Some Rules of Derivatives

1. If $y = f(x) = c$, where c is constant, then $\frac{dy}{dx} = 0$
2. Power rule for positive integer (power of x) $y = x^n$ if n is positive integer then $\frac{d}{dx}(x^n) = nx^{n-1}$
3. The constant multiple rule.

$$\frac{d}{dx}\big(cf(x)\big) = c\frac{df(x)}{dx} = cf'(x)$$

4. The sum rulr

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx} = u' + v'$$

5. The product rule

$$\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

6. Positive integer power of a diff. fun.
   If u is a diff. fun., n is power of u then
   $\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$
7. Negative integer power of a diff. fun.
   If u is a diff. fun., n is power of u then
   $\frac{d}{dx}u^{-n} = -nu^{-n-1}\frac{du}{dx}$

8. The quotient rule.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Ex.s

1) If $y = 5$ then $\frac{dy}{dx} = 0$

2) If $y = x$ then $\frac{dy}{dx} = 1x^{1-1} = 1$

3) If $y = -8x^4$ then $\frac{dy}{dx} = -8(4x^3) = -32x^3$

4) If $y = -2x^5 - 3x^2 + 6$ then $\frac{dy}{dx} = -10x^4 - 6x + 0$

$= -10x^4 - 6x$

5) If $y = (x^2 + 2)(x^3 + 3x + 1)$ then

$\frac{dy}{dx} = (x^2 + 2)(3x^2 + 3) + (x^3 + 3x + 1)(2x)$

6) If $y = \left(x^3 - \frac{x}{2}\right)^6$ then $\frac{dy}{dx} = 6\left(x^3 - \frac{x}{2}\right)^5 (3x^2 - \frac{1}{2})$

7) If $y = \frac{2x^3+3x-1}{x^2-1}$ then $\frac{dy}{dx} = \frac{(x^2-1)(6x^2+3)-(2x^3+3x-1)(2x)}{(x^2-1)^2}$

8) If $y = (2x^2 - 5x^{-2})^{-5}$ then

$\frac{dy}{dx} =$ -5$(2x^2 - 5x^{-2})^{-6}(4x + 10x^{-3})$

Implicit Differentiation (الأشتقاق الضمني)

Ex. If $y^2 = x$ Find $\frac{dy}{dx}$

$2y\frac{dy}{dx} = 1 \qquad \therefore \frac{dy}{dx} = \frac{1}{2y}$

Ex. Find the derivative of the implicit fun. $x^3 - xy + y^3 = 1$

Sol.

$3x^2 - \left(x\frac{dy}{dx} + y.1\right) + 3y^2\frac{dy}{dx} = 0$

$3x^2 - x\frac{dy}{dx} - y + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$

$\frac{dy}{dx} = \frac{y-3x^2}{3y^2-x}$

Ex.

If $x = y\sqrt{1-y^2}$ Find $\frac{dy}{dx}$?

Sol.

$$\frac{dx}{dy} = \frac{1}{2}y(1-y^2)^{-\frac{1}{2}}(-2y) + \sqrt{1-y^2}$$

$$= \frac{-y^2}{\sqrt{1-y^2}} + \sqrt{1-y^2} = \frac{-y^2+1-y^2}{\sqrt{1-y^2}} = \frac{1-2y^2}{\sqrt{1-y^2}}$$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{1-2y^2}{\sqrt{1-y^2}}} = \frac{\sqrt{1-y^2}}{1-2y^2}$$

Derivatives of higher order

$y' = \frac{dy}{dx}$ First derivative

$y'' = \frac{dy'}{dx'} = \frac{d}{dx}(\frac{dy}{dx})$ Second derivative

$y^n = f^n(x) = \frac{dy^n}{dx^n}$

Ex. If $y = f(x) = x^4 - 3x^3 + 1$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = 4x^3 - 9x^2 = y^-$ and $\frac{d^2y}{dx^2} = 12x^2 - 18x = y''$

Ex. If $y = 3x^4 - 5x^3 + 6x - 7$ find $\frac{d^4y}{dx^4}$

Sol.

$$\frac{dy}{dx} = 12x^3 - 15x^2 + 6$$

$$\frac{d^2y}{dx^2} = 36x^2 - 30x$$

$$\frac{d^3y}{dx^3} = 72x - 30$$

$$\frac{d^4y}{dx^4} = 72$$

Ex. Find y^n to $y = \frac{(x+1)}{(x-1)}$ if we use $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Sol.

$$y = \frac{(x-1)+2}{(x-1)} = 1 + \frac{2}{(x-1)} = 1 + 2(x-1)^{-1}$$

$$y' = -2(x-1)^{-2}$$

$$y'' = 4(x-1)^{-3}$$

$$y''' = -12(x-1)^{-4}$$

$$y'''' = 48(x-1)^{-5}$$

$$y^n = (-1)^n 2(n!)(x-1)^{-(n+1)} = (-1)^n (n!)\frac{2}{(x-1)^{(n+1)}}$$

1) Ex. find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 7$

Sol.

To start , we differentiate both side of the eq. w.r.t. x to find $y' = \frac{dy}{dx}$

$$6x^2 - 6y\frac{dy}{dx} = 0$$

$\frac{dy}{dx} = \frac{x^2}{y}$ when $y \neq 0$

We now apply the Quotient Rule to find y''

2) $y'' = \frac{d^2y}{dx^2}$

$$= \frac{d}{dx}(y') = \frac{y(2x) - x^2y'}{y^2} = \frac{2x}{y} - \frac{x^2y'}{y^2}$$

Finally, we substitute $y' = \frac{x^2}{y}$ in the eq.

$y'' = \frac{2x}{y} - \frac{x^4}{y^3}$ when $y \neq 0$

Rule9:- power rule for fractional exponents

If U is a differentiable function of x and p and q are integers with $q > 0$ then

$$\frac{d}{dx}U^{\frac{p}{q}} = \frac{p}{q}U^{\frac{p}{q}-1}\frac{du}{dx} \quad \text{provided } U \neq 0$$

Ex.s

a) $y = \sqrt{x} = x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ when $x > 0$

b) $y = (3x^{\frac{1}{2}} + 5)^{-\frac{1}{4}}$

$$\frac{dy}{dx} = -\frac{1}{4}\left(3x^{\frac{1}{2}} + 5\right)^{-\frac{5}{4}}\left(\frac{3}{2}x^{-\frac{1}{2}}\right)$$

The Chain Rule (قاعدة السلسلة)

(Short form)

If y is a differentiable function of x and x is differentiable function of t , then y is differentiable function of t and $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

Ex.

find $\frac{dy}{dt}$ at $t = -1$ if $y = x^3 + 5x - 4$ and $x = t^2 - 1$ Sol.

At $t = -1 \Rightarrow x = 0$

$$\left.\frac{dy}{dt}\right|_{t=-1} = \left.\frac{dy}{dx}\right|_{x=0} \cdot \left.\frac{dx}{dt}\right|_{t=-1} = (3x^2 + x).2t = 5(-2) = -10$$

The Chain Rule

(First form)

Suppose that $h = g \circ f$ is the composite of differential function $y = g(x)$ and $x = f(t)$, then h is a differential function of t whose derivative at each value of t is

$$\underset{\text{at } t}{(g \circ f)'} = \underset{\text{at } x = f(t)}{g'} \ . \ \underset{\text{at } t}{f'}$$

In short, at each value of t

$$h'(t) = g'\big(f(t)\big).f'(t)$$