ULTIMATE STRENGTH DESIGN METHOD (S.D.M)

The assumptions which are used in this method:

1- **Stress in reinforcement** varies linearly with strain up to the specified yield strength. The stress remains constant beyond this point as strains continue increasing. This implies that the strain hardening of steel is ignored.
2- **Concrete sections** are considered to have reached their flexural capacities when they develop 0.003 strain in the extreme compression fiber.
3- **Strains in reinforcement** and concrete are directly proportional to the distance from neutral axis. This implies that the variation of strains across the section is linear, and unknown values can be computed from the known values of strain through a linear relationship.
4- **Tensile strength of concrete is neglected.**
5- **Compressive stress** distribution of concrete can be represented by the corresponding stress-strain relationship of concrete.

Safety Factors: S.F.

a- S.F. = Max. Stress / Allowable Stress (W.S.D.M)
b- S.F. = Max Load / Service Load (S.D.M)

Load Factors:

U= 1.2 D + 1.6 L

U= 1.2 D + 1.6 L +0.5 (Lr or S or R)

U= 1.2 D + 1.6 (Lr or S or R) + (1.0 L or 0.5 W)

U= 1.2 D + 1.0 W +1.0 L + 0.5 (Lr or S or R)

D: Dead Load, L: Live Load, W: Wind Load, S: Snow Load, Lr: Roof Load, R: Rain Loaf.

Strength Reduction Factors:

⋆ Tension .... Ǿ = 0.9 ⟹ Mu = Ǿ Mn

Mu: Ultimate moment capacity.

Mn: Nominal (Actual) moment capacity.

⋆ Shear, Torsion .... Ǿ = 0.75 ⟹ Vu = Ǿ Vn

Vu: Ultimate shear capacity.

Vn: Nominal shear capacity.

⋆ Compression:

a- Ǿ = 0.70 for spiral reinforced member like column.
b- Ǿ = 0.65 for other reinforced member like column.

Stress and Strain Distribution:

Resultant of concrete compressive force :

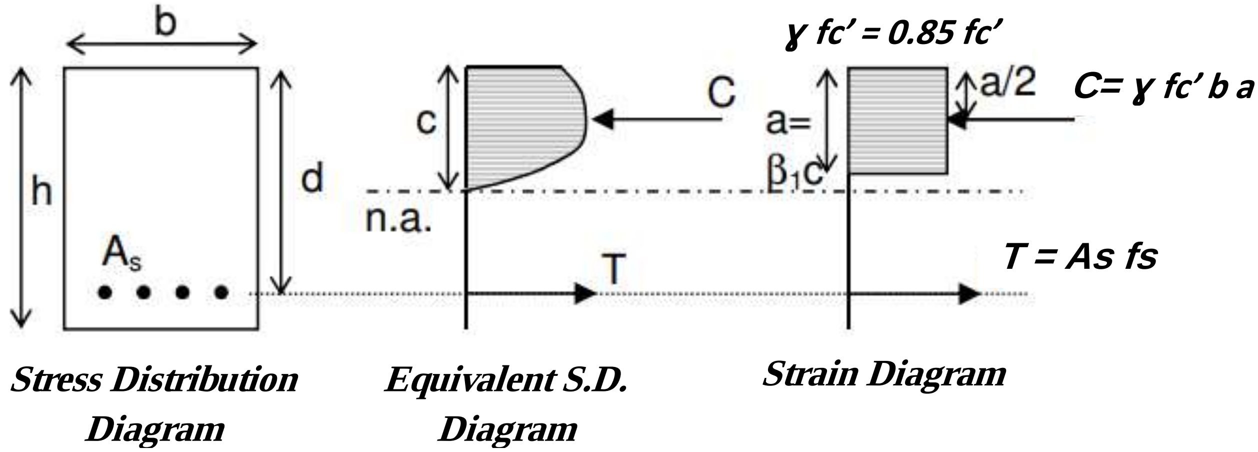
C = fav . b . c

Where:

fav: average compressive stress.

b: the width of section.

c: the depth of Neutral Axis.



$$C = \alpha\ fc'\ b\ c$$

Where:

$$\alpha = \frac{avaerage\ concrtet\ stress}{concrete\ compressive\ stress}$$

The location of the resultant is usually represented by βc.

Where:

$$\beta = \frac{compressive\ resultant\ depth}{N.A.\ depth}$$

α: 0.72 for fc' ≤ 30 MPa

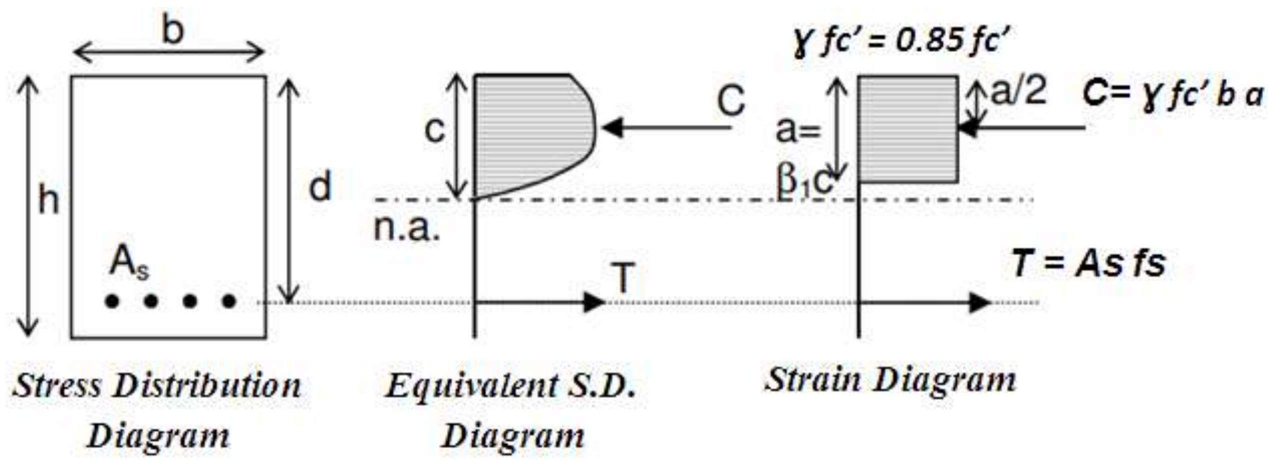
α: decreased by (0.04) for every (7 MPa) increasing in compressive strength of concrete.

α: Value must not be less than (0.56).

β: 0.425 for fc'≤ 30MPa

β: decreased by (0.025) for every (7 MPa) increasing in compressive strength of concrete.

β: value must be less than (0.325).



Equivalent rectangular stress block is used for analysis of reinforced concrete sections:

$$C = \alpha fc' b c = \gamma fc' a b \text{ ....(1)}$$

Let $a = \beta 1 . c$ .... (2)

We can find γ , $\beta 1$, α , β

$a/2 = \beta . c$ .... $a = 2 (\beta . c)$

from eq. (2) .... $\beta 1 . c = 2 (\beta . c)$ .... $\beta 1 = 2 \beta$ ...... (3)

sub. in eq. (1) :

$$\alpha fc' b c = \gamma fc' a b \Longrightarrow \gamma = \alpha c/a \Longrightarrow \gamma = \alpha c/2(\beta c) \Longrightarrow$$

$$\gamma = \alpha / 2(\beta_1/2) \Longrightarrow \gamma = \alpha / \beta_1 \ldots\ldots (4)$$

From the above equation and from the value of (β, γ) we can find the value of (β_1, γ):

$\beta_1 = 2\ \beta \ \ldots\ \beta_1 = 2 * 0.425 = 0.85$

$\gamma = (\alpha / \beta_1) = (0.72/0.85) = 0.85 \ \ldots\ (5)$

$\beta_1 = 0.85$ *for* $f_c' \leq 30$ MPa,

β_1: decreased by (0.05) for every (7 MPa) increasing in compressive strength of concrete.

β_1: value must not be less than (0.65).

$\beta_1 = 0.85 - [0.05(f_c'-30)/7]$

Analysis and Design of Singly Reinforced Rectangular Beam:

a- **Balance or Under Reinforced.**

$$\rho \leq \rho_b \Rightarrow f_s = f_y$$

from the equilebrium conditions

$$C = T$$

$$0.85 f_c' * b * a = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \text{ —(a)}, \quad A_s = \rho b d \text{ —(b)}$$

$$a = \frac{\rho b d f_y}{0.85 f_c' b} \rightarrow a = \frac{\rho f_y d}{0.85 f_c'} \text{ —(1)}$$

$$M_n = A_s f_y * (d - \frac{a}{2}) \text{ —(2)}$$

$$M_n = 0.85 f_c' \cdot a \cdot b (d - \frac{a}{2}) \text{ —(3)}$$

sub a & b in equ (2)

$$M_n = \rho b d f_y \left[d - \frac{\rho f_y d}{2\ 0.85 f_c'}\right]$$

$$M_n = \rho b d^2 f_y \left[1 - \frac{0.59 \rho f_y}{f_c'}\right]$$

$$M_u = \phi M_n$$

$$M_u = \phi \rho b d^2 f_y \left[1 - \frac{0.59 \rho f_y}{f'_c}\right] \quad \text{—— (4)}$$

b- **Over Reinforced Beam:**

b- Over Reinforced Beams $\therefore \rho > \rho_b$

f_s = unknown $> f_y$

$$A_s f_s = 0.85 f'_c \cdot a \cdot b$$

$$A_s f_s = 0.85 f'_c \cdot (\beta_1 c) \cdot b$$

There are (2) unknowns f_s and c

After many steps:-

$$m = \frac{600}{0.85 \beta_1 f'_c} \quad , \quad k_u = \sqrt{\frac{(\rho m)^2}{2} + \rho m} - \frac{\rho m}{2}$$

Then we can find the nominal strength by the following procedure:-

1- find ρ, m were $\rho = \frac{A_s}{bd}$, $m = \frac{600}{0.85 \beta_1 f'_c}$

2- submit in $K_u = \sqrt{\frac{(\rho m)^2}{2} + \rho m} - \frac{\rho m}{2}$

3- calculate c value were $c = k_u \cdot d$

4- calculate a value were $a = \beta_1 c$

5- find f_s were $f_s = 600 - \frac{(d-c)}{c}$

6- find the nominal bending moment M_n by using on of the three equations (2), (3) and (4)

متطلبات الكود الامريكي للعتبات ناقصة التسليح:

Maximum Steel Ratio

- Tension failure occurs when $f_s = f_y$ before concrete strain reachs Max. Strain = 0.003, and this failure occurs gradually.
- Compression failure occurs when the strain of concrete reach max. strain = 0.003 before steel stress reach the yeild strength = f_y
- Tension failure hapend when $\rho < \rho_b$
- Tension failure is better than compression failure.

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \cdot \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

($\varepsilon_s = 0.004$ according to ACI code 2002)

Determination of Reduction Factors (ϕ)

a- For members with tension controlled

$$\varepsilon_t \geq 0.005 \Rightarrow \phi = 0.9$$

$$\rho_t = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005}$$

b- For compression controlled members

$$\varepsilon_t \leq 0.002 \Rightarrow \phi = 0.7 \text{ (spiral Reinforcement)}$$

$$\phi = 0.65 \text{ (other kind of Reinforcement)}$$

C- Transition between tension and compression

- Spiral reinforcement

$$\phi = 0.7 + \frac{0.2}{0.003}(\varepsilon_t - 0.002) = 0.567 + 66.7\,\varepsilon_t$$

- Other reinforcement

$$\phi = 0.65 + \frac{0.25}{0.003}(\varepsilon_t - 0.002) = 0.483 + 83.3\,\varepsilon_t$$

ACI-Code encourages the designers to reduce the (ρ) value to increase the magnitude of (ϕ)

Minimum Steel Ratio

$$\rho_{min} = \frac{A_{s\,min}}{b_w d} = \frac{\sqrt{f'_c}}{4 f_y} \geq \frac{1.4}{f_y}$$

Design by Ultimate Design Method:

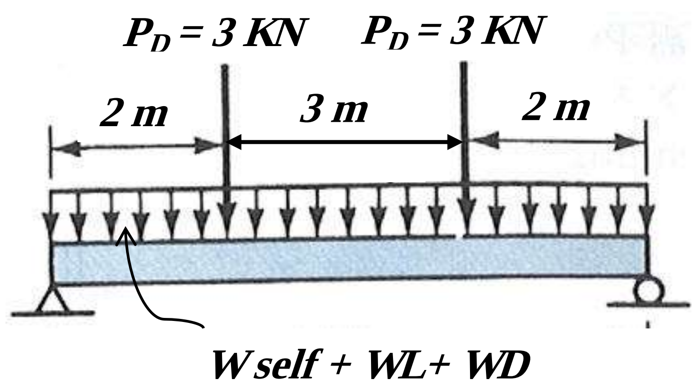
1- The design of R.C. members means finding the adequate dimensions for these members and the reinforcement magnitude to enable the member to withstand maximum loads applied on it safety.
2- Sometime, all dimensions or some of them are determined by architectures.
3- Complete design for the beam requires determine the shear reinforcement, torsion reinforcement and check deflections; check development lengths and points of cuts or bend of steel reinforcement. All these details must be put on the beam sketch or diagram.

Ex.: Design the beam shown for the following data:

fc' = 20 N/mm2 and fy= 300 N/mm2

WL= 6 KN/m and WD= 12 KN/m

γc = 24 KN/m3



Solution:

1- Find Moment (Mu):

* for cantilever, assume $h = \frac{L}{5}$

for simply supported beam & continous beam, assume $h = \frac{L}{10}$

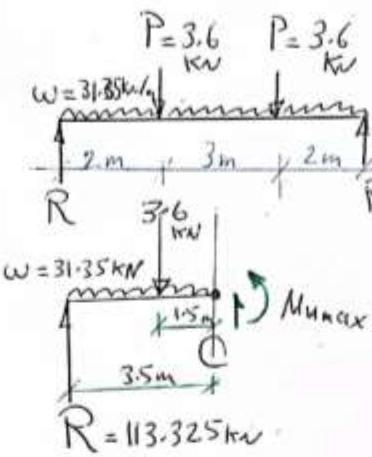
* $b = \frac{h}{2}$, $\therefore b = \frac{L}{10}/2 = \frac{L}{20}$

$$\omega_{self} = b * h * 1 * \gamma_c = \frac{L}{20} * \frac{L}{10} * 1 * 24 = \frac{L^2}{8.33} \simeq \frac{L^2}{8} (kN/m)$$

$$\omega_{total} = \omega_{self} + \omega_D + \omega_L$$

$$= 1.2\left(\frac{(7)^2}{8} + 12\right) + 1.6(6) = 31.35 \text{ kN/m}$$

$$P = 1.2 P_D = 1.2 * 3 = 3.6 \text{ kN}$$



$$\Sigma F_y = 0$$

$$R = \frac{3.6*2 + 31.35*7}{2} = 113.325 \text{ kN}$$

$$\Sigma M_{(c)} = 0$$

$$M_{umax} = R*3.5 - P*1.5 - \frac{\omega(3.5)^2}{2}$$

$$= 113.325*3.5 - 3.6*1.5 - \frac{31.35(3.5)^2}{2}$$

$$M_{umax} = 199.219 \text{ kN}\cdot\text{m}$$

2- ρ_{min}, ρ_{max} & ρ_t

* from Table(3), $\rho_{min} = 0.0047$

or $\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{300} = 0.00467$ ✓

$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{20}}{4*300} = 0.00373$

use the bigst value

* ρ_{max} :- from Table(3) :- $\rho_{max} = 0.0206$

or by the eq. :- $\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003+\varepsilon_t}$ $(\varepsilon_t = 0.004)$

$(\beta_1 = 0.85 \text{ for } f'_c \leq 30MPa)$ $\rho_{max} = 0.85*(0.85)*\frac{20}{300}*\frac{0.003}{0.003+0.004}$

$\rho_{max} = 0.02064$

• We must use $\rho \Longrightarrow \rho_{min} \leq \rho \leq \rho_{max}$

3- • for use $\phi = 0.9$ ρ_{must} be $\leq \rho_t$

$\rho_t = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{0.003}{0.003+\varepsilon_t}$ $(\varepsilon_t = 0.005)$

$\rho_t = 0.85(0.85)*\frac{20}{300}*\frac{0.003}{0.003+0.005} = 0.01806$

or from Table(3) $\rho_t = 0.0180$

∴ USE $\rho = 0.0170$

4- $M_u = \phi \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f'_y}{f'_c}\right)$

$199.22 \times 10^6 = 0.9 \times 0.0170 \times bd^2 \times 300 \left(1 - 0.59 \times 0.017 \times \frac{300}{20}\right)$

$199.22 \times 10^6 = 4.59\, bd^2 - 0.6906\, bd^2$

$bd^2 = 51089459.3$

assume $b = \frac{d}{2}$

$\frac{d}{2} \times d^2 = 51089459.3$

$d^3 = 102.179 \times 10^6$

$d = \sqrt[3]{102.179 \times 10^6} = 467.5$ mm $\Rightarrow$ USE $d = 470$ mm

5- $A_s = \rho b d = 0.0170 \times 240 \times 470 = 1917.6 \simeq 1918\,mm^2$

Use $\phi_{bar} = 22\,mm$ $A_{s\,bar} = \frac{\pi}{4} \times (22)^2 = 380.13\,mm^2$

(or from Table (1)):-

No of bars $= \frac{A_s}{A_{s\,bar}} = \frac{1918}{380} = 5.047 \simeq 5$

6- • $S \geq \begin{cases} 25\,mm \\ \phi_{bar} \\ \frac{4}{3} \times \text{max. Size of aggregate} \end{cases}$ distance between bars

- Thickness of the cover:-

a- cover $\geq$ 75mm (on the ground)

b- cover $\geq$ 25mm (concrete in contact with soil or with invironment conditions)

(for slabs & أسس & wall slabs)

* for other concrete members = 40 mm

c- cover $\geq$ 20mm (concret is not in contact with soil or other conditions)

(slabs, أسس & walls)

* for beams & columns = 40mm & for secondary = 25mm

$$S = \left(b - 2(\overset{40}{\text{cover}} + \phi_{s_{bar}}) - n\phi_{bar}\right)/(n-1)$$

n = No of bars

b = width of the section

ϕ_{bar} = bar diameter

ϕ_s or $\phi_{s_{bar}}$ = diameter of shear reinf.

assume the steel bars are distrebuted in one layer.

$$S_{actuale} = [240 - 2*(40+10) - 5*22] / (5-1) = 7.5 \text{ mm}$$

$$S = \begin{cases} 25 \text{ mm} \\ \phi_b = 22 \text{ mm} \\ \frac{4}{3} * \text{max. of agg.} \end{cases} \quad \therefore S = 25 \text{ mm}$$

$\because S_{act} < S_{min} = 25$ mm

$\therefore$ let us use the reinf. in two layers

5 bars → 3 bars, 2 bars

$$S_{act} = [240 - 2(40+10) - 3*22] / (3-1) = 37 \text{ mm} > 25 \text{ mm}$$

$\therefore$ o.k.

$h = d + 70$ mm (one layer)
$h = d + 100$ mm (two layers)
$h = d + 130$ mm (3 layers)

or actual $h = d + \frac{S}{2} + d_{bar} + d_s + \text{cover}$

$$h = 470 + 25/2 + 22 + 10 + 40$$
$$= 554.5 \text{ mm}$$

Use $h = 560$ mm

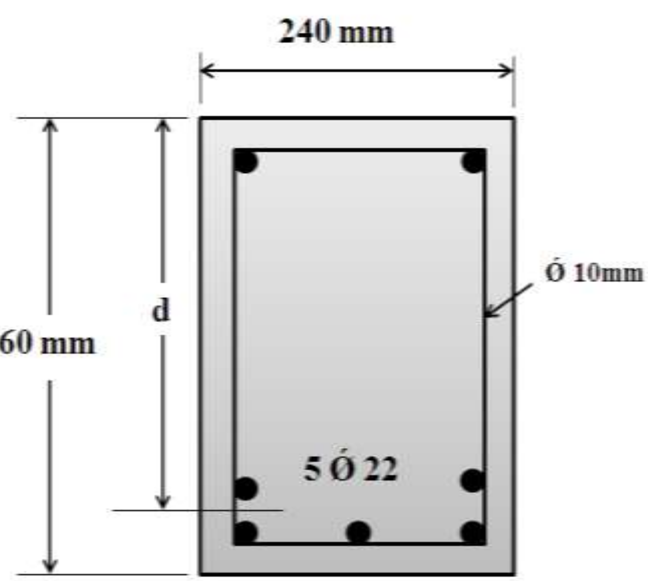
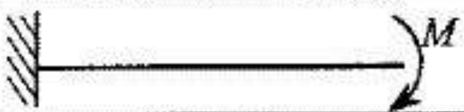
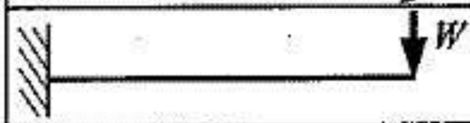
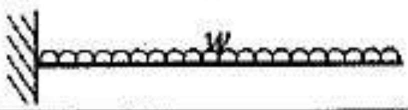
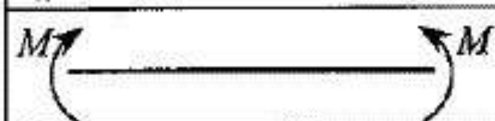
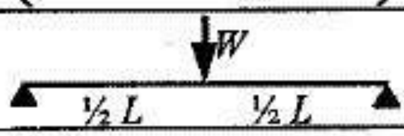
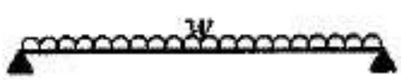
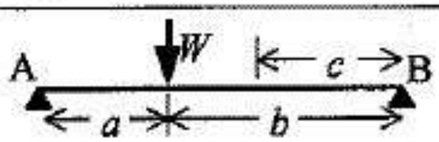


Table (3) : ρ_{min} and ρ_{max} values

f_y (Mpa)	f'_c (Mpa)	β_1	ρ_b	ρ_{max}	ρ_t	$\rho_{min} = \frac{1.4}{f_y}$	$\rho_{min} = \frac{\sqrt{f'_c}}{4f_y}$
300	20	0.85	0.0321	0.0206	0.018	0.0047	0.0037
	25	0.85	0.0401	0.0258	.0226	0.0047	0.0042
	30	0.85	0.0482	0.031	0.0271	0.0047	0.0046
	35	0.814	0.0538	0.0346	.0303	0.0047	0.0049
	40	0.779	0.588	0.0378	.0331	0.0047	0.0053
350	20	0.85	0.0261	0.0177	.0155	0.004	0.0032
	25	0.85	0.0326	0.0221	0.0193	0.004	0.0036
	30	0.85	0.0391	0.0265	0.0232	0.004	0.0039
	35	0.814	0.0437	0.0296	0.0259	0.004	0.0042
	40	0.779	0.0478	0.0324	0.0284	0.004	0.0045
400	20	0.85	0.0217	0.0155	0.0136	0.0035	0.0028
	25	0.85	0.0271	0.0194	0.017	0.0035	0.0031
	30	0.85	0.0325	0.0232	0.0203	0.0035	0.0034
	35	0.814	0.0363	0.026	0.0228	0.0035	0.0036
	40	0.779	0.0397	0.0284	0.0249	0.0035	0.0039

BEAM BENDING

L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \le b, \quad c = \sqrt{\frac{1}{3} b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)