2.4 Addition of a System of Concurrent Coplanar Forces

When a force is resolved into two components along the x and y axes, the components are then called *rectangular components*. For analytical work we can represent these components in one of two ways, using either scalar notation or Cartesian vector notation.

Scalar Notation: The rectangular components of force **F** shown in Fig.2–15*a* are found using the parallelogram law, so that: $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Because these components form a right triangle, they can be determined from,

$$F_x = F \cos \theta \qquad \text{and} \qquad F_y = F \sin \theta$$

Instead of using the angle θ, however, the direction of **F** can also be defined using a small "*slope*" triangle, as in the example shown in Fig. 2–15 *b* . Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives:

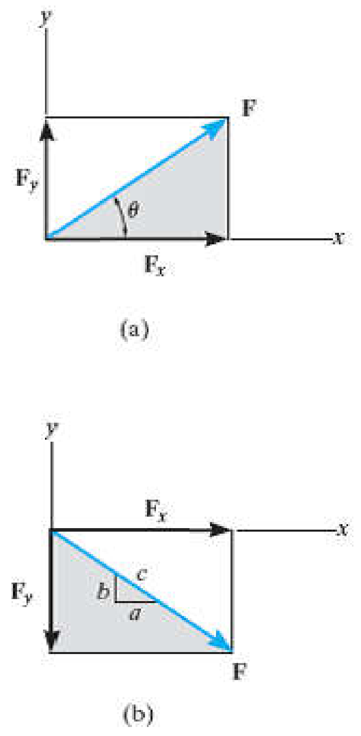


Fig. 2–15

$$\frac{F_x}{F} = \frac{a}{c} \quad or \quad F_x = F\left(\frac{a}{c}\right)$$

and

$$\frac{F_y}{F} = \frac{b}{c} \quad or \quad F_y = -F\left(\frac{b}{c}\right)$$

Here the y component is a *negative scalar* since F_y is directed along the negative y axis.

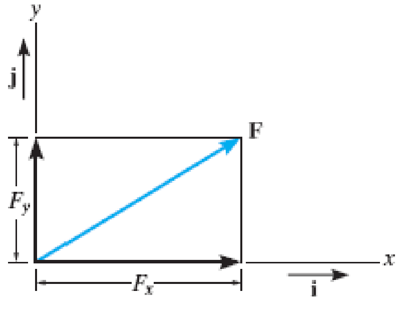


Fig. 2–16

Cartesian Vector Notation: It is also possible to represent the ***x*** and ***y*** components of a force in terms of Cartesian unit vectors **i** and **j** . They can be used to designate the *directions* of the ***x*** and ***y*** axes, respectively, Fig. 2–16.

Since the *magnitude* of each component of **F** is *always a positive quantity,* which is represented by the (positive) scalars F_x and F_y , then we can express **F** as a *Cartesian vector* ,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

Concurrent Coplanar Force Resultants: We can use the method just described to determine the resultant of several *Concurrent coplanar forces*. To do this, each force is first resolved into its *x* and *y* components, and then the respective components are added using *scalar algebra* since they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law.

For example, consider the three concurrent forces in Fig. 2–17 *a* , which have *x* and *y* components shown in Fig. 2–17 *b* . Using *Cartesian vector notation* , each force is first represented as a Cartesian vector, i.e.,

$$F_1 = F_{1x}\,i + F_{1y}\,j$$
$$F_2 = -F_{2x}\,i + F_{2y}\,j$$
$$F_3 = F_{3x}\,i - F_{3y}\,j$$

The vector resultant is therefore,

$$F_R = F_1 + F_2 + F_3$$
$$= F_{1x}\,i + F_{1y}\,j - F_{2x}\,i + F_{2y}\,j + F_{3x}\,i - F_{3y}\,j$$
$$= (F_{1x} - F_{2x} + F_{3x})\,i + (F_{1y} + F_{2y} - F_{3y})\,j$$
$$= (F_{Rx})i + (F_{Ry})j$$

If *scalar notation* is used, then from Fig. 2–17 *b* , we have:

$$(\rightarrow+) \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$(\uparrow+) \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

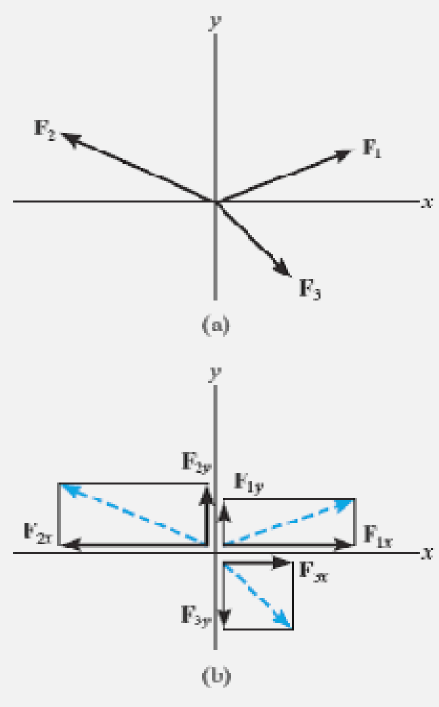


Fig. 2–17

These are the *same* results as the **i** and **j** components of $\mathbf{F}_R$ determined above.

We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the *x* and *y* components of all the forces, i.e.,

$$(F_R)_x = \Sigma F_x \qquad (2\text{–}1)$$
$$(F_R)_y = \Sigma F_y$$

Once these components are determined, they may be sketched along the *x* and *y* axes with their proper sense of direction, and the resultant force can be determined from vector addition, as shown in Fig. 2–17c .

From this sketch, the magnitude of $\mathbf{F}_R$ is then found from the Pythagorean Theorem; that is,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

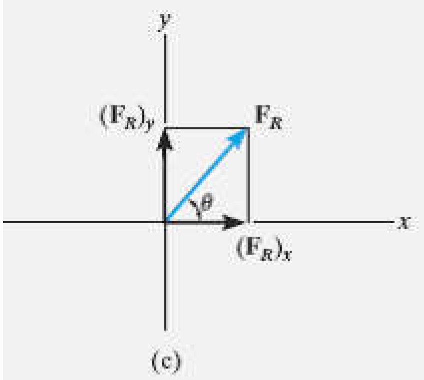


Fig. 2–17 (cont.)

Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

EXAMPLE 2.5

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig. 2–18*a*. Express each force as a Cartesian vector.

SOLUTION

Scalar Notation. By the parallelogram law, $\mathbf{F}_1$ is resolved into x and y components, Fig. 2–18*b*. Since $\mathbf{F}_{1x}$ acts in the $-x$ direction, and $\mathbf{F}_{1y}$ acts in the $+y$ direction, we have

$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow \qquad \textit{Ans.}$$

$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow \qquad \textit{Ans.}$$

The force $\mathbf{F}_2$ is resolved into its x and y components, as shown in Fig. 2–18*c*. Here the *slope* of the line of action for the force is indicated. From this "slope triangle" we could obtain the angle θ, e.g., $\theta = \tan^{-1}(\frac{5}{12})$, and then proceed to determine the magnitudes of the components in the same manner as for $\mathbf{F}_1$. The easier method, however, consists of using proportional parts of similar triangles, i.e.,

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

Similarly,

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$

Notice how the magnitude of the *horizontal component*, $\mathbf{F}_{2x}$, was obtained by multiplying the force magnitude by the ratio of the *horizontal leg* of the slope triangle divided by the hypotenuse; whereas the magnitude of the *vertical component*, F_{2y}, was obtained by multiplying the force magnitude by the ratio of the *vertical leg* divided by the hypotenuse. Hence, using scalar notation to represent these components, we have

$$F_{2x} = 240 \text{ N} = 240 \text{ N} \rightarrow \qquad \textit{Ans.}$$

$$F_{2y} = -100 \text{ N} = 100 \text{ N} \downarrow \qquad \textit{Ans.}$$

Cartesian Vector Notation. Having determined the magnitudes and directions of the components of each force, we can express each force as a Cartesian vector.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{N} \qquad \textit{Ans.}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\} \text{N} \qquad \textit{Ans.}$$

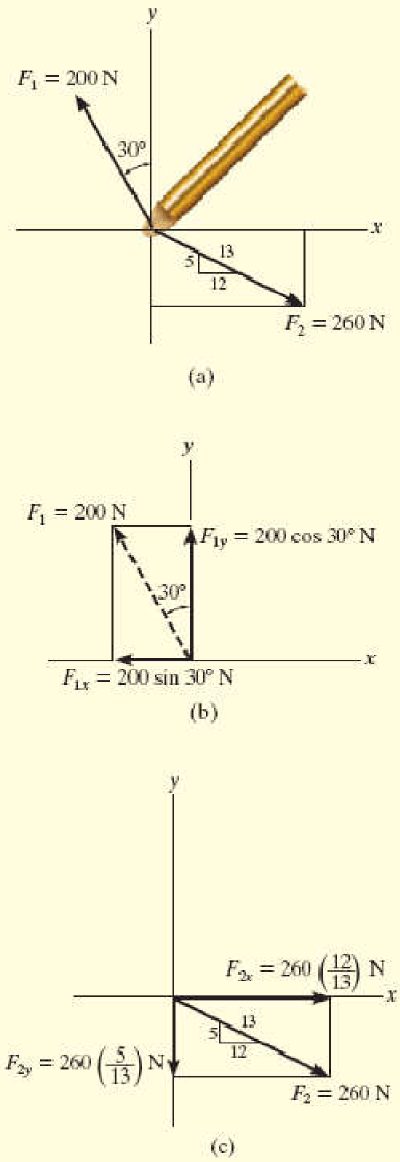


Fig. 2–18

EXAMPLE 2.6

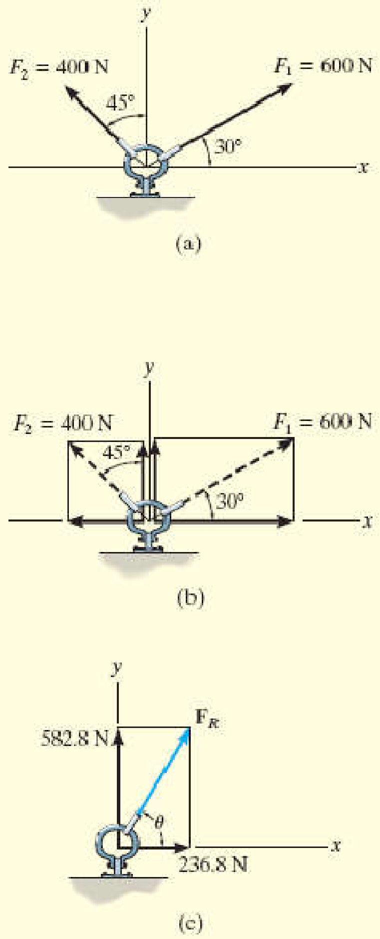


Fig. 2–19

The link in Fig. 2–19*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.

SOLUTION I

Scalar Notation. First we resolve each force into its x and y components, Fig. 2–19*b*, then we sum these components algebraically.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N} = 236.8 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N} = 582.8 \text{ N} \uparrow$$

The resultant force, shown in Fig. 2–18*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2} = 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition,

$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9^\circ \qquad \textit{Ans.}$$

SOLUTION II

Cartesian Vector Notation. From Fig. 2–19*b*, each force is first expressed as a Cartesian vector.

$$\mathbf{F}_1 = \{600 \cos 30^\circ \mathbf{i} + 600 \sin 30^\circ \mathbf{j}\} \text{ N}$$
$$\mathbf{F}_2 = \{-400 \sin 45^\circ \mathbf{i} + 400 \cos 45^\circ \mathbf{j}\} \text{ N}$$

Then,

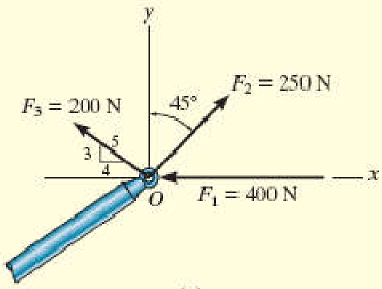
$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (600 \cos 30^\circ \text{ N} - 400 \sin 45^\circ \text{ N})\mathbf{i} + (600 \sin 30^\circ \text{ N} + 400 \cos 45^\circ \text{ N})\mathbf{j} = \{236.8\mathbf{i} + 582.8\mathbf{j}\} \text{ N}$$

The magnitude and direction of $\mathbf{F}_R$ are determined in the same manner as before.

NOTE: Comparing the two methods of solution, notice that the use of scalar notation is more efficient since the components can be found *directly*, without first having to express each force as a Cartesian vector before adding the components. Later, however, we will show that Cartesian vector analysis is very beneficial for solving three-dimensional problems.

EXAMPLE 2.7

The end of the boom O in Fig. 2–20a is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



(a)

SOLUTION

Each force is resolved into its x and y components, Fig. 2–20b. Summing the x components, we have

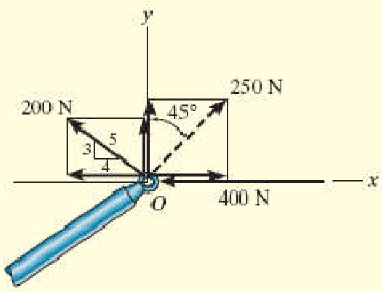
$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400\text{ N} + 250\sin 45^\circ\text{ N} - 200\left(\tfrac{4}{5}\right)\text{ N}$$

$$= -383.2\text{ N} = 383.2\text{ N} \leftarrow$$

The negative sign indicates that F_{Rx} acts to the left, i.e., in the negative x direction, as noted by the small arrow. Obviously, this occurs because F_1 and F_3 in Fig. 2–20b contribute a greater pull to the left than F_2 which pulls to the right. Summing the y components yields

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250\cos 45^\circ\text{ N} + 200\left(\tfrac{3}{5}\right)\text{ N}$$

$$= 296.8\text{ N}\uparrow$$



(b)

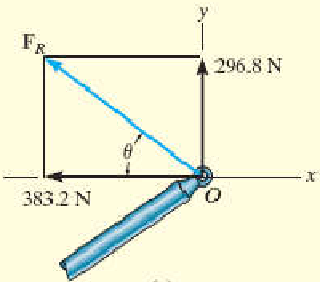
The resultant force, shown in Fig. 2–20c, has a *magnitude* of

$$F_R = \sqrt{(-383.2\text{ N})^2 + (296.8\text{ N})^2}$$

$$= 485\text{ N} \qquad \textit{Ans.}$$

From the vector addition in Fig. 2–20c, the direction angle θ is

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ \qquad \textit{Ans.}$$



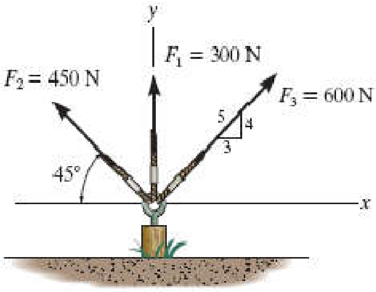
(c)

Fig. 2–20

NOTE: Application of this method is more convenient, compared to using two applications of the parallelogram law, first to add $\mathbf{F}_1$ and $\mathbf{F}_2$ then adding $\mathbf{F}_3$ to this resultant.

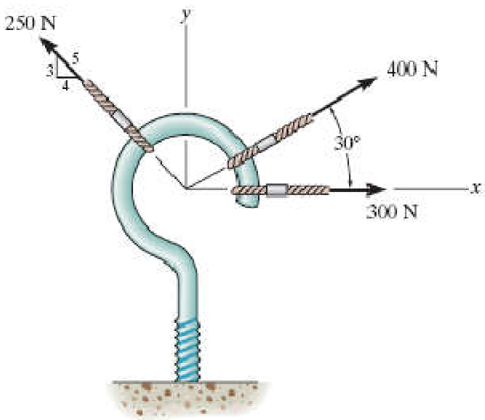
FUNDAMENTAL PROBLEMS

F2–7. Resolve each force acting on the post into its x and y components.



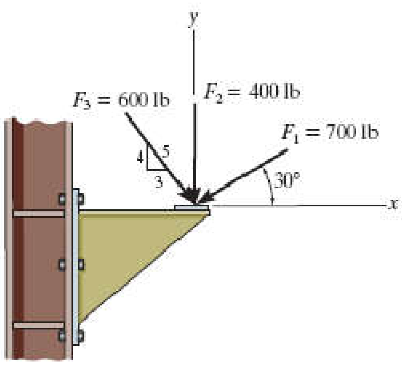
F2–7

F2–8. Determine the magnitude and direction of the resultant force.



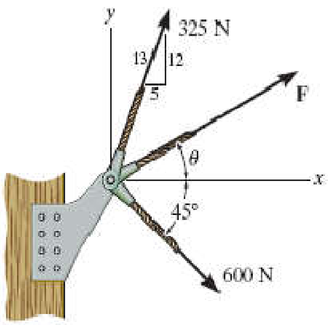
F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



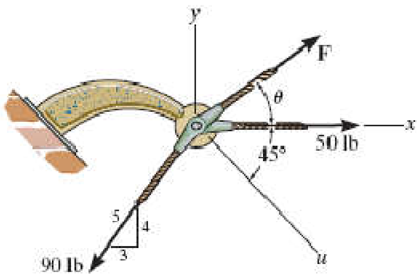
F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of **F** and its direction θ.



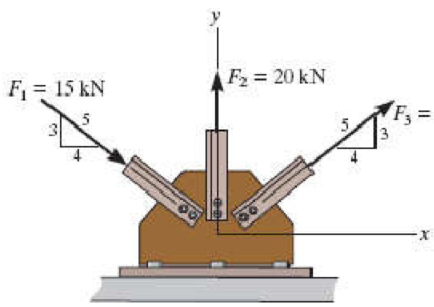
F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of **F** and its direction θ.



F2–11

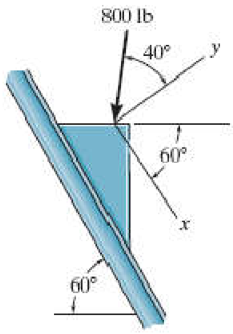
F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



F2–12

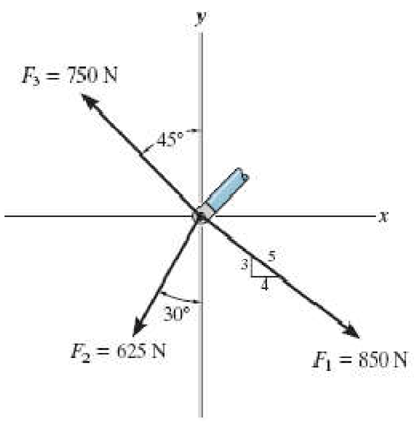
PROBLEMS

***2–32.** Determine the x and y components of the 800-lb force.



Prob. 2–32

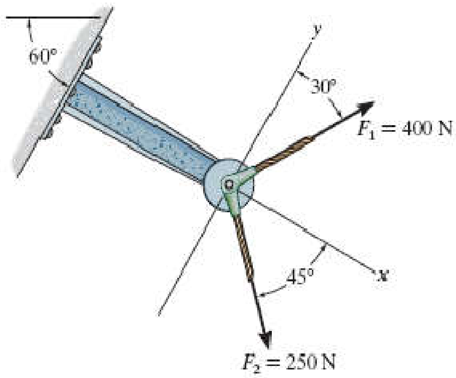
2–33. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Prob. 2–33

2–34. Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.

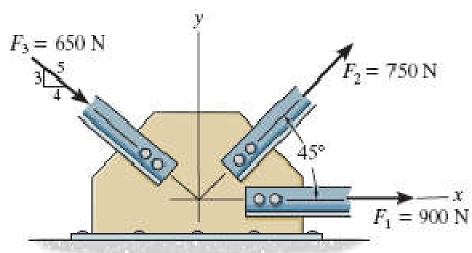
2–35. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Probs. 2–34/35

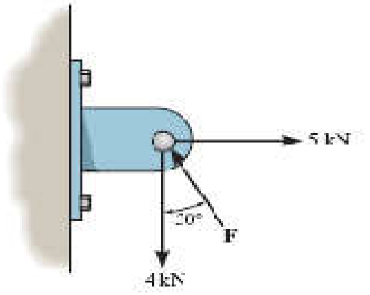
***2–36.** Resolve each force acting on the gusset plate into its x and y components, and express each force as a Cartesian vector.

2–37. Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



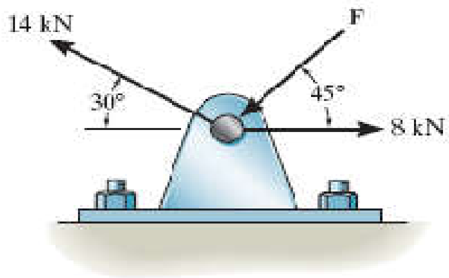
Probs. 2–36/37

***2–52.** Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?



Prob. 2–52

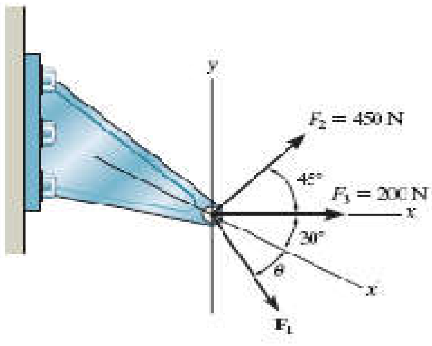
2–53. Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



Prob. 2–53

2–54. Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 1 kN.

2–55. If $F_1 = 300$ N and $\theta = 20°$, determine the magnitude and direction, measured counterclockwise from the x' axis, of the resultant force of the three forces acting on the bracket.



Probs. 2–54/55