Relationship Between Square Root and Absolute Value (العلاقة بين الجذر التربيعي و القيمة المطلقة)

Recall from algebra that a number is called a ***square root*** of a if its square is. Recall also that every positive real number has two square roots, one positive and one negative; the positive square root is denoted by $\sqrt{a}$ and the negative square root by $-\sqrt{a}$. For example, the positive square root of 9 is $\sqrt{9} = 3$, and the negative square root of 9 is $-\sqrt{9} = -3$.

Students who may have been taught to write $\sqrt{9}$ as ± 3 should stop doing so, since it is incorrect.

It is a common error to replace $\sqrt{a^2}$ by a .Although this is correct when a is nonnegative, it is false for negative a. For example, if $a = -4$, then

$$\sqrt{a^2} = \sqrt{(-4)^2} = \sqrt{16} = 4 \neq a$$

A result that is correct for all a is given in the following theorem

Theorem.

For any real number a, $\sqrt{a^2} = |a|$

Theorem:-

Let a , b two no.s then

1- $|a| = |-a|$
2- $|a.b| = |a||b|$
3- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|} \qquad b \neq 0$

Functions (الدوال)

Note:-

From the definition of the Intervals Note that $a < x$

- Variable:- symbol x is represented to any no. from a set of numbers is called a variable.
- Constat:- a is represented only one number is called constant.

The Cartesian product (الضرب الديكارتي)

Def:- the **Cartesian product** of two sets *A* and *B*, denoted by $A \times B$, is the set of all ordered pairs (*a*, *b*) where *a* is in *A* and *b* is in *B*. in terms of set-builder notation, that is

$A \times B = \{(a, b) : a \in A, b \in B\}$

This definition implies that

$$A \times B = \emptyset \quad \text{iff} \quad A = \emptyset \quad \text{or} \quad B = \emptyset$$

The sets $A \times B$ and $B \times A$ are not identical $A \times B \neq B \times A$

$A \times B = B \times A$ iff $A = B$ and $A = \emptyset$ or $B = \emptyset$

Functions and its Algebra (الدوال وجبرها)

Def:- A function of from a set D to a set R is a rule that assigns a single element $y \in R$ to each element $x \in D$

$$f = \{(x, f(x)/x \in D\}$$

- نقول عن المتغير y أنه دالة لمتغير أخر x إذا أعطية قاعدة أو وسيلة بحيث أن تقابل كل قيمة لـ x في مداها قيمة لـ y.

i.e

Let $f: D \rightarrow R$ is function iff for each $\in D$, there is one and only one $y \in R$ satisfying $f(x) = y$.

If (x, y_1) and (x, y_2) are elements in F then $y_1 = y_2$

1- The Domain (منطلق الدالة) مجال الدالة

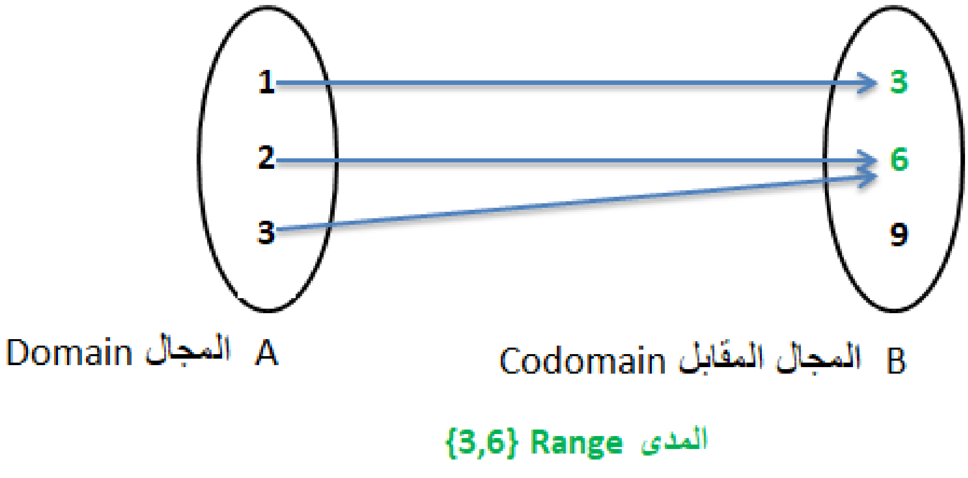
The set D is called the domain of f is the set of all x....... Components occurring in the ordered pairs of f.

Denoted by dom(f) =$D_f = \{x$: F is defined$\}$

$$= \{x \in D: (x, y) \in f \text{ for some } y \in R\}$$

2- The Codomain المجال المقابل

The codomain is for all element in too set R and denoted by Cod_F such that $Cod_F = \{y: y \in R\}$

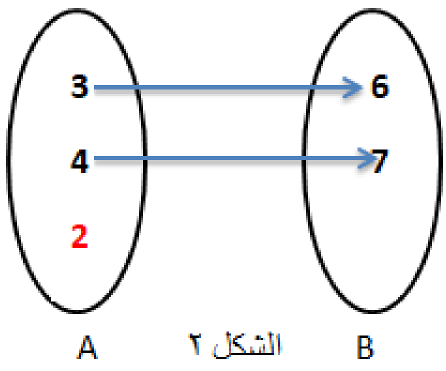


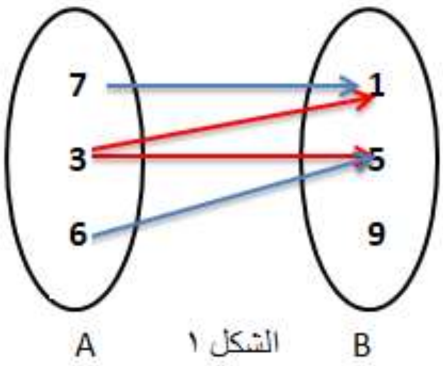
3- The Range (المدى)

Is the set of all second components of element of f denoted by.

$$\text{Rang}_f = R_f = \{f(x): x \in \text{Dom}(f)\}$$

$$= \{y: (x, y) \in F \text{ for som } x \in D\}$$

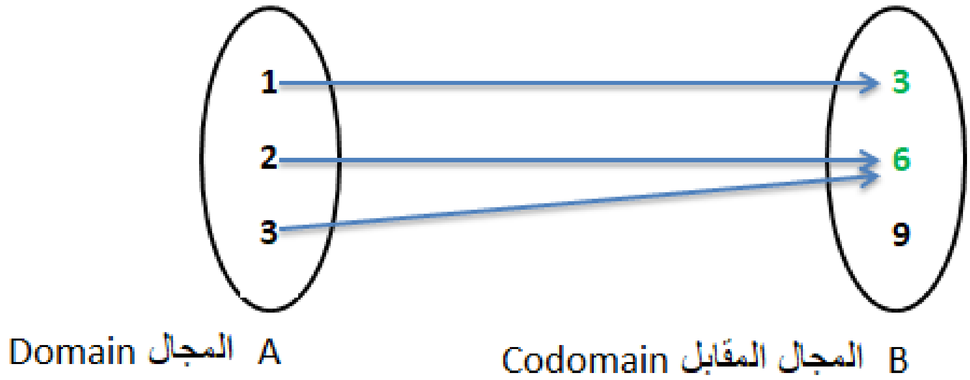




في الشكلين اعلاه (2&1) لا تمثل دوال لان في الشكل 1 هناك العنصر 3 له علاقة في عنصرين من عناصر المجال المقابل وهذا يعارض شرط ان لكل عنصر من عناصر المجال له علاقة بعنصر واحد فقط في المجال المقابل ، اما الشكل 2 فأن العنصر 2 ليس له علاقة مع اي عنصر في المجال المقابل.

How can write the function as ordered pairs?

Let try about the previous example.



$$f: A \rightarrow B$$

$$f = \{(1, 3), (2, 6), (3, 6)\}$$

Ex. Determine which of the following sets is a function. If it is a function, what is its domain (D_f) and rang (R_F)?

a) $f = \{(1,4),(3,6),(-3,5),(0,0),(5,0)\}$

Sol.

$$D_f = \{1,3,-3,0,5\} \quad \& \quad R_f = \{4,6,5,0\}$$

b) $g = \{(1,3),(3,6),(3,5),(4,4),(5,0)\}$

Sol.

g is not a function $(\mathbf{3},6)$ and $(\mathbf{3},5)$ have the same x-coordinate.

- The variable x is called the independent variable of f (or argument) and the variable y is called the dependent variable of f.

Def:- Let f be fun. then the graph of fun. f is the set of all points (x, y) is the ordered pairs in

Ex.

Let f be the fun. $f(x) = \sqrt{5-x}$ find its domain and range?

$$D_f = \{x: 5 - x \geq 0\} = \{x: x \leq 5\} = (-\infty, 5]$$

$$R_f = \{f(x): x \in D_f\} = \{f(x): x \leq 5\} = [0, \infty)$$

Ex. Let g be fun. defined as follows

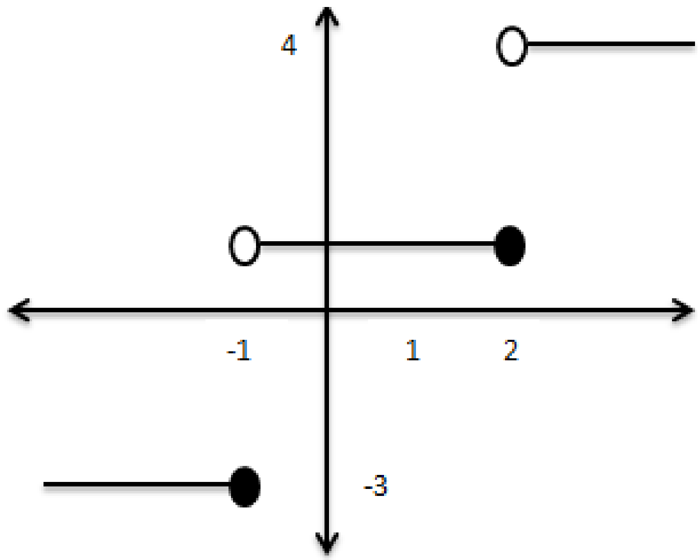
$$y = g(x) = \begin{cases} -3 & \text{if} \quad x \leq -1 \\ 1 & \text{if} \quad -1 < x \leq 2 \\ 4 & \text{if} \quad 2 < x \end{cases}$$

Find the domain, range, and sketch the fun.

Sol.

$D_g = R$

$R_g = \{-3,1,4\}$



Kinds of Functions (انواع الدوال)

Functions are divided into two types.

1- Algebraic function (الدالة الجبرية)

i. Polynomial function (دالة متعددة الحدود)

$$y = a_\circ + a_1 x + a_2 x^2 + \cdots \ldots \ldots \ldots \ldots + a_n x^n$$

Such that $a_\circ, a_1, a_2, \ldots . a_n \in R$

Ex. 1. $y = x^3 + 2x + 7$

2. $y = x + 5$

3. $y = 2x$

ii. Regular function (دالة قياسية أو الكسرية) $y = \frac{\text{متعددة الحدود}}{\text{متعددة الحدود}}$

$$y = \frac{2x+1}{x^2+3x} \quad , \quad y = \frac{1}{x^2}$$

iii. Radical function (دالة جذرية)

1. $y = \frac{\sqrt{x^2-1}}{x+2}$ 2. $y = \sqrt{x^2+1}$

2- Non algebraic function (الدالة غير الجبرية)

i. Trigonometric function (دوال مثلثية)

1. $y = \sin x$ 2. $y = \cos x$ 3. $y = \tan x$

ii. Exponential function (دالة أسية)

1. $y = 2^x$ 2. $y = (\frac{1}{2})^x$ 3. $y = e^x$

iii. Logarithmic function (دالة لوغارتمية)

1. $y = log_{10}^x$ (اللوغاريتم العشري)

2. $y = lnx$ (اللوغاريتم الطبيعي)

iv. Absolute value function (دالة القمية المطلقة)

$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$