

جامعة الانبار

كلية العلوم

قسم الرياضيات التطبيقية

نظرية البيانات / الفصل الاول

Traversability in Graphs

Eulerian Graphs

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Lecture (6)

Traversability in Graphs Eulerian Graphs

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Eulerian Graphs

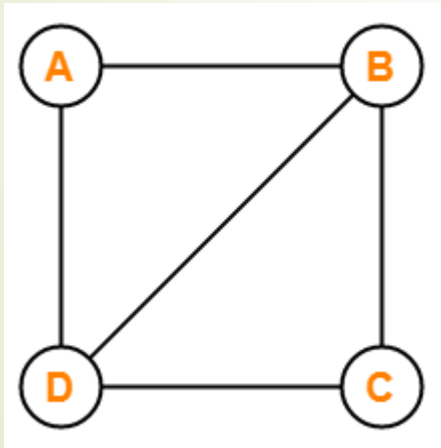
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A trail is a walk that does not pass over the same edge twice. A trail might visit the same vertex twice, but only if it comes and goes from a different edge each time.

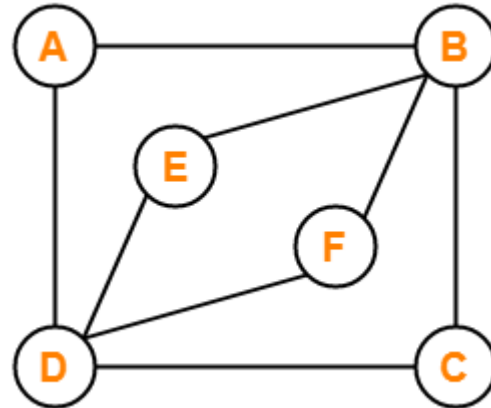
Definition: An Eulerian trail is a trail that visits every edge of the graph once and only once. It can end on a vertex different from the one on which it began. A graph of this kind is said to be traversable (semi-Eulerian) graph.

Definition: An Eulerian circuit is an Eulerian trail that is a circuit. That is, it begins and ends on the same vertex (closed Eulerian trail).

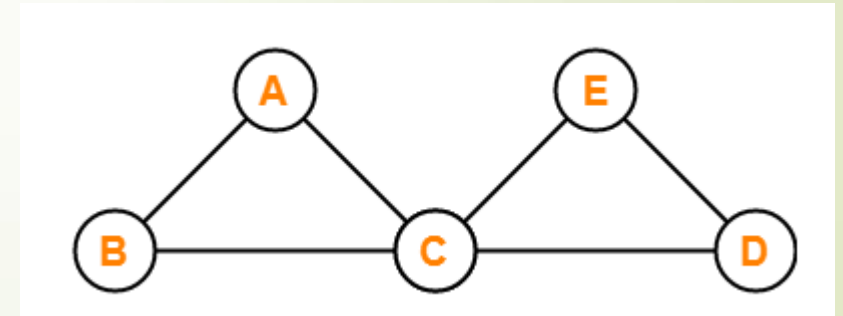
Definition: A graph is called Eulerian when it contains an Eulerian circuit.



Eulerian trail
BCDBAD



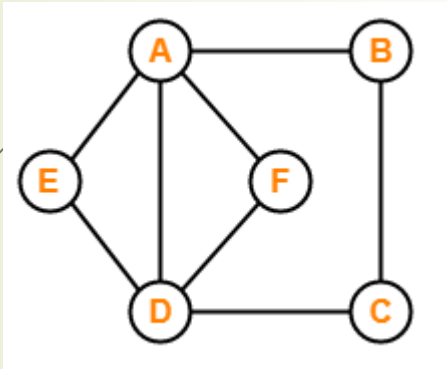
Eulerian circuit
ABCDFBEDA



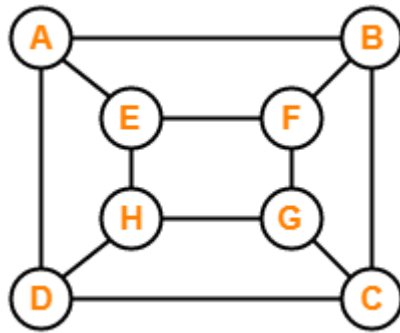
Eulerian graph
Eulerian circuit: BCDECAB

The following theorem characterizes the class of Eulerian graphs:

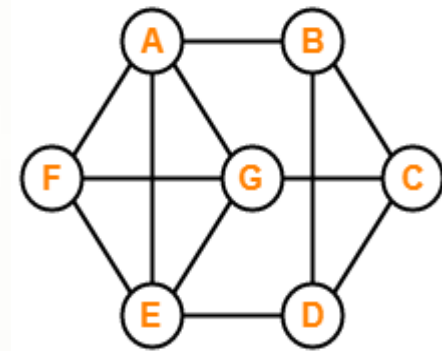
Theorem 1: (Euler Theorem) A connected graph G is Eulerian if and only if every vertex in G is of even degree.



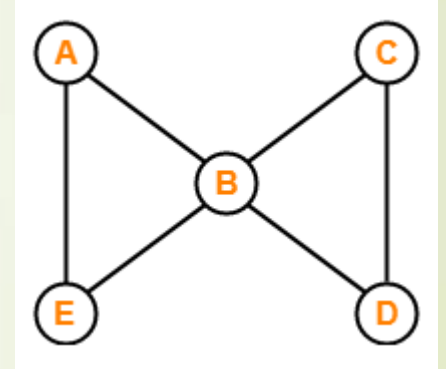
Eulerian graph
ABCDFADEA



Not Eulerian graph



Not Eulerian graph

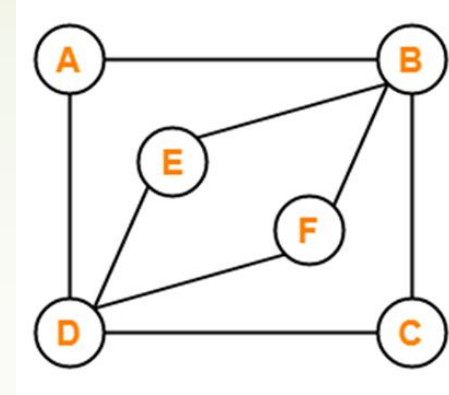


Eulerian graph
ABDCBEA

Note: i) Cycles C_n are Eulerian graphs.

ii) Paths P_n have no circuits at all $\Rightarrow P_n$ are not Eulerian graphs.

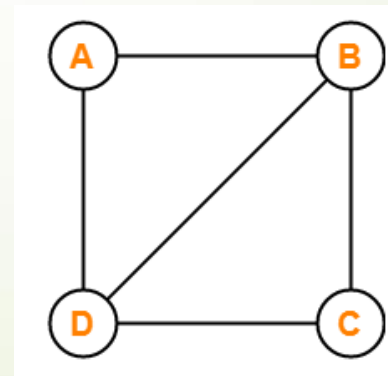
Theorem 2: A connected graph is Eulerian if and only if each of its edges lies on an odd number of cycles.



Theorem 3: A connected graph G is Eulerian if and only if it can be decomposed into edge-disjoint cycles.

Corollary: A graph is Eulerian if and only if it has an odd number of cycle decompositions.

Theorem 4: A connected graph G is semi-Eulerian if and only if it has exactly two odd degree vertices.



Important Notes

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Note 1: To check whether any graph is an Euler graph or not, any one of the following two ways may be used:

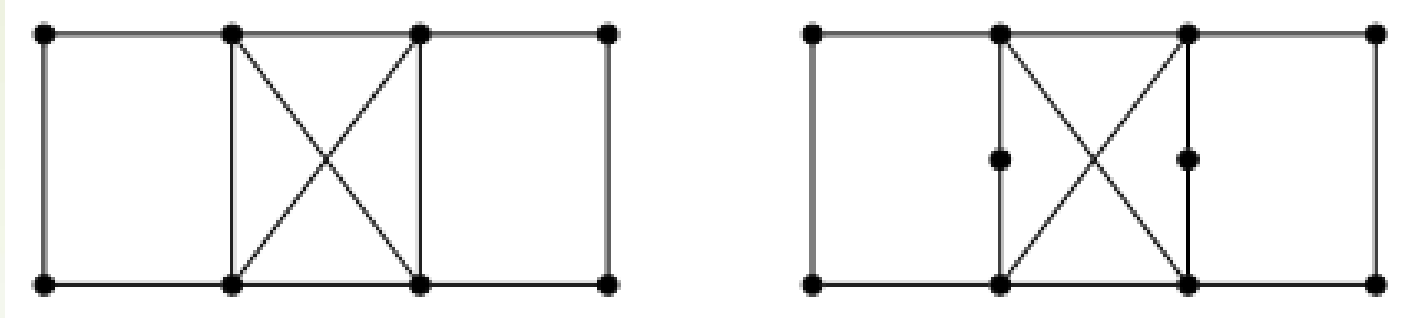
- If the graph is connected and contains an Euler circuit, then it is an Euler graph.
- If all the vertices of the graph are of even degree, then it is an Euler graph.

Note 2: To check whether any graph contains an Euler circuit or not,

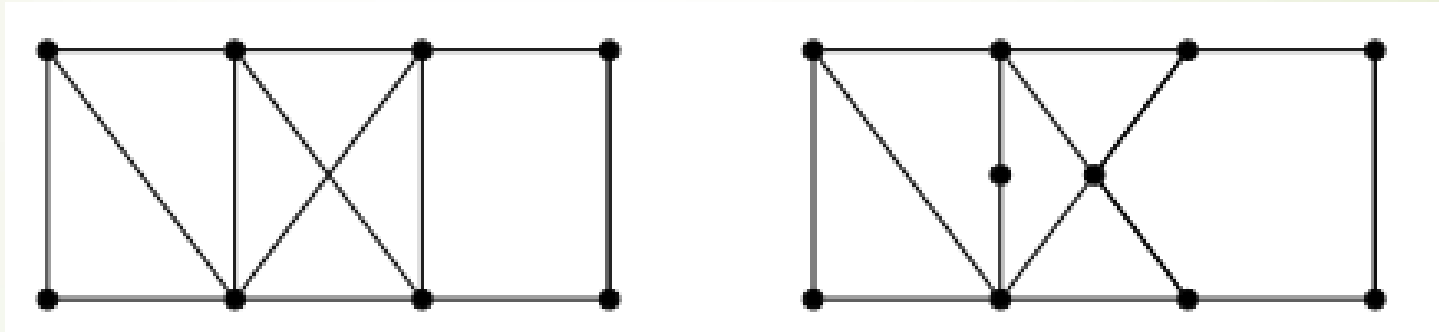
- Just make sure that all its vertices are of even degree.
- If all its vertices are of even degree, then graph contains an Euler circuit otherwise not.

Note 3: To check whether any graph is a semi-Euler graph or not,

- Just make sure that it is connected and contains an Euler trail.
- If the graph is connected and contains an Euler trail, then graph is a semi-Euler graph otherwise not.



Eulerian Graphs



Not Eulerian Graphs

Thank You

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