Def:-

1- $f(x)$ is an **even** fun. if $f(-x) = f(x)$ for every x in the domain.

2- $f(x)$ is an **odd** fun. if $f(-x) = -f(x)$ for every x in the domain.

Exs.

1) $\sin\theta$

$\sin(-\theta) = -\sin(\theta)$ is odd fun.

2) $\cos\theta$

$\cos(\theta) = \cos(-\theta)$ is even fun.

3) $f(x) = x^2 \cos x$

$$f(-x) = (-x)^2 \cos(-x) = x^2 \cos x = f(x)$$

$\therefore f(x)$ is even fun.

4) $f(x) = \frac{x^2-1}{\sin x}$

$$f(-x) = \frac{(-x)^2 - 1}{\sin(-x)} = \frac{x^2 - 1}{-\sin x} = -\left(\frac{x^2 - 1}{\sin(x)}\right) = -f(x)$$

$\therefore f(x)$ is odd fun.

Sum, Differences, product and quotient of Functions (الجمع، الطرح، الضرب، و القسمة للدوال)

If $f(x)$ and $g(x)$ are two functions then :-

1- The sum $f+g$ is the fun.

$(f+g)(x) = f(x) + g(x)$

Domain $(f+g) = \text{domain}(f) \cap \text{domain}(g)$

2- The differences $f-g$ is the fun.

$(f-g)(x) = f(x) - g(x)$

Domain $(f-g) = \text{domain}(f) \cap \text{domain}(g)$

3- The product $f.g$ is the fun.

$(f.g)(x) = f(x).g(x)$

Domain $(f.g) = \text{domain}(f) \cap \text{domain}(g)$

4- The quotient $\frac{f}{g}$ is the fun.

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0$

Domain $\left(\frac{f}{g}\right) = \text{domain}(f) \cap \text{domain}(g) \setminus \{x: g(x) = 0\}$

Ex. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$

Find $(f+g)$, $(g-f)$, $(f-g)$, $\left(\frac{f}{g}\right)$, $\left(\frac{g}{f}\right)$, $(f.g)$ and its domain.

Sol.

$\text{domain}(f) = [0,\infty)$, $\text{domain}(g) = (-\infty, 1]$

$1 - x \geq 0$

$-x \geq -1$

$x \leq 1$

$\text{Domain}(f) \cap \text{Domain}(g) = [0,1]$

- $(f+g)(x) = \sqrt{x} + \sqrt{1-x}$ $\text{Domain}(f+g) = [0,1]$
- $(f-g)(x) = \sqrt{x} - \sqrt{1-x}$ $\text{Domain}(f-g) = [0,1]$
- $(g-f)(x) = \sqrt{1-x} - \sqrt{x}$ $\text{Domain}(g-f) = [0,1]$
- $(f.g)(x) = f(x).g(x) = \sqrt{x}\,\sqrt{1-x} = \sqrt{x-x^2}$

$$D_{f.g} = [0,1]$$

- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{1-x}} = \sqrt{\frac{x}{1-x}}$ $D_{\frac{f}{g}} = [0,1)$
- $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{1-x}}{\sqrt{x}} = \sqrt{\frac{1-x}{x}} = \sqrt{\frac{1}{x}-1}$ $D_{\frac{g}{f}} = (0,1]$

Composition of Functions (تركيب الدوال)

Composition is another method for combining functions. In this operation the output from one function becomes the input to a second function.

Def. :- If ***f*** and ***g*** are functions, the **composite** function ***f* ∘ *g*** ("***f*** composed with ***g***") is defined by:

$$(f \circ g)(x) = f(g(x))$$

The domain of ***f* ∘ *g*** consists of the numbers x in the domain of ***g*** for which **g(*x*)** lies in the domain of ***f***.

Figure (1)

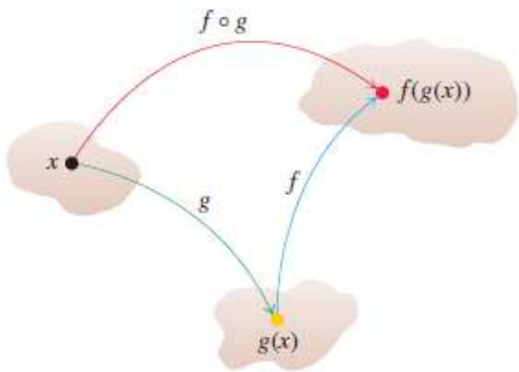

Figure (2)

The definition implies that $\boldsymbol{f \circ g}$ can be formed when the range of $\boldsymbol{g}$ lies in the domain of $\boldsymbol{f}$. To find $\boldsymbol{(f \circ g)(x)}$, *first* find $\boldsymbol{g(x)}$ and *second* find $\boldsymbol{f(g(x))}$. Figure (1) pictures $f \circ g$ as a machine diagram, and Figure (2) shows the composition as an arrow diagram.

To evaluate the composite function $g \circ f$ (when defined), we find $f(x)$ first and then find $g(f(x))$. The domain of $g \circ f$ is the set of numbers x in the domain of f such that $f(x)$ lies in the domain of g.

The functions $f \circ g$ and $g \circ f$ are usually quite different.

Ex. If $f(x) = \sqrt{x}$ and $g(x) = x + 1$

Find **(a)** $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$.

Sol.	**Composition**	**Domain**
(a)	$(f \circ g)(x) = f\big(g(x)\big) = \sqrt{g(x)} = \sqrt{x+1}$	$[-1, \infty)$
(b)	$(g \circ f)(x) = g\big(f(x)\big) = f(x) + 1 = \sqrt{x} + 1$	$[0, \infty)$
(c)	$(f \circ f)(x) = f\big(f(x)\big) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$	$[0, \infty)$
(d)	$(g \circ g)(x) = g\big(g(x)\big) = g(x) + 1 = (x+1) + 1 = x + 2$	$(-\infty, \infty)$

To see why the domain of $f \circ g$ is $[-1, \infty)$, notice that $g(x) = x + 1$ is defined for all real x but $g(x)$ belongs to the domain of f only if $x + 1 \geq 0$,that is to say, when $x \geq -1$.

Notice that if $f(x) = x^2$ and $g(x) = \sqrt{x}$, then

$(f \circ g)(x) = (\sqrt{x})^2 = x$. However,the domain of $f \circ g$ is $[0, \infty)$, not (∞, ∞), since $\sqrt{x}$ requires $x \geq 0$.

Ex. Let $f(x) = 2x + 1$ find the fun. $g(x)$ in which

$(f \circ g)(x) = x^3$.

Sol.

$(f \circ g)(x) = f(g(x)) = 2g(x) + 1 = x^3 \Leftrightarrow 2g(x) = x^3 - 1$

$$g(x) = \frac{x^3 - 1}{2}$$

Exercises (تمارين)

1) Let $f(x) = -3x + 2$ find the fun. $g(x)$ in which

$(g \circ f)(x) = x$.

2) Find the domain and rang of the following fun.s

a) $y = x^2 + 1$ d) $y = \sqrt{8 - \sqrt{x}}$

b) $y = \sqrt{6 - x}$ e) $y = \frac{x-1}{x}$

c) $y = \frac{1}{\sqrt{x}}$ f) $y = \frac{1-|x|}{|x|}$

3) Find $f + g$, $f - g$, $f . g$, $\frac{f}{g}$, $\frac{g}{f}$ and its domain

a) $f(x) = \frac{x}{2}$, $g(x) = \sqrt{x + 1}$

b) $f(x) = \frac{1}{x-2}$, $g(x) = \frac{1}{\sqrt{x-1}}$

4) Find $f \circ g$ and $g \circ f$ and its domain of

a) $f(x) = \sqrt{2 - x}$ $g(x) = \sqrt{x - 2}$

b) $f(x) = \sqrt{3 + x^2}$ $g(x) = \frac{1}{x}$