كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة الرابعة باللغة العربية: بعض الخواص والنظريات حول المجاميع المفتوحة

اسم المحاضرة الرابعة باللغة الإنكليزية: **some properties and theorems about open sets**

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Mathematical Analysis

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$[\boldsymbol{a}, \boldsymbol{b}], [\boldsymbol{a}, \boldsymbol{b}), [\boldsymbol{a}, \infty)$ $\boldsymbol{and}$ $(-\infty, \boldsymbol{b}]$ are not open set

Proof:

If S=[a,b] ,then S is not open set ?

Since, if $x = a \rightarrow \forall r > 0 \,, B(a, r) = (a - r, a + r) \not\subset [a, b]$

The intersection of any tow open set is open set

i.e ((the intersection of any finite family of open set is open))

Proof:

Let $A = \{ S_k : S_k \; is \; open \; set \; k = 1,2, \dots, n\}$

$T.p \; \bigcap_{k=1}^{n} S_k$ is open set

Let $x \in \bigcap_{k=1}^{n} S_k \rightarrow x \in S_k, \forall k$, but S_k is open set $\forall k$, then $\exists r_k > 0$ s.t $B(x, r_k) \subset S_k$

Let $r = \min \{r_1, r_2, \dots, r_n\}$

Then $B(x, r) \subset S_k, \forall k$.

$\therefore B(x, r) \subset \bigcap_{k=1}^{n} S_k$, therefore $\bigcap_{k=1}^{\infty} S_k$ is open set.

Theorem 2.1: the infinite intersection of open sets is not necessary open set.

Ex: let $S_n = \left(x - \frac{1}{n}, x + \frac{1}{n}\right)$ $\forall x \in R$, open interval.

$n = 1 \rightarrow s_1 = (x - 1, x + 1)$

$n = 2 \rightarrow S_2 = (x - \frac{1}{2}, x + \frac{1}{2})$

$n = 3 \rightarrow S_3 = (x - \frac{1}{3}, x + \frac{1}{3})$

. .

. .

When $n \rightarrow \infty \; \bigcap_{k=1}^{\infty} S_k = \{x\}$ is not open

Theorem 2.2: the union of any family (finite or infinite) – (countable or uncountable) of open set is open

Proof:

Let $A = \{S_\alpha , S_\alpha \text{ } is \text{ } open \text{ } set \text{ } \alpha \in \wedge\}$

T.P: $\bigcup_{\alpha\in\wedge} S_\alpha$ is open set

Let $x \in \bigcup_{\alpha\in\wedge} S_\alpha \rightarrow \exists\alpha \in\wedge s.t \; x \in S_\alpha$

Since S_α is open set $\rightarrow \exists\alpha > 0 \; s.t$

$B(x, r_\alpha) \subset S_\alpha$, then $x \in B(x, r_\alpha) \subset S_\alpha \subset \bigcup_{\alpha\in\wedge} S_\alpha$

This is true $\forall x \in \bigcup_{\alpha\in\wedge} S_\alpha$, therefore $\bigcup_{\alpha\in\wedge} S_\alpha$ is open set

Theorem 2.3: S is open iff S is the Union of balls

Definition 2.4: let X be anon-empty set and τ is a family of subsets of X, if τ satisfy the following

1- $\phi , X \in \tau$

2- If $G , H \in \tau \rightarrow G \cap H \in \tau$

3- If $\{G_\lambda\} \in \tau \rightarrow \bigcup_{\lambda\in\wedge} G_\lambda \in \tau$

Then, the order pair (X, τ) is called topological Space.

Theorem 2.4: every metric space is topological space.

Proof:

Let (X, d) be a metric space and $\tau =$ the family of all open subsets of X, then

1- ϕ , X open sets $\rightarrow \phi, X \in \tau$

2- $G_1 , G_2 \in \tau \rightarrow G_1, G_2$ are open sets

$\rightarrow G_1 \cap G_2 \in \tau$

3- If $G_\lambda \in \tau , \lambda \in\wedge \rightarrow \forall\lambda , G_\lambda$ open subset of X

$\rightarrow \bigcup_{\lambda\in\wedge} G_\lambda$ open set of

$\rightarrow \bigcup_{\lambda\in\wedge} G_\lambda \in \tau$

$\therefore (X, \tau)$ is a topological space

Definition 2.5: let d_1 and d_2 be two metric mapping in the set X, then d_1, d_2 are called Equivalent if every open set in (X, d_1) is open in (X, d_2) and Vice Versa

Definition 2.6: let (X, d) be a metric space and $S \subseteq X$, S is called closed set if S^c is open Set where $S^c = X - s$ (Complement of S)

Example 2.5:

1- $S = X$ is closed set.
Solution:
Since $S^c = X^c = \phi$ open set

2- $S = \phi$ is closed set
Solution:
since $S^c = \phi^c = X$ is open set

3- $S = [a, b], [a, b), S = (-\infty, b]$ are closed set in R
Solution:
if $S = [a, b] \to S^c = (-\infty, a) \cup (b, \infty)$ open set $\to S$ is closed set

4- In R , $S = \{x\}$ is closed set
Since :
$S^c = (-\infty, x) \cup (x, \infty) \to S^c$ is open, So S is closed set.

5- Any finite set in R is closed set
Solution:
let $S = \{x_1, x_2, \dots, x_n\} \subseteq R$.
$S^c = (-\infty, x_1) \cup (x_1, x_2) \cup \dots \cup (x_{n-1}, x_n) \cup (x_n, \infty)$
So, S^c is open , then S is closed set

6- If $S = N$, $S = Z$, then S is Closed set
Solution:
let $S = N$
then $S^c = (-\infty, 1) \cup (1,2) \cup (2,3) \dots (\cup_{n=4}^{\infty}(n, n+1))$
$\to S^c$ is open $\to$ S is closed
if $S = Z \to S^c = (\cup_{n=1}^{\infty}(-(n+1), -n)) \cup (-1,0) \cup (0,1) \cup (\cup_{n=1}^{\infty}(n, n+1))$

S^c is open, then S is closed

7- The Union of finite number of closed sets is closed.
Solution:
let $A = \{S_i, ;\ S_i\ closed\ set\ in\ X\ , i = 1,2, \dots , n\}$
T.P: $\bigcup_{i=1}^{n} S_i$ is closed set
i.e. T.P $(\bigcup_{i=1}^{n} S_i)^c$ is open set
Since S_i is closed, $\forall i$ then S_i^c is open $\forall i$
and $\bigcap_{i=1}^{n} S_i^c$ is open
So, $(\bigcup_{i=1}^{n} S_i)^c$ is open $\qquad ((\bigcup_{i=1}^{n} S_i)^c = \bigcap_{i=1}^{n} S_i^c)$
therefore $\bigcup_{i=1}^{n} S_i$ is closed.