جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات / الفصل الاول

Graph Theory

Introduction to Graphs (1)

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UNIVERSITY OF ANBAR

Lecture (1)

Graph Theory

Introduction to Graphs (1)

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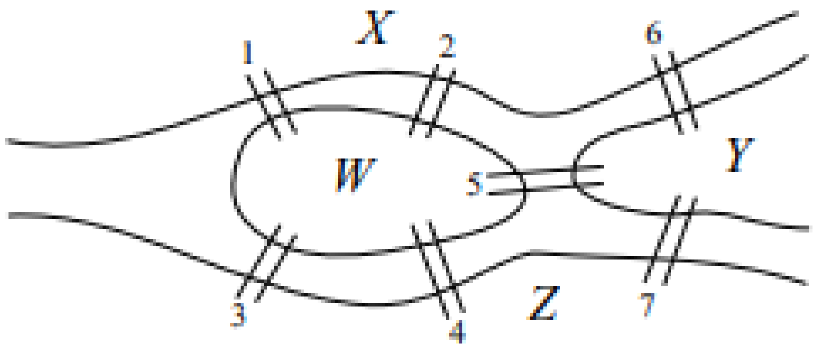
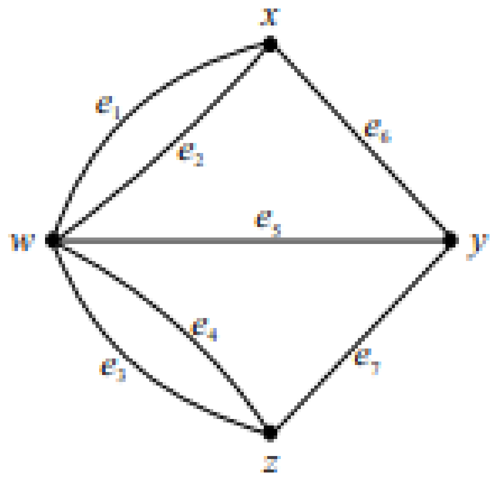
The Königsberg Bridge Problem

The city of Königsberg was located on the Pregel river in Prussia. The river divided the city into four separate landmasses, including the island of Kneiphopf. These four regions were linked by seven bridges as shown in the diagram. Residents of the city wondered if it were possible to leave home, cross each of the seven bridges exactly once, and return home. The Swiss mathematician Leonhard Euler (1707-1783) thought about this problem and the method he used to solve it is considered by many to be the birth of graph theory.





See if you can find a round trip through the city crossing each bridge exactly once, or try to explain why such a trip is not possible.

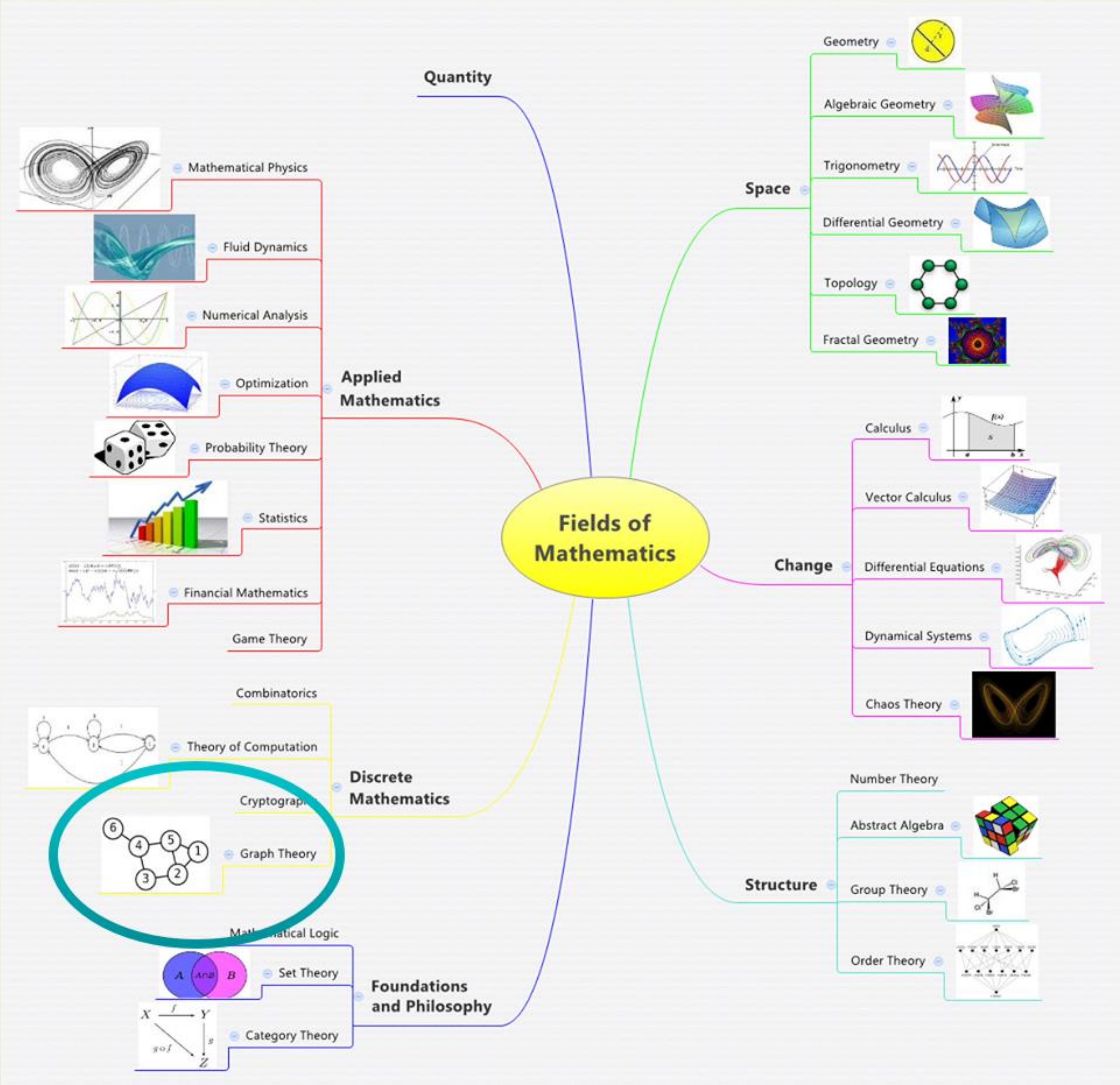
Outlines

- ✓ Basic Definitions.
- ✓ Degree of Vertex.
- ✓ Minimum and Maximum Degree of a Graph.

Basic Definitions:

Graph Theory is a well-known area of discrete mathematics which deals with the study of graphs. A graph may be considered as a mathematical structure that is used for modelling the pairwise relations between objects.

Graph Theory has many theoretical developments and applications not only to different branches of mathematics, but also to various other fields of basic sciences, technology, social sciences, computer science etc.


Fields of Mathematics
Quantity
Space
Geometry
Algebraic Geometry
Trigonometry
Differential Geometry
Topology
Fractal Geometry
Change
Calculus
Vector Calculus
Differential Equations
Dynamical Systems
Chaos Theory
Structure
Number Theory
Abstract Algebra
Group Theory
Order Theory
Applied Mathematics
Mathematical Physics
Fluid Dynamics
Numerical Analysis
Optimization
Probability Theory
Statistics
Financial Mathematics
Game Theory
Discrete Mathematics
Combinatorics
Theory of Computation
Cryptograp
Graph Theory
Foundations and Philosophy
Mat atical Logic
Set Theory
Category Theory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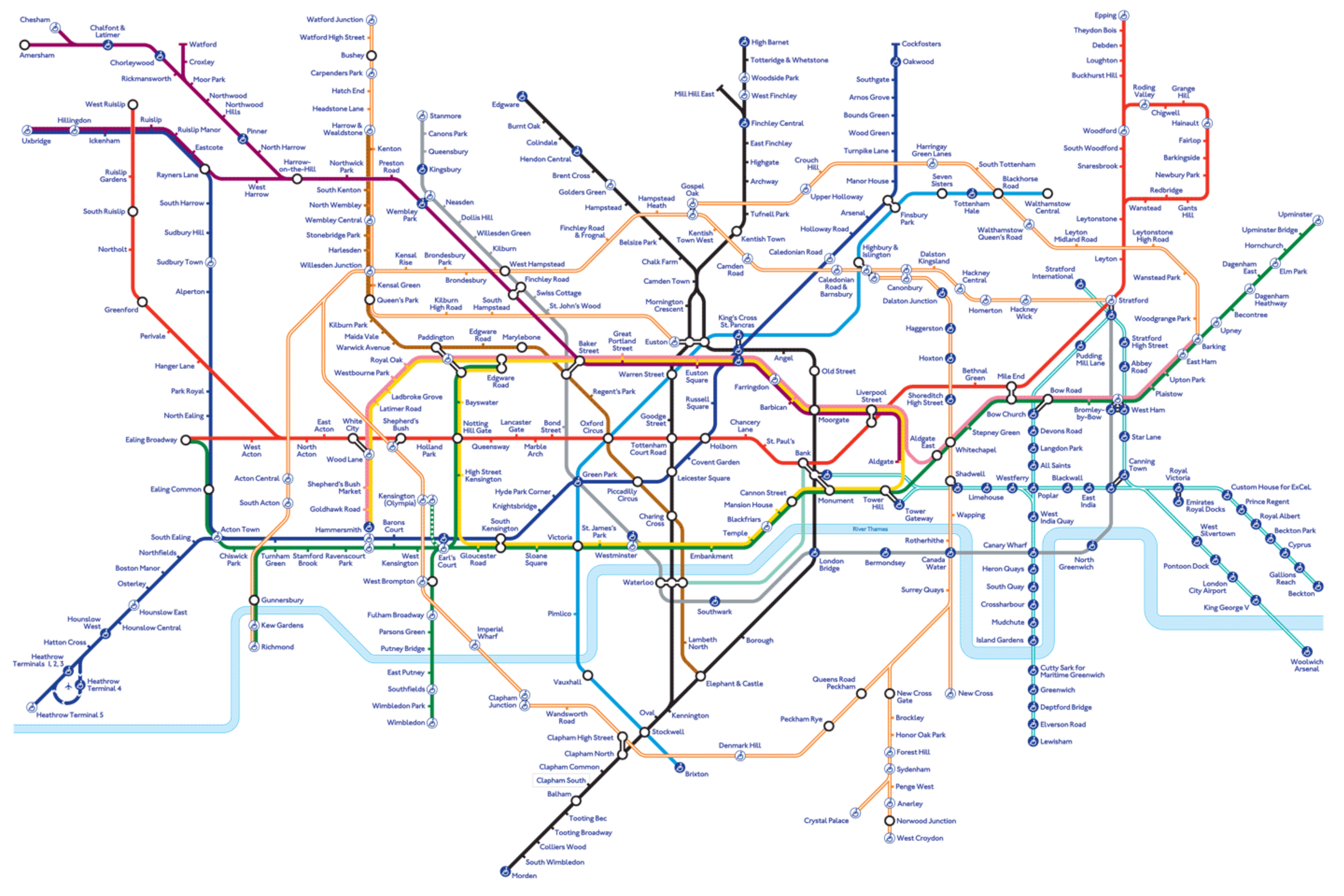
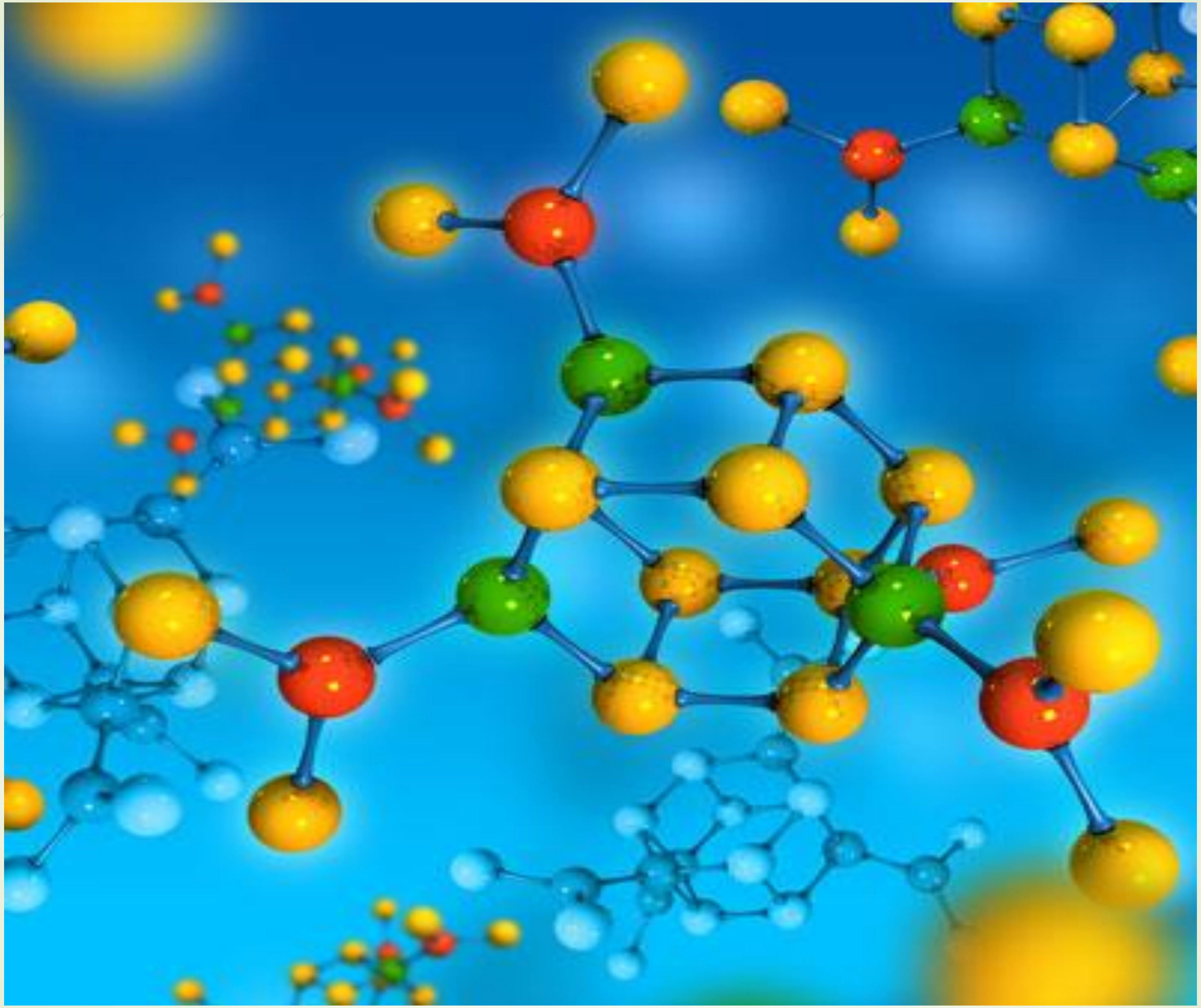


Chesham
Chalfont & Latimer
Amersham
Chorleywood
Rickmansworth
Watford
Croxley
Moor Park
Northwood
Northwood Hills
Pinner
North Harrow
Harrow-on-the-Hill
West Harrow
West Ruislip
Ruislip
Ruislip Manor
Eastcote
Hillingdon
Uxbridge
Ickenham
Rayners Lane
Ruislip Gardens
South Ruislip
Northolt
Greenford
Perivale
Hanger Lane
South Harrow
Sudbury Hill
Sudbury Town
Alperton
Park Royal
North Ealing
Ealing Broadway
West Acton
North Acton
East Acton
White City
Wood Lane
Ealing Common
Acton Central
South Acton
Acton Town
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Hounslow West
Hounslow Central
Hatton Cross
Heathrow Terminals 1, 2, 3
Heathrow Terminal 4
Heathrow Terminal 5
Chiswick Park
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
Watford Junction
Watford High Street
Bushey
Carpenders Park
Hatch End
Headstone Lane
Harrow & Wealdstone
Kenton
Northwick Park
Preston Road
South Kenton
North Wembley
Wembley Central
Stonebridge Park
Harlesden
Willesden Junction
Kensal Green
Queen's Park
Kilburn Park
Maida Vale
Warwick Avenue
Royal Oak
Westbourne Park
Ladbroke Grove
Latimer Road
Shepherd's Bush
Shepherd's Bush Market
Goldhawk Road
Hammersmith
Barons Court
West Kensington
Kensington (Olympia)
Holland Park
Stanmore
Canons Park
Queensbury
Kingsbury
Wembley Park
Neasden
Dollis Hill
Willesden Green
Kilburn
Kensal Rise
Brondesbury Park
Brondesbury
West Hampstead
Finchley Road
Swiss Cottage
St. John's Wood
Kilburn High Road
South Hampstead
Edgware
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Finchley Road & Frognal
Hampstead Heath
Gospel Oak
Belsize Park
Chalk Farm
Kentish Town West
Camden Road
Camden Town
Mornington Crescent
Paddington
Edgware Road
Marylebone
Baker Street
Great Portland Street
Euston
King's Cross St. Pancras
Bayswater
Notting Hill Gate
Queensway
Lancaster Gate
Marble Arch
Bond Street
Oxford Circus
High Street Kensington
Earl's Court
Gloucester Road
South Kensington
Sloane Square
Knightsbridge
Hyde Park Corner
Green Park
Piccadilly Circus
Victoria
St. James's Park
Westminster
West Brompton
Fulham Broadway
Parsons Green
Putney Bridge
East Putney
Southfields
Wimbledon Park
Wimbledon
Imperial Wharf
Clapham Junction
Pimlico
Vauxhall
Wandsworth Road
Clapham High Street
Clapham North
Clapham Common
Clapham South
Balham
Tooting Bec
Tooting Broadway
Colliers Wood
South Wimbledon
Morden
Stockwell
Oval
Kennington
Brixton
Warren Street
Regent's Park
Euston Square
Russell Square
Goodge Street
Tottenham Court Road
Holborn
Chancery Lane
Covent Garden
Leicester Square
Charing Cross
Embankment
Waterloo
Southwark
Lambeth North
Elephant & Castle
Borough
High Barnet
Totteridge & Whetstone
Woodside Park
West Finchley
Mill Hill East
Finchley Central
East Finchley
Highgate
Archway
Crouch Hill
Upper Holloway
Tufnell Park
Kentish Town
Caledonian Road
Holloway Road
Arsenal
Caledonian Road & Barnsbury
Highbury & Islington
Angel
Old Street
Farringdon
Barbican
Moorgate
Liverpool Street
St. Paul's
Bank
Aldgate
Monument
Cannon Street
Mansion House
Blackfriars
Temple
Tower Hill
Tower Gateway
London Bridge
Bermondsey
Cockfosters
Oakwood
Southgate
Arnos Grove
Bounds Green
Wood Green
Turnpike Lane
Harringay Green Lanes
Manor House
Finsbury Park
Seven Sisters
South Tottenham
Tottenham Hale
Blackhorse Road
Walthamstow Central
Walthamstow Queen's Road
Leyton Midland Road
Leytonstone High Road
Dalston Kingsland
Hackney Central
Canonbury
Dalston Junction
Homerton
Hackney Wick
Haggerston
Hoxton
Shoreditch High Street
Bethnal Green
Aldgate East
Whitechapel
Stepney Green
Mile End
Bow Road
Bow Church
Devons Road
Langdon Park
All Saints
Shadwell
Limehouse
Westferry
Poplar
Blackwall
East India
Wapping
Rotherhithe
Canada Water
Surrey Quays
West India Quay
Canary Wharf
Heron Quays
South Quay
Crosshardour
Mudchute
Island Gardens
Cutty Sark for Maritime Greenwich
Greenwich
Deptford Bridge
Elverson Road
Lewisham
North Greenwich
New Cross Gate
New Cross
Queens Road Peckham
Peckham Rye
Denmark Hill
Brockley
Honor Oak Park
Forest Hill
Sydenham
Penge West
Anerley
Crystal Palace
Norwood Junction
West Croydon
River Thames
Epping
Theydon Bois
Debden
Loughton
Buckhurst Hill
Roding Valley
Grange Hill
Chigwell
Hainault
Fairlop
Barkingside
Newbury Park
Redbridge
Woodford
South Woodford
Snaresbrook
Wanstead
Gants Hill
Leytonstone
Leyton
Stratford International
Stratford
Wanstead Park
Woodgrange Park
Stratford High Street
Pudding Mill Lane
Abbey Road
Bromley-by-Bow
West Ham
Star Lane
Plaistow
Upton Park
East Ham
Barking
Upney
Becontree
Dagenham Heathway
Dagenham East
Elm Park
Hornchurch
Upminster Bridge
Upminster
Canning Town
Royal Victoria
Custom House for ExCeL
Prince Regent
Royal Albert
Beckton Park
Cyprus
Gallions Reach
Beckton
Emirates Royal Docks
West Silvertown
Pontoon Dock
London City Airport
King George V
Woolwich Arsenal



Definition : A graph G can be considered as an ordered triple (V, E, φ), where (i) $V = \{v_1, v_2, \dots, v_n\}$ is called the vertex set of G and the elements of V are called the vertices (or points or nodes);

(ii) $E = \{e_1, e_2, \dots, e_n\}$ is the called the edge set of G and the elements of E are called edges (or lines or arcs); and

(iii) φ is called the adjacency relation, defined by $\varphi : E \to V \times V$, which defines the association between each edge with the vertex pairs of G.

Usually, the graph is denoted as $\mathrm{G} = (V, E)$. The vertex set and edge set of a graph G are also written as $V(G)$ and $E(G)$ respectively.

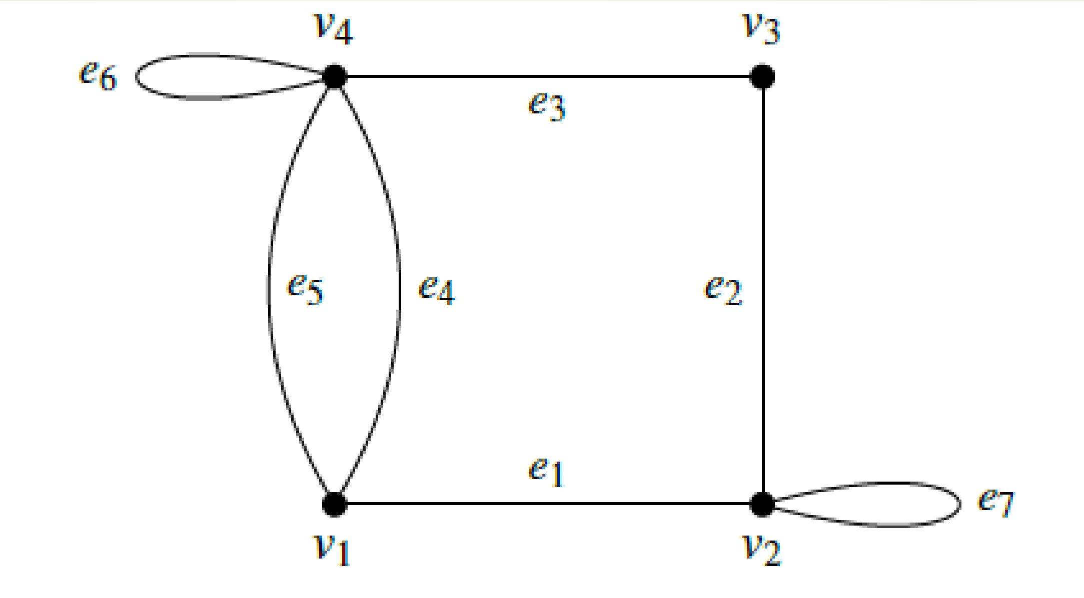


Fig. 1: An example of a graph

If two vertices u and v are the (two) end points of an edge e, then we represent this edge by uv or vu. If e $= uv$ is an edge of a graph G, then we say that u and v are adjacent vertices in G and that e joins u and v . In such cases, we also say that u and v are adjacent to each other.

Given an edge $e = uv$, the vertex u and the edge e are said to be incident with each other and so are v and e. Two edges e_i and e_j are said to be adjacent edges if they are incident with a common vertex.

Definition: (Order and Size of a Graph) The order of a graph G, denoted by $V(G)$, is the number of its vertices and the size of G, denoted by $E(G)$, is the number of its edges.

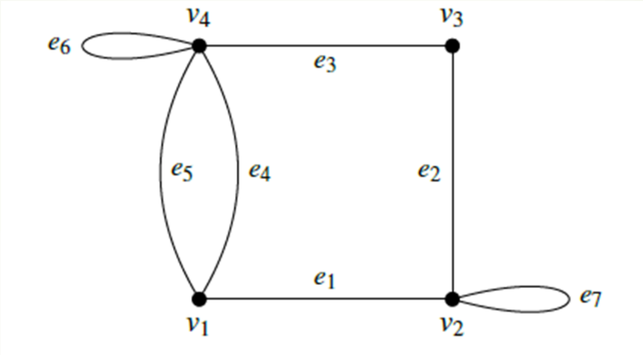
A graph with $p-$ vertices and $q-$ edges is called a $(p, q)-$graph. The $(1,0)$ $-$ graph is called a trivial graph. That is, a trivial graph is a graph with a single vertex. A graph without edges is called an empty graph or a null graph.

Definition: (Finite and Infinite Graphs) A graph with a finite number of vertices as well as a finite number of edges is called a finite graph. Otherwise, it is an infinite graph.

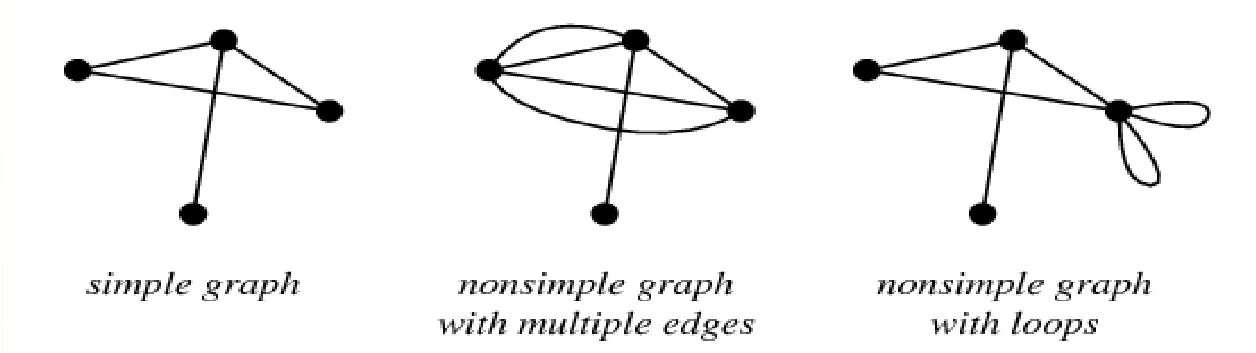
Definition: (Self-loop) An edge of a graph that joins a node to itself is called loop or a self-loop. That is, a loop is an edge uv, where $u = v$.

Definition: (Parallel Edges) The edges connecting the same pair of vertices are called multiple edges or parallel edges.

In Fig. 1., the edges e_6 and e_7 are loops and the edges e_4 and e_5 are parallel edges.



Definition: (Simple Graphs and Multigraphs) A graph G which does not have loops or parallel edges is called a simple graph. A graph which is not simple is generally called a multigraph.



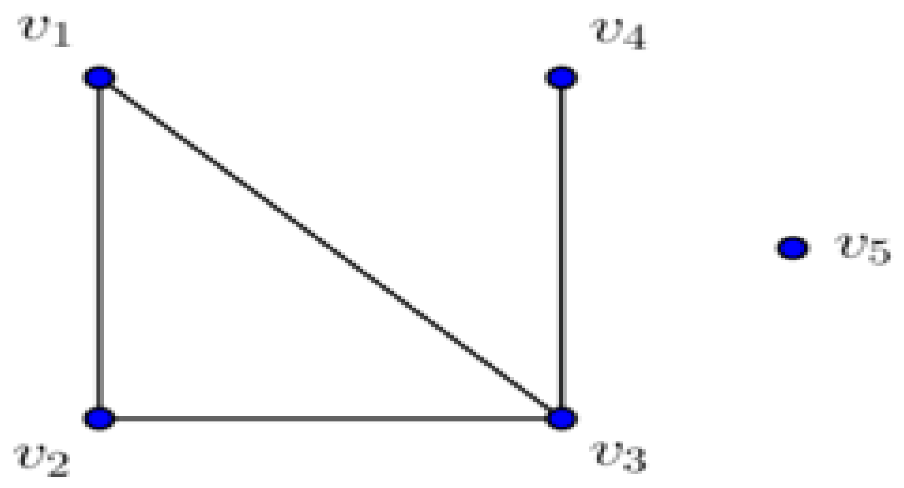
Degrees & Minimum and Maximum Degree

Definition: (Degree of a vertex) The number of edges incident on a vertex v, with self-loops counted twice, is called the degree of the vertex v and is denoted by $deg_G(v)$ or simply $d(v)$.

Definition: (Isolated vertex) A vertex having no incident edge is called an isolated vertex. In other words, isolated vertices are those with zero degree.

Definition: (Pendant vertex) A vertex of degree 1, is called a pendent vertex or an end vertex.

Definition: (Internal vertex) A vertex, which is neither a pendent vertex nor an isolated vertex, is called an internal vertex.



The degrees of the vertices of the above graph are:

$$\deg(v_1) = 2$$
$$\deg(v_2) = 2$$
$$\deg(v_3) = 3$$
$$\deg(v_4) = 1$$
$$\deg(v_5) = 0$$

Internal vertex

Pendant vertex

Isolated vertex

Definition: (Minimum and Maximum Degree of a Graph) The maximum degree of a graph G, denoted by $\Delta(G)$, is defined to be $\Delta(G) = max\,\{d(v){:}\ v \in V(G)\}$. Similarly, the minimum degree of a graph G, denoted by $\delta(G)$, is defined to be $\delta(G) = min\ \{d(v) : v \in V(G)\}$. Note that for any vertex v in G, we have $\delta(G) \leq d(v) \leq \Delta(G)$.

The following theorem (first theorem on graph theory) is a relation between the sum of degrees of vertices in a graph G and the size of G.

Theorem: In a graph G, the sum of the degrees of the vertices is equal to twice the number of edges. That is, $\sum_{v \epsilon V(G)} d(v) = 2E$.

The following two theorems are immediate consequences of the above theorem.

Theorem: For any graph G, $\delta(G) \leq \frac{2|E|}{|V|} \leq \Delta(G)$.

Theorem: For any graph G, the number of odd degree vertices is always even.

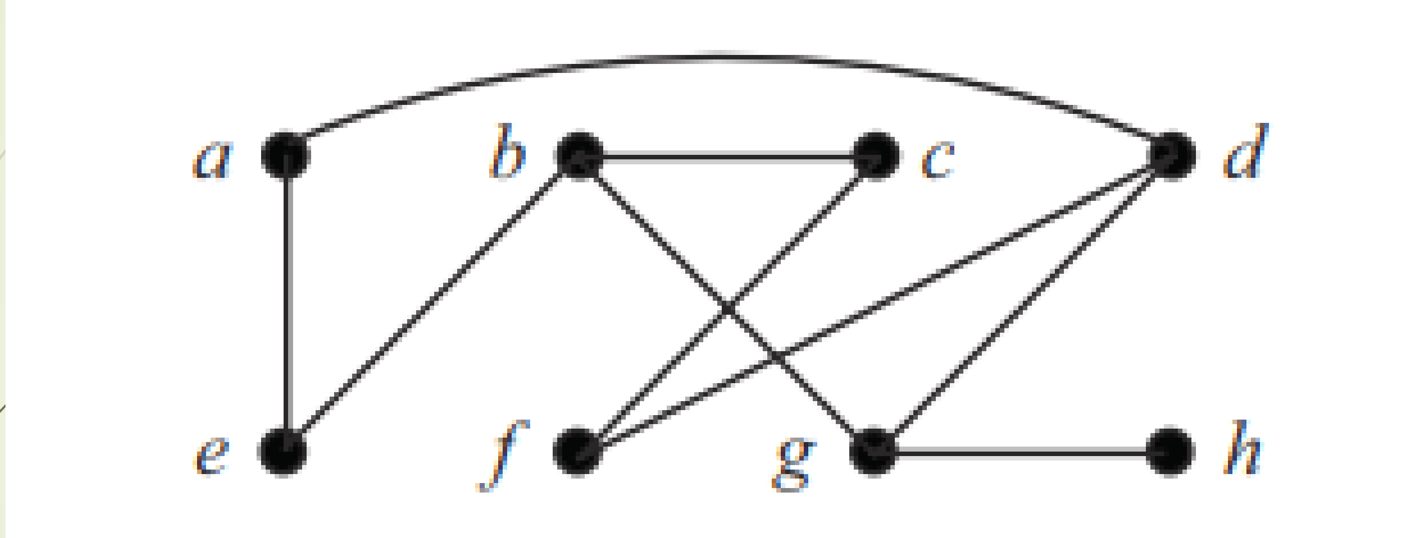
Theorem: $\sum_{v \epsilon V(G)} d(v) = 2E$.

$\sum_{v \epsilon V(G)} d(v) = 18\,, 2E = 18$.

Theorem: $\delta(G) \leq \frac{2|E|}{|V|} \leq \Delta(G)$.

$|V| = 8\ ,|E| = 9\,, 2|E|/|V| = 18/8 = 2.25$

$1 \leq 2.25 \leq 3$



3, 3, 3, 2, 2, 2, 2, 1

$\Delta(G) = 3,\ \delta(G) = 1$

Theorem: For any graph G, the number of odd degree vertices is always even.

$\{\,b, d\,, g\,, h\}$

Thank You

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