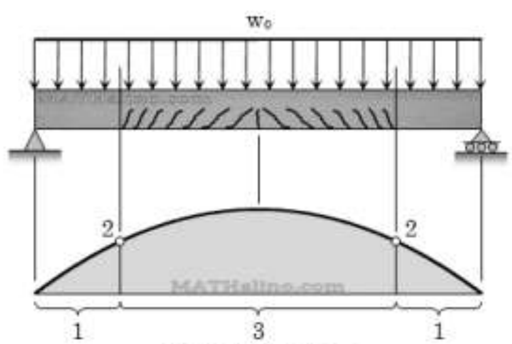
Flexural Analysis of Beam by Working Stress Method

Behaviour of Reinforced Concrete Beam under Loading:

Working Stress Analysis for Concrete Beams Consider a relatively long simply supported beam shown below. Assume the load (Wo) to be increasing progressively until the beam fails.

The beam will go into the following three stages:

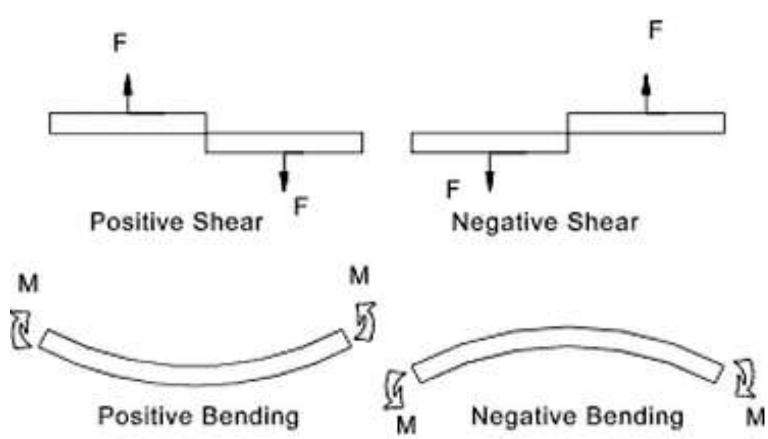
1. **Uncrack Concrete Stage.**
2. **Crack Concrete Stage (Elastic).**
3. **Ultimate Stress Stage – Beam Failure.**



Moment Diagram

At section 1: Uncrack stage:

1. Actual moment, (M) < Cracking moment (Mcr).
2. No cracking occur.
3. The gross section resists bending.
4. The tensile stress of concrete is below rupture.



$f_c < 0.5 f_c'$ .... Concrete is Elastic

$f_s < f_y$ .... Steel is Elastic

$f_{ct} < f_r$ .... Un-cracked

$$n = \frac{Es}{Ec} = \frac{200000}{4700\sqrt{fc'}}$$

<u>Where:</u>

fc: Actual compressive Strength for Concrete.

fc': Maximum compressive Strength for Concrete.

fs: Actual tensile strength for steel.

fy: Yield strength for steel.

fr: Modulus of rupture.

n: Modulus ratio.

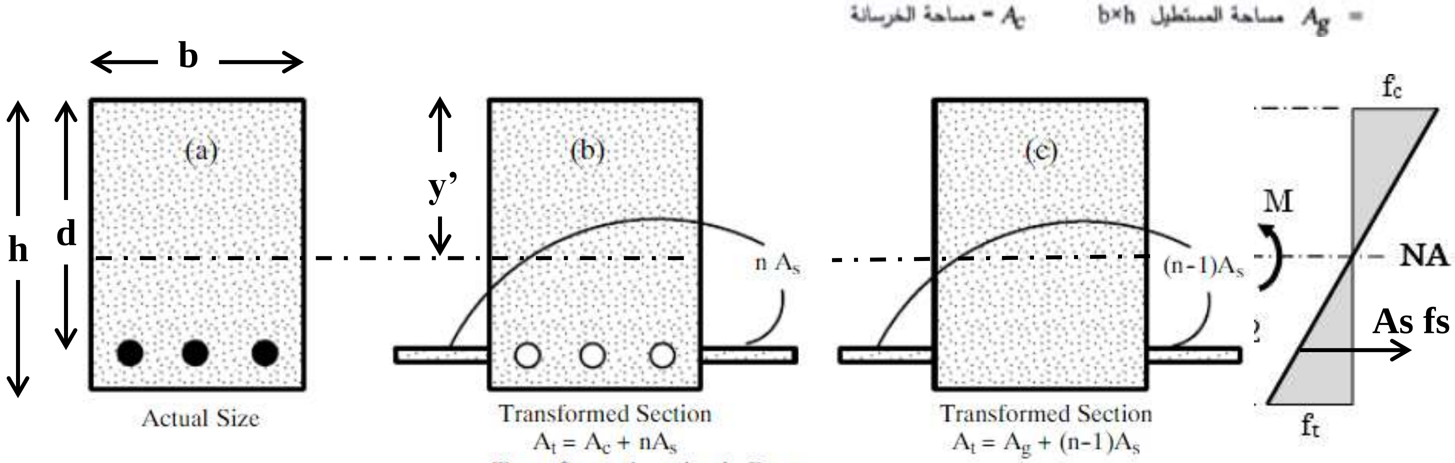
$$\varepsilon_{cs} = \varepsilon_s$$
$$\frac{f_{cs}}{E_c} = \frac{f_s}{E_s}$$
$$f_s = \frac{E_s}{E_c} f_{cs}$$
$$n = \frac{E_s}{E_c} \text{..........}$$
$$\therefore\ f_s = n f_{cs}$$

$$F_c = F_s$$
$$A_t f_{cs} = A_s f_s$$
$$A_t f_{cs} = A_s n f_{cs}$$
$$\therefore A_t = nA_s \text{ ......}$$

$$A_t = A_c + A_t = A_g - A_s + nA_s$$
$$A_t = A_g + (n-1)A_s \text{ ....(3.3)}$$

حيث A_t = مساحة المقطع المحول .

A_g = مساحة المستطيل b×h A_c = مساحة الخرسانة



Transformed section in Beams

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{(bh)(h/2) + (n-1)A_s * (d)}{bh + (n-1)A_s}$$

$$I_{N.A.} = \frac{b\bar{y}^3}{3} + \frac{b(h-\bar{y})^3}{3} + (n-1)A_s(d-\bar{y})^2$$

$$f_{ct} = \frac{M.C}{I_{N.A.}} = \frac{M_{max}(h-\bar{y})}{I_{N.A.}} \quad \text{check} < f_r = 0.7\sqrt{f'_c}$$

$\therefore$ Uncracked ok

$$f_c = \frac{M.C}{I_{N.A.}} = \frac{M_{max}(\bar{y})}{I_{N.A.}} \quad \text{check} < f_c \text{ allowable} = 0.5 f'_c$$

$\therefore$ ok Elastic

$$f_s = \frac{n.M(d-\bar{y})}{I_{N.A.}} \quad \text{check} < f_y$$

$\therefore$ ok Elastic

At Section 2 : Crack concrete stage:

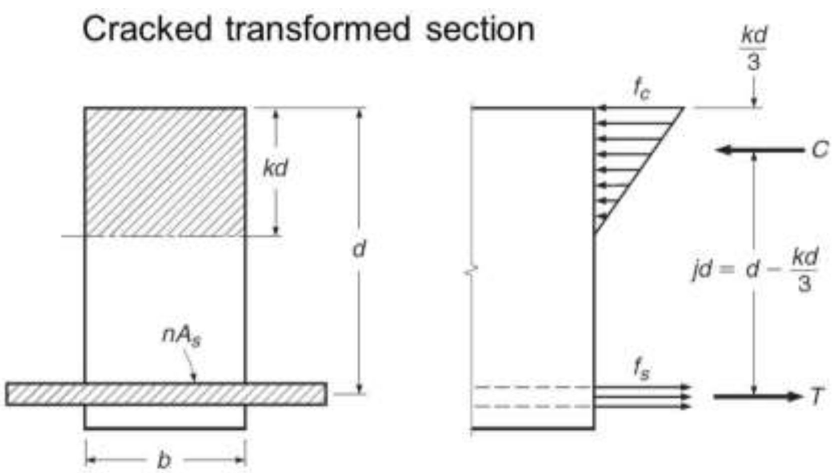
1- Actual moment, (M) > Cracking moment (Mcr).
2- Elastic stress stage.
3- Cracks developed at the tension fiber of the beam and spreads quickly to the neutral axis.
4- The tensile stress of concrete is higher than the rupture strength.
5- Ultimate stress stage can occur at failure.

$f_c < 0.5 f_c'$ .... Concrete is Elastic

$f_s < f_y$ .... Steel is Elastic

$f_{ct} > f_r$ .... Cracked

$$n = \frac{Es}{Ec} = \frac{200000}{4700\sqrt{fc'}}$$



Find N.A Position

Area of Compression about N.A = Area of Tension about N.A

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = n A_s (d - kd) \quad \text{—— (2)}$$

$$\text{Steel Ratio} = \frac{A_s}{bd} \Longrightarrow A_s = \rho b d$$

Sub. in eq.(2)

$$b \cdot kd \cdot \left(\frac{kd}{2}\right) = n \rho b d (d - kd)$$

$$\frac{k^2}{2} = n \rho (1 - k)$$

$$k^2 = 2 n \rho - 2 k \rho n$$

$$k^2 + 2 \rho n k - 2 \rho n = 0 \quad \text{—— (3)}$$ حل بالدستور

$$k = \sqrt{(\rho n)^2 + 2(\rho n)} - \rho n \quad \text{—— (4)}$$

$$I_{N.A} = \frac{b (kd)^3}{3} + n A_s (d - kd)^2$$

$$f_c = \frac{MC}{I_{N.A}} = \frac{M_{max}\, kd}{I_{N.A}} < 0.5 f'_c \quad \therefore (\text{Elastic})$$

$$f_s = n \frac{MC}{I_{N.A}} = \frac{n M_{max} (d - kd)}{I_{N.A}} < f_y \quad \therefore (\text{Elastic})$$

Allowable Stresses of Materials according to ACI-Code

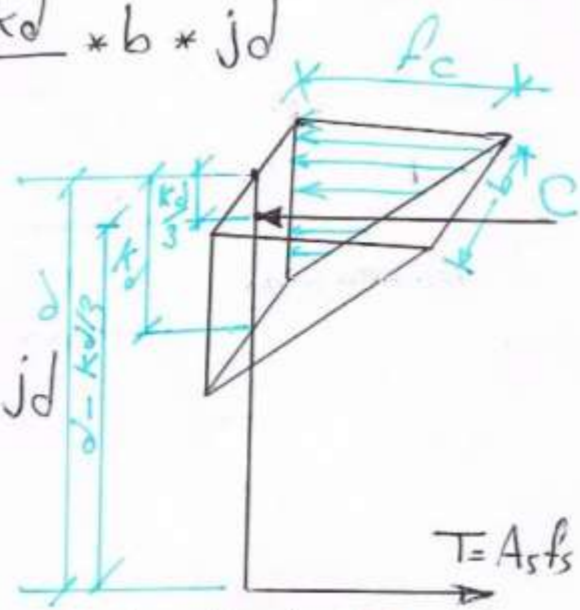
- Concrete $f_c = 0.45 f'_c$
- Steel Reinforcement $f_y = 300\,MPa \Rightarrow f_s = 140\,MPa$

$f_y = 400\,MPa \Rightarrow f_s = 170\,MPa$

- Method of Internal Moment أسلوب العزم الداخلي

$$M = C \cdot jd = \frac{f_c \cdot kd}{2} * b * jd$$

$$f_c = \frac{2M}{kjbd^2}$$



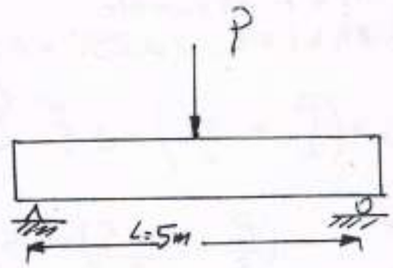
$$M = T \cdot jd = A_s f_s \cdot jd$$

$$\therefore f_s = \frac{M}{A_s j d}$$

$$\cancel{d} = \frac{k\cancel{d}}{3} + j\cancel{d} \Longrightarrow$$ (للمقطع المستطيل (فقط))

$$j = 1 - \frac{k}{3}$$

Ex:- Find Maximum Load (P) Can be applied at the Center of the beam Shown below for these information:-

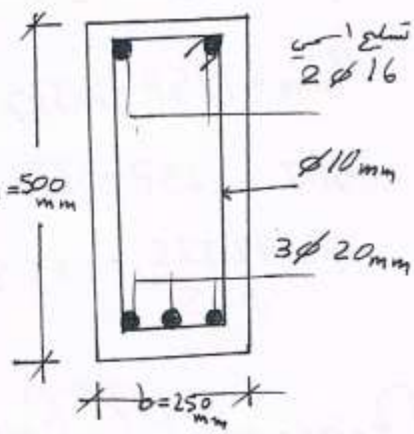


$b = 250\ mm$, $h = 500\ mm$

$A_s = 3\phi\ 20mm$, $E_s = 200\,000\ N/mm^2$

$E_c = 22\,000\ N/mm^2$, $\gamma_c = 24\ kN/m^3$

$f_y = 300\ MPa$, $f'_c = 20\ MPa$



Solution:- $d = 500 - (40 + 10 + \frac{20}{2}) = 440\ mm$

$$A_s = 3 * \frac{\pi}{4}(20)^2 = 942\ mm^2$$

$$\rho = \frac{A_s}{bd} = \frac{942}{250 * 440} = 0.0088$$

$$n = \frac{E_s}{E_c} = \frac{200\,000}{22\,000} \simeq 9.09$$

$$\rho n = 0.0088 * 9.09 = 0.08$$

$$K = \sqrt{(0.08)^2 + 2(0.08)} - (0.08) = 0.328$$

$$j = 1 - \frac{K}{3} = 0.891$$

$$jd = d - \frac{kd}{3} = 440 - \frac{0.328 * 440}{3} = 391.9\ mm$$

$$f_c = \frac{2M}{Kjbd^2} \quad , \quad f_{c_{all}} = 0.45 * 20 = 9.0\ MPa$$

$$M = 0.5 * f_{c_{all}} Kjbd^2 = 0.5 * 9 * 0.328 * 0.891 * 250 * (440)^2$$
$$= 63.652 * 10^6\ N.mm$$

$$f_s = \frac{M}{A_s jd} \quad (f_y = 300\ MPa) \quad f_{s_{all}} = 140\ MPa$$

$$M = f_{s_{all}} A_s j d = 140 * 942 * 0.891 * 440 = 51.7 * 10^6\ N.mm$$
$$= 51.7\ kN.m$$

M allowable is the smaller of the moments that make $f_c = f_{c_{all}}$

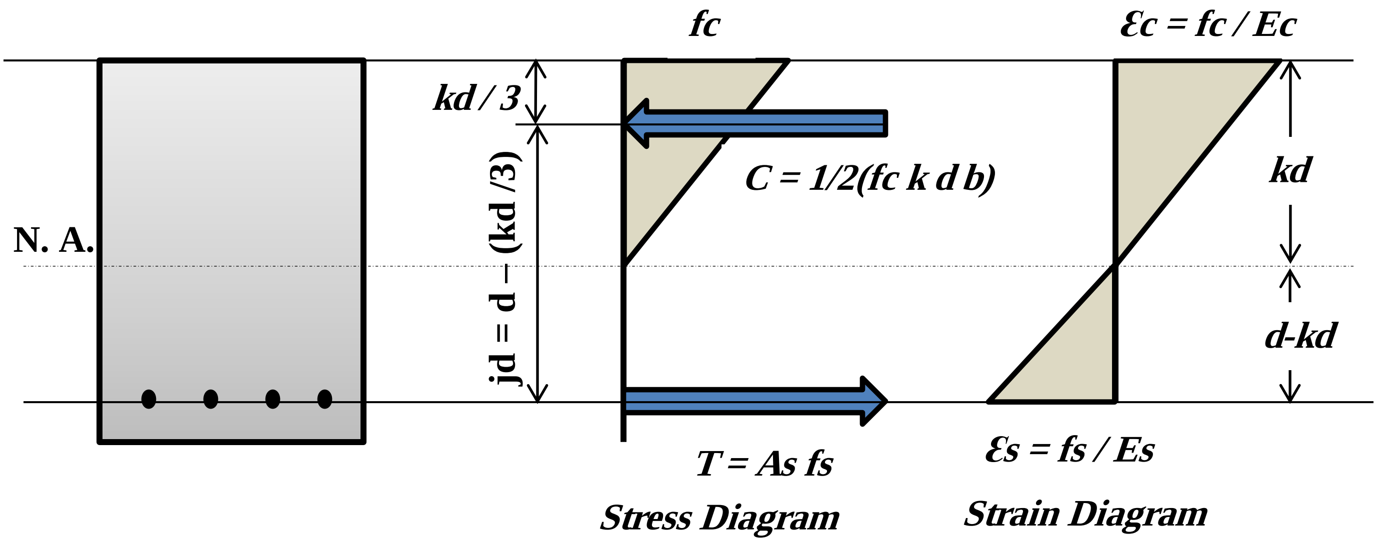
Design of R.C. Rectangular Beam by W.D. Method:

Notes:

1- **Analysis:** Given a cross section, concrete strength, reinforcement size and location, and yield strength, compute the resistance or strength. In analysis there should be one unique answer.

2- **Design**: Given a factored design moment, normally designated as select a suitable cross section, including dimensions, concrete strength, reinforcement, and so on. In design there are many possible solutions.

3- **Balance Section**: is economical section because it is used both of steel and concrete properties in high level.



From Strain Diagram:

$$\frac{\dfrac{f_c}{E_c}}{kd} = \frac{\dfrac{f_s}{E_s}}{d-kd} \Longrightarrow \frac{\dfrac{E_s}{E_c}}{k} = \frac{\dfrac{f_s}{f_c}}{1-k}$$

Let $r = \dfrac{f_{s\,all}}{f_{c\,all}}$, $\dfrac{n}{k} = \dfrac{\dfrac{f_s}{f_c}}{1-k}$

$\dfrac{f_s}{f_c} = \dfrac{n(1-k)}{k}$, in balance conditions $r = \dfrac{n(1-k_b)}{k_b}$

$$r k_b = n - n k_b \Longrightarrow r k_b + n k_b = n$$

$$k_b(r+n) = n \Longrightarrow \boxed{k_b = \frac{n}{n+r}}$$

From Stress Diagram

$$T = C \Longrightarrow A_s f_s = \frac{1}{2} f_c\, k\, d\, b$$

$$\frac{A_s}{bd} * \frac{f_s}{f_c} = \frac{1}{2} * k \Longrightarrow \rho \frac{f_s}{f_c} = \frac{k}{2}$$

in balance conditions $\rho_b \dfrac{f_{s\,all}}{f_{c\,all}} = \dfrac{k_b}{2} \Rightarrow \boxed{\rho_b = \dfrac{k_b}{2r}}$

$$\boxed{\rho_{min} = \frac{1.4}{f_y}}$$ according to ACI-code

$$\rho_b > \rho > \rho_{min}$$

E.X.

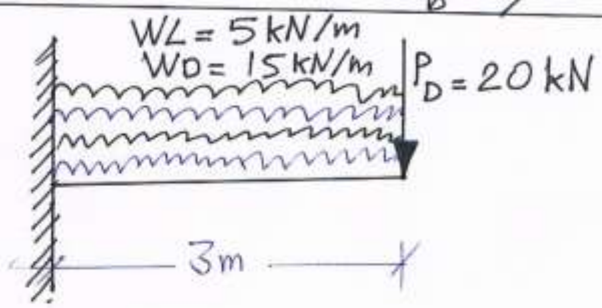


Fig.

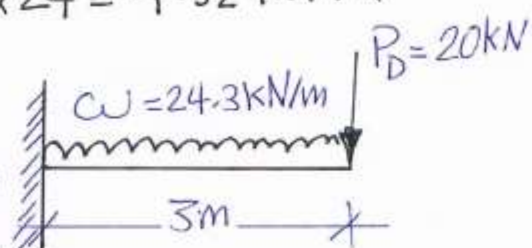
Design the cantilever shown in the Fig. below using the following data. $f'_c = 20\,N/mm^2$, $f_y = 275\,N/mm^2$, $E_s = 200\,000\,N/mm^2$, $\gamma_c = 24\,kN/m^3$

Solution:-

- Assume depth of cantilever $= \frac{L}{5} = h$
- Assume width of cantilever $(b) = \frac{h}{2} = \frac{L}{5}/2 = \frac{L}{10}$

$$\omega_{self} = b \times h \times 1 \times \gamma = \frac{h}{2} \times h \times 24$$

$$= \frac{L}{5} \times \frac{L}{10} \times 24 = 4.32\ kN/m$$



$$\omega_{total} = \omega_L + \omega_D + \omega_{self}$$

$$= 5 + 15 + 4.3 = 24.3\,kN/m$$

$$M_{max} = P_D \cdot L + \frac{\omega L^2}{2} = 169.44\,kN.m$$

$$\rho_b = \frac{k_b}{2r} \Rightarrow \quad k_b = \frac{n}{n+r} \quad , \quad r = \frac{f_{s\,all}}{f_{c\,all}} = \frac{140}{0.45 * 20} = 15.55$$

$$n = \frac{200\,000}{4700\sqrt{20}} = 9.52$$

$$k_b = \frac{9.52}{9.52 + 15.55} \simeq 0.38 \quad , \quad j = 1 - \frac{k}{3} = 1 - \frac{0.38}{3} = 0.8733$$

$\rho_b = \frac{0.38}{2*15.55} = \underline{\underline{0.0122}}$, $\rho_{min} = \frac{1.4}{275} = \underline{\underline{0.005}}$

Use $\rho = 0.01$

$M = M_s = \rho f_s j b d^2 \Longrightarrow 169.44*10^6 = 0.01*140*0.8733 * b*(2b)^2$

$4b^3 = 138.587*10^6$

$b = \sqrt[3]{34.64\times10^6} = 326$ mm USE $b = 330$mm

$d = 2b = 2*330 = 660$mm

$A_s = \rho b d = 0.01*330*660 = 2178$ mm^2

Use ϕ 22mm , $A_b = \frac{\pi}{4}*22^2 = 380$ mm^2

No of bars $= \frac{2178}{380} = 5.73$

Use $6\phi22$

- The distanse between each bar must not be less than 25mm
- The concrete cover from each sides must not be less than 100mm (i.e. 50mm for each side)
- If we put all bars in the same layer ∴ the width of the beam will be equal to
  $6\times22+5\times25+100 = 357$mm $> b = 330$ mm
- ∴ We distribute the bars into (2) layer one of them contains $4\phi22$ & the other contains $2\phi22$
- $4\times22+3\times25+100 = 263$mm $< b = 330$mm ∴ ok

$$h = d + \frac{\text{the distance between 2 bars}}{2} + d_{bar} + d_{stirup} + \text{Cover}$$

$$h = 660 + \frac{25}{2} + 22 + 10 + 40$$

$$= 744.5\,mm \Longrightarrow \text{USE } h = 750\,mm$$