جامعة الانبار
كلية العلوم
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نظرية البيانات / الفصل الاول

Isomorphic Graphs

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Lecture (3 Part 2)

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23 December 2020

Isomorphic Graphs

Graph isomorphism is a phenomenon of existing the same graph in more than one form. Such graphs are called an isomorphic graphs .

Definition: An isomorphism of two graphs G and H is a bijective function (one-to-one and onto) $f : V(G) \to V(H)$ such that any two vertices u and v of G are adjacent in G if and only if $f(u)$ and $f(v)$ are adjacent in H.

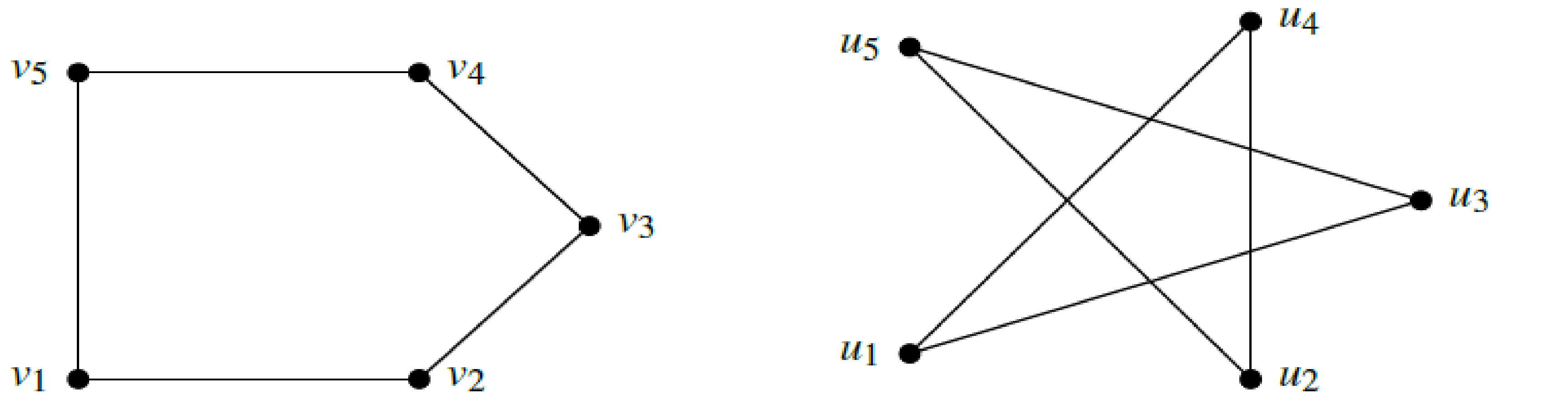
That is, two graphs G and H are said to be isomorphic if

$i)$ $|V(G)| = |V(H)|$ $ii)$ $|E(G)| = |E(H)|$

$iii)$ If $v_i v_j \in E(G)$ then $f(v_i)\, f(v_j) \in E(H)$.

This bijection is commonly described as edge-preserving bijection.

If an isomorphism exists between two graphs, then the graphs are called isomorphic graphs and denoted as $G \cong H$.

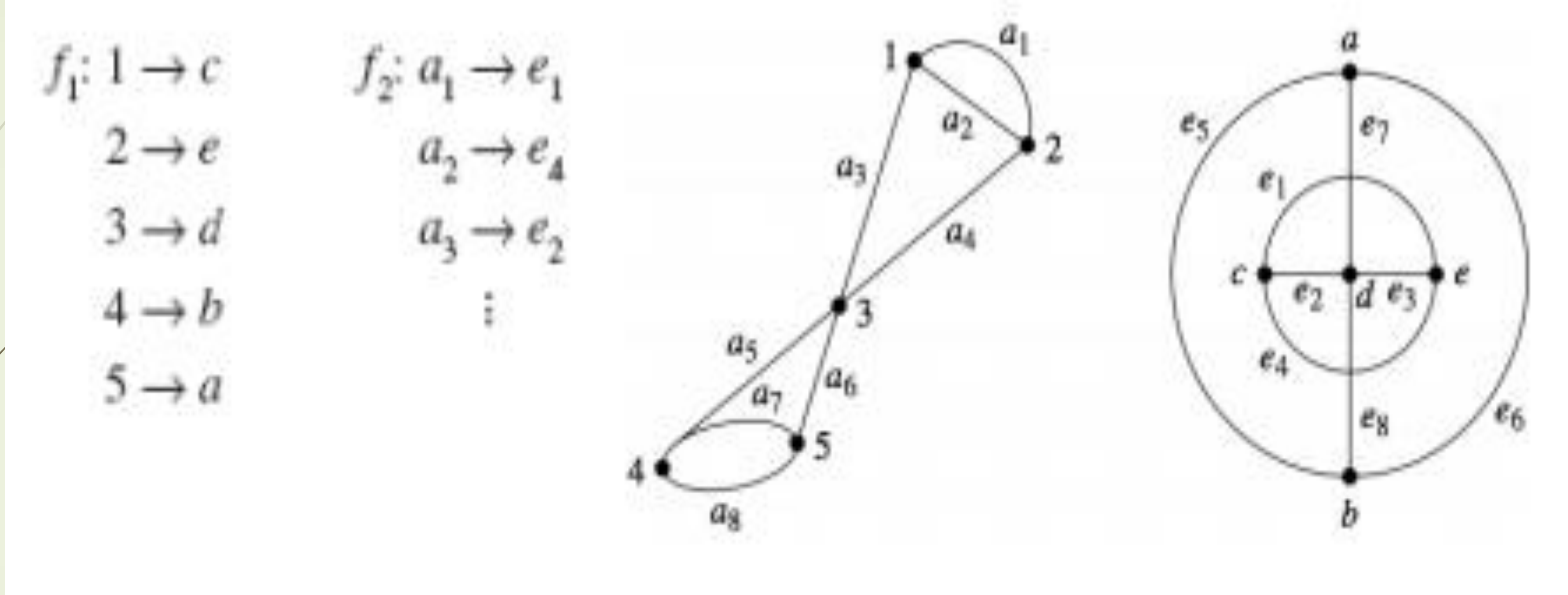


(a) G_1.　　(b) G_2.

$i)\ |V(G)| = |V(H)|$　　$ii)\ |E(G)| = |E(H)|$

$iii)$ If $v_i v_j \in E(G)$ then $f(v_i)\, f(v_j) \in E(H)$.

In the above graphs, we can define an isomorphism f from G_1 to G_2 such that $f\,(v_1) = u_1, f\,(v_2) = u_3, f\,(v_3) = u_5\,, f\,(v_4) = u_2$ and $f\,(v_5) = u_4$. Hence, these two graphs are isomorphic $G_1 \cong G_2$.



Hence, these two graphs are isomorphic.

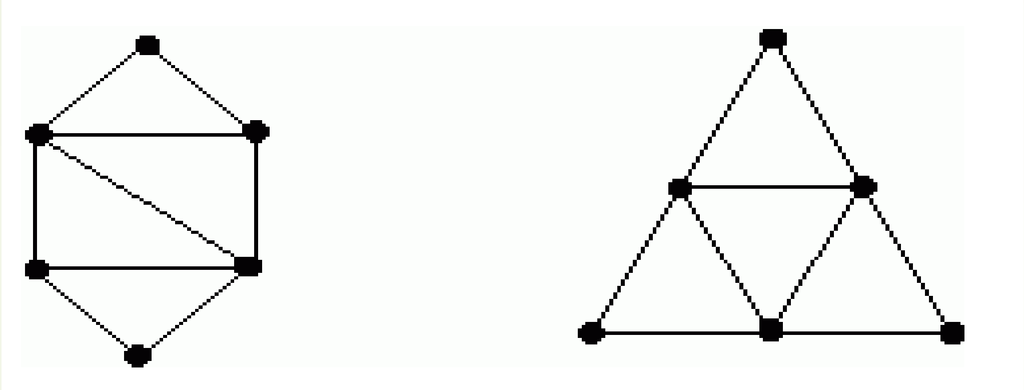
Note: It is easier to check non-isomorphism than isomorphism. If any of these following conditions not occurs, then two graphs are non-isomorphic.

Here's a partial list of ways you can show that two graphs are not isomorphic if the way not satisfy.

1) Two isomorphic graphs must have the same number of vertices.
2) Two isomorphic graphs must have the same number of edges.
3) Two isomorphic graphs must have the same number of vertices of degree n.
4) Two isomorphic graphs have the same number of connected components.
5) Two isomorphic graphs have the same number of cycles of the same length.

Example 1:

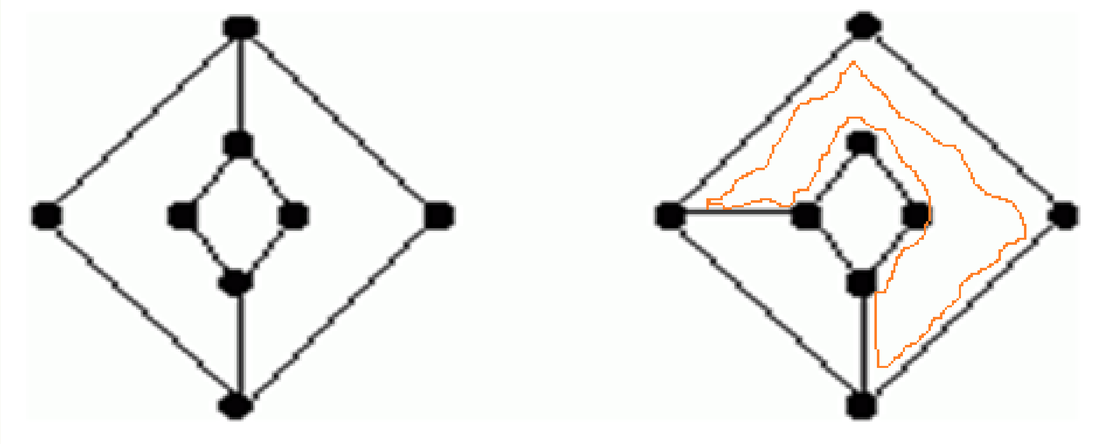
1) Two isomorphic graphs must have the same number of vertices.
2) Two isomorphic graphs must have the same number of edges.
3) Two isomorphic graphs must have the same number of vertices of degree n.
4) Two isomorphic graphs have the same number of connected components.
5) Two isomorphic graphs have the same number of cycles of the same length.



Each of the two graphs has 6 vertices and each of them has 9 edges, they are still not isomorphic. To see this, count the number of vertices of each degree. The graph on the left has 2 vertices of degree 2, while the one on the right has 3 vertices of degree 2. The following two graphs are also not isomorphic.

Example 2:

1) Two isomorphic graphs must have the same number of vertices.
2) Two isomorphic graphs must have the same number of edges.
3) Two isomorphic graphs must have the same number of vertices of degree n.
4) Two isomorphic graphs have the same number of connected components.
5) Two isomorphic graphs have the same number of cycles of the same length.

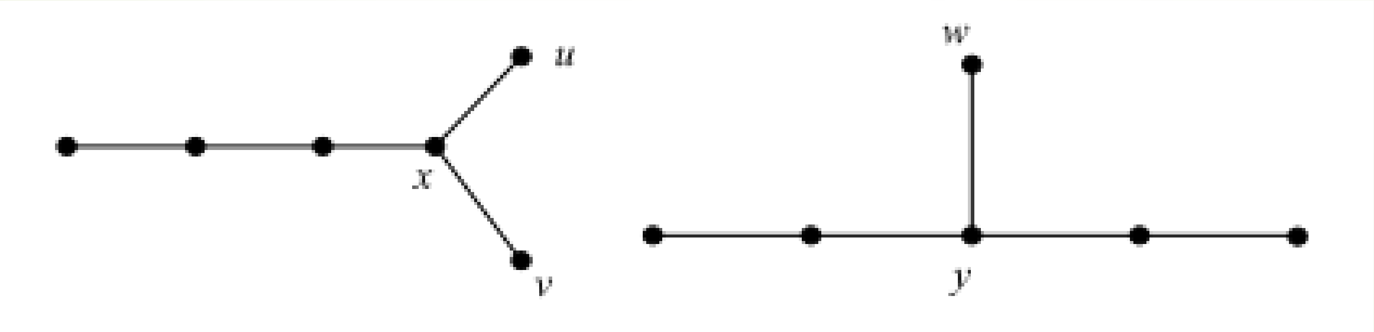


In this example, each graph has 8 vertices and 10 edges. Furthermore, each has 4 vertices of degree 2 and 4 vertices of degree 3. Still, the graphs are not isomorphic. There are many ways to show this, but the easiest is probably to note that the graph on the right has a cycle of length 8 (The cycle is a path that ends at the vertex it began and repeats no edges), while the graph on the left does not.

Example 3:

1) Two isomorphic graphs must have the same number of vertices.
2) Two isomorphic graphs must have the same number of edges.
3) Two isomorphic graphs must have the same number of vertices of degree n.
4) Two isomorphic graphs have the same number of connected components.
5) Two isomorphic graphs have the same number of cycles of the same length.

1, 2, 3 are necessary for the graphs G_1 and G_2 to be isomorphic, but not sufficient to prove that the graphs are isomorphic.



The two graphs have the same number of vertices and edges, also have the same number of vertices of degree 1, 2 and 3. But there are not isomorphism (Why ???)

Thank You

References:

1. S. Arumugam and S. Ramachandran, (2015), Invitation to graph theory, Scitech Publ., Kolkata, India.
2. G. S. Singh, (2013). Graph theory, Prentice Hall of India, New Delhi.
3. R. Balakrishnan and K. Ranganathan, (2012). A textbook of graph theory, Springer, New York.
4. J.A. Bondy and U.S.R Murty, (2008). Graph theory, Springer.
5. G. Agnarsson and R. Greenlaw, (2007). Graph theory: Modeling, applications & algorithms, Pearson Education, New Delhi.
6. G. Chartrand and P. Zhang, (2005). Introduction to graph theory, McGraw-Hill Inc.
7. G. Sethuraman, R. Balakrishnan, and R.J. Wilson, (2004). Graph theory and its applications, Narosa Pub. House, New Delhi.
8. D.B. West, (2001). Introduction to graph theory, Pearson Education Inc., Delhi.
9. V.K. Balakrishnan, (1997). Graph theory, McGrawhill, New York.
10. G. Chartrand and L. Lesniak, (1996). Graphs and digraphs, CRC Press.
11. J.A. Bondy and U.S.R Murty, (1976). Graph theory with applications, North-Holland, New York.