جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات / الفصل الاول

Graph Theory

Introduction to Graphs (2)

م. د. امين شامان امين

UNIVERSITY OF ANBAR

Lecture (2)

Graph Theory
Introduction to Graphs (2)

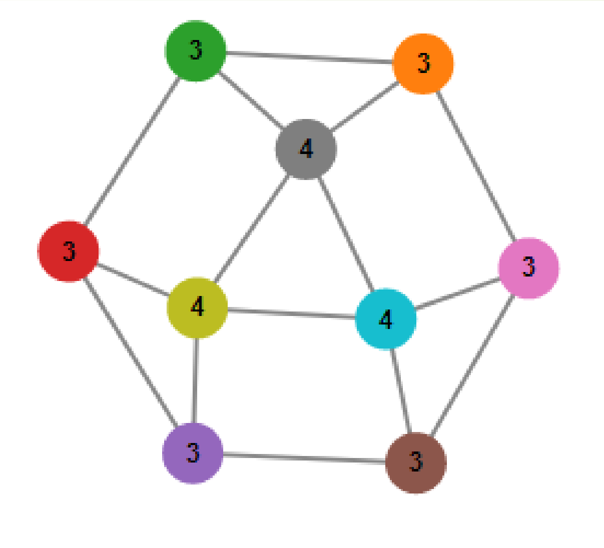
Dr. Ameen Sh. Ameen
Dept. of Applied Mathematics.
College of Science \ University of Anbar.

Outlines

- ✓ Degree Sequences in Graphs.
- ✓ Havel Hakimi Algorithm.
- ✓ Neighborhoods.
- ✓ H. W.

Definition: (Degree Sequence) The degree sequence of a graph of order n is the n −term sequence (usually written in descending order) of the vertex degrees.

$$\text{Degree Sequence} = (4, 4, 4, 3, 3, 3, 3, 3, 3)$$



Definition: (Graphical Sequence) An integer sequence is said to be graphical if it is the degree sequence of some graphs.

Example 1: Is the sequence $S = (9,9,8,7,7,6,6,5,5)$ graphical? Justify your answer.

Solution: The sequence $S = (a_i)$ is graphical if every element of S is the degree of some vertex in a graph. For any graph, we know that $\sum_{v\epsilon V(G)} d(v) = 2E$, an even integer. Here, $\sum a_i = 62$, an even number. But note that the maximum degree that a vertex can attain in a graph of order n is $n - 1$. If S were graphical, the corresponding graph would have been a graph on 9 vertices and have $\Delta(G) = 9$. Therefore, the given sequence is not graphical.

Example 2: Is the sequence $S =(9,8,7,6,6,5,5,4,3,3,2,2)$ graphical? Justify your answer.

Solution: The sequence $S = (a_i)$ is graphical if every element of S is the degree of some vertex in a graph. For any graph, we know that $\sum_{v\epsilon V(G)} d(v) = 2E$, an even integer. Here, we have $\sum a_i = 60$, an even number. Also, note that the all elements in the sequence are less than the number of elements in that sequence. Therefore, the given sequence is graphical.

Example 3: Is the sequence $S = (5,4,3,3,2,2,2,1,1,1,1)$ graphical? Justify your answer.

Solution: The sequence $S = (a_i)$ is graphical if every element of S is the degree of some vertex in a graph. For any graph, we know that $\sum_{v \epsilon V(G)} d(v) = 2E$, an even integer. Here, $\sum a_i = 25$, not an even number. Therefore, the given sequence is not graphical.

Havel Hakimi Theorem: The non-negative integer sequence $D = [d_i]_1^n$ is graphic if and only if $\acute{D}$ is graphic, where $\acute{D}$ is the sequence (having $n-1$ elements) obtained from D by deleting its largest element Δ and subtracting 1 from its Δ next largest elements.

Havel Hakimi Algorithm (HHA)

The Havel Hakimi algorithm gives a systematic approach to answer the question of determining whether it is possible to construct a simple graph from a given degree sequence.

Take as input a degree sequence S and determine if that sequence is graphical
That is, can we produce a graph with that degree sequence?

Assume the degree sequence is S

$$S = d_1, d_2, d_3, \cdots, d_n$$

$$d_i \geq d_{i+1}$$

1. If any $d_i \geq n$ then fail
2. If there is an odd number of odd degrees then fail
3. If there is a $d_i < 0$ then fail
4. If all $d_i = 0$ then report success
5. Reorder S into non-increasing order
6. Let $k = d_1$
7. Remove d_1 from S.
8. Subtract 1 from the first k terms remaining of the new sequence
9. Go to step 3 above

Example 1:

Consider the degree sequence: S = 7, 5, 5, 4, 4, 4, 4, 3

Where n = 8 (no. of vertices)

Step 1. Degree of all vertices is less than n (no.of vertices)

Step 2. Odd number vertices are four.

Step 3. There is no degree less than zero.

Step 4. Remove '7' from the sequence and subtract 1 from the remaining new sequence and arrange again in non-increasing order to get

S = 4, 4, 3, 3, 3, 3, 2

Step 5. Now remove the first '4 ' from the sequence and subtract 1 from the remaining new sequence to get:

S = 3, 2, 2, 2, 3, 2

rearrange in non-increasing order to get:

S = 3, 3, 2, 2, 2, 2

Repeating the above step we get following degree sequences:

S = 2, 2, 2, 1, 1

S = 1, 1, 1, 1

S = 1, 1, 0

S = 0, 0

Step 6. Since all the deg remaining in the sequence is zero, the given sequence is graphical (or in other words, it is possible to construct a simple graph from the given degree sequence).

Example 2:

8

S = 4, 3, 3, 3, 1

Where n = 5 (no. of vertices)

Step 1. Degree of all vertices is less than n (no.of vertices)

Step 2. Odd number vertices are four.

Step 3. There is no degree less than zero.

Step 4. Remove '4' from the sequence and subtracting 1 from the remaining new sequence and arrange again in non-increasing order we get

S = 2,2,2,0

Step 5. Again Remove '2 ' from the sequence and subtracting 1 from the remaining new sequence and arrange in non-increasing order we get

S= 1,1,0

Repeating the above step

S= 0,0

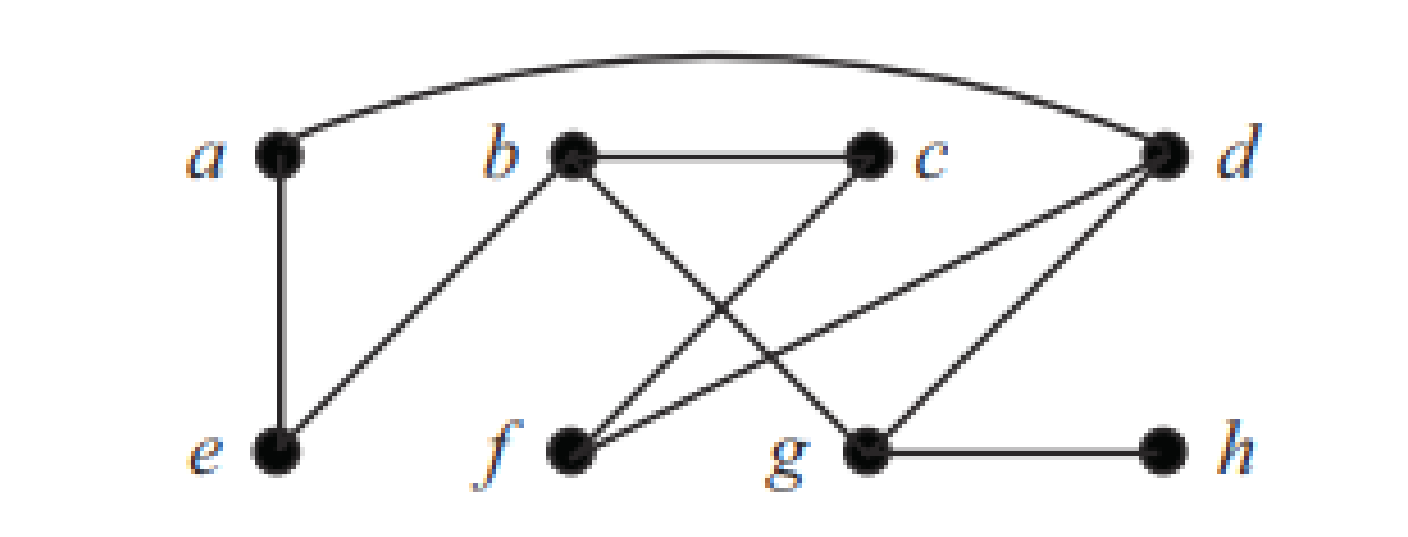
Step 6. Since all the deg remaining in the sequence is zero, the given sequence is graphical.

Neighbourhoods

Definition: (Neighbourhood of a Vertex) The neighbourhood (or open neighbourhood) of a vertex v, denoted by $N(v)$, is the set of vertices adjacent to v. That is, $N(v) = \{x \in V : vx \in E\}$. The closed neighbourhood of a vertex v, denoted by $N[v]$, is simply the set $N(v) \cup \{v\}$.

Then, for any vertex v in a graph G, we have $d(v) = |N(v)|$. A special case is a loop that connects a vertex to itself; if such an edge exists, the vertex is said to belong to its own neighbourhood.

Given a set S of vertices, we define the neighbourhood of S, denoted by $N(S)$, to be the union of the neighbourhoods of the vertices in S. Similarly, the closed neighbourhood of S, denoted by $N[S]$, is defined to be $S \cup N(S)$.

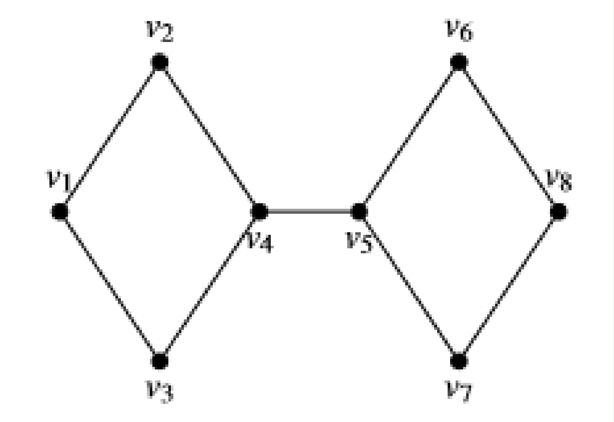


$N(f) = \{c, d\}$ & $N[f] = \{c, d, f\}$

Let $S = \{a, b, c\}$, $then\ N(S) = \{e, d\} \cup \{c, e, g\} \cup \{b, f\}$

H. W.

1) Verify whether the integer sequences (7,6,5,4,3,3,2) and (6,6,5,4,3,3,1) are graphical. (Hint: Use Havel Hakimi Algorithm)
2) For the following graph G , find:
$\delta(G)$, $\Delta(G)$, $N[v_5]$ and degree sequence .



Thank You

References:

1. S. Arumugam and S. Ramachandran, (2015), Invitation to graph theory, Scitech Publ., Kolkata, India.
2. G. S. Singh, (2013). Graph theory, Prentice Hall of India, New Delhi.
3. R. Balakrishnan and K. Ranganathan, (2012). A textbook of graph theory, Springer, New York.
4. J.A. Bondy and U.S.R Murty, (2008). Graph theory, Springer.
5. G. Agnarsson and R. Greenlaw, (2007). Graph theory: Modeling, applications & algorithms, Pearson Education, New Delhi.
6. G. Chartrand and P. Zhang, (2005). Introduction to graph theory, McGraw-Hill Inc.
7. G. Sethuraman, R. Balakrishnan, and R.J. Wilson, (2004). Graph theory and its applications, Narosa Pub. House, New Delhi.
8. D.B. West, (2001). Introduction to graph theory, Pearson Education Inc., Delhi.
9. V.K. Balakrishnan, (1997). Graph theory, McGrawhill, New York.
10. G. Chartrand and L. Lesniak, (1996). Graphs and digraphs, CRC Press.
11. J.A. Bondy and U.S.R Murty, (1976). Graph theory with applications, North-Holland, New York.