector in $S_1$ is a linear combination of those in $S_2$, and each vector in $S_2$ is a linear combination of those in $S_1$.

## Linear independent and linear dependent:

**Definition (5):** If $S= \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r\}$ is a nonempty set of vectors, then the vector equation

$$k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \ldots + k_r\mathbf{v}_r= \mathbf{0},$$

has at least one solution, namely

$$k_1= 0, k_2= 0, \ldots, k_r= 0$$

If this is the only solution, then $S$ is called a **linearly independent set**. If there are other solutions, then $S$ is called a **linearly dependent set**.

**Example (8):** Show whether the vectors $\mathbf{v}_1= <2, -1, 0, 3>$, $\mathbf{v}_2= <1, 2, 5, -1>$ and $\mathbf{v}_3= <7, -1, 5, 8>$ are linearly independent.

Solution:

We must show that the vectors equation $k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3= \mathbf{0}$ for $k_1= k_2= k_3=0$.

$k_1<2, -1, 0, 3> + k_2<1, 2, 5, -1> + k_3<7, -1, 5, 8> = <0, 0, 0, 0>$

$<2k_1, -k_1, 0, 3k_1> + < k_2, 2k_2, 5k_2, -k_2> + <7k_3, -k_3, 5k_3, 8k_3> = <0, 0, 0, 0>$

$<2k_1 + k_2 + 7k_3, -k_1 + 2k_2 - k_3, 5k_2 + 5k_3, 3k_1 - k_2 + 8k_3> = <0, 0, 0, 0>$

$2k_1 + k_2 + 7k_3 = 0$

$-k_1 + 2k_2 - k_3 = 0$

$5k_2 + 5k_3 = 0$

$3k_1 - k_2 + 8k_3 = 0$

We have system of linear equations and we can solve this system using Gauss-Jordan elimination method. At first, using augmented matrix we obtain;

$$\begin{bmatrix} 2 & 1 & 7 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & 5 & 5 & 0 \\ 3 & -1 & 8 & 0 \end{bmatrix} \tfrac{1}{2}R_1 \rightarrow R_1 \sim \begin{bmatrix} 1 & \frac{1}{2} & \frac{7}{2} & 0 \\ -1 & 2 & -1 & 0 \\ 0 & 5 & 5 & 0 \\ 3 & -1 & 8 & 0 \end{bmatrix} R_1 + R_2 \rightarrow R_2 \text{ and } -3R_1 + R_4 \rightarrow R_4$$

$$\sim \begin{bmatrix} 1 & \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & \frac{5}{2} & \frac{5}{2} & 0 \\ 0 & 5 & 5 & 0 \\ 0 & \frac{-5}{2} & \frac{-5}{2} & 0 \end{bmatrix} \tfrac{2}{5}R_2 \rightarrow R_2 \sim \begin{bmatrix} 1 & \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & \frac{-5}{2} & \frac{-5}{2} & 0 \end{bmatrix} -\tfrac{1}{2}R_2 + R_1 \rightarrow R_1,\ -5R_2 + R_3 \rightarrow R_3 \text{ and } \tfrac{5}{2}R_2 + R_4 \rightarrow R_4$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$k_1 + 3k_3 = 0 \rightarrow k_1 = -3k_3$

$k_2 + k_3 = 0 \rightarrow k_2 = -k_3$, and if $k_3 = 1 \rightarrow k_1 = -3, k_2 = -1$

The system has infinity many solutions. Thus, the set of vectors is linearly dependent, since

$-3\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0}$.

**Example (9):** Show whether the set $S = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ of the vector space $\mathbb{R}^4$, is a linearly independent or linearly dependent, where $\mathbf{x}_1 = <1, 0, 1, 2>$, $\mathbf{x}_2 = <0, 1, 1, 2>$, and $\mathbf{x}_3 = <1, 1, 1, 3>$.

Solution:

$k_1\mathbf{x}_1 + k_2\mathbf{x}_2 + k_3\mathbf{x}_3 = \mathbf{0}$

$k_1<1, 0, 1, 2> + k_2<0, 1, 1, 2> + k_3<1, 1, 1, 3> = <0, 0, 0, 0>$

$< k_1, 0, k_1, 2k_1> + <0, k_2, k_2, 2k_2> + < k_3, k_3, k_3, 3k_3> = <0, 0, 0, 0>$

$<k_1 + k_3, k_2 + k_3, k_1 + k_2 + k_3, 2k_1 + 2k_2 + 3k_3> = <0, 0, 0, 0>$

$$\begin{aligned} k_1 + \qquad k_3 &= 0 \\ k_2 + k_3 &= 0 \\ k_1 + k_2 + k_3 &= 0 \\ 2k_1 + 2k_2 + 3k_3 &= 0 \end{aligned}$$

Using Gauss-Jordan elimination method, we solve the system of linear equations.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow k_1 = k_2 = k_3 = 0$$

Thus, the set $S$ is a linearly independent.

**Example (10):** Determine whether the polynomials $\boldsymbol{p}_1=1-x$, $\boldsymbol{p}_2= 5 + 3x - 2x^2$ and $\boldsymbol{p}_3= 1 + 3x - x^2$ form a linearly dependent set in $P_2$. (**Homework**)

**Example (11): Determine whether the set** $S= \{\boldsymbol{p}_1(t), \boldsymbol{p}_2(t), \boldsymbol{p}_3(t)\}$is a linearly independent, where $\boldsymbol{p}_1(t)= t^2 + t + 2$, $\boldsymbol{p}_2(t)= 2t^2 + t$, and $\boldsymbol{p}_1(t)= 3t^2 + 2t + 2$. (**Homework**)



## References

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