Energy Dissipation below Spillways:

Water flowing over a spillway has a very high kinetic energy because of the conversion of the entire potential energy to the kinetic energy. If the water flowing with such a high velocity is discharged directly into the channel downstream, serious scour of the channel bed may occur. If the scour is not properly controlled, it may extend backward and may endanger the spillway and the dam. In order to protect the channel bed against scour, the kinetic energy of the water should be dissipated before it is discharged into the *d/s* channel. The energy-dissipating devices can be broadly classified into two types:
1. Devices using a hydraulic jump for the dissipation of energy.
2. Devices using a bucket for the dissipation of energy.
The choice of the energy-dissipating device at a particular spillway is governed by the tail water depth and the characteristics of the hydraulic jump, if formed, at the toe. If the tail water depth at the site is not approximately equal to that required for a perfect hydraulic jump, a bucket-type energy dissipating device is usually provided. The characteristics of the hydraulic jump are discussed in the following section. The sequent depth (conjugate depth or post-jump depth) y_2 is determined for different values of the discharge, and a jump height curve (*JHC*) is plotted between the conjugate depth y_2 as ordinate and the discharge (Q) as abscissa. The tail water rating curve (*TWRC*) at the spillway site is determined by stream gauging, (Q) as abscissa. As discussed later, the correct choice of the energy-dissipating device is made after comparing the relative positions of the jump height curve (*JHC*) and the tail water rating curve (*TWRC*). For the design of spillways, the discharge per unit length (q) is usually taken as abscissa instead of Q. Different types of stilling basins have been developed which are quite effective for the formation of stable hydraulic jumps and for confining the hydraulic jump. Stilling basins are commonly used for spillways and other hydraulic structures, such as weir and barrages. In a stilling basin, chute blocks, basin blocks (baffle blocks) and an end sill are usually provided. Chute blocks are triangular blocks installed at the upstream end of the basin. An end sill is constructed at the downstream end of the basin. It may be a solid sill or a dentate sill. Baffle blocks are installed on the basin floor between the chute blocks and the end sill. These are also known as baffle blocks or baffle piers.

Characteristics of a Hydraulic Jump:

Hydraulic jump is a sudden and turbulent rise of water which occurs in an open channel when the flow changes from the supercritical flow state to the subcritical state. It is accompanied by the formation of extremely turbulent rollers and considerable dissipation of energy. Thus a hydraulic jump is a very effective means of dissipation of energy below spillways.

Types of jumps

The type of jump and its characteristics depend mainly upon the Froude number of the incoming flow or the initial Froude number (F_1), given by

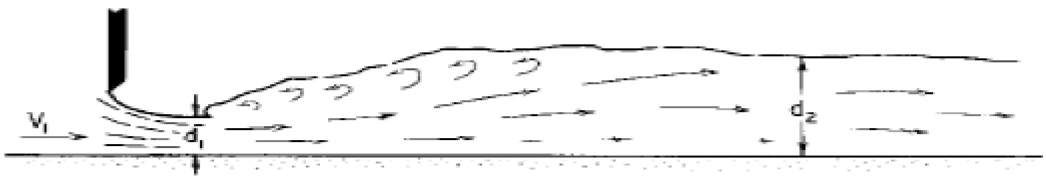
$$F_1 = V_1/\sqrt{gy_1}$$

Where V_1 is the mean velocity of flow before the hydraulic jump, g is the acceleration due to gravity and y_1 is the pre-jump depth (or the initial depth of flow).
For the formation of a hydraulic jump, the initial Froude number F_1 should be greater than unity. Different types of hydraulic jump are as follows:
1. Undular Jump An undular jump is formed when F_1 = 1.0 to 1.70. In an undular jump, the water surface shows some undulation. The energy dissipation is about 5%.
2. Weak Jump When F_1 = 1.70 to 2.50, a weak hydraulic jump occurs. In this case, a series of small rollers develops on the surface of the jump, but the downstream water surface remains quite smooth. The velocity is uniform throughout. The energy dissipation is about 20%.

3. Oscillating Jump An oscillating hydraulic jump occurs when F_1 = 2.50 to 4.50. There is an oscillating jet entering the jump bottom to surface and back again without any periodicity. The energy dissipation is between 20 to 40 %.

4. Steady Jump A steady jump occurs when F_1 = 4.50 to 9.0. The jump is quite stable and balanced. This jump is not much sensitive to variations in the tail water depth. The steady jump has very good performance, and most of the hydraulic structures utilize this type of jump for the dissipation of energy. The energy dissipation is between 45 to 70 %.

5. Strong Jump A strong jump occurs when F_1 > 9.0. The jump action is quite rough but effective. It causes a rough water surface with strong surface waves downstream. The energy dissipation is between 70 to 85 %. Because of rough action, a strong jump is avoided in spillways, as far as possible.



F1 BETWEEN 1.7 and 2.5
FORM A-PREJUMP STAGE

F1 BETWEEN 2.5 and 4.5
FORM B-TRANSITION STAGE

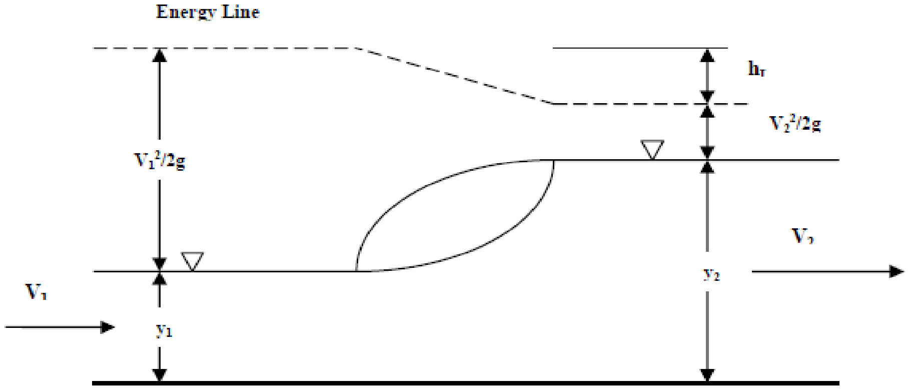
F1 BETWEEN 4.5 and 9.0
FORM C-RANGE OF WELL-BALANCED JUMPS

F1 GREATER THAN 9.0
FORM D-EFFECTIVE JUMP BUT ROUGH SURFACE DOWNSTREAM

Mathematical Derivation of Hydraulic Jump

In the mathematical derivation of hydraulic jump, the following assumptions are made,

- Rectangular channel with horizontal bottom slope,
- Before and after the hydraulic jump, velocity distributions are uniform and the pressure distribution over the cross sections are hydrostatic,
- Friction losses are neglected.



Hydraulic Jump

Momentum equation will be applied to the control volume taken at the hydraulic jump section for a unit width perpendicular to the control volume,

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} = \rho Q V_2 - \rho Q V_1$$

$$q = y_1 V_1 = y_2 V_2 \Rightarrow V_1 = \frac{q}{y_1}, V_2 = \frac{q}{y_2}, \gamma = \rho g$$

$$\frac{\rho g}{2}(y_1^2 - y_2^2) = \rho(\frac{q^2}{y_2^2} y_2 - \frac{q^2}{y_1^2} y_1)$$

$$\frac{g}{2}(y_1 - y_2)(y_1 + y_2) = q^2(\frac{1}{y_2} - \frac{1}{y_1}) = \frac{q^2(y_1 - y_2)}{y_1 y_2}$$

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g} = \frac{2y_1^2 V_1^2}{g}$$

$$y_1^2 y_2 (1 + \frac{y_2}{y_1}) = \frac{2y_1^2 V_1^2}{g}$$

Multiplying both side of the above equation with ($1/y_1^3$) yields,

$$[y_1^2 y_2 (1 + \frac{y_2}{y_1}) = \frac{2y_1^2 V_1^2}{g}] * \frac{1}{y_1^3}$$

$$\frac{y_2}{y_1}(1 + \frac{y_2}{y_1}) = 2\frac{V_1^2}{g y_1}$$

Since for rectangular channels,

$$Fr = \frac{V}{\sqrt{gy}}$$

$$(\frac{y_2}{y_1})^2 + \frac{y_2}{y_1} - 2Fr_1^2 = 0$$

Solution of this equation and taking the positive sign of the square root gives,

$$\frac{y_2}{y_1} = \frac{1}{2}(\sqrt{1 + 8Fr_1^2} - 1)$$

The ratio of flow depths after and before the hydraulic jump (y_2/y_1) is a function of the Froude number of the subcritical flow before hydraulic jump.

Hydraulic Jump as an Energy Dissipater

If we write the difference of the specific energies before after the hydraulic jump,

$$\Delta E = E_1 - E_2 = (y_1 + \frac{V_1^2}{2g}) - (y_2 + \frac{V_2^2}{2g})$$

$$\Delta E = (y_1 - y_2) + (\frac{V_1^2}{2g} - \frac{V_2^2}{2g})$$

Since,

$$q = Vy \Rightarrow V_1 = \frac{q}{y_1}, V_2 = \frac{q}{y_2}$$

$$\Delta E = (y_1 - y_2) + \frac{q^2}{2g}(\frac{1}{y_1^2} - \frac{1}{y_2^2})$$

It has been derived that,

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g}$$

$$\frac{q^2}{2g} = \frac{1}{4} y_1 y_2 (y_1 + y_2)$$

Substituting above equation in equation of ΔE,

$$\Delta E = (y_1 - y_2) + \frac{1}{4} y_1 y_2 (y_1 + y_2)(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2})$$

$$\Delta E = (y_1 - y_2) + \frac{1}{4} \frac{(y_2 - y_1)(y_1 + y_2)^2}{y_1 y_2}$$

$$\Delta E = \frac{4 y_1 y_2 (y_1 - y_2) + (y_2 - y_1)(y_1 + y_2)^2}{4 y_1 y_2}$$

$$\Delta E = \frac{(y_2 - y_1)[-4 y_1 y_2 + (y_1 + y_2)^2]}{4 y_1 y_2}$$

$$\Delta E = \frac{(y_2 - y_1)(y_2 - y_1)^2}{4 y_1 y_2}$$

The analytical equation of the energy dissipated with the hydraulic jump is,

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

The power lost by hydraulic jump can be calculated by,

$$P = \gamma_w Q \Delta E$$

Where:

γ_w = Specific weight of water = 9.81 kN/m^3

Q = Discharge (m^3/sec)

ΔE = Energy dissipated as head (m)

P=Power dissipated (kW)

Length of Hydraulic Jump:

Some empirical equations were given to calculate the length of hydraulic jump as,

$L = 5.2 y_2$	Safranez equation
$L = 5(y_2 - y_1)$	Bakhmetef equation
$L = 6(y_2 - y_1)$	Smetana equation
$L = 5.6 y_2$	Page equation

Example: If the Froude number at the drop of a hydraulic jump pool is 6 and the water depth is 0.50 m, find out the length of the hydraulic jump. Calculate the power dissipated with the hydraulic jump if the discharge on the spillway is 1600 m³/sec.

Sol:

$$\frac{y_2}{y_1} = \frac{1}{2}(\sqrt{1+8Fr_1^2} - 1)$$

$$\frac{y_2}{y_1} = \frac{1}{2}(\sqrt{1+8*6^2} - 1) = 8$$

$$y_2 = y_1 * 8 \Rightarrow y_2 = 0.5*8 = 4$$

The length of hydraulic jump by different equations,

L = 5.2y_2 =5.2*4=20.8m Safranez equation
L = 5(y_2 – y_1) =5(4-0.5)=17.5m Bakhmetef equation
L = 6(y_2 – y_1)=6(4-0.5)=21m Smetana equation
L = 5.6y_2 =5.6*4=22.4m Page equation

It is preferred to be on the safe side with the hydraulic structures. Therefore, the longest result will be chosen. The length of the hydraulic jump will be taken as L = 22.4 m for design purposes.

Energy dissipated as head,

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$\Delta E = \frac{(4-0.5)^3}{4*4*0.5} = 5.36m$$

The power dissipated with the hydraulic jump,

$$P = \gamma_w Q \Delta E$$

$$P = 9.81*1600*5.36$$

$$P = 84131kW$$