كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة التاسعة باللغة العربية: المتسلسلات اللانهائية

اسم المحاضرة التاسعة باللغة الإنكليزية: **Infinite sequences**

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Mathematical Analysis

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Chapter four

Infinite Series

Definition 3.1: Let $\langle x_n \rangle$ be a real seq the series of the form, if $x_1 + x_2$ then itis Called infinite series, and it is written as $\sum_{n=1}^{\infty} x_n$.

If the series of the form $x_1 + x_2 + \cdots + x_n$, then itis Called finite Series.and written as $\sum_{k=1}^{n} x_k$

Definition 3.2: Let $\sum_{n=1}^{\infty}$ an be a finite series, the seq $\langle S_n \rangle$ is called the sequence of Partial sums of $\sum_{n=1}^{\infty} \mathrm{a_n}$

where $S_1 = a_1$

$$
\begin{aligned}
&S_2 = a_1 + a_2 \\
&S_3 = a_1 - a_2 + a_3 \\
&\vdots \\
&S_n = a_{1+}a_2 + \cdots + a_n
\end{aligned}
$$

Definition 3.3: let $\sum_{n=1}^{\infty} a_n$ be infinite series, then it is said toble

1. Converge, if $\langle S_n \rangle$ converge
2. diverge, if $\langle s_n \rangle$ diverge.
3. If ⟨ Sn ⟩ Converge to b. then $\sum_{n=1}^{\infty} a_n = S_n$.

Example 3.1:

let $a_n = 1, \forall n$, then

$\sum_{n=1}^{\infty} a_n = 1 + 1 + 1 + \cdots$

$S_1 = a_1 = 1$

$S_2 = a_1 + a_2 = 1 + 1 = 2$

$S_3 = a_1 + a_2 + a_3 = 1 + 1 + 1 = 3$

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$S_n = a_1 + a_2 + a_3 + \cdots + a_n = 1 + 1 + 1 + \cdots + 1 = n$
The seq of partial sum is $\langle S_n \rangle = \langle n \rangle$ is divergent Since it is unbounded
$\Rightarrow \sum_{n=1}^{5}$ an is diverge.

Example 3.2: let $\sum_{n=1}^{\infty} a_n = 3 - 3 + 3 - 3 + \cdots$

$S_1 = a_1 = 3$
$S_2 = a_1 + a_2 = 3 - 3 = 0$
$S_3 = \mathrm{a}_1 + \mathrm{a}_2 + a_3 = 3 - 3 + 3 = 3$
$\vdots$

$$S_n = a_1 + a_2 + \cdots + a_n = \begin{cases} 3 & , if\ n\ odd \\ 0 & , if\ n\ even \end{cases}$$

The Sequence of partial Sum $\langle S_n \rangle$ is divergent

$\therefore \sum_{n=1}^{\infty} a_n$ is divergent

Example 3.3:
Let: $\sum_{n}^{\infty} a_n = 2 + 4 + 2 + 4 + 24$
$n = 1$
$S_1 = a_1 = 2$
$S_2 = a_1 + a_2 = 2 + 4 = 6$
$S_3 = a_1 + a_2 + a_3 = 2 + 4 + 2 = 8$
$Sn = a_1 + a_2 + a_3 + a_4 = 2 + 4 + 2 + 4 + \cdots = ?$
The sequence of partial sums $\langle S_n \rangle$ is unbownded, then $\langle s_n \rangle$ is diverg ent
so $\sum_{n=1}^{\infty} a_n$ is diverge

Exercises

let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

Harmonic Series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \text{ divergent}$$

proof:

$S_1 = a_1 = 1$

$s_2 = a_1 + a_2 = 1 + \frac{1}{2}$

$S_3 = a + a_2 + a_3 = 1 + \frac{1}{2} + \frac{1}{3}$

$S_n = a_1 + a_2 + \cdots + a_{n-1} + \frac{1}{2} + \cdots + \frac{1}{n}$

$S_{n+1} = a_1 + a_2 + \cdots + a_n - a_{n-1} = 1 + \frac{1}{2} + \cdots + \frac{1}{n} + \frac{1}{n+1}$

$S_{n+n} = \frac{1}{2n} + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$

let $m = 2n$

$$\begin{aligned}
(S_m - S_n) \quad &= \left| \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} + \cdots + \frac{1}{2n}\right) - \left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right) \right| \\
&= \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \\
&> \frac{1}{2n} + \frac{1}{2n} + \cdots + \frac{1}{2n} \\
&= n \cdot \frac{1}{2n} = \frac{1}{2}
\end{aligned}$$

If $\epsilon = \frac{1}{2}$, then $|S_m - S_n| > \epsilon$

$\therefore \langle S_n \rangle$ is not Caushy sequence $\Rightarrow \langle S_n \rangle$ is not Convergent.

So $\sum_{n=1}^{2} a_n$ is diverge.