كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

أستاذ المادة : م.د. ناديه علي ناظم

اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة الثانية باللغة العربية: بعض النظريات حول الاعداد الحقيقية

اسم المحاضرة الثانية باللغة الإنكليزية: **some theorems of real numbers**

...

Mathematical Analysis

Dr. Nadia Ali

Teaching at the University of Anbar
College of Education for Pure Sciences
Department of Mathematics

2022 – 2023

Proposition 1.2:

If $\emptyset \neq S \subset R$ and $\sup(S) = M$, then $\forall p < M \;\; \exists x \in S$ s.t $p < x \leq M$
i.e.: if $\sup(S) = M$ then $\forall \epsilon > 0 \,, \exists x \in S$ s.t $M - \epsilon < x \leq M$

proof:

let $\sup(S) = M$ then $\forall x \in S, \;\; x \leq M$

T.P $\;\; \forall x \in S \,, p < x$?

Suppose that $x \leq p \,, \forall x \in S$

$\rightarrow$ p is upper bounded for S, but by hypothesis $p < M = \sup(S)$

………. C!

$\therefore \;\; \exists \, x \in S \ni p < x \leq M.$

Theorem 1.5: The set N of natural numbers is unbounded above in R

Proof:

Suppose N is bounded above.

By completeness axiom

N has a supreme M

Let $\sup(N) = M$

From proposition above $\exists n \in N$ s.t $M - 1 < n < M.$

Then $M - 1 < n \rightarrow M < n + 1$,

But $n + 1 \in N$

And $n + 1 > M = \sup(N) \rightarrow C!$

Therefore, N is unbounded above

Theorem 1.6: Archimedan property

If $x \in R^{++}$ then for any $y \in R$, there exists $n \in N$ s.t $n > y$

Detention 1.2: let F a field, F is called Archimedean filed, if for any $x \in F, \exists n \;\; \in N$ s.t $n > x$

i.e.: N is abounded above in F

Example 1.1:

1. R is Archimedean field
2. Q is Archimedean field
3. $s = \{a + b\sqrt{2} : a, b \in Q\}$ is Archimedean field

Theorem 1.7: Denseness property

Between any two distinct reals, there exists infinitely many rationales and irrationals

Detention 1.3: (irrational numbers Q')

Let Q' be a complement of Q in the real number R.

i.e.: $Q' = R - Q$, we called is set of irrational numbers

remark: $R = Q \cup Q'$

Theorem 1.8: prove that $\sqrt{2}$ is irrational number

i.e.: There are no rational numbers whose square is 2

i.e.: $\nexists\ x \in Q \ni x^2 = 2$

proof:

suppose $\sqrt{2}$ is rational number i.e. $\sqrt{2} = \frac{m}{n}$

So $2 = \frac{m^2}{n^2}$, then $m^2 = 2n^2$

Case 1:

m and n are odd.

Since m is odd $\rightarrow m^2$ is odd

Since n is odd $\rightarrow n^2$ is odd

But $2n^2$ is even $\rightarrow\ m^2 = 2n^2 \rightarrow C!$

Case 2:

m is even and n is odd, then $m = 2p$

and $m^2 = 4p^2$,$\rightarrow 4p^2 = 2n^2 \rightarrow 2p^2 = n^2 \rightarrow C!$

Case 3:

m is odd and n is even, then, since m is odd

$\rightarrow m^2$ is odd, and $2n^2$ is even $\rightarrow m^2 = 2n^2 \rightarrow C!$

$\therefore\ \sqrt{2}$ is irrational number

Theorem 1.9: Q is not Complete field

Theorem 1.10: for every real $x > 0$ and every integer $n > 0$ there is one and only one positive real y such that $y^n = x$

i.e.:$\forall x > 0 , \forall n \in N , \exists! , y \in R^+ \ s.t\ y = \sqrt[n]{x}$

Theorem 1.11: if $\frac{m}{n}$ and $\frac{p}{q}$ are rationales and $q \neq 0$ then $\frac{m}{n} + \sqrt{2}\frac{p}{q}$ is irrational number

Proof:

Suppose $\frac{m}{n} + \sqrt{2}\,\frac{p}{q}$ is rational

Then there is $r, s \in Z\,, s \neq 0\ s.t\ \frac{m}{n} + \sqrt{2}\frac{p}{q} = \frac{r}{s}$

So $\sqrt{2}\frac{p}{q} = \frac{r}{s} - \frac{m}{n} \rightarrow\ \sqrt{2} = \frac{p}{q}\left(\frac{rn-sm}{sn}\right) \in Q$

So $2 = \left(\frac{q(nr-sm)}{psn}\right)^2$ $\rightarrow !$ with theorem: $\nexists\ x \in Q \ni x^2 = 2$

Theorem 1.12: Between any two distinct rationales there is an irrational number.

Example 1.2:

1. Prove $x^2 \geq 0\,, \forall\ x \in R$
2. Let a, b be tow real s.t $a \leq b + \epsilon\ \forall\ \epsilon > 0$ then $a \leq b$

   Proof (2):

   Suppose $a > b$

   Then $a + a > b + a$

   $\frac{2a}{2} > \frac{b+a}{2}$

   $a > \frac{b+a}{2}$ ………….(1)

   Take $\epsilon = \frac{a-b}{2} > 0$ (Since $> b$, then $a - b > 0 \rightarrow \frac{a-b}{2} > 0$)

   $a \leq b + \epsilon \rightarrow a \leq\ b + \frac{a-b}{2} = \frac{2b+a-b}{2} = \frac{a+b}{2} < a$

   From (1) ……………. C!

   $a \leq b$

Example 1.3:

1. Q is order field $(A_1 \rightarrow\ A_{14})$

2. C is field but not order
since: if $x = 1 \rightarrow x = \sqrt{1} \rightarrow x^2 = -1 < 0 \rightarrow C!$
since: $(x^2 \geq 0 , \forall x \in R)$