

كلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم المحاضرة العاشرة باللغة العربية: بعض انواع المتسلسلات

اسم المحاضرة العاشرة باللغة الإنكليزية: **some types of sequences**

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Mathematical Analysis

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GEOMETRIC SERIES

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \dots$$

where $a > 0$, r is called the base of Series. the sequence of partial, Sum is

$$s_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$$

(1) if $|r| = 1$

$$\therefore S_n = a + a^{-1} + a + \dots + a = n \cdot a$$

$\langle S_n \rangle = \langle n_a \rangle$ diverg $\Rightarrow \sum_{n=1}^{\infty} ar^{n-1}$ diverge.

(2) if $|r| > 1$

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + \dots + ar^n$$

$$\rightarrow S_n - rS_n = a - ar^n$$

$$S_n(1 - r) = a(1 - r^n)$$

$$\therefore S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\begin{aligned} \text{When } n \rightarrow \infty \Rightarrow \lim_{n \rightarrow \infty} S_n &= \lim_{n \rightarrow \infty} \frac{a(1-r^n)}{(1-r)} \\ &= \frac{a(1-0)}{1-r} = \frac{a}{1-r} \end{aligned}$$

$$\therefore \sum ar^{n-1} = \frac{a}{1-r} \text{ .Converge}$$

(3) if $|r| > 1$

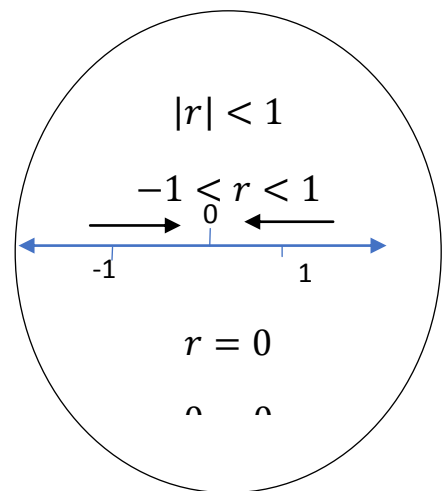
$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\text{when } n \rightarrow \infty; r^n = \mp \infty \Rightarrow S_n \rightarrow \infty$$

$\therefore S_n$ diverge.

$\therefore \sum_{n=1}^{\infty} ar^{n-1}$ diverge.

$$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \text{diverge} & \text{if } |r| > 1 \\ \text{Ganverge} & \text{if } |r| < 1 \\ & = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \end{cases}$$



Example 3.4:

Let $\sum_{n=1}^{\infty} a_n = 1 + \frac{5}{2} + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^3 + \dots$ Geometric series.

$$a = 1, v = \frac{5}{2} \Rightarrow |r| = \left|\frac{5}{2}\right| = \frac{5}{2} > 1$$

$\therefore \sum_{n=1}^{\infty} a_n$ an diverge:

$\sum_{n=1}^{\infty} a_{n+1} = 1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \dots$ Geometric Series.

$$\sum_{n=1}^{\infty} ar^{n-1} = 1 + \left(\frac{-3}{4}\right) + \left(\frac{-3}{4}\right)^2 + \left(\frac{-3}{4}\right)^3 + \dots$$

$$a = 1, v = \frac{3}{4} \Rightarrow$$

$$|r| = \left|-\frac{3}{4}\right| = \frac{3}{4} < 1$$

$\therefore \sum_{n=1}^{\infty} a_n$ is Converger and

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} = \frac{15}{1+\frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$$

Theorem 3.1: If $\sum_{n=1}^{\infty} a_n$ an Convergent, then $\lim_{n \rightarrow \infty} a_n = 0$
(that is, $\forall \epsilon > 0, \exists k \in \mathbb{N}, s. t. |a_n - 0| < \epsilon, \forall n > k$)

Proof:

Suppose $S_n = a_1 + a_2 + \dots + a_n$

$\sum_{n=1}^{\infty} a_n$ an convergent, then $\langle S_n \rangle$ Convergent

$\Rightarrow \langle S_n \rangle$ canshy sequence.

$\therefore \forall \epsilon > 0, \exists k \in \mathbb{N}$, sit $|S_m - s_n| < \epsilon, \forall n, m > k$

let $m = n + 1$

So $|S_m - S_n| < \epsilon \rightarrow |S_{n+1} - S_n| = |a_{n+1}| < \epsilon, \forall n > k$

$\rightarrow |a_n| < \epsilon, \forall n > k$, So $|a_n - 0| < \epsilon, \forall n > k$.

then $\lim_{n \rightarrow \infty} a_n = 0$

Example:

$$\langle a_n \rangle = \left\langle \frac{1}{n} \right\rangle \rightarrow 0$$

and $\lim_{n \rightarrow \infty} a_n = 0$ bul $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diveroe

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Corollary:

If $\lim_{n \rightarrow \infty} a_n \neq 0$ diverge.

proof:

Suppose that $\sum_{n=1}^{\infty} a_n$ Convergent.

then, $\lim_{n \rightarrow \infty} a_n = 0$, by theorem, $\rightarrow C!$

Example 3.5:

$$\sum_n^{\infty} a_n = \sum (\sqrt{m} - \sqrt{n-1})$$

$\sum_{n=1}^{\infty} a_n$ Diverge, but $\lim_{n \rightarrow \infty} a_n = 0$

Exercises

(1) $\sum_{n^3}^{\infty} \frac{1}{\sqrt{n}}$

$$(2) \sum_{n=1}^{\infty} a_n \sqrt{\frac{n}{3n+5}}$$

$$(3) \sum_{n=1}^{\infty} \frac{n^3+2}{2n(n+5)}$$

Theorem 3.2: If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are Convergent Series and $k \in \mathbb{R}$, then

$$(1) \sum_{n=1}^{\infty} (a_n + b_n) \text{ convergent and } \sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$$

$$(2) \sum_{n=1}^{\infty} k a_n \text{ convergent and } \sum_{n=1}^{\infty} k a_n = k \sum_{n=1}^{\infty} a_n$$

proof: (1)

let $\langle s_n \rangle$ be a sequence of partial sums of

$$\sum_{n=1}^{\infty} a_n \text{ and}$$

$\langle t_n \rangle$ be a seq of partial sum of $\sum_{n=1}^{\infty} b_n$

$\sum_{n=1}^{\infty} a_n$ convergent, so $\exists s \in \mathbb{R}$ s.t $\sum_{n=1}^{\infty} a_n = s$

and $\langle s_n \rangle \rightarrow s \Rightarrow \lim_{n \rightarrow \infty} s_n = s$.

also, $\sum_{n=1}^{\infty} b_n$ Convergent, then $\exists t \in \mathbb{R}$, s.t $\sum_{n=1}^{\infty} b_n = t$ and $\langle t_n \rangle \rightarrow t \rightarrow$

$$\lim_{n \rightarrow \infty} t_n = t$$

$\lim_{n \rightarrow \infty} (s_n + t_n) \rightarrow s + t$, but $\langle s_n + t_n \rangle$ is the seq of partial sum of

$$\sum_{n=1}^{\infty} (a_n + b_n) \rightarrow \sum_{n=1}^{\infty} a_n + b_n = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n = s + t$$

(2) let $\langle s_n \rangle$ be a seq of partial sums of $\sum_{n=1}^{\infty} a_n$ but $\sum_{n=1}^{\infty} a_n$ Convergent
 $\Rightarrow \exists s \in \mathbb{R}$ s.t $\sum_{n=1}^{\infty} a_n = s$ and $\langle s_n \rangle \rightarrow s, \lim_{n \rightarrow \infty} s_n = s$.

$$\lim_{n \rightarrow \infty} k s_n = k s, \lim_{n \rightarrow \infty} s_n = s \rightarrow \langle k s_n \rangle \rightarrow k s$$

$$\text{then } \sum_{n=1}^{\infty} k a_n = k s = k \sum_{n=1}^{\infty} a_n$$

$$\text{then } \sum_{n=1}^{\infty} k a_n = k \sum_{n=1}^{\infty} a_n$$

