جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

المادة: التفاضل والتكامل2

للطلبة المرحلة الاولى

الفصل الخامس: المحاضرة الخامسة

(التكامل غير المحدد وطريقة التعويض)

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CH. 5: Integration

5.5 Indefinite Integrals and Substitution Rule

If f is continuous on an interval I and n is real number then

$$\int (f(x))^n \; f'(x)dx = \int u^n \, du = \frac{u^{n+1}}{n+1} + c$$

$$\boxed{u = f(x) \to \frac{du}{dx} = f'(x) \to du = f'(x)dx}$$

In general, $\int f\big(g(x)\big)\, g'(x)dx = \int f(u)\, du$ $\boxed{u = g(x) \to du = g'(x)dx}$

Ex.(1) **Evaluate the following integrals.**

1) $\int (2x+3)^8 \, dx$, $t = 2x+3 \to dt = 2dx \to \frac{1}{2}dt = dx$

$$= \int t^8 \, \frac{1}{2} \, dt = \frac{1}{2}\int t^8 \, dt = \frac{1}{2}\frac{t^9}{9} + c = \frac{1}{18}(2x+3)^9 + c$$

2) $\int x^2 \sqrt{2x^3+3}\, dx$, let $w = 2x^3+3 \to dw = 6x^2dx \to \frac{1}{6}dw = x^2dx$

$$= \int \sqrt{w}\, \frac{1}{6}dw = \frac{1}{6}\int w^{\frac{1}{2}}dw = \frac{1}{6}\frac{w^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{6\cdot 3}(2x^3+3)^{\frac{3}{2}} + c = \frac{1}{9}(2x^3+3)^{\frac{2}{3}} + c$$

3) $\int x\sqrt{4-x}\, dx$, let $y = 4-x \to dy = -dx \to -dy = dx$, $(x = 4-y)$

$$= \int (4-y)\sqrt{y}(-dy) = \int (y-4)\, y^{\frac{1}{2}}\, dy = \int \left(y^{\frac{3}{2}} - 4y^{\frac{1}{2}}\right) dy$$

$$= \frac{2}{5}y^{\frac{5}{2}} - 4\cdot\frac{2}{3}y^{\frac{3}{2}} + c = \frac{2}{5}(4-x)^{\frac{5}{2}} - \frac{8}{3}(4-x)^{\frac{3}{2}} + c$$

4) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}dx$, $z = 1+\sqrt{x} \to dz = \frac{1}{2\sqrt{x}}dx \to 2dz = \frac{1}{\sqrt{x}}dx$

$$= \int \frac{1}{z^2}2dz = 2\int z^{-2}dz = 2\frac{z^{-1}}{-1} + c = -2\left(\frac{1}{z}\right) + c = \frac{-2}{1+\sqrt{x}} + c$$

5) $\int 3x^5\sqrt{x^3+1}dx$, $t = x^3+1 \to dt = 3x^2dx$, $(x^3 = t-1)$

$$\int 3x^2 \cdot x^3\sqrt{x^3+1}\,dx = \int (t-1)\sqrt{t}\;dt = \int \left(t^{\frac{3}{2}} - t^{\frac{1}{2}}\right) dt$$

$$= \frac{2}{5}t^{\frac{5}{2}} - \frac{2}{3}t^{\frac{3}{2}} + c = \frac{2}{5}(x^3+1)^{\frac{5}{2}} - \frac{2}{3}(x^3+1)^{\frac{3}{2}} + c$$

6) $\int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\,dx$, $\theta = \frac{1}{x} \to d\theta = \frac{-1}{x^2}dx \to -d\theta = \frac{1}{x^2}dx$

$$= -\int \sin\theta\cos\theta\,d\theta, \quad y = \sin\theta \to dy = \cos\theta\,d\theta$$

$$= -\int y\,dy = -\frac{y^2}{2} + c = -\frac{1}{2}\sin^2\theta + c = -\frac{1}{2}\sin^2\left(\frac{1}{x}\right) + c$$

or $= -\frac{1}{2}\int 2\sin\theta\cos\theta\,d\theta = -\frac{1}{2}\int \sin 2\theta\,d\theta = -\frac{1}{2}\left(\frac{-\cos 2\theta}{2}\right) + c$

$$= \frac{1}{4}\cos 2\theta + c = \frac{1}{4}\cos\left(\frac{2}{x}\right) + c$$

7) $\int \sin^5\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)dx = 3\int \sin^5\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)\frac{1}{3}dx$

$$= 3\frac{\sin^6\left(\frac{x}{3}\right)}{6} + c = \frac{1}{2}\sin^6\left(\frac{3}{x}\right) + c$$

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