Chapter One

Vectors

1.1 Introduction to Vectors

Many physical quantities, such as area, length, mass and temperature, are completely described once the magnitude of the quantity is given. Such quantities are called **scalars**.

Scalars:- كمية غير متجهه او عدد عادي تعرف بمقدارها وليس لها اتجاه مثل كتلة الجسم وحجمه وكثافته.

Other physical quantities, called **vectors**, are not completely determined until both a magnitude and a direction are specified. For example , wind movement , say **20 mph northeast.**

Vectors can be represented geometrically as directed line segment or **arrows** in 2-space or 3-space , the direction of the arrow specifies the direction of the vectors , and the length of the arrow describes its magnitude.

Initial point (The tail of arrow)

Terminal point (The tip of the arrow)

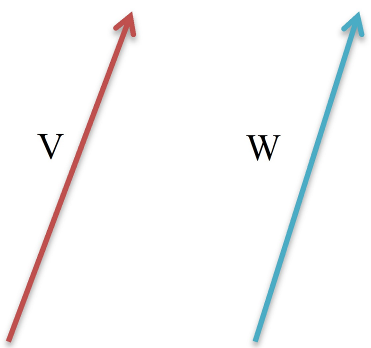
(**arrow**) قطعة من مستقيم تشير الى اتجاه معين (سهم)

If the initial point of a vectors **V** is **A** and the terminal point is **B** , we write

$$\mathbf{V}=\overrightarrow{\mathbf{AB}}$$

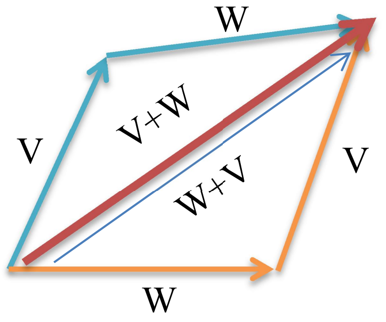
Vectors having the same length and the same direction , are called **equivalent vectors** .

V=W



1.1.1 Sum of Vectors

If **V** and **W** are any two vectors , the sum **V+W** is the vector determined as follows. The vectors **V+W** is represented by the arrow from the initial point of **V** to the terminal point of **W**

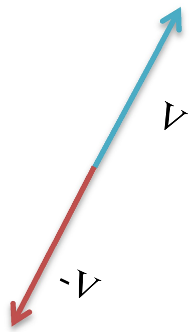


V+W = W+V

The vectors of Length **Zero** is called **zero vectors** and denoted by **O**.

O+V = V+O = V

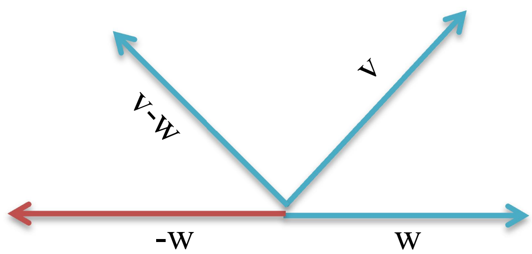
V+(-V) = O

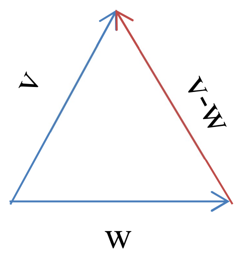


1.1.2 Subtraction of vectors

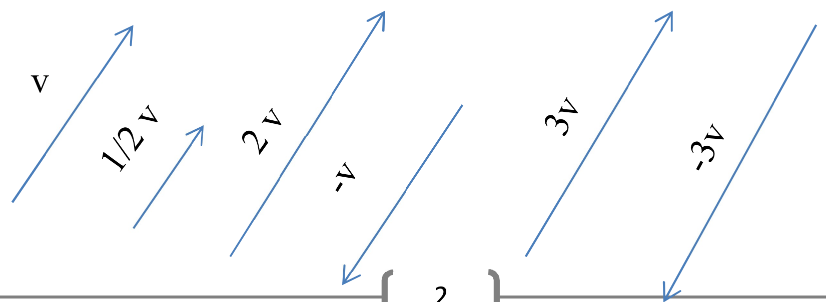
If V and W are any two vectors , then subtraction of W from V is defined by :

$$\mathbf{v-w = v + (-w\ )}$$





If V is nonzero vectors and K is nonzero scalar, then KV is defined to be the vectors whose length is $|k|$ is times the length of V and whose direction is the same



1.1.3 Components of Vectors

Let **V** be any vectors in the plane (2-space) , its initial point at the origin of rectangular (Cartesian) coordinate system . coordinates ($\mathbf{V_1}$, $\mathbf{V_2}$) of the terminal point of **V** are called **the components of V** .

$$\mathbf{V = (V_1 , V_2)}$$

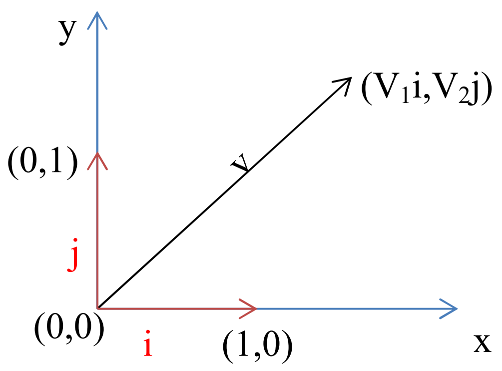
The most algebra vectors of vectors is based on representing each vectors in terms of components parallel to the cartesian coordinate axes

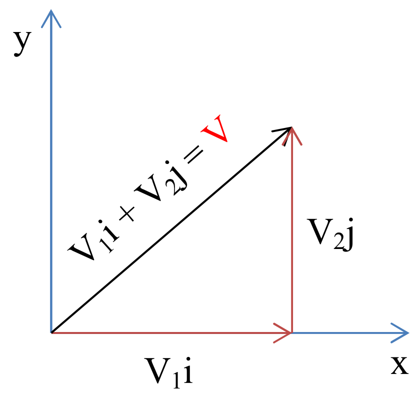
- The basic vector in the positive **X** direction is the vector (**i**) that runs from (0, 0) to (1 , 0).
- The basic vector in positive **Y** direction is the vector (**j**) that runs from (0, 0) to (0, 1).

Then …

$\mathbf{V_1 i}$ represents a vectors of length $|\mathbf{V_1}|$ parallel to the X – axis , pointing to the right if $\mathbf{V_1}$ is positive and to the left if $\mathbf{V_1}$ is negative.

$\mathbf{V_2 j}$ represent a vector of length $|\mathbf{V_2}|$ parallel to the Y – axis , pointing up if $\mathbf{V_2}$ is positive and down if $\mathbf{V_2}$ is negative .



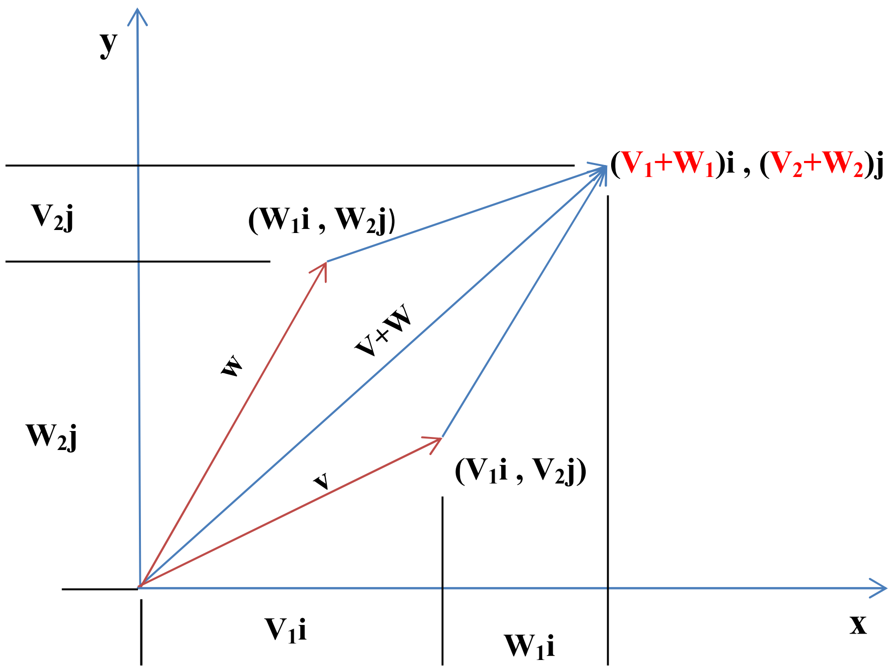


$\mathbf{V = v_1 i + v_2 j}$ and $\mathbf{W = w_1 i + w_2 j}$ are equivalent if and only if :

$$\mathbf{v_1 = w_1} \quad \text{and} \quad \mathbf{v_2 = w_2}$$

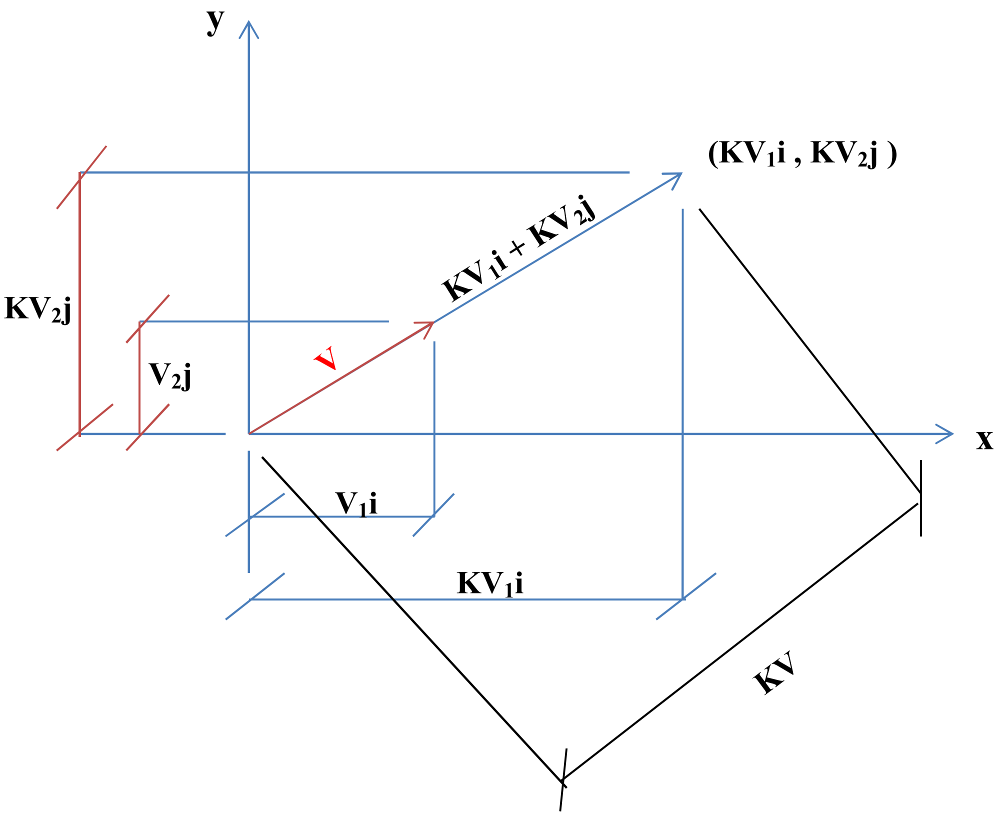
- If $V = v_1 i + v_2 j$ and $W = w_1 i + w_2 j$ then …

$V+W = (v_1+w_1)\, i + (v_2 + w_2)\, j$ ……………………….(1.1)



- If $V= v_1 i + v_2 j$ and K is any scalar , then :

$KV= KV_1 i + KV_2 j$



Example :

If V = (1 , -2) and W = (7, 6) , find V+W & 4V & V-W .

Solution :

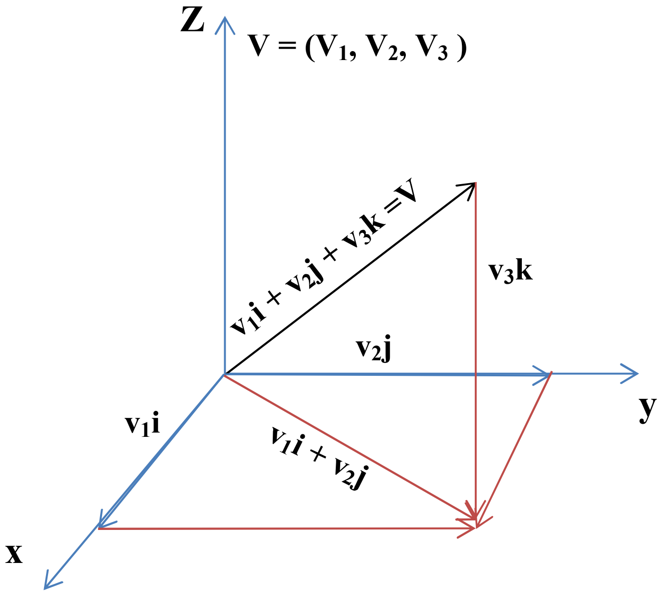
V+W = (1, -2) + (7 , 6) = (1+7 , -2+6)= (8, 4)

4V = 4 (1, -2) = (4, -8)

V-W = V + (-1) W

= (1, -2) + (-1) (7, 6) = (1, -2) + (-7, -6) = (-6 , -8)

If vector in 3- space , the coordinates of the terminal point are …



If $V=V_1i+V_2j+VjK$ and $W=W_1i+W_2j+W_3K$ then V=W if and only if

$$V_1=W_1 \quad , \quad V_2=W_2 \quad , \quad V_3=W_3$$

$$V+W = (V_1+W_1)i + (V_2+W_2)j + (V_3+W_3)k$$

$$KV = KV_1i + KV_2j + KV_3k$$

Example : if V= (1,-3,2) and W=(4,2,1) ,then find V+W, -W, V-W,& 2V.

Solution : **V+W= (5,-1,3)**

-W=(-4,-2,-1)

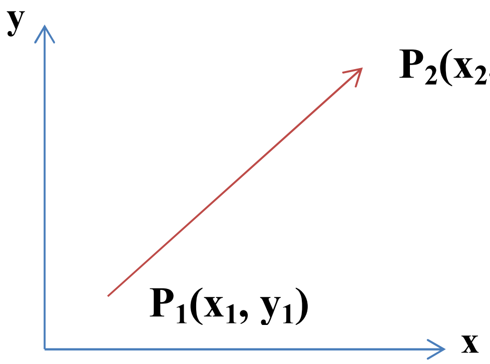
V-W= V+(-W)= (-3,-5,1)

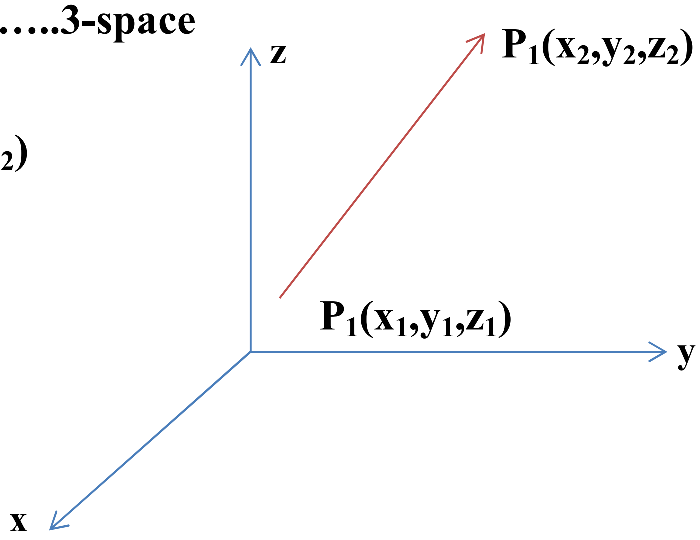
2V+ (2,-6,4)

If the vector $\overrightarrow{P_1P_2}$ has initial point $\mathbf{P_1}$ $(\mathbf{x_1}, \mathbf{y_1})$ and terminal point $\mathbf{P_2}$ $(\mathbf{x_2}, \mathbf{y_2})$ then

$\overrightarrow{P_1P_2} = (x_2-x_1, y_2-y_1)$ **………2-Space**

$\overrightarrow{P_1P_2} = (x_2-x_1, y_2-y_1, z_2-z_1)$**……….3-space**





Example : find the components of the vector $V=\overrightarrow{P_1P_2}$ with initial point $P_1(2,1,4)$ and terminal point P_2 $(7,5,-8)$.

Solution : $\mathbf{V=P_1P_2=[\ 7-2,\ \ 5-(-1)\ ,\ (-8,\ -4)]\ =\ (5,\ 6,\ -8)}$

1.2 Basic Rules of Vectors arithmetic

1- $u + v = v + u$

2- $(u + v) + w = u + (v + w)$

3- $v + 0 = 0 + v$

4- $u + (-u) = 0$

5- $k(Lu) = (KL)u$

6- $k(u + v) = ku + kv$

7- $(k + L)u = ku + lu$

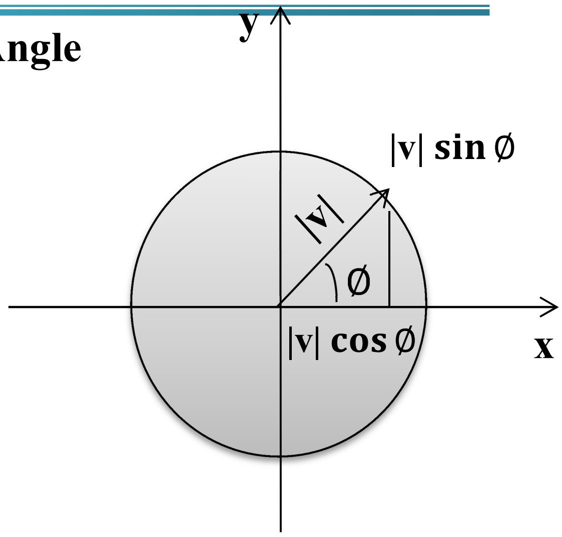
8- $1u = u$

<u>**Note :**</u> we have developed two approaches to vectors :

Geometric : in which vectors are represented by arrows or directed line segments .

Analytic : in which vectors are represented by pairs or triples of numbers called components.

❖ Vectors Determined by Length and Angle



$V = |V| (\cos\emptyset , \sin\emptyset)$

$\quad = |V| \cos\emptyset\ i + |V| \sin\emptyset\ j$

In the special case of a unite vector u

$u = (\cos\emptyset , \sin\emptyset)$

or

$u = \cos\emptyset\ i + \sin\emptyset\ j$

Example :

a) Find the vector of length L that makes an angle of $\pi/4$ with the positive x – axis

b) Find the angle that the vector $V = -\sqrt{3}\ i + j$ makes with the positive x - axis

Solution :

a) $V = 2\cos\frac{\pi}{4}\ i + 2\sin\frac{\pi}{4}\ j = \sqrt{2}\ i + \sqrt{2}\ j$

b)

$$\frac{v}{|v|} = \frac{-\sqrt{3}\ i + j}{\sqrt{(-\sqrt{3}) + 1}} = \frac{\sqrt{3}}{2}\ i + \frac{1}{2}\ j$$

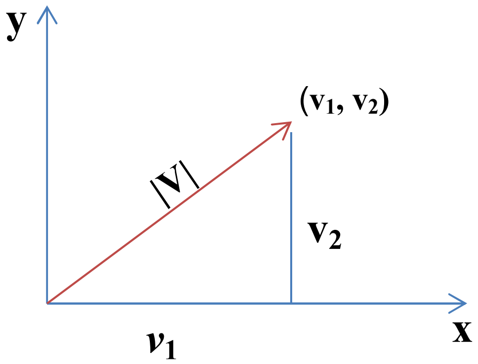
Thus , $\cos\emptyset = \frac{-\sqrt{3}}{2}$ and $\sin\emptyset = \frac{1}{2}$ that is mean $\emptyset = 5\pi/6$

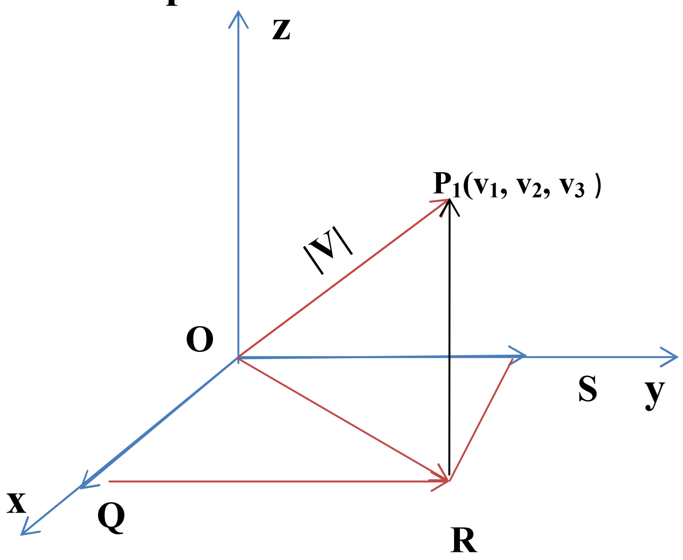
1.3 Length of Vectors (Norm of Vector)

The length of a vectors is denoted by **|v|**

$|v| = (v_1^2 + v_2^2)^{1/2}$ **in 2- space**

$|v| = (v_1^2 + v_2^2 + v_3^2)^{1/2}$ **in 3- space**





$|V|^2 = (OR)^2 + (RP)^2$

$= (OQ)^2 + (OS)^2 + (RP)^2$

$= V_1^2 + V_2^2 + V_3^2$

$|V| = (v_1^2 + v_2^2 + v_3^2)^{1/2}$

If P_1 (x_1, y_1, z_1) and P_2 (x_2, y_2, z_2) are two points in 3-space then the distance (**d**) between them is the length of vector

$\overrightarrow{P_1P_2} = (x_2-x_1, y_2-y_1, z_2-z_1)$

$d = [(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2]^{1/2}$……………….3- space

$d = [(x_2-x_1)^2 + (y_2-y_1)^2]^{1/2}$ ……………………….2 – space

Example: find the norm of the vector V = (-3,2,1)

Solution : $|v| = [(-3)^2 + (2)^2 + (1)^2]^{1/2} = \sqrt{14}$

Example : find the distance (d) between the points P_1 (2,-1,-5) and P_2 (2,-1,-5) and P_3 (4, -3, 1)

Solution :$d = [(4-2)^2 + (-3+1)^2 + (1+5)^2]^{1/2} = \sqrt{44} = 2\sqrt{11}$

Unit Vectors

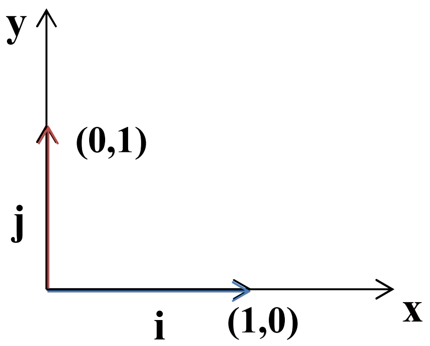
A vector of length 1 is called a unit vector.

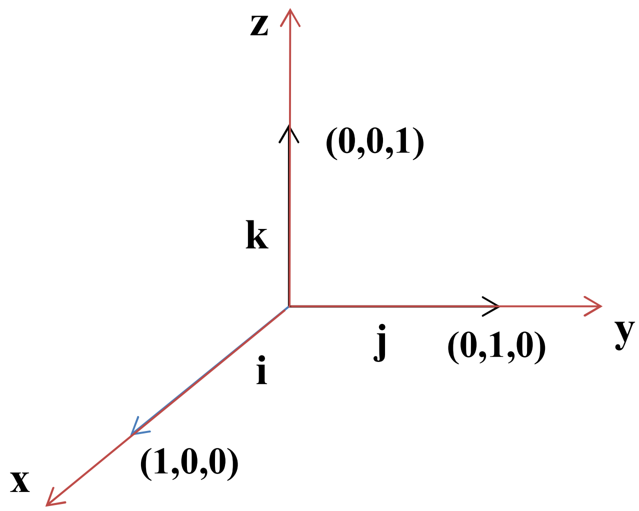
i = (1,0), j = (0,1) ……………………in 2- space

i = (1,0,0) , j = (0,1,0) , k = (0,0,1) ……...in 3 –pace

$v = (v_1,v_2) = (v_1,0) + (0,v_2) = v_1(1,0) + v_2(0,1) = v_1 i + v_2 j$

$v = (v_1,v_2,v_3) = v_1(1,0,0) + v_2(0,1,0) + v_3 (0,0,1) = v_1 i + v_2 j + v_3 k$





Normalizing a vector

A common problem in applications is to find a unit vector u that has the same direction as some given nonzero vector v

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\,\mathbf{v} = \frac{\mathbf{v}}{|\mathbf{v}|}$$, u is a unit vector with the same direction as v

the process of multiplying a vector (v) by the reciprocal of its length to obtain a unit vector with the same direction is called normalizing V

Example : find the unit vector that has the same direction as V=2i+2j-k

Solution : $|\mathbf{v}| = [2^2+2^2+(-1)^2]^{1/2} = 3$

$$\mathbf{u} = \frac{1}{|\mathbf{v}|}\cdot \mathbf{v} = \frac{1}{3}(2i+2j-k) = \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$$

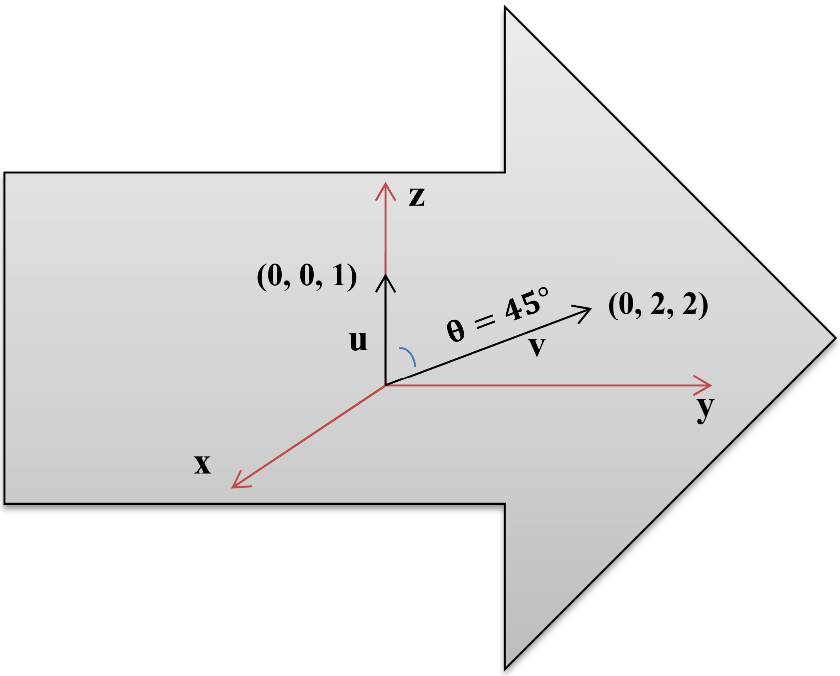
1.4 Dot product

If **u** and **v** are vectors in 2 – space or 3 – space and θ is the angle between them , then the dot product is :

$$u \cdot v = \begin{cases} |u|\,|v|\cos\theta & \text{if } u \neq 0 \,\&\, v \neq 0 \\ 0 & \text{if } u = 0 \text{ or } v = 0 \end{cases}$$

Example : the angle θ between the vectors u = (0, 0, 1) and v = (0, 2, 2) is 45° , find the dot product .

Solution : $u \cdot v = |u|\ |v| \cos\theta = (0^2 + 0^2 + 1^2)^{1/2} (0^2 + 2^2 + 2^2)^{1/2} (1/\sqrt{2}) = 2$



$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$u \cdot v = u_1 v_1 + u_2 v_2$, $\cos\theta = \frac{u1\,v1 + u2\,v2}{|u||v|}$

Example : u = (2,-1,1) and v =(1,1, 2) find u . v and determined the angle θ

Solution :

u . v = u1 . v1 + u2 . v2 + u3 .v3 = (2*1) + (-1*1) + (1*2) =3

$|u| = (u_1^2 + u_2^2 + u_3^2)^{1/2} = \sqrt{6}$

$|v = (v_1^2 + v_2^2 + v_3^2)^{1/2} = \sqrt{6}$

$\cos\theta = \frac{u \cdot v}{|u||v|} = 1/2$ that is mean $\theta = 60°$

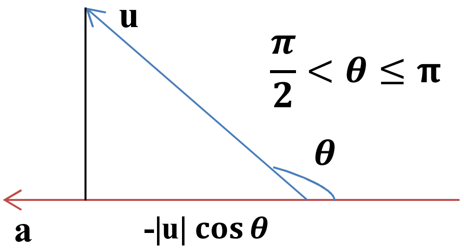
Theorems

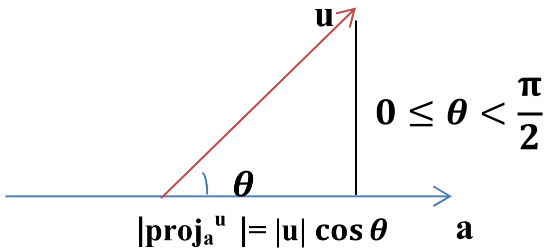
1. $v \cdot v = (v)^2$ $\qquad |v| = (v \cdot v)^{1/2}$
2. θ is acute if and only if $u \cdot v > 0$
   θ is obtuse if and only if $u \cdot v < 0$
   θ $\pi/2$ if and only if $u \cdot v = 0$
3. $u \cdot v = v \cdot u$
4. $u \cdot (v + w) = u \cdot v + u \cdot w$
5. $k\,(u \cdot v) = (ku) \cdot v = u \cdot (kv)$
6. $v \cdot v > 0$ if $v \neq 0$ and $v \cdot v = 0$ if $v = 0$

If **u** and **a** are vectors in 2- space or 3 – space and if $a \neq 0$ then :

$\mathbf{Proj}_{\mathbf{a}}^{\mathbf{u}} = \dfrac{\mathbf{u \cdot a}}{|\mathbf{a}|} * \mathbf{a}$ vector component of u along a

$\mathbf{u} - \mathbf{proj}_{\mathbf{a}}^{\mathbf{u}} = \mathbf{u} - \dfrac{\mathbf{u \cdot a}}{|\mathbf{a}|} * \mathbf{a}$ vector component of u along a





$$|\mathbf{Proj}_{\mathbf{a}}^{\mathbf{u}}| = |\mathbf{u}| \cos\theta = |\mathbf{u}| \frac{\mathbf{u \cdot a}}{|\mathbf{u}||\mathbf{a}|} = \frac{\mathbf{u \cdot a}}{|\mathbf{a}|}$$

$$|\mathbf{u}| = \cos\theta = \frac{\mathbf{u \cdot a}}{|\mathbf{a}|}$$

$$\mathbf{proj}_{\mathbf{a}}^{\mathbf{u}} = |\mathbf{u}| \cos\theta \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a \cdot u}}{\mathbf{a \cdot a}} * \mathbf{a}$$

Example : let w = (2, -1 , 3) and a = (4 , -1 , 2), find the vector component of w along a & the vector component of w orthogonal to a and the length of projection .

Solution :

$$\mathbf{Proj}_{\mathbf{a}}^{\mathbf{w}} = \frac{\mathbf{w\,.\,a}}{|\mathbf{a}|} \;.\,\mathbf{a}$$

w. a = (2) (4) +(-1) (-1) + (3) (2) = 15

$|a|^2 = a\,.a = (4)\,(4) + (-1)^2 + (2)^2 = 21$

$\text{Proj}_a^w = 15/21\ (4, -1, 2) = (\frac{20}{7}, \frac{-5}{7}, \frac{10}{7})$

$$\mathbf{w - proj}_{\mathbf{a}}^{\mathbf{w}} = \mathbf{w} - \frac{\mathbf{w.a}}{|\mathbf{a}|} * \mathbf{a}$$

$\text{w- proj}_a^w = (2,-1, 3) - (\frac{20}{7}, \frac{-5}{7}, \frac{10}{7})$

$= (\frac{-6}{7}, \frac{-2}{7}, \frac{11}{7})$

$\text{Proj}_a^w = |w| \cos\theta = \frac{w\,.a}{|a|}$

$|a| = (a_1^2 + a_2^2 + a_3^2)^{1/2} = [(4^2 + (-1)^{2} + 2^2)]^{1/2} = \sqrt{21}$

$|\text{proj}_a^w| = \frac{15}{\sqrt{21}}$

Direction cosines of a vector in 3 – space

They are the number $\cos\propto$, $\cos\beta$, $\cos\gamma$, where $\propto$, β , and γ are the angle between v and the positive x , y and z axes

$\cos\propto = a / (a^2 + b^2 + c^2)^{1/2}$ v = (a,b,c)

$\cos\beta = b / (a^2 + b^2 + c^2)^{1/2}$

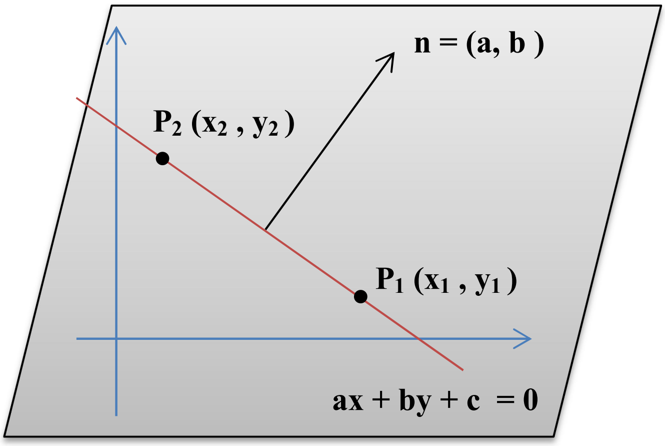
$\cos\gamma = c / (a^2 + b^2 + c^2)^{1/2}$

$\cos^2\propto + \cos^2\beta + \cos^2\gamma = 1$

$v = i\cos\propto + j\cos\beta + k\cos\gamma$ ⟶ unit vector

Example : show that in 2 –space the nonzero vector n = (a+b) is perpendicular to the line ax + by + c =0

Solution : let $P_1\ (x_1\ ,\ y_1\ )$ and $P_2\ (x_2\ ,\ y_2\ )$ be distance points on the line so that :



$ax_1 + by_1 + c = 0$

$ax_2 + by_2 + c = 0$

$P_1P_2 =(x_2 – x_1\ ,\ y_2 – y_1\ )$, by subtracting

$a(x_2 – x_1\ ) + b(y_2- y_1) = 0$

$n\ .\ P_1P_2 = a(x_2 – x_1\ ) + b\ (y_2 – y_1\ ) =0$

So that n and P_1P_2 are perpendicular

Example : find a formula for the distance D between the point $P_o\ (x_o\ ,\ y_o\ )$ and the line $ax +by + c = 0$

Solution :

$D = |proj_{\ n}\ QP_o\ | = QP_o\ .\ n\ /\ |n|$

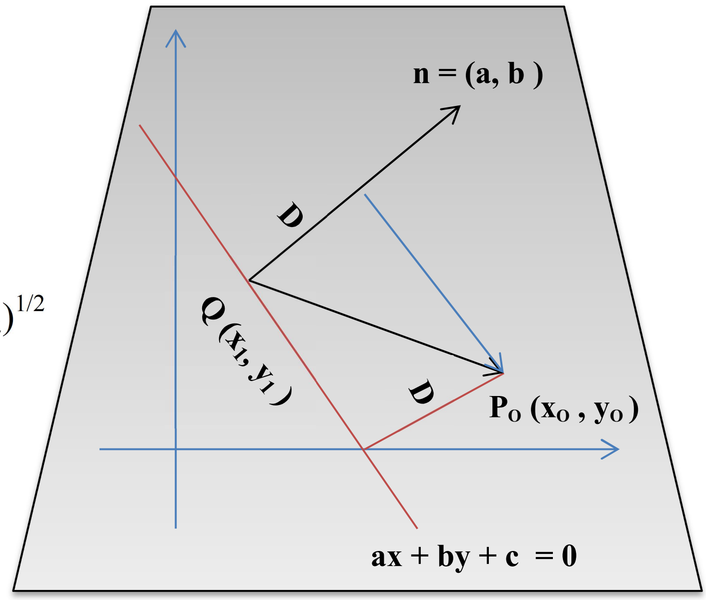
$QP_o = (x_o – x_1\ ,\ y_o – y_1\ )$

$n = (a\ ,\ b\ )$

$QPO\ .\ n = a(x_o – x_1) + b(y_o– y_1)$

$|n\ | = (a_2 + b_2\ )^{1/2}$

$D = [\ a\ (x_o – x_1\ ) + b\ (\ y_o– y_1\ )]\ \ /\ (a_2 + b_2)^{1/2}$



$ax_1 + by_1 +c = 0$

$C = -ax_1 – by_1$

$D = (ax_o + by_o +c\ )\ /\ (\ a_2 + b_2\ )^{1/2}$

1.5 Cross Product :

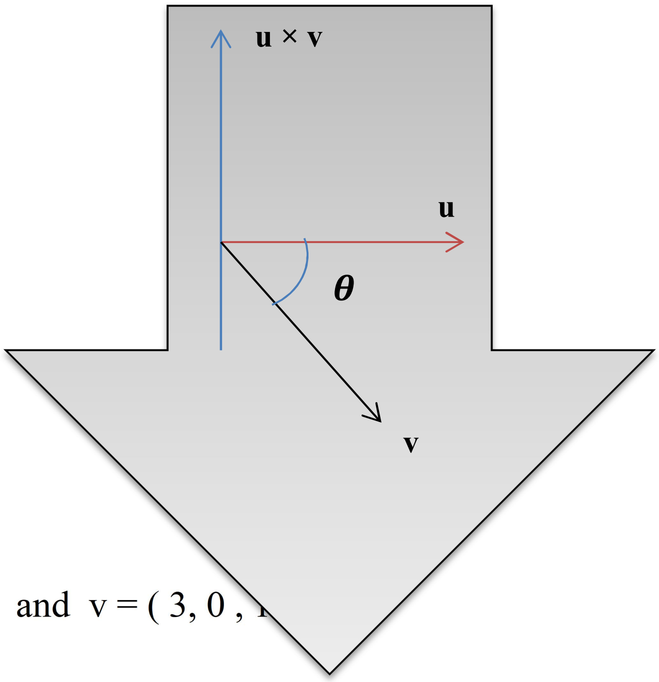
If $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are vectors in 3 – space , then the cross product

$u \times v = (u_2v_3 - u_3v_2 , u_3v_1 - u_1v_3 , u_1v_2 - u_2v_1)$ or in determined notation

$$u \times v = \begin{vmatrix} u2 & u3 \\ v2 & v3 \end{vmatrix} , - \begin{vmatrix} u1 & u3 \\ v1 & v3 \end{vmatrix} , \begin{vmatrix} u1 & u2 \\ v1 & v2 \end{vmatrix}$$

if we form the 2×3 matrix

$$\begin{bmatrix} u1 & u2 & u3 \\ v1 & v2 & v3 \end{bmatrix}$$



Example : find u × v where u = (1, 2 ,-2) and v = (3, 0 , 1

Solution :
$$\begin{bmatrix} 1 & 2 & -2 \\ 3 & 0 & 1 \end{bmatrix}$$

$$u \times v = \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} , - \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} , \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}$$

$= (2, -7 , -6)$

Note : The dot product of two vector is **scalar**

The cross product of two vector is **vector**

1.5.1 An important relationship between dot and cross product

a) $u \cdot (u \times v) = 0$ — $u \times v$ is orthogonal to u
b) $v \cdot (u \times v) = 0$ — $u \times v$ is orthogonal to v
c) $|u \times v|^2 = |u|^2 |v|^2 - (u \cdot v)^2$ — Lagrange's identity

Example : consider the vectors $u = (1, 2, -2)$ and $v = (3, 0, 1)$
find $u \times v$, $u \cdot (u \cdot v)$, $v \cdot (u \cdot v)$.

Solution :

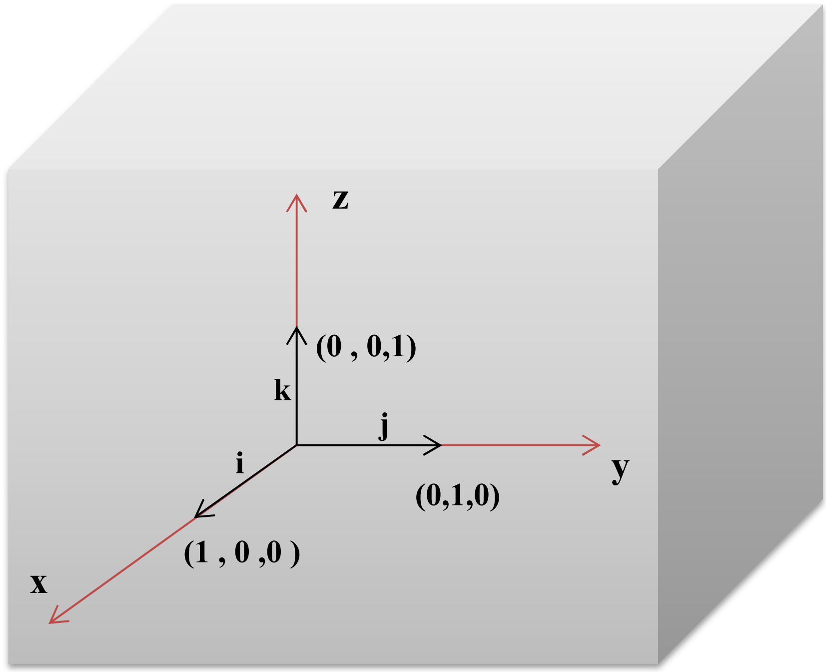
$(u \times v) = (2, -7, -6)$

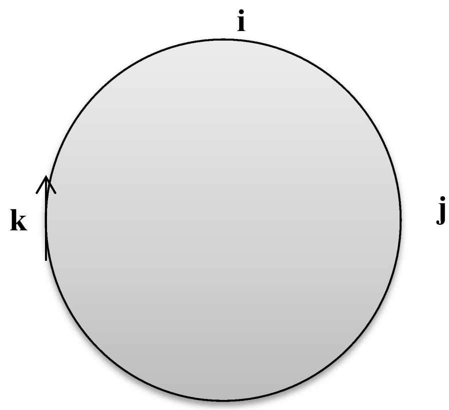
$u \cdot (u \times v) = (1)(2) + (2)(-7) + (-2)(-6) = 0$

$v \cdot (u \times v) = (3)(2) + (0)(-7) + (1)(-6) = 0$

$u \times v$ is orthogonal to both u and v .

1.5.2 Standard unit vectors in 3 – space





$$i \times j = \left(\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} , - \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} , \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right) = (0, 0, 1) = k$$

$i \times i = i \times j = k \times k = 0$

$i \times j = k , j \times k = i , k \times i = j$

$j \times i = -k , k \times j = -i , i \times j = -j$

$|u \times v| = |u|^2 \; |v|^2 = |u|^2 \; |v|^2 - (u.v)^2$

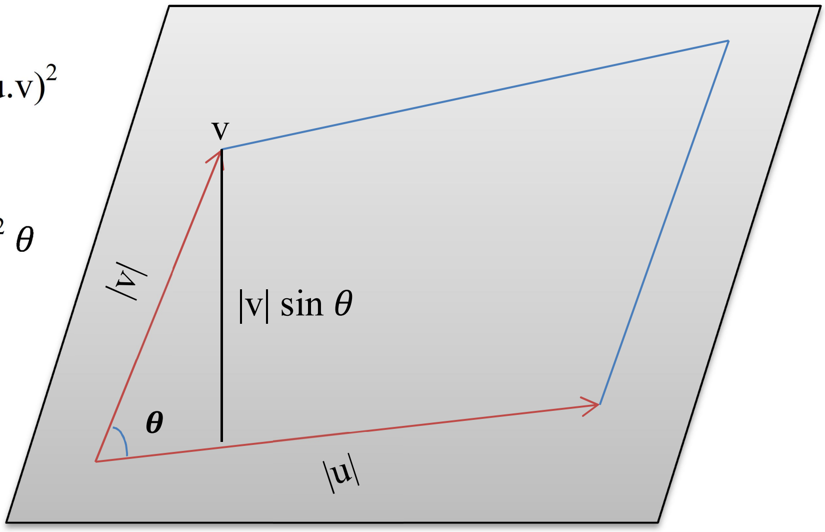
$u \,.\, v = |u| \;\; |v| \cos\theta$

$|u \times v|^2 = |u|^2 \; |v|^2 - |u|^2 |v|^2 \; \cos^2\theta$

$= |u|^2 \; |v|^2 \,(1-\cos^2\theta)$

$= |u|^2 \; |v|^2 \; \sin^2\theta$

$|u\times v| = |u| \; |v| \sin\theta$



In other words , the norm of $u\times v$ is equal to area of parallelogram determined by u and v

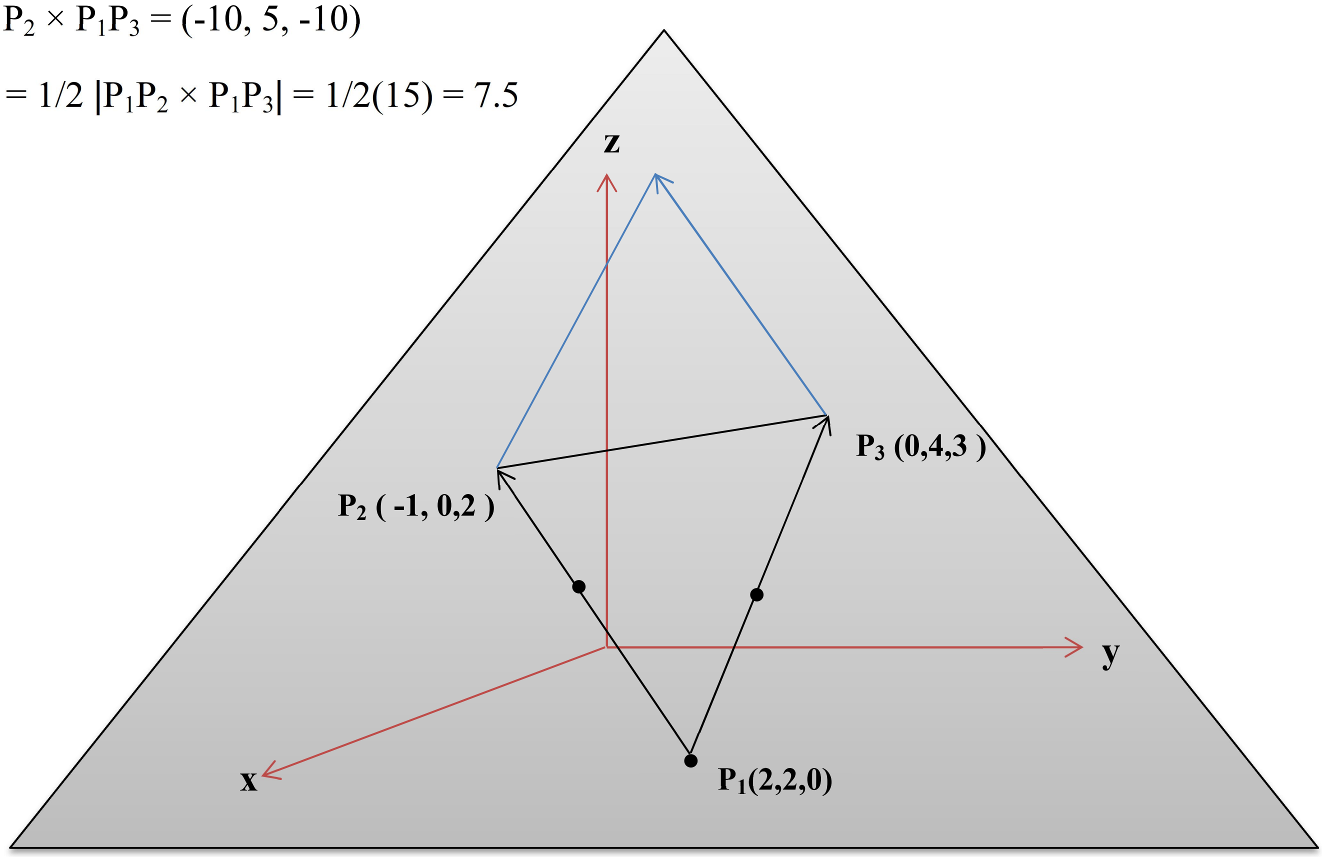
Example : find the area of the triangle determined by the points P_1 (2,2,0) , P_2 (-1, 0,2) and P_3 (0,4,3)

Solution : The area of the triangle is 1/2 the area of the parallelogram .

$P_1P_2 = (-3, -2, 2)$, $P_1P_3 = (-2, 2, 3)$

$P_1P_2 \times P_1P_3 = (-10, 5, -10)$

$A = 1/2\, |P_1P_2 \times P_1P_3| = 1/2(15) = 7.5$



Triple dot product (scalar triple product)

Let u = (u1, u2, u3) , v =(v1, v2, v3) and w = (w1, w2, w3) then :

$$u \cdot (v \times w) = \begin{vmatrix} u1 & u2 & u3 \\ v1 & v2 & v3 \\ w1 & w2 & w3 \end{vmatrix} = u.(\begin{vmatrix} v2 & v3 \\ w2 & w3 \end{vmatrix} i - \begin{vmatrix} v1 & v3 \\ w2 & w3 \end{vmatrix} j + \begin{vmatrix} v1 & v2 \\ w1 & w2 \end{vmatrix} k)$$

$$= u_1 \begin{vmatrix} v2 & v3 \\ w2 & w3 \end{vmatrix} - u_2 \begin{vmatrix} v1 & v3 \\ w1 & w3 \end{vmatrix} + u_3 \begin{vmatrix} v1 & v2 \\ w1 & w2 \end{vmatrix}$$

$$= u_1 (v_2w_3 - v_3w_2) + u_2(v_3w_1 - v_1w_3) + u_3(v_1w_2 - v_2w_1)$$

Example : calculate the scalar triple product u. (v×w) of the vectors u = 3i-2j-5k , v = i+4j-4k , w = 3j+ 2k

Solution :

$$u \cdot (v \times w) = \begin{vmatrix} 3 & -2 & -5 \\ 1 & 4 & -4 \\ 0 & 3 & 2 \end{vmatrix} = 49$$

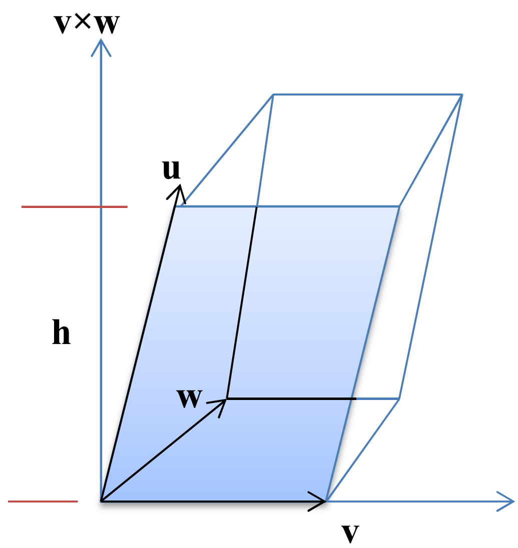
let u, v and w be nonzero vectors in 3 –space

a) The volume v of the parallelepiped that has u,v, and w as adjacent edges is v = |u .(v×w)|
b) u . (v×w) = 0 if and only if u, v, and w lie in the same plane

v = (area of base)(height) = $|v \times w|\ h$

$$h = |proj_{v \times w}{}^{u}| = \frac{|u.(v\times w)|}{|v\times w|}$$

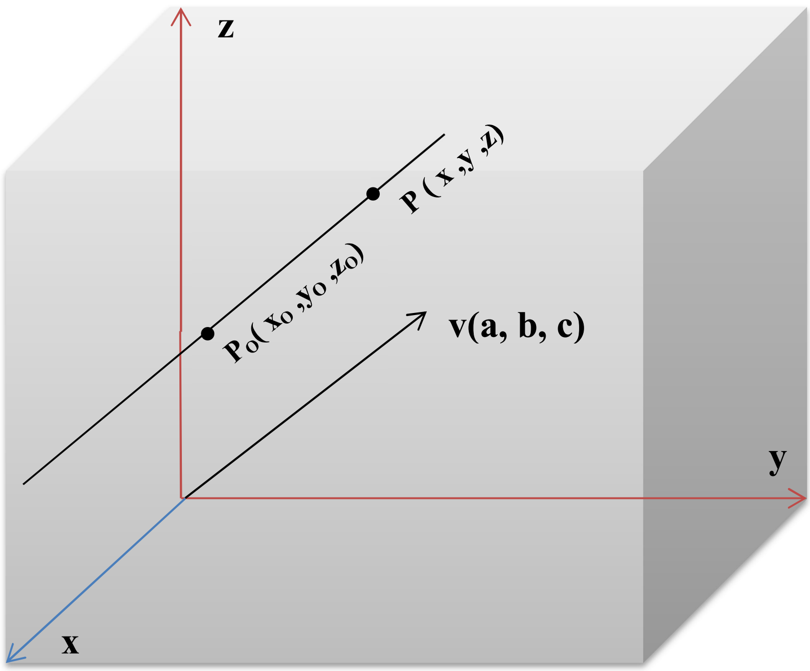
$$= |v\times w| \frac{|u.(v\times w)|}{|v\times w|} = |u.(v\times w)|$$



1.6 lines and planes in 3 –spaces

1.6.1 lines

Suppose the line in 3- space through the point $P_o(x_o, y_o, z_o)$ and parallel to the nonzero vector v= (a, b, c)



$\overrightarrow{P.P}$ is parallel to v , for which there is a scalar (t)

$\overrightarrow{P.P} = tv$

In terms of components

$(x-x_o , y-y_o, z-z_o) = (ta, tb, tc)$

$x = x_o + ta$

$y = y_o + tb$ } parametric equations

$z = z_o + tc$

Example: find parametric equation for the line (L) passing through the points P_o (1, 2, -3) and parallel to the vector v= (4, 5, -7)

Solution :

x = 1+4t

y =2+5t

z =-3-7t

Example : find parametric equation for the line (L) passing through the points P_1 (2, 4, -1), and P_2 (5,0,7) , where dose the line intersect the xy- plane?

Solution :

$\overrightarrow{PP}$ = (3, -4, 8) is parallel to (L) and P_1 (2, 4, -1) lies on it

x = 2+3t

y = 4-4t

z = -1 +8t

The line intersect the xy –plane at the point where

z = -1+8t =0

-1 +8t =o

t = 1/8

x= 19/8

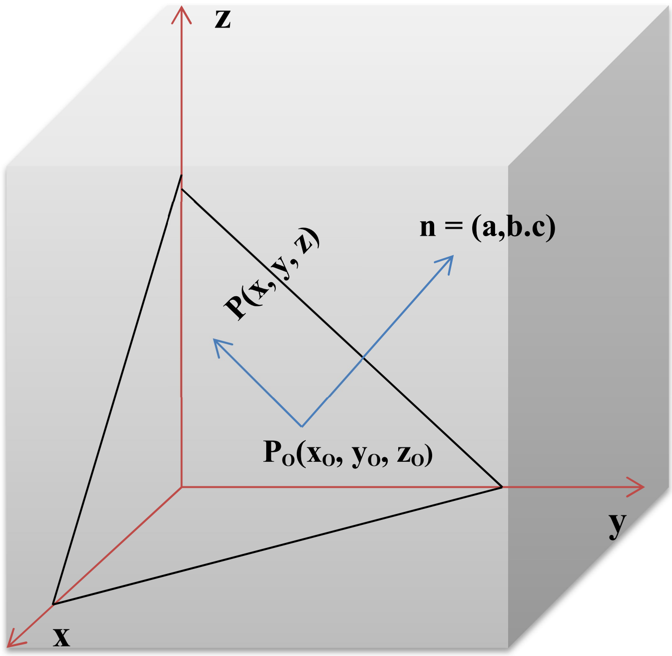
y= 7/2

z= 0

(x, y, z) = (19/8, 7/2, 0)

1.6.2 Planes

Suppose we want the equation of the plane passing through the point $P_o(x_o, y_o, z_o)$ and having the nonzero vector n=(a,b,c) as a normal



$$n.\overrightarrow{P_oP} = 0, \quad \overrightarrow{P_oP} = (x - x_o , y - y_o , z - z_o)$$

$a(x-x_o) + b(y-y_o) + c(z-z_o) = 0$ ⟶ point normal form

Example : find an equation of the plane passing through point (3,-1,7) and perpendicular to the vector n= (4,2,-5)

Solution : $4(x-3) + 2(y+1) - 5(z-7) = 0$

$$4x+2y-5z+25 = 0$$

$ax + by + cz + d = 0$ is the equation of a plane having the vector n n=(a,b,c) as a normal .

"There is no way to happiness. Happiness is the way."