Surveying 2

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المحاضرة التاسعة

Global position system (GPS)

FUNDAMENTALS OF SATELLITE POSITIONING

The precise travel time of the signal is necessary to determine the distance, or so-called range, to the satellite. Since the satellite is in an orbit approximately 20,200 km above the Earth, the travel time of the signal will be roughly 0.07 sec after the receiver generates the same signal. If this time delay between the two signals is multiplied by the signal velocity (speed of light in a vacuum) c, the range to the satellite can be determined from

$$r = c \times t \qquad (13.11)$$

where r is the range to the satellite and t the elapsed time for the wave to travel from the satellite to the receiver.

Satellite receivers in determining distances to satellites employ two fundamental methods: code ranging and carrier phase-shift measurements. Those that employ the former method are often called mapping grade receivers; those using the latter procedure are called survey-grade receivers. From distance observations made to multiple satellites, receiver positions can be calculated. Descriptions of the two methods and their mathematical models are presented in the subsections that follow. These mathematical models are presented to help students better understand the underlying principles of GPS operation. Computers that employ software provided by manufacturers of the equipment perform solutions of the equations.

Code Ranging

The code ranging (also called code matching) method of determining the time it takes the signals to travel from satellites to receivers was the procedure briefly described in Section 13.3. With the travel times known, the corresponding distances to the satellites can then be calculated by applying Equation (13.11). With one range known, the receiver would lie on a sphere. If the range were determined from two satellites, the results would be two intersecting spheres. As shown in Figure 13.8(a), the intersection of two spheres is a circle. Thus, two ranges from two satellites would place the receiver somewhere on this circle. Now if the range for a third satellite is added, this range would add an additional sphere, which when intersected with one of the other two spheres would produce another circle of intersection. As shown in Figure 13.8(b), the intersection of two circles would leave only two possible locations for the position of the receiver. A "seed position" is used to quickly eliminate one of these two intersections.

For observations taken on three satellites, the system of equations that could be used to determine the position of a receiver at station A is

$$
\begin{aligned}
\rho_A^1 &= \sqrt{(X^1 - X_A)^2 + (Y^1 - Y_A)^2 + (Z^1 - Z_A)^2} \\
\rho_A^2 &= \sqrt{(X^2 - X_A)^2 + (Y^2 - Y_A)^2 + (Z^2 - Z_A)^2} \\
\rho_A^3 &= \sqrt{(X^3 - X_A)^2 + (Y^3 - Y_A)^2 + (Z^3 - Z_A)^2}
\end{aligned}
\qquad (13.12)
$$

where ρ_A^n are the *geometric ranges* for the three satellites to the receiver at station A, (X^n, Y^n, Z^n) are the geocentric coordinates of the satellites at the time of the signal transmission, and (X_A, Y_A, Z_A) are the geocentric coordinates of the receiver at transmission time. Note that the variable n pertains to superscripts and takes on values of 1, 2, or 3.

However, in order to obtain a valid time observation, the systematic error (known as *bias*) in the clocks, and the refraction of the wave as it passes through the Earth's atmosphere, must also be considered. In this example, the receiver clock bias is the same for all three ranges since the same receiver is observing

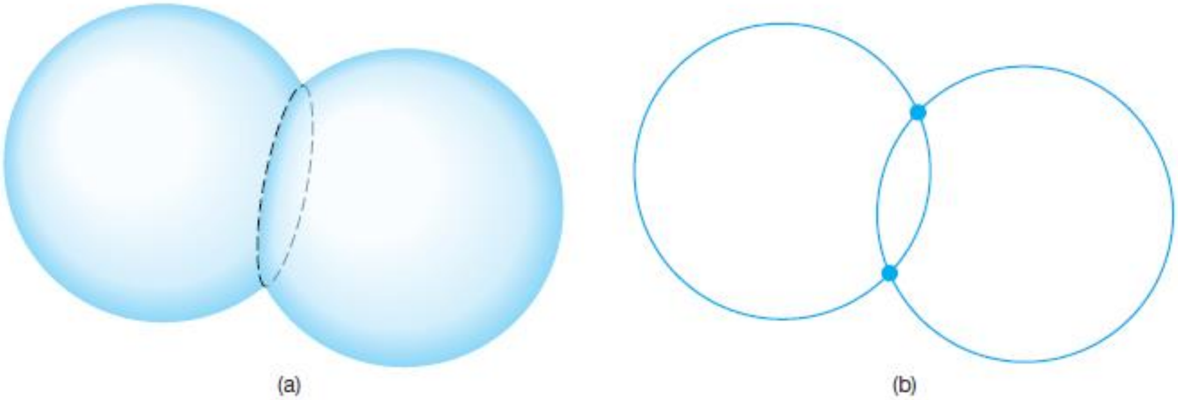

Figure 13.8 (a) The intersection of two spheres and (b) the intersection of two circles.

each range. With the introduction of a fourth satellite range, the receiver clock bias can be mathematically determined. This solution procedure allows the receiver to have a less accurate (and less expensive) clock.Algebraically, the system of equations used to solve for the position of the receiver and clock bias are:

$$\begin{aligned}
R_A^1(t) &= \rho_A^1(t) + c(\delta^1(t) - \delta_A(t)) \\
R_A^2(t) &= \rho_A^2(t) + c(\delta^2(t) - \delta_A(t)) \\
R_A^3(t) &= \rho_A^3(t) + c(\delta^3(t) - \delta_A(t)) \\
R_A^4(t) &= \rho_A^4(t) + c(\delta^4(t) - \delta_A(t))
\end{aligned} \qquad (13.13)$$

where $R_A^n(t)$ is the observed *range* (also called *pseudorange*) from receiver A to satellites 1 through 4 at epoch (time) t, $\rho_A^n(t)$ the geometric range as defined in Equation (13.12), c the speed of light in a vacuum, $\delta_A(t)$ the receiver clock bias, and $\delta^n(t)$ the satellite clock bias, which can be modeled using the coefficients supplied in the broadcast message. These four equations can be simultaneously solved yielding the position of the receiver (X_A, Y_A, Z_A), and the receiver clock bias $\delta_A(t)$. Equations (13.13) are known as the *point positioning equations* and as noted earlier they apply to code-based receivers.

THE UNIVERSAL TRANSVERSE MERCATOR PROJECTION

The Universal Transverse Mercator Projection (UTM) is a worldwide system of transverse Mercator projections. It comprises 60 zones, each 6◦ wide in longitude, with central meridians at 3◦, 9◦, etc. The zones are numbered from 1 to 60, starting with 180◦ to 174◦Was zone 1 and proceeding eastwards to zone 60. Therefore the central meridian (CM) of zone n is given by CM = 6n◦ – 183◦. In latitude, the UTM system extends from 84◦ N to 80◦ S, with the polar caps covered by a polar stereographic projection.

The scale factor at each central meridian is 0.9996 to counteract the enlargement ratio at the edges of the strips. The false origin of northings is zero at the equator for the northern hemisphere and 106 m at the equator for the southern hemisphere. The false origin for eastings is 5 × 105 m west of the zone central meridian.

2- **Carrier Phase-Shift Measurements**

Better accuracy in measuring ranges to satellites can be obtained by observing phase-shifts of the satellite signals. In this approach, the phase-shift in the signal that occurs from the instant it is transmitted by the satellite until it is received at the ground station, is observed. This procedure, which is similar to that used by EDM instruments, yields the fractional cycle of the signal from satellite to receiver. However, it does not account for the number of full wavelengths or cycles that occurred as the signal traveled between the satellite and receiver. This number is called the integer ambiguity or simply ambiguity. Unlike EDM instruments, the satellites utilize one-way communication, but because the satellites are moving and thus their ranges are constantly changing, the ambiguity cannot be determined by simply transmitting additional frequencies. There are different techniques used to determine the ambiguity. All of these techniques require that additional observations be obtained. Once the ambiguity is determined, the mathematical model for carrier phase-shift, corrected for clock biases, is

$$\Phi_i^j(t) = \frac{1}{\lambda}\rho_i^j(t) + N_i^j + f^j[\delta^j(t) - \delta_i(t)] \qquad (13.14)$$

where for any particular epoch in time, t, $\Phi_i^j(t)$ is the carrier phase-shift measurement between satellite j and receiver i, f^j the frequency of the broadcast signal generated by satellite j, $\delta^j(t)$ the clock bias for satellite j, λ the wavelength of the signal, $\rho_i^j(t)$ the range as defined in Equations (13.12) between receiver i and satellite j, N_i^j the integer ambiguity of the signal from satellite j to receiver i, and $\delta_i(t)$ the receiver clock bias.

ERRORS IN OBSERVATIONS

Electromagnetic waves can be affected by several sources of error during their transmission. Some of the larger errors include (1) satellite and receiver clock biases and (2) ionospheric and tropospheric refraction. Other errors in satellite surveying work stem from (a) satellite ephemeris errors, (b) multipathing, (c) instrument miscentering, (d) antenna height measurements, (e) satellite geometry, and (f) before May 1, 2000, selective availability. All of these errors contribute to the total error of satellite-derived coordinates in the ground stations.

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