UNIVERSITY OF ANBAR

كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

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اسم المحاضرة الأولى باللغة الإنكليزية: **Axioms of real numbers**

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Mathematical Analysis

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Axioms of real numbers

1. The axioms arithmetics
2. The axioms of ordered
3. The complete Axioms

* Let R be a real number and $a, b, c \in R$. Then

$A_1 : \forall a, b, c \in R \;\; a + (b + c) = (a + b) + c.$

$A_2 : a + b = b + a$

$A_3 : for\ any\ a \in R, \exists!\ element\ 0 \in R\ \ s.t$

$a + (-a) = -a + a = 0$

$A_4 : Ther\ exists\ an\ element\ 0 \in R\ , S.t$

$a + 0 = 0 + a = a$

Then $(R, +)$ is a commutative group.

$A_5 : a.(b.c) = (a.b).c$

$A_6 : a.b = b.a$

$A_7 :$ $\exists!$ Element in $R(1 \in R\ )\ s.t\ a.1 = 1.a = a$

$A_8 :$ $\forall a \in R\ , \exists!\ a^{-1} \in R\ , s.t\ a.a^{-1} = a^{-1}.a = 1$

Form $A_5 \rightarrow A_8$. $(R, .)$ commutive ring

$A_9 : a.(b + c) = (a.b) + (a.c)$

$A_1 \rightarrow A_9$ $(R, +, .)$ Is a field

* Subtraction $a - b = a + (-b)\ , \forall a, b \in R$
* Division $a \div b = a.b^{-1} \ni b \neq 0$

The Axioms of order:

$A_{10}: a \leq b\ or\ b \leq a$

$A_{11}: a \leq b\ and\ b \leq c \rightarrow a = b$

$A_{12}: a \leq b\ and\ b \leq c \rightarrow a \leq c$

$A_{13}: a \leq b\ , c \in R \rightarrow a + c \leq b + c$

$A_{14}: a \leq b\ , c\ is\ not\ negative \rightarrow a.c < -b.c$

$A_1 \rightarrow A_{14}\ , (R, +\ , ., \leq)$ order field.

Remark 1.1:

$R^+ = \{x \in R\ ; x > 0\}$

$R^- = \{x \in R\ ; x < 0\}$

Propositions 1.1: Let $(R, +, .)$ be a field, then prove the following

1. $\forall a, b, c \in R\ , if\ a + b = b + c\ , then\ a = c$
2. $\forall a, b, c \in R\ , if\ a.b = c.b\ \ , then\ a = c$
3. $\forall a, b \in R$,prove that:

1. $-(-a) = a$

2. $(a^{-1})^{-1} = a$
3. $(-a) + (-b) = -(a + b)$
4. $(-a).b = -a, b$
5. $if\ a.b = 0$ then either $a = 0\ or\ b = 0$

Proof (5):

Let $a \neq 0$, T.P $b = 0$

Since $\neq 0$, then $\exists a^{-1} \in R\ s.t\ \ a.a^{-1} = 1$

$a^{-1}(a.b) = 0$

$(a^{-1}.a).b = 0$

$1.b = 0 \rightarrow b = 0$

Let $b \neq 0$, T.P $a = 0$

Since $\neq 0$, then $\exists b^{-1} \in R\ s.t\ \ b.b^{-1} = 1$

$(a.b)b^{-1} = 0$

$a.(b.b^{-1}) = 0$

$a.1 = 0 \rightarrow a = 0$

Absolute Value:

let $a \in R$, the absolute value of a is:

$$|a| = \begin{cases} a & if\ a > 0 \\ 0 & if\ a = 0 \\ -a & if\ a < 0 \end{cases}$$

$|a|: R \rightarrow R^+ \cup \{0\}$ is the function of absolute value.

Properties of absolute value.

Theorem 1.1: let a be a real number, then

1. $|x| < a \ \leftrightarrow -a < x < a$
2. $|X| > a \ \leftrightarrow x > a\ or\ x < -a$

Corollary 1.2: let $a \in R^+$ and $b \in R$, then

1. $|x - b \leq a\ iff\ b - a \leq x \leq b + a$
2. $|x - b| \geq a\ iff\ x \geq b + a\ \ or\ \ x \leq b - a$

Let $a, b \in R$ and k be areal number, then

1. $|a| \geq 0$
2. $|a| = 0$ iff $a = 0$
3. $a^2 = |a|^2$
4. $|ab| = |a|.|b|$
5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
6. $|ka| = |k|.|a|$

Example 1.1: $\forall a \in R, \sqrt{a^2} = |a|$

Proof:

If $a > 0$ then $\sqrt{a^2} = a$

If $a < 0$ then $\sqrt{a^2} = -a$

by def absolute value to a we have

$$|a| = \begin{cases} a = \sqrt{a^2} \ \ if \ \ a \geq 0 \\ -a = \sqrt{a^2} \ \ if \ \ a < 0 \end{cases}$$

وفي كلتا الحالتين يكون لدينا $|a| = \sqrt{a^2}$

The triangle inequality

Theorem 1.3: if $a, b \in R$, then $|a + b| \leq |a| + |b$

Proof:

$$|a + b|^2 = (a + b)^2 \leq a^2 + 2ab + b^2$$
$$\leq |a|^2 + 2|ab| + |b|^2$$
$$\leq (|a| + |b|)^2$$
$$\therefore |a + b| \leq |a| + |b|$$

Corollary 1.4: if $a, b \in R$, then $|a - b| \geq |a| - |b|$

Definition 1.1: let $S \subset R$ S is said to be bounded above if there is some real numbers m s.t $x \leq m \ \forall x \in S$, m is called upper bounded of S