Moment of a couple

A couple is defined as a two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d (fig 4-25).The moment produced by a couple is called a couple moment.

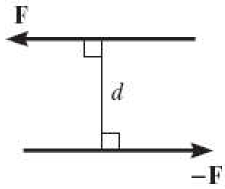


Fig. 4–25

Scalar Formulation: The moment of a couple **M** (fig 4-27), is defined as having a magnitude of: $M = F\,d$

Where ***F*** is the magnitude of one of the forces and ***d*** is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right hand rule. **M** will act perpendicular to the plane containing these forces.

Vector Formulation: The moment of a couple can also be expressed by the vector Cross product as:

$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However, $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that:

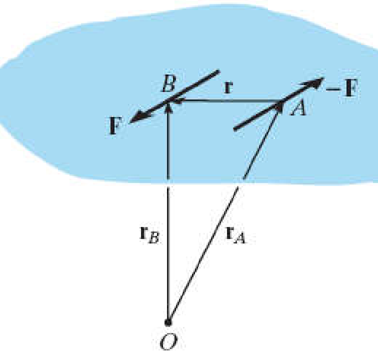


Fig. 4–26

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Note: *This result indicates that a couple moment is a free vector, i.e., it can act at any point since **M** depends only upon the position vector **r** directed between the forces and not the position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$, directed from the arbitrary point O to the forces.*

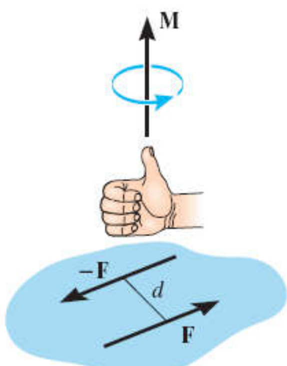
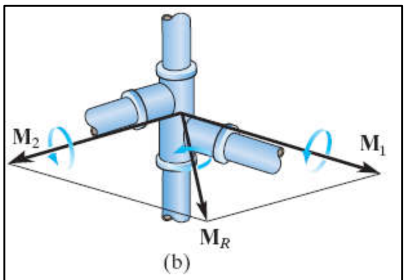


Fig. 4–27

Equivalent Couples. If two couples produce a moment with the *same magnitude and direction,* then these two couples are *equivalent*.

Resultant Couple Moment. Since couple moments are vectors, their resultant can be determined by vector addition. For example, consider the couple moments M_1 and M_2 acting on the pipe in Fig. 4–29 a. Since each couple moment is a free vector, we can join their tails at any arbitrary point and find the resultant couple moment, $M_R = M_1 + M_2$ as shown in Fig. 4–29 b.



(b)

Fig. 4–29

Equivalent Couples: If two couples produce a moment with the *same magnitude and direction* , then these two couples are ***equivalent*** . For example, the two couples shown in Fig. 4–28 are *equivalent* because each couple moment has a magnitude of:

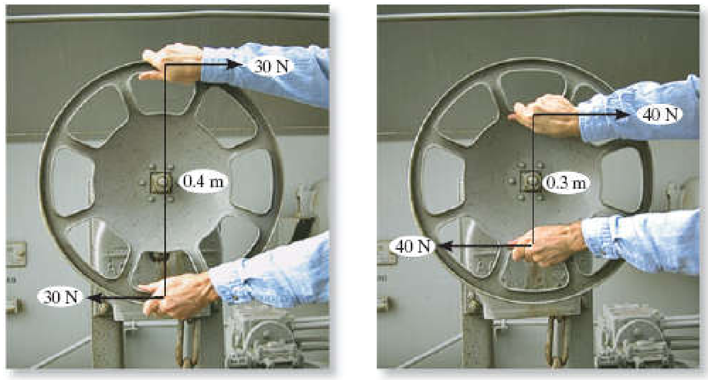


Fig. 4–28

$M = 30\text{ N}(0.4\text{ m})$
$= 40\text{ N}(0.3\text{ m}) = 12\text{ N.m}$, and each is directed into the plane of the page. Notice that larger forces are required in the second case to create the same turning effect because the hands are placed closer together.

Resultant Couple Moment: Since couple moments are vectors, their resultant can be determined by vector addition. For example, consider the couple moments $\mathbf{M}_1$ and $\mathbf{M}_2$ acting on the pipe in Fig. 4–29 *a*. Since each couple moment is a free vector, we can join their tails at any arbitrary point and find the resultant couple moment, $\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$ as shown in Fig. 4–29 *b*. If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as:

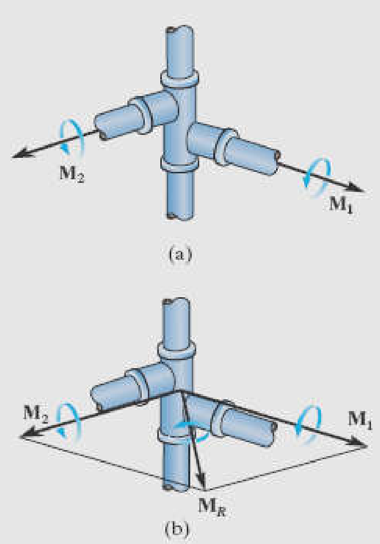


Fig. 4–29

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$

EXAMPLE 4.10

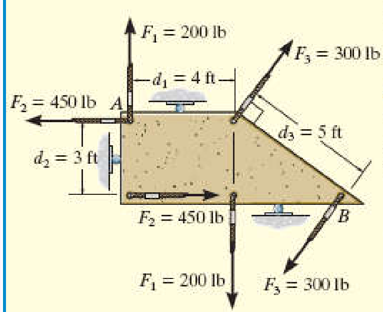


Fig. 4–30

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

SOLUTION

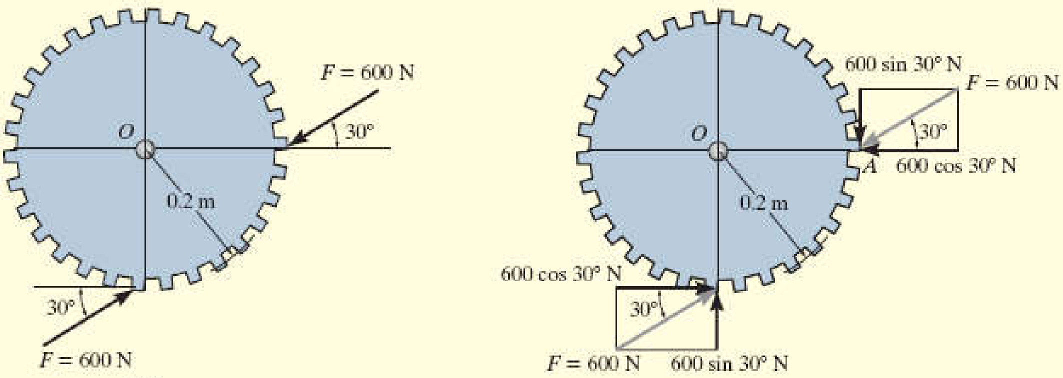
As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\circlearrowleft + M_R = \Sigma M;\quad M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$
$$= -(200\text{ lb})(4\text{ ft}) + (450\text{ lb})(3\text{ ft}) - (300\text{ lb})(5\text{ ft})$$
$$= -950\text{ lb}\cdot\text{ft} = 950\text{ lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

EXAMPLE 4.11

Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.



(a) (b)

SOLUTION

The easiest solution requires resolving each force into its components as shown in Fig. 4–31*b*. The couple moment can be determined by summing the moments of these force components about any point, for example, the center O of the gear or point A. If we consider counterclockwise moments as positive, we have

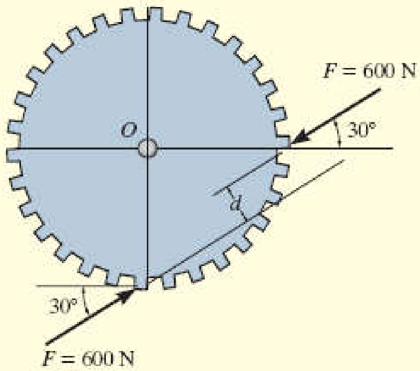
$$\circlearrowleft + M = \Sigma M_O;\; M = (600 \cos 30^\circ\text{ N})(0.2\text{ m}) - (600 \sin 30^\circ\text{ N})(0.2\text{ m})$$
$$= 43.9\text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

or

$$\circlearrowleft + M = \Sigma M_A;\; M = (600 \cos 30^\circ\text{ N})(0.2\text{ m}) - (600 \sin 30^\circ\text{ N})(0.2\text{ m})$$
$$= 43.9\text{ N}\cdot\text{m} \circlearrowleft \qquad \textit{Ans.}$$

This positive result indicates that **M** has a counterclockwise rotational sense, so it is directed outward, perpendicular to the page.

NOTE: The same result can also be obtained using $M = Fd$, where d is the perpendicular distance between the lines of action of the couple forces, Fig. 4–31*c*. However, the computation for d is more involved. Realize that the couple moment is a free vector and can act at any point on the gear and produce the same turning effect about point O.

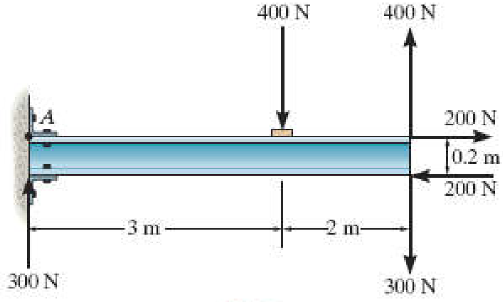


(c)

Fig. 4–31

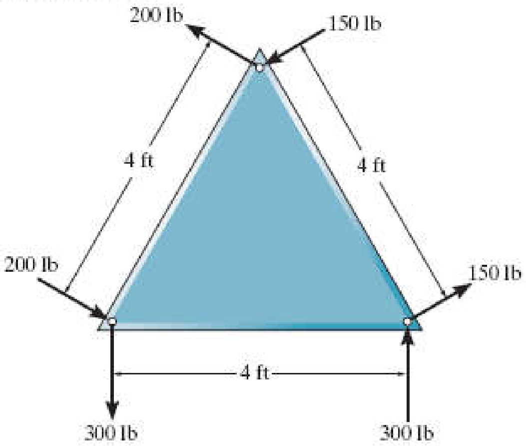
FUNDAMENTAL PROBLEMS

F4–19. Determine the resultant couple moment acting on the beam.



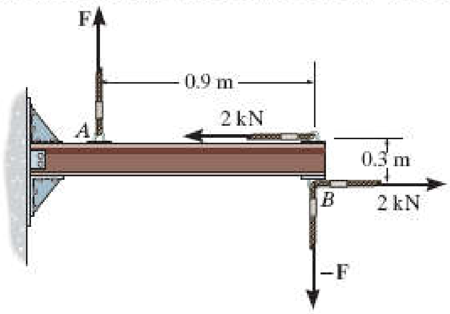
F4–19

F4–20. Determine the resultant couple moment acting on the triangular plate.



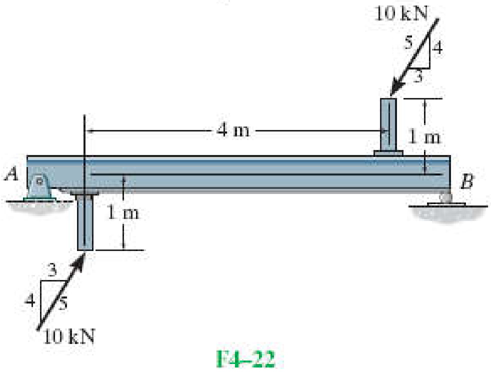
F4–20

F4–21. Determine the magnitude of **F** so that the resultant couple moment acting on the beam is 1.5 kN · m clockwise.



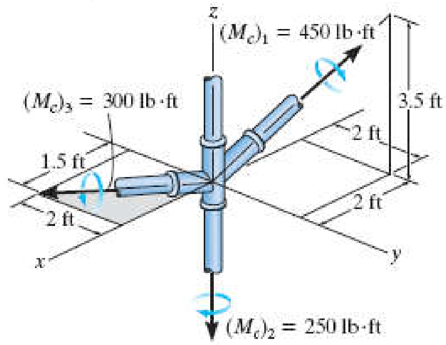
F4–21

F4–22. Determine the couple moment acting on the beam.



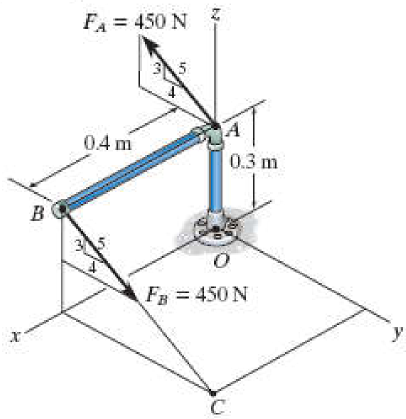
F4–22

F4–23. Determine the resultant couple moment acting on the pipe assembly.



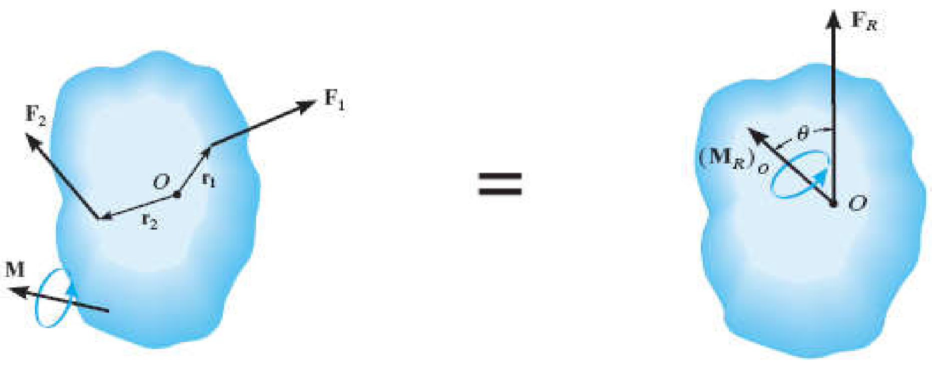
F4–23

F4–24. Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector.



F4–24

System of Forces and Couple Moments: Using the above method, a system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force (F_R) acting at a point O and a resultant couple moment $(M_R)_O$.



We can generalize the above method of reducing a force and couple system to an equivalent resultant force $\mathbf{F}_R$ acting at point O and a resultant couple moment $(\mathbf{M}_R)_O$ by using the following two equations:

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

If the force system lies in the $x - y$ plane and any couple moments are perpendicular to this plane, then the above equations reduce to the following three scalar equations:

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$

Where:

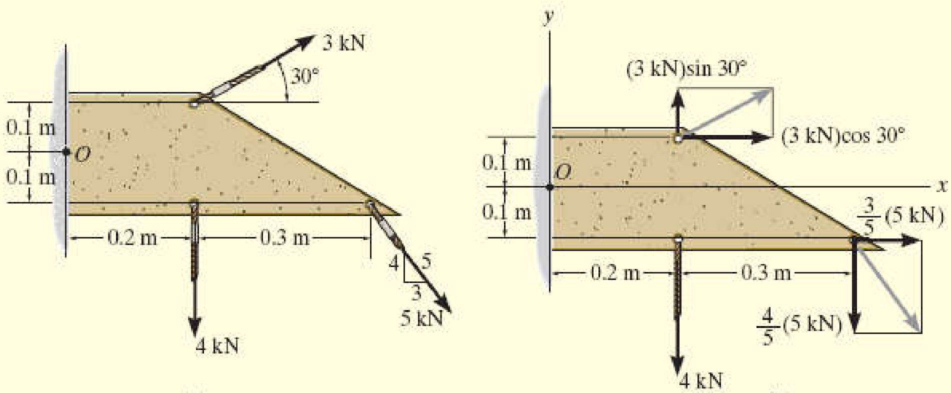
ΣM = the sum of all the couple moments.

ΣM_O = the sum of the moments of all the forces about point O.

- ***Here the resultant force is determined from the vector sum of its two components*** $(F_R)_x$ and $(F_R)_y$.

EXAMPLE 4.14

Replace the force and couple system shown in Fig. 4–37*a* by an equivalent resultant force and couple moment acting at point *O*.



(a) (b)

SOLUTION

Force Summation. The 3 kN and 5 kN forces are resolved into their *x* and *y* components as shown in Fig. 4–37*b*. We have

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

Using the Pythagorean theorem, Fig. 4–37*c*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN} \qquad \textit{Ans.}$$

Its direction θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3^\circ \qquad \textit{Ans.}$$

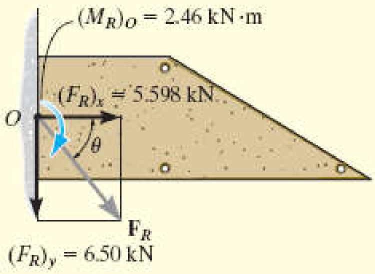
Moment Summation. The moments of 3 kN and 5 kN about point *O* will be determined using their *x* and *y* components. Referring to Fig. 4–37*b*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3\text{ kN})\sin 30^\circ(0.2\text{ m}) - (3\text{ kN})\cos 30^\circ(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})\,(0.1\text{ m}) - \left(\tfrac{4}{5}\right)(5\text{ kN})\,(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$

$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–37*c*.



(c)

Fig. 4–37

NOTE: Realize that the resultant force and couple moment in Fig. 4–37*c* will produce the same external effects or reactions at the supports as those produced by the force system, Fig. 4–37*a*.

EXAMPLE 4.15

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.

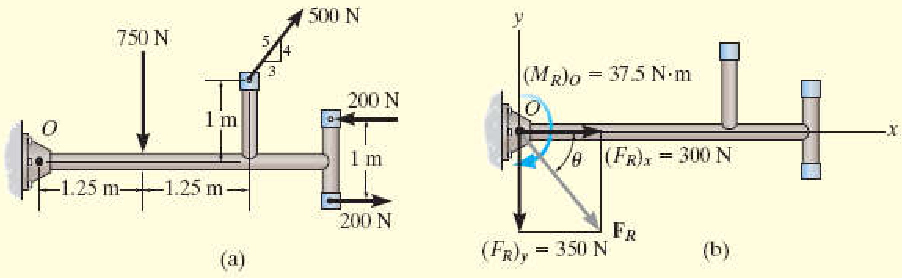


Fig. 4–38

SOLUTION

Force Summation. Since the couple forces of 200 N are equal but opposite, they produce a zero resultant force, and so it is not necessary to consider them in the force summation. The 500-N force is resolved into its *x* and *y* components, thus,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x;\ (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;\ (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N} \qquad \textit{Ans.}$$

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4^\circ \qquad \textit{Ans.}$$

Moment Summation. Since the couple moment is a free vector, it can act at any point on the member. Referring to Fig. 4–38*a*, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

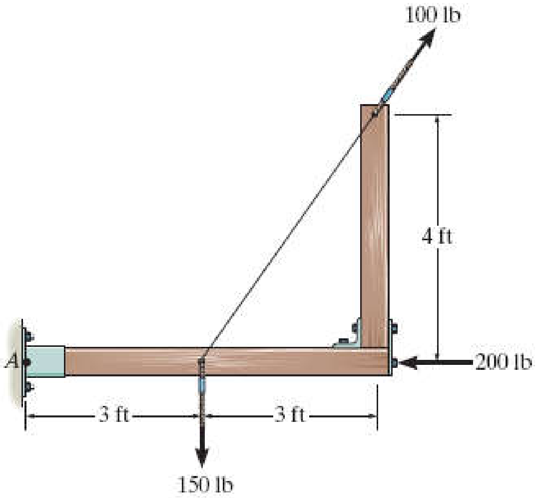
$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$

This clockwise moment is shown in Fig. 4–38*b*.

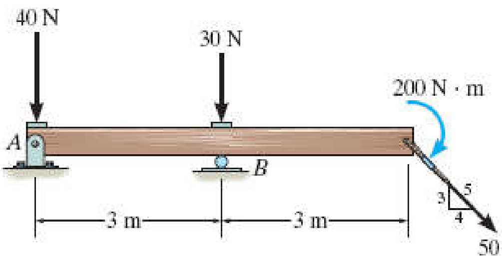
FUNDAMENTAL PROBLEMS

F4–25. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



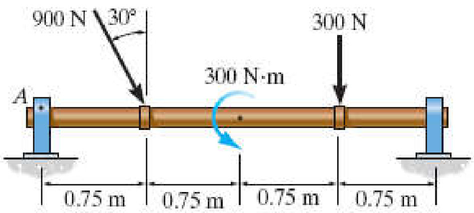
F4–25

F4–26. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



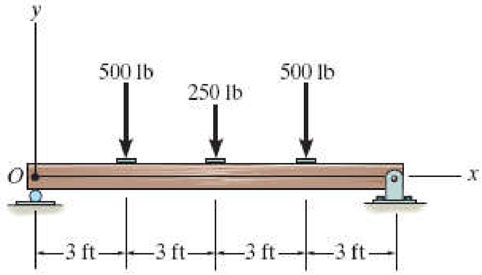
F4–26

F4–27. Replace the loading system by an equivalent resultant force and couple moment acting at point A.



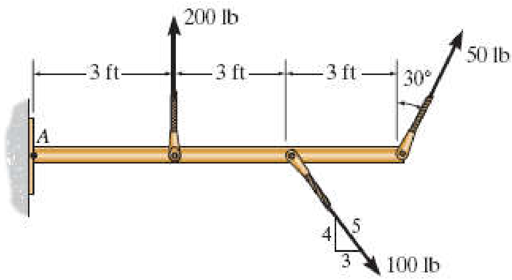
F4–27

F4–31. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O.



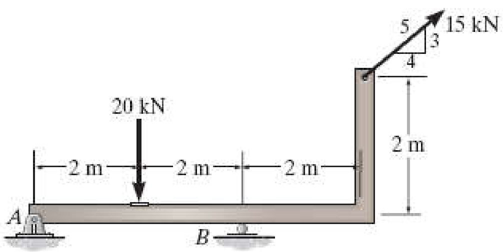
F4–31

F4–32. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from A.



F4–32

F4–33. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the horizontal segment of the member measured from A.



F4–33