UNIVERSITY OF ANBAR
COLLEGE OF ENGINEERING
CIVL ENG.DEPT.

Hydrology Eng. (3rd class)

HYDROGRAPH, PART II

Lecture (2)

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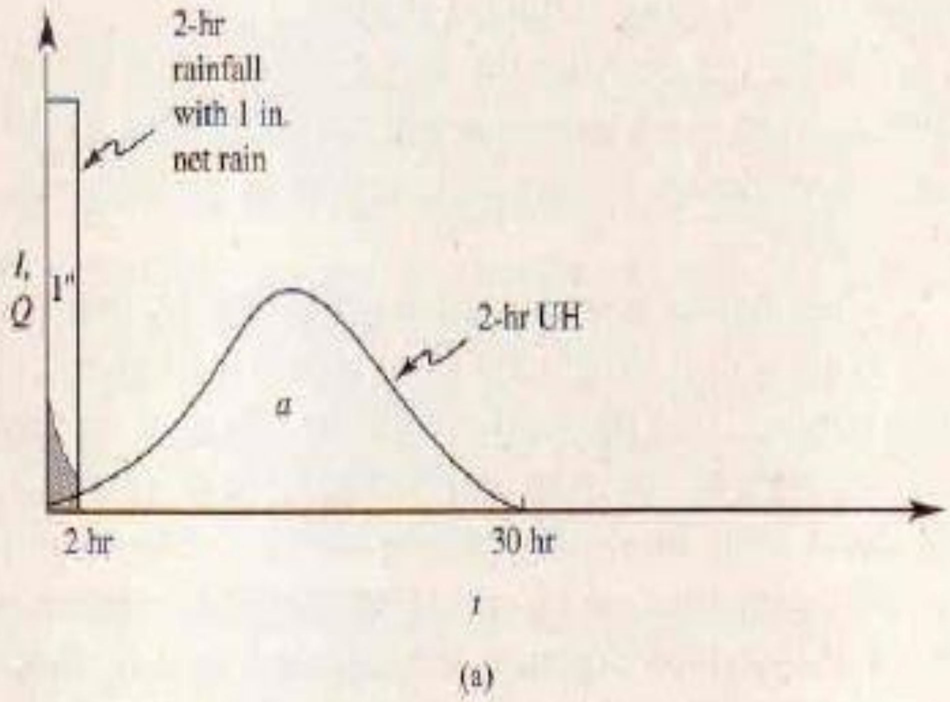
- ***Outline*** :

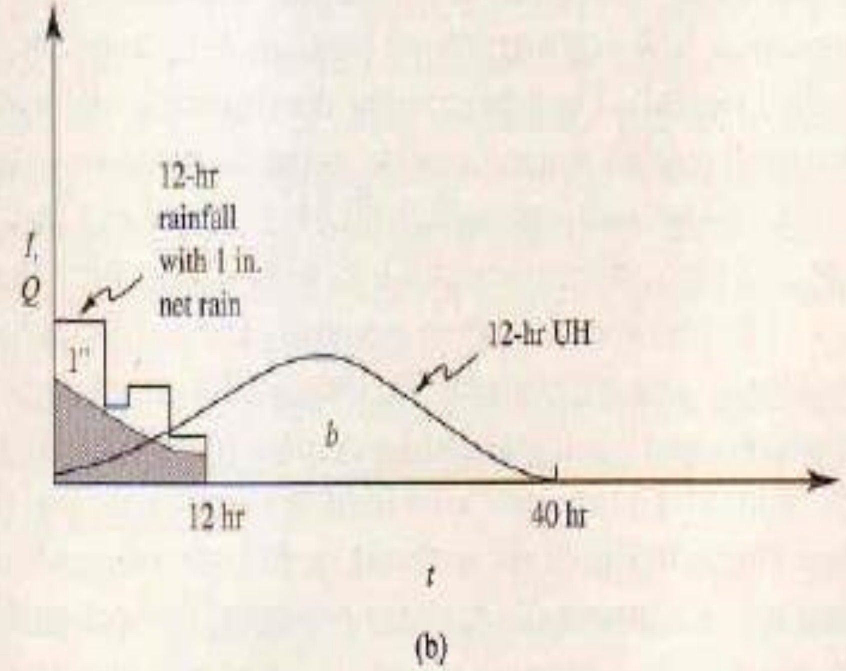
1. Unit Hydrograph Definitions .
2. Assumptions in Derivation of UH Theory .
3. Limitation and Uses of UH Theory .
4. Unit Hydrograph Derivation :
5. Unit Hydrograph of different durations :
6. Synthetic Unit Hydrograph (SUH) .
7. Dimensionless SCS Unit Hydrograph .
8. References .

➢ *Unit Hydrograph Definitions :*

The concept of a unit hydrograph(UH) was first introduced by Sherman in 1932 . He defined a (UH) as follows :

(is a direct runoff hydrograph (DRH) resulting from one inch or one cm of excess rainfall generated uniformly over the drainage area at a constant rate for an effective duration)





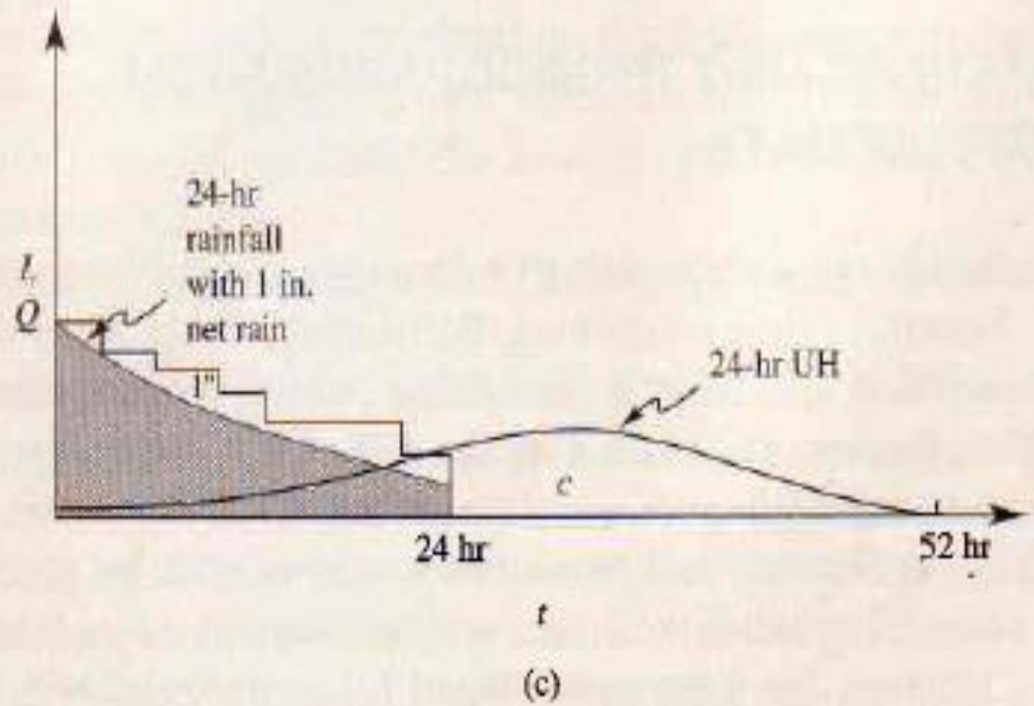


Fig. (1) Unit hydrograph for 2-hr ,12 hr and 24 hr for the same watershed

➢ *Assumptions in Derivation of (UH) Theory :*

1. The rainfall is of spatially uniform intensity within its specified duration .
2. The effective rainfall is uniformly distributed throughout the whole area of drainage basin .
3. A unit hydrograph reflected all the combined physical characteristics of the basin .
4. The ordinates DSR hydrograph due to net rains of different intensities but same duration are proportional .
5. The base of time duration of hydrograph of direct runoff due to effective rainfalls of unit duration is constant . Base period of hydrograph of different rainfall intensities remain approximately same .

Example:(1) The following runoff from a 6 hr rainfall was recorded on a 4300 km watershed. Find the unit hydrograph.

Date	Hour	Total flow m^3/s	Base flow m^3/s	DRO	6hr UHG (divided by 10)	6hr DRO for 30 cm x 30
Feb 16	0600	500	500	0	0	0
	0800	5600	450	5150	515	15450
	1000	9200	400	8800	880	26400
	1200	10100	400	9700	976	29100
	1400	7800	450	7350	735	22050
	1600	6600	450	6150	615	18450
	1800	5550	500	5050	505	15150
	2000	4700	550	4150	415	12450
	2200	4000	600	3400	340	10200
	2400	3300	600	2700	270	8100
Feb 17	0200	2700	600	2100	210	6300
	0400	2300	650	1650	165	4950
	0600	1950	650	1300	130	3900
	0800	1650	700	950	95	2850
	1000	1400	700	700	70	2100
	1200	1200	750	450	45	1350
	1400	1000	750	250	25	750
	1600	800	800	0	0	0

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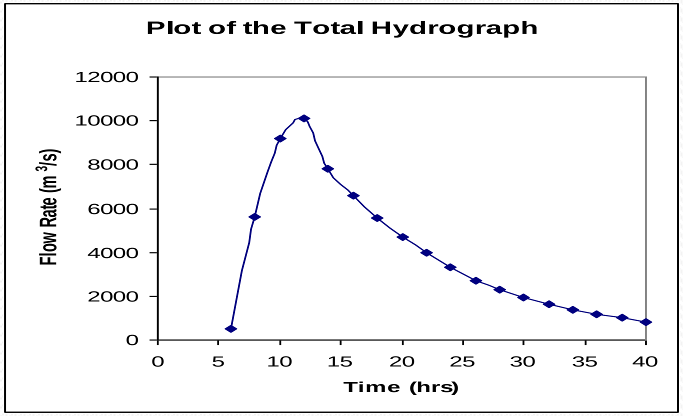


Limitation and Uses of UH Theory :

- Limitations:

1. Similar rainfall distribution from storm to storm over a large area is rare . Hence UH theory is suited to catchment area under about 500 km^3
2. Odd shaped basins particularly those which are long and narrow , commonly have very uneven rainfall distribution and hence UH theory for such basins is not much suitable .
3. The catchment having large storage like reservoir , lake , low areas , etc. affect the linear relationship and hence theory cannot be applied.
4. The unit hydrograph method cannot be applied when an appreciable portion of the storm precipitation falls as snow.
5. If the variation of base period and peak flow very more than $\mp$ 20% and +10% , theory applied is not generally accepted .
6. In mountainous regions , subject to orographic rainfall, aerial distribution is very uneven, but the pattern tends to remain the same from storm to storm and UH theory may not be successfully applied .

➢ *Limitation and Uses of UH Theory :*

- Uses :

1. As the UH is a linear model of the catchment , it is used to determine runoff hydrograph of the catchment even for extreme magnitude to determine peak flow for design of hydrograph structure .
2. It can be used for flood forecasting and waring .
3. Based on rainfall records, it is used for extension of flood flow records .

➢ Unit Hydrograph Derivation :

A-From a simple flood hydrograph of isolated storm:

Different steps required to drive UH are :

l. Analyze the stream flow hydrograph to permit separation of surface runoff from groundwater flow.

2. Measure the total volume of surface runoff (direct runoff) from the storm producing the original hydrograph. This is the area under the hydrograph after groundwater base flow has been removed.

3. Divide the ordinates of the direct runoff hydrograph by total direct runoff volume in inches, and plot these results versus time as a unit graph for the basin.

4. Finally, the effective duration of the runoff-producing rain for this unit graph must be found from the hyetograph (time history of rainfall intensity) of the storm event used.

EXAMPLE (2) : *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km² due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.*

Time from start of storm (h)	−6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5
Time from start of storm (h)	54	60	66	72	78	84	90	96	102	
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0	

SOLUTION: The storm hydrograph is plotted to scale(Figure 2) the denoting the time from beginning of storm as t, by inspection of Fig. 2

$$A = \text{beginning of DRH} \qquad t = 0$$

$$B = \text{end of DRH} \qquad t = 90\text{ h}$$

$$P_m = \text{peak} \qquad t = 20\text{ h}$$

Hence

$$N = (90 - 20) = 70\text{ h} = 2.91\text{ days}$$

By Eq.

$$N = 0.83\,(423)^{0.2} = 2.78\text{ days.}$$

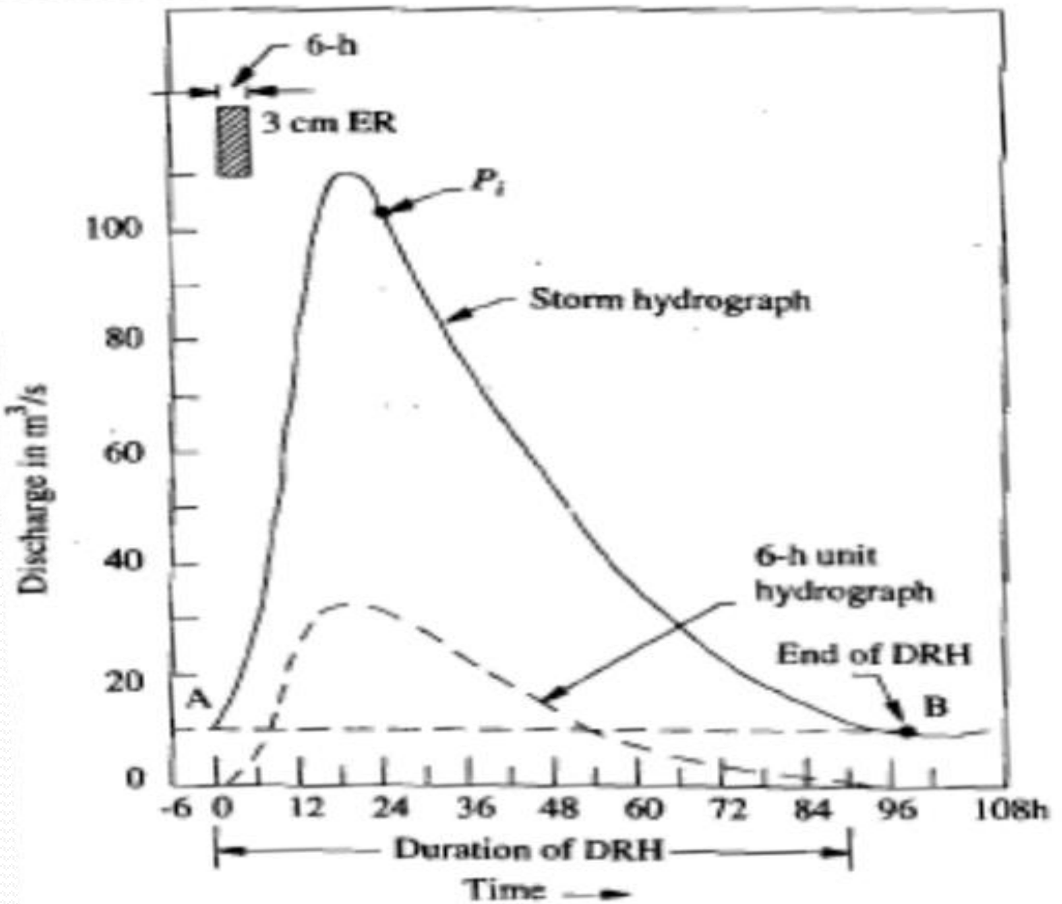


Fig (2) : Derivation of unit hydrograph from a storm hydrograph

$$\text{Volume of DRH} = 60 \times 60 \times 6 \times (\text{sum of DRH ordinates})$$

$$= 60 \times 60 \times 6 \times 587 = 12.68 \text{ Mm}^3$$

$$\text{Drainage area} = 423 \text{ km}^2 = 423 \text{ Mm}^2$$

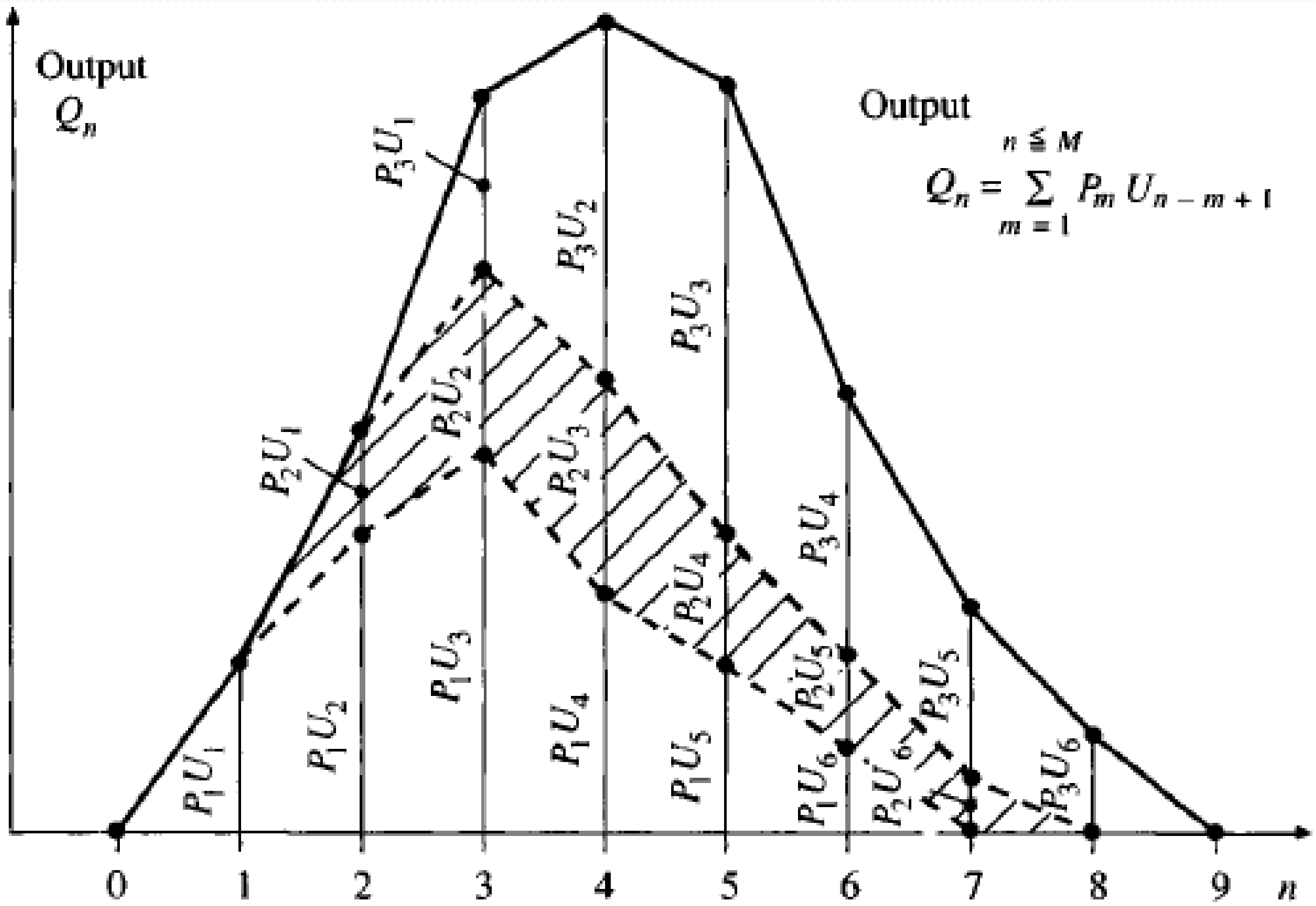
$$\text{Runoff depth} = \text{ER depth} = \frac{12.68}{423} = 0.03 \text{ m} = 3 \text{ cm}.$$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph,, see Table (1).

Table (1):Calculation of the ordinates of 6 hr unit hydrograph, see Example (2)

Time from beginning of storm (h)	Ordinate of storm hydrograph (m^3/s)	Base flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph (Col. 4 + 3)
1	2	3	4	5
–6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	101.0	33.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0
54	39.0	[illegible]		

B-From complex storm :



The set of equations for discrete time convolution $Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$;

$n = 1, 2, \ldots, N$

$$
\begin{array}{lllllllllll}
Q_1 & = P_1 U_1 & & & & & & & & & \\
Q_2 & = P_2 U_1 & + P_1 U_2 & & & & & & & & \\
Q_3 & = P_3 U_1 & + P_2 U_2 & + P_1 U_3 & & & & & & & \\
\ldots & & & & & & & & & & \\
Q_M & = P_M U_1 & + P_{M-1} U_2 & + & + P_1 U_M & & & & & & \\
Q_{M+1} & = \quad 0 & + P_M U_2 & + \ldots & + P_2 U_M & + P_1 U_{M+1} & & & & & \\
\ldots & & & & & & & & & & \\
Q_{N-1} & = \quad 0 & + \quad 0 & + \ldots & + \quad 0 & + \quad 0 & + \ldots & + P_M U_{N-M} & + P_{M-1} U_{N-M+1} & & \\
Q_N & = \quad 0 & + \quad 0 & + \ldots & + \quad 0 & + \quad 0 & + \ldots & + \quad 0 & + P_M U_{N-M+1} & &
\end{array}
$$

Example(3): Find the half-hour unit hydrograph using the excess rainfall hyetograph and direct runoff hydrograph given in Table (2)

Table (2) Excess rainfall hydrograph and direct runoff hydrograph for Example (3)

Time ($\frac{1}{2}$ h)	Excess rainfall (in)	Direct runoff (cfs)
1	1.06	428
2	1.93	1923
3	1.81	5297
4		9131
5		10625
6		7834
7		3921
8		1846
9		1402
10		830
11		313

$$U_1 = \frac{Q_1}{P_1} = \frac{428}{1.06} = 404 \text{ cfs/in}$$

$$U_2 = \frac{Q_2 - P_2 U_1}{P_1} = \frac{1923 - 1.93 \times 404}{1.06} = 1079 \text{ cfs/in}$$

$$U_3 = \frac{Q_3 - P_3 U_1 - P_2 U_2}{P_1} = \frac{5297 - 1.81 \times 404 - 1.93 \times 1079}{1.06} = 2343 \text{ cfs/in}$$

and similarly for the remaining ordinates

$$U_4 = \frac{9131 - 1.81 \times 1079 - 1.93 \times 2343}{1.06} = 2506 \text{ cfs/in}$$

$$U_5 = \frac{10625 - 1.81 \times 2343 - 1.93 \times 2506}{1.06} = 1460 \text{ cfs/in}$$

$$U_6 = \frac{7834 - 1.81 \times 2506 - 1.93 \times 1460}{1.06} = 453 \text{ cfs/in}$$

$$U_7 = \frac{3921 - 1.81 \times 1460 - 1.93 \times 453}{1.06} = 381 \text{ cfs/in}$$

$$U_8 = \frac{1846 - 1.81 \times 453 - 1.93 \times 381}{1.06} = 274 \text{ cfs/in}$$

$$U_9 = \frac{1402 - 1.81 \times 381 - 1.93 \times 274}{1.06} = 173 \text{ cfs/in}$$

Unit Hydrograph of different durations :

- Unit hydrograph are derived from simple isolated storms and if the duration of the various storms do not differ two much duration of D h.
- In practice the unit hydrographs of different duration are needed (nD).
- Two methods are available

1.Method of Superposition

2. the S-Curve method

1-Method of Superposition

Example (4) :Given the ordinates of 4 hr unit hydrograph as below, derive the ordinates of 12 hr unit hydrograph for the same catchment.

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4-h UH	0	20	80	130	150	130	90	52	27	15	5	0

Solution: The calculations are performed in table (3)

The 12 hr unit hydrograph is shown in fig.(3)

Table(3) Calculation of a 12 hr unit hydrograh from a 4 hr unit hydrograph ,Example 4 .

Time (h)	Ordinates of 4-h UH (m^3/s) A	B Lagged by 4 h	C Lagged by 8 h	DRH of 3 cm in 12-h (m^3/s) (Col.2+3+4)	Ordinate of 12-h UH (m^3/s) (Col. 5)/3
1	2	3	4	5	6
0	0	—	—	0	0
4	20	0	—	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

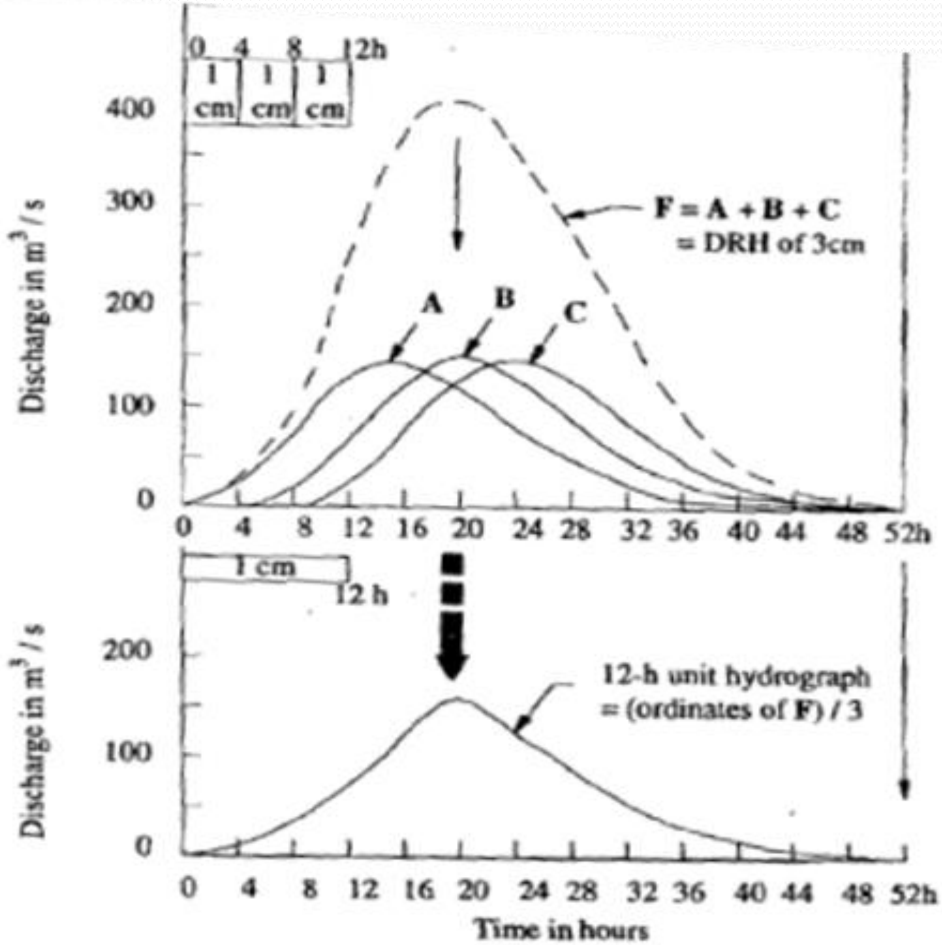


Fig.(3) Construction of a 12 hr unit hydrograph from a 4 hr unit hydrograph .

2-THE S-CURVE METHOD :

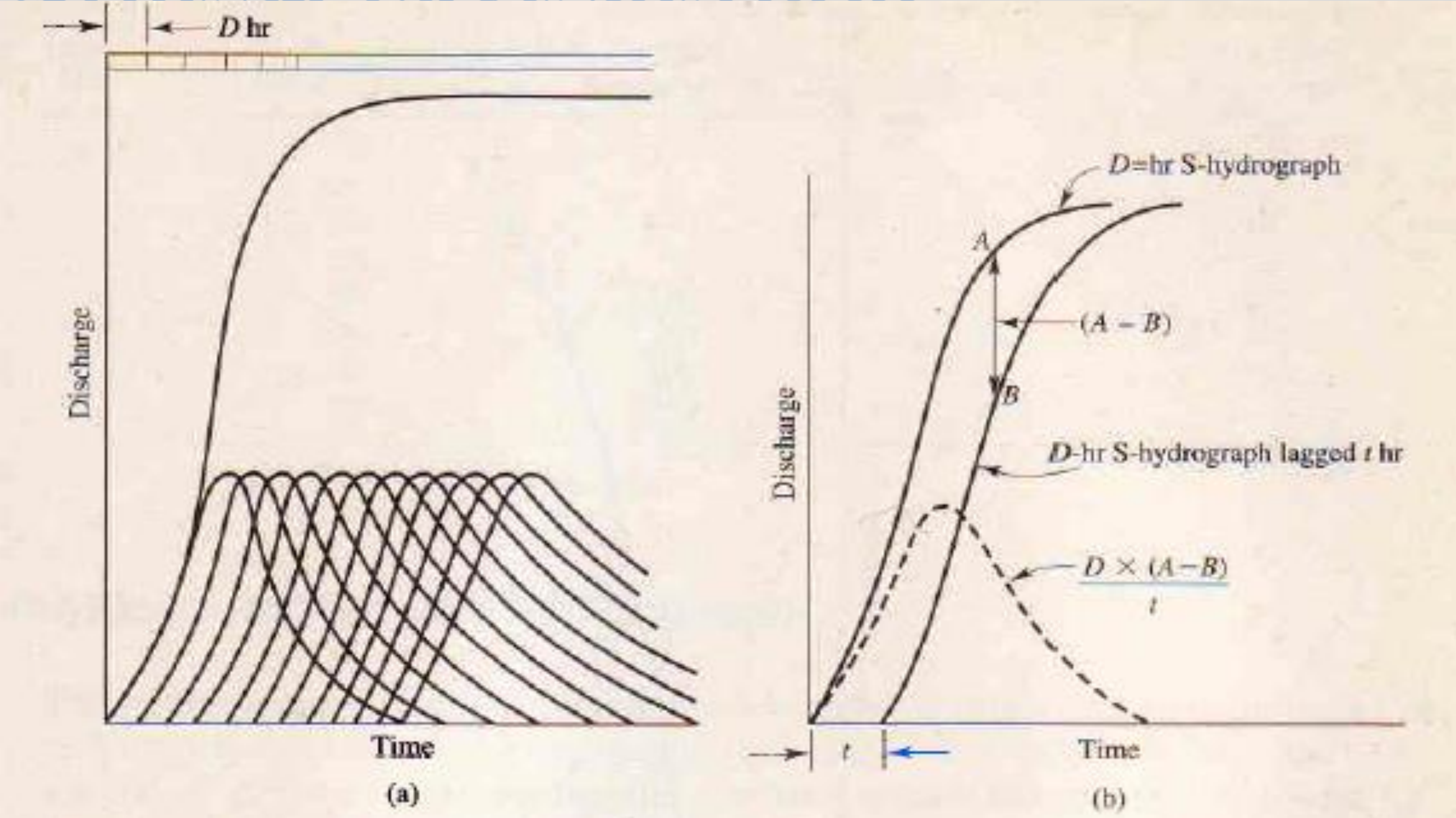


Fig.4 S curve hydrograph lagged t hr

Example 5. *The ordinates of a 4-hour unit hydrograph for a particular basin are given below. Derive the ordinates of (i) the S-curve hydrograph, and (ii) the 2-hour unit hydrograph, and plot them, area of the basin is 630 km^2.*

Time (hr)	*Discharge (cumec)*	*Time (hr)*	*Discharge (cumec)*
0	*0*	*14*	*70*
2	*25*	*16*	*30*
4	*100*	*18*	*20*
6	*160*	*20*	*6*
8	*190*	*22*	*1.5*
10	*170*	*24*	*0*
12	*110*		

Solution See Table (4)

*Slight adjustment is required to the tail of the 2-hour unit hydrograph.

Col (5): lagged S-curve is the same as col (4) but lagged by $t_r' = 2$ hr.

Col (7): col (6) $\times \frac{t_r}{t_r'}$, $t_r = 4$ hr. $t_r' = 2$ hr.

Col (3): No. of unit storms in succession = $T/t_r = 24/4 = 6$, to produce a constant outflow.

$Q_e = \frac{2.78\text{ A}}{t_r} = \frac{2.78 \times 630}{4} = 437$ cumec, which agrees very well with the tabulated *S*-curve terminal value of 436. The *S*-curve additions can be written in one column, without having to write in 5 columns successively lagged by 4 hours (= t_r), as is illustrated in the example 5.5.

Time (hr)	4-hr UGO (cumec)	S-curve additions (cumec) (unit storms after every 4 hr = t_r)					S-curve ordinates (cumec) (2) + (3)	lagged S-curve (cumec)	S-curve difference (cumec) (4) – (5)	2-hr UGO (6) × 4/3 (cumec)
1	2	3					4	5	6	7
0	0	—	—	—	—	—	0	—	0	0
2	25	—	—	—	—	—	25	0	25	50
4	100	0	—	—	—	—	100	25	75	150
6	160	25	—	—	—	—	185	100	85	170
8	190	100	0	—	—	—	290	185	105	210
10	170	160	25	—	—	—	335	290	65	130
12	110	190	100	0	—	—	400	355	45	90
14	70	170	160	25	—	—	425	400	25	50
16	30	110	190	100	0	—	430	425	5	10*
18	20	70	170	160	25	—	445	430	15	30
20	6	30	110	190	100	0	436	445	– 9	– 18*
22	1.5	20	70	170	160	25	446.5	436	10.5	21
24	0	6	30	110	190	100	436	446.6	– 10.5	– 21*

➢ Synthetic Unit Hydrograph (SUH)

- Synder 1938 .

- the peak discharge per unit of watershed area q_{PR}
- the basin lag t_{PR} (time difference between the centroid of the excess rainfall hyetograph and the unit hydrograph peak).
- The base time t_b.
- the widths W (in time units) of the unit hydrograph at 50 and 75 percent of the peak discharge.
- Snyder defined a standard unit hydrograph as one whose rainfall duration t_r is related to the basin lag t_b by :

$$t_p = 5.5t_r$$

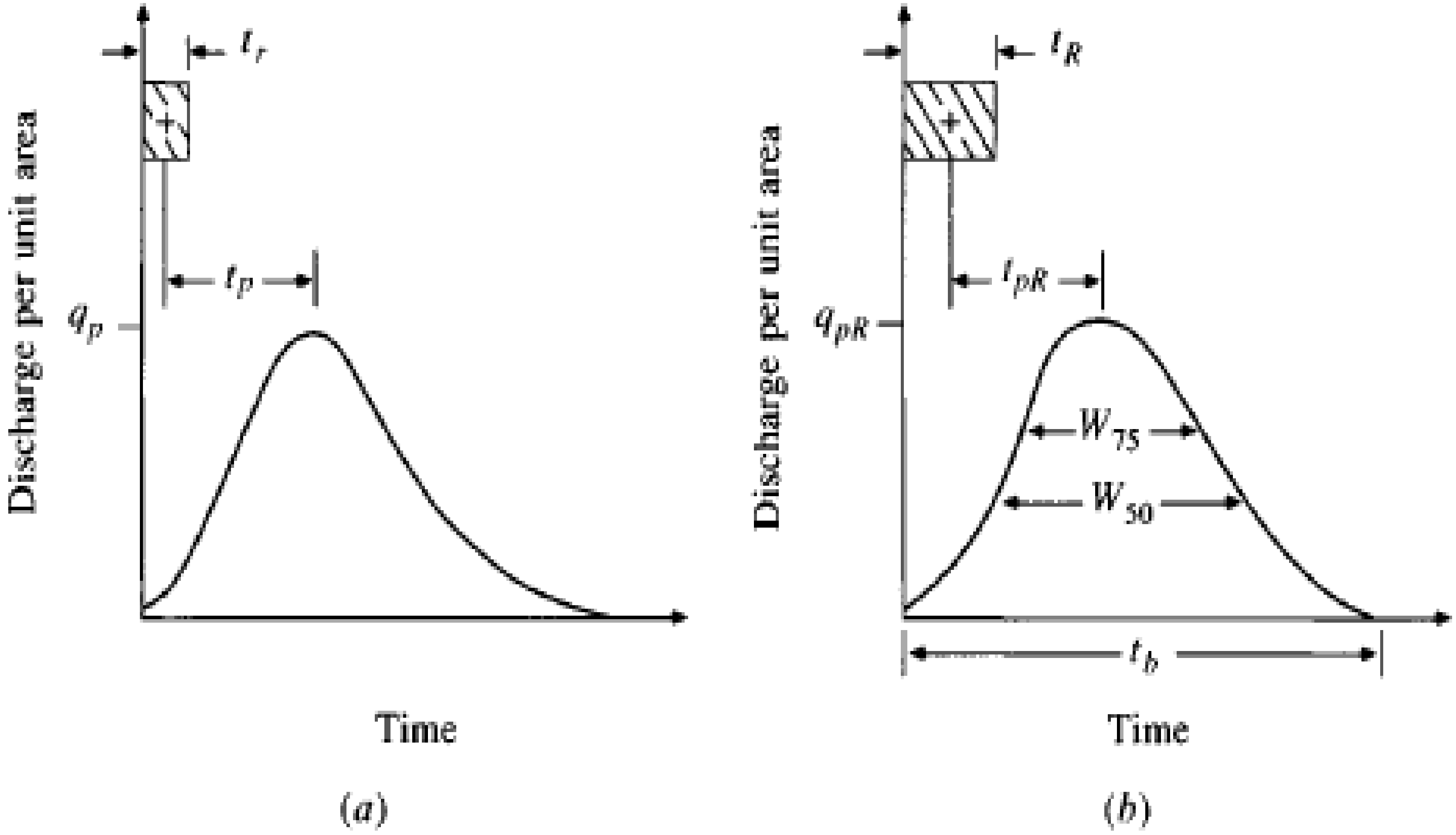


Fig. (5)

Snyder's synthetic unit hydrograph. (*a*) Standard unit hydrograph ($t_p = 5.5t_r$). (*b*) Required unit hydrograph ($t_{pR} \neq 5.5t_R$).

1. The basin lag is

$$t_p = C_1 C_t (L L_c)^{0.3}$$

$C_1 = 0.75$ (1.0 for the English system) C_t is a coefficient derived from gaged watersheds in the same region

2. The peak discharge per unit drainage area in $m^3/s \cdot km^2$ (cfs/mi^2) of the standard unit hydrograph is

$$q_p = \frac{C_2 C_p}{t_p}$$

where $C_2 = 2.75$ (640 for the English system) and C_p is a coefficient derived from gaged watersheds in the same region.

$$t_p = t_{pR} + \frac{t_r - t_R}{4}$$

3. The relationship between q_p and the peak discharge per unit drainage area q_{pR} of the required unit hydrograph is

$$q_{pR} = \frac{q_p t_p}{t_{pR}}$$

4. The base time t_b in hours of the unit hydrograph can be determined using the fact that the area under the unit hydrograph is equivalent to a direct runoff of 1 cm (1 inch in the English system). Assuming a triangular shape for the unit hydrograph, the base time may be estimated by

$$t_b = \frac{C_3}{q_{pR}}$$

where $C_3 = 5.56$ (1290 for the English system).

5. The width in hours of a unit hydrograph at a discharge equal to a certain percent of the peak discharge q_{pR} is given by

$$W = C_w q_{pR}^{-1.08}$$

where $C_w = 1.22$ (440 for English system) for the 75-percent width and 2.14 (770, English system) for the 50-percent width. Usually one-third of this width is distributed before the unit hydrograph peak time and two-thirds after the peak.

Example (6) : From the basin map of a given watershed, the following quantities are measured: $L = 150$ km, $L_c = 75$ km, and drainage area $= 3500$ km^2. From the unit hydrograph derived for the watershed, the following are determined: $t_R = 12$ h, $t_{pR} = 34$ h, and peak discharge $= 157.5$ m^3/s·cm. Determine the coefficients C_t and C_p for the synthetic unit hydrograph of the watershed.

Solution. From the given data, $5.5t_R = 66$ h, which is quite different from t_{pR} (34 h). Equation :

$$t_p = t_{pR} + \frac{t_r - t_R}{4}$$

$$= 34 + \frac{t_r - 12}{4}$$

$$t_p = C_1 C_t (L L_c)^{0.3}$$

$$32.5 = 0.75 C_t (150 \times 75)^{0.3}$$

$$C_t = 2.65$$

The peak discharge per unit area is $q_{pR} = 157.5/3500 = 0.045$ m^3/s·km^2·cm. The coefficient C_p is calculated by Eq. below with $q_p = q_{pR}$, and $t_p = t_{pR}$:

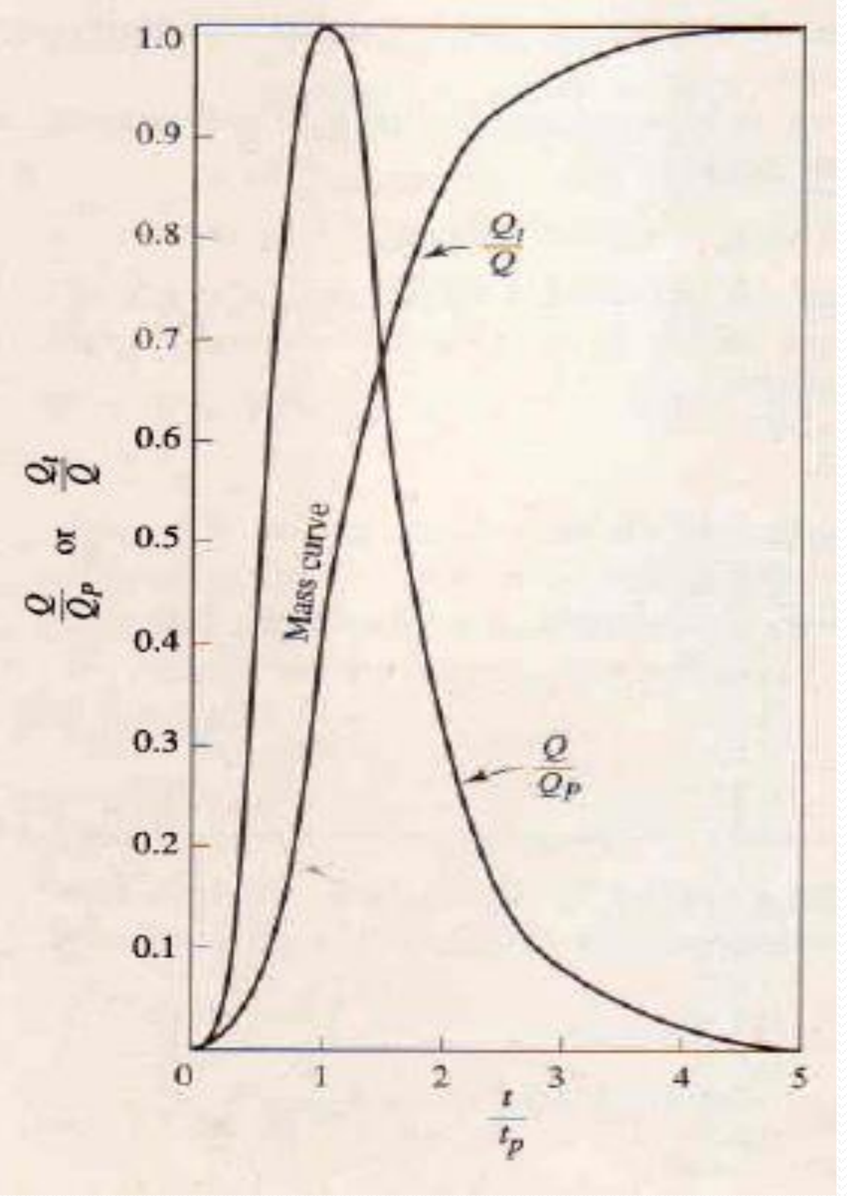
$$q_{pR} = \frac{C_2 C_p}{t_{pR}}$$

$$0.045 = \frac{2.75 C_p}{34.0}$$

$$C_p = 0.56$$

Dimensionless SCS Unit Hydrograph :

- A method developed by the Soil Conservation Service for constructing synthetic unit hydrographs is based on a dimensionless hydrograph



Fig(6) Dimensionless unit hydrograph and mass curve

$$t_p = \frac{D}{2} + t_l$$

$$Q_p = \frac{484A}{t_p}$$

Q_p = peak discharge (cfs)

A = drainage area (mi^2)

t_p = the time to peak (hr)

t_p = the time from the beginning of rainfall to peak discharge (hr)

D = the duration of rainfall (hr)

t_l = the lag time from the centroid of rainfall to peak discharge (hr)

COORDINATES OF SCS DIMENSIONLESS UNIT HYDROGRAPH OF FIGURE (6).

t/t_p	Q/Q_p	t/t_p	Q/Q_p
0	0	1.4	0.75
0.1	0.015	1.5	0.66
0.2	0.075	1.6	0.56
0.3	0.16	1.8	0.42
0.4	0.28	2.0	0.32
0.5	0.43	2.2	0.24
0.6	0.60	2.4	0.18
0.7	0.77	2.6	0.13
0.8	0.89	2.8	0.098
0.9	0.97	3.0	0.075
1.0	1.00	3.5	0.036
1.1	0.98	4.0	0.018
1.2	0.92	4.5	0.009
1.3	0.84	5.0	0.004

Example (7): For a drainage area of 70 mi^2 having a lag time of $8\frac{1}{2}$ hr, derive a unit hydrograph of duration 2 hr. Use the SCS dimensionless unit hydrograph.

Solution

1. Using Eq. $t_p = \frac{2}{2} + 8\frac{1}{2} = 9\frac{1}{2}$ hr

2. From Eq. $Q_p = \dfrac{484 \times 70}{9.5}$

   $Q_p = 3560$ cfs occurring at $t = 9\frac{1}{2}$ hr

3. Using Fig. 12.13, we find the following:
   - **a.** The peak flow occurs at $t/t_p = 1$ or at $t = 9\frac{1}{2}$ hr.
   - **b.** The time base of the hydrograph $= 5t_p$ or 47.5 hr.
   - **c.** The hydrograph ordinates are:
     1. At $t/t_p = 0.5$, $Q/Q_P = 0.43$; thus at $t = 4.75$ hr, $Q = 1531$ cfs.
     2. At $t/t_p = 2$, $Q/Q_P = 0.32$; thus at $t = 19$ hr, $Q = 1139$ cfs.
     3. At $t/t_p = 3$, $Q/Q_P = 0.07$; thus at $t = 28.5$ hr, $Q = 249$ cfs.

REFERENCES :

1. Ven Te Chow , David R.Maidment and Larry W. Mays, APPLIED HYDROLOGY , INDIAN EDITION , Mc Graw Hill Education .
2. H.M.Raghunath, Hydrology principles . Analysis . Design . Revised second Edition .
3. Warren Viessman , Jr.Gary L. Lewis. Introduction to hydrology . Fourth Edition .
4. P.K. Bhattacharya. Elements of Applied Hydrology . KHANNA PUBLISHERS DELHI-6 .
5. Das,M MOHAN .Hydrology .