التفاضل و التكامل

CALCULUS

مساحة الدائرة

$$A = \pi r^2$$

قانون الدستور

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

متسلسلة تايلور

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots , \qquad -\infty < x < \infty$$

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كلية التربية الاساسية – حديثة

قسم العلوم العامة

كلية التربية الأساسية / حديثة التفاضل والتكامل المستوى 2 م. صالح محمد حسين
Calculus - Level 2nd Collage of Basic Education/Haditha
(2020-2021)

مصطلحات ورموز

رسم الرمز	اسم الرمز	رسم الرمز	اسم الرمز
α	Alpha	w.r.t	With respect to
β	Beta	no.	Number
θ	Theta	Ex.	Example
δ or Δ	Delta	Sol.	Solution
ε	Epsilon	Def.	Definition
γ or Γ	Gamma	i.e	Which mean
θ	Theta	∴	So that
λ	Lamda	∵	Since
ζ	Zeta	→	Approach
η	Eata	⟹	Implies
μ	Mu	≡	Identical
φ or Φ	Phi	=	Equal
ψ	Psi	-ve	Negative
π	Pi	+ve	Positive
σ	Sigma	∃	There exist
τ	Tau	∀	For all
ρ	Rho		
∞	Infinity	∧	And
Iff	If and only if	∨	Or
fun.	function	∈	belongs to

Real Numbers: R (الأعداد الحقيقية)

R=$\{x: -\infty < x < \infty\}$

- If **a** and **b** are two real no.s then one of the following is true.
  some properties of R
  1- If $a > b$ then $-a < -b$
  2- If $a > b$ then $\frac{1}{a} < \frac{1}{b}$
  3- If $a < b$, $b < c$ then $a < c$
  4- If $a < b$ then $a + c < b + c$ $\forall$ real no. c
  5- If $a < b$, $c < d$ then $a + c < b + d$
  6- If $a < b$, c any + ve real no. then $a.c < b.c$
  7- If $a < b$, c any − ve real no. then $a.c > b.c$
  8- If $0 < a < b$, $0 < c < d$ then $a.c < b.d$
  9- If $a > b > 0$, $c > d > 0$ then $a.c > b.d$

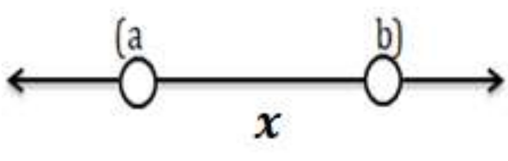
Intervals (الفترات)

Def. :- An interval is a set of real no.s x having one of the following forms:-

1- Open interval (a, b) (الفترة المفتوحة)

All real no.s x $\ni$ $a < x < b$

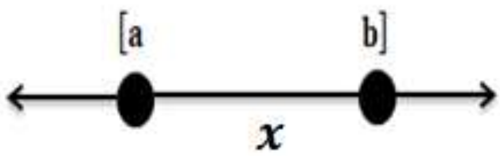
$(a, b) = \{x: a < x < b\}$



2- Closed interval [a, b] [الفترة المغلقة]

All real no.s x $\ni$ $a \leq x \leq b$

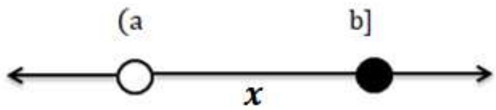
$[a, b] = \{x: a \leq x \leq b\}$



3- Half-open from the left or Half closed from the right.
$(\mathbf{a},\mathbf{b}]$ **(الفترة نصف مفتوحة من اليسار أو نصف مفتوحة من اليمين]**

All real no.s $\boldsymbol{x}$ ∋ $a < x \le b$

$(a,b] = \{x: \ a < x \le b\}$



4- Half-open from the right or Half closed from the left.
$[\mathbf{a},\mathbf{b})$ **[الفترة نصف مفتوحة من اليسار أو نصف مفتوحة من اليمين)**

All real no.s $\boldsymbol{x}$ ∋ $a \le x < b$

$[a,b) = \{x: \ a \le x < b\}$

Notes:- **(ملحوظات)**

❖ $(a,\infty) = \{x: \ x > a, x \in R\}$

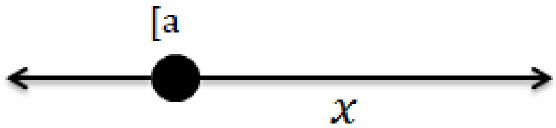
$\equiv a < x < \infty \equiv x > a$

❖ $(-\infty,a) = \{x: \ x < a, x \in R\}$

$\equiv -\infty < x < a \equiv x < a$

❖ $[a,-\infty) = \{x: \ x \ge a, x \in R\}$

$\equiv a \le x < \infty \equiv x \ge a$



❖ $(-\infty, a] = \{x: \ x \le a, x \in R\}$

$\equiv a \geq x > -\infty \equiv x \leq a$

Inequalities (متباينات)

$2x - 3 > 0, \quad x^2 - 5x - 24 \leq 0$

Examples:- Find the set of the following Inequalities.

1) $\mathbf{2 + 3x < 5x + 8}$

$2 - 8 < 5x - 3x$

$-6 < 2x$

$x > -3$

Sol. Set = $\{x: x > -3\} \equiv x > -3 \equiv (-3, \infty)$

2) $\mathbf{4 < 3x - 2 \leq 10}$

$4 + 2 < 3x - 2 + 2 \leq 10 + 2$

$6 < 3x \leq 12$

$2 < x \leq 4$

Sol. Set = $\{x: 2 < x \leq 4\} \equiv (2, 4]$

3) $\mathbf{\frac{7}{x} > 2 \quad , \quad x \neq 0 \quad , \quad x \in R}$

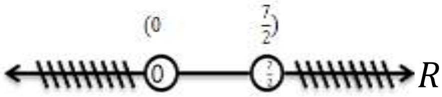
1st case:- $\mathbf{x > 0}$

$7 > 2x \Leftrightarrow \frac{7}{2} > x \Leftrightarrow x < \frac{7}{2}$

Sol. set 1st case = $(0\,,\frac{7}{2})$ R

2nd case:- $\boldsymbol{x < 0}$

$7 < 2x \Leftrightarrow \frac{7}{2} < x \Leftrightarrow x > \frac{7}{2}$



Sol. set 2nd case = $\boldsymbol{\phi}$

Sol. Set $= \left(\mathbf{0}\,,\frac{\mathbf{7}}{\mathbf{2}}\right) \cup \boldsymbol{\phi} = (\mathbf{0}\,,\frac{\mathbf{7}}{\mathbf{2}})$

4) $\boldsymbol{\frac{x}{x-3} < 4 \quad , \quad x \neq 3}$

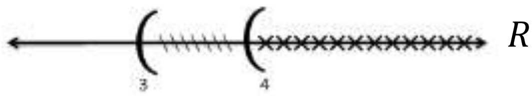
1st case:- $\boldsymbol{x > 3} \Leftrightarrow x - 3 > 0$

Multiply by $(x - 3)$

$x < 4(x-3) \Leftrightarrow x < 4x - 12 \Leftrightarrow x - 4x < -12$

$\Leftrightarrow -3x < -12 \Leftrightarrow -\frac{1}{3}(-3x < -12) \Leftrightarrow x > 4$

Sol. set 1st case = $(4\,,\infty)$

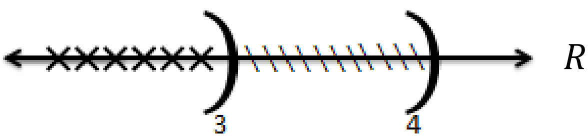


2nd case:- $\boldsymbol{x < 3}$

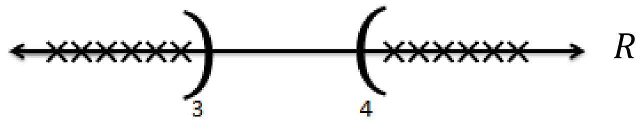
$x > 4(x-3) \Leftrightarrow x > 4x - 12 \Leftrightarrow x - 4x > -12$

$\Leftrightarrow -3x > -12 \Leftrightarrow -\frac{1}{3}(-3x > -12) \Leftrightarrow x < 4$

Sol. set 2nd case = $(-\infty\,,3)$



Sol. Set $=(4,\infty) \cup (-\infty,3) = R/[3,4]$

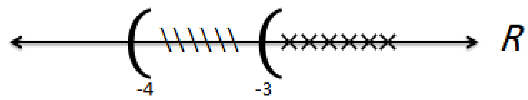


5) $\boldsymbol{(x+3)(x+4) > 0}$

1st case:- $(x+3) > 0 \;\wedge\; (x+4) > 0$

$\Leftrightarrow \; x > -3 \;\wedge\; x > -4$

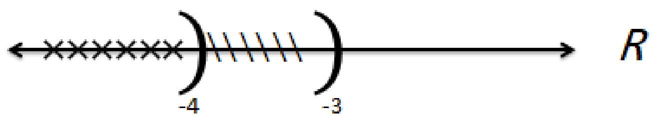
Sol. set 1st case $= (-3,\infty)$



2nd case :- $(x+3) < 0 \;\wedge\; (x+4) < 0$

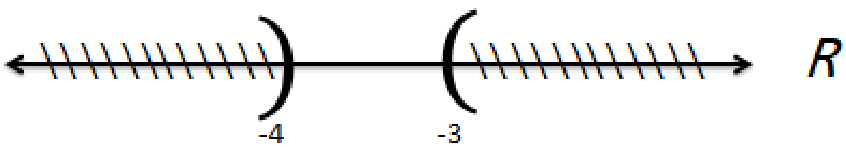
$\Leftrightarrow \; x < -3 \;\wedge\; x < -4$

Sol. set 2nd case $=(-\infty,-4)$



Sol. Set for both of the cases

$=(-3,\infty) \cup (-\infty,-4) = R/[-4,-3]$



6) $\boldsymbol{\frac{x-1}{x^2+x-6} < 0}$

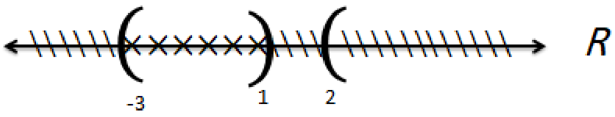
$x^2 + x - 6 \neq 0 \Leftrightarrow (x+3)(x-2) \neq 0$

1st case:- $(x+3)(x-2)>0$

$x-1<0 \Leftrightarrow x<1$

i. $(x+3)>0 \;\wedge\; (x-2)>0$

$x>-3 \;\wedge\; x>2$

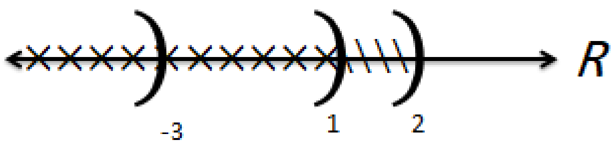


Sol. set i.1st case = ϕ

ii. $(x+3)<0 \;\wedge\; (x-2)<0$

$x-1<0 \Leftrightarrow x<1$

Sol. set ii.1st case $=(-\infty\,,-3)$



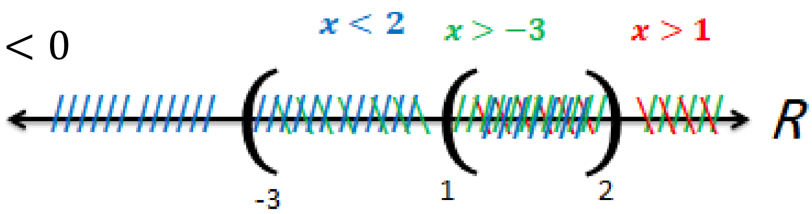
Sol. set 1st case = $\phi \cup (-\infty\,,-3) = (-\infty\,,-3)$

2nd case:- $(x+3)(x-2)<0$

$x-1>0 \Leftrightarrow x>1$

I. $(x+3)>0 \;\wedge\; (x-2)<0$

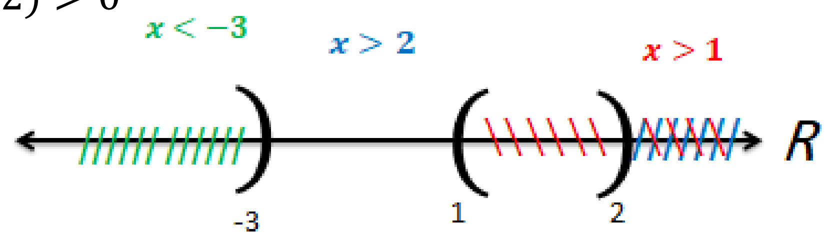
$x>-3 \;\wedge\; x<2$



Sol. set I. **2nd** case $=(1,2)$

II. $(x+3)<0 \;\wedge\; (x-2)>0$

$x<-3 \;\wedge\; x>2$



Sol. set II. **2nd** case $=\phi$

Sol. set **2nd** case = $=(1\,,2) \cup \phi = (1\,,2)$

Sol. Set of all cases $=(-\infty\,,-3) \cup (1,2)$

Absolute Value (القيمة المطلقة)

Def. :- The absolute value of a real no. $\boldsymbol{x}$ denoted by $|\boldsymbol{x}|$ is defined as follows:-

$$|\boldsymbol{x}| = \begin{cases} x & \text{if } x \geq 0 \\ x & \text{if } x < 0 \end{cases}$$

Ex.

$|0| = 0$, $|5| = 5$, $|-3| = 3$, $|1.8| = 1.8$,

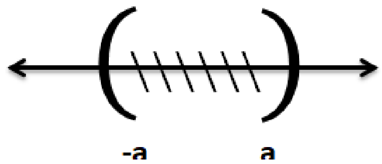
$|8-5| = 3$, $|4+2| = 6$

Note:-

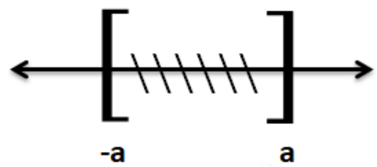
$$|\boldsymbol{x} - \boldsymbol{a}| = \begin{cases} x - a & \text{if } x \geq a \\ a - x & \text{if } x < a \end{cases}$$

Properties of Absolute Value (خواص القيمة المطلقة)

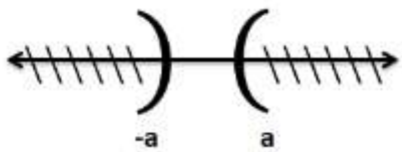
1- $|x| < a$ iff $-a < x < a$

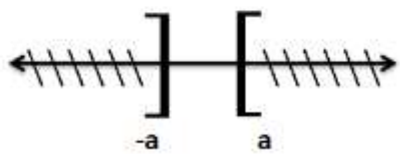


2- $|x| \leq a$ iff $-a \leq x \leq a$



3- $|x| > a$ iff $\mathrm{x} > a$ or $x < -a$





4- $|x| \geq a$ iff $x \geq a$ or $x \leq -a$

Note:-

$|a| = |b|$

1- $a = b$

2- $a = -b$

3- $-a = b \Leftrightarrow a = -b \Rightarrow 2$

4- $-a = -b \Leftrightarrow a = b \Rightarrow 1$

Ex. Find sol. Set of the following:-

1- $|x - 4| < 5$

$-5 < x - 4 < 5$ (properties no. 1)

$\mathbf{-5 + 4 < x - 4 + 4 < 5 + 4}$

$-1 < x < 9$

Sol. Set = $\{x: -1 < x < 9\}$ = (1 , 9)

2- $|2x - 5| = 7$

This satisfies in two cases either

$2x - 5 = 7$ **or** $2x - 5 = -7$

$2x = 12$ $\quad$ $2x = -2$

$x = 6$ $\quad$ $x = -1$

Sol. Set = {-1 , 6}

3- $|x - 2| = |3x + 4|$

$x - 2 = 3x + 4$ **or** $x - 2 = -(3x + 4)$ **(Nots 1&2)**

$x - 3x = 2 + 4$ $\quad$ $x - 2 = -3x - 4$

$-2x = 6$ $\quad$ $4x = 2 - 4$

$x = -3$ $\qquad$ $x = \frac{-1}{2}$

Sol. Set = {-3 , $\frac{-1}{2}$}

Exercises (تمارين)

1- $-3 < \frac{3x-4}{5} \leq 2$

2- $\frac{2}{3} < \frac{10}{x}$

3- $|2x - 5| > 4$

4- $|6x| = |4x|$

5- $\left|\frac{2x-8}{2x-3}\right| \leq 4$