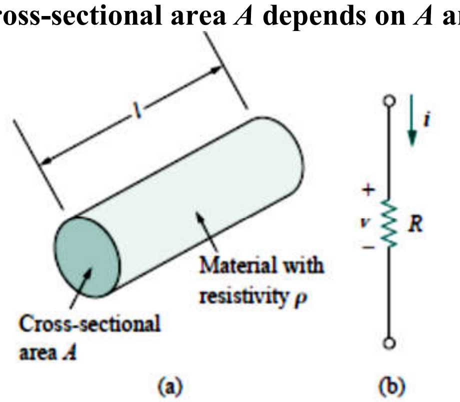
WEEK 3

Basic Laws

OHM'S LAW

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist current, is known as *resistance* and is represented by the symbol *R*. The resistance of any material with a uniform cross-sectional area *A* depends on *A* and its length *l*, as shown in Fig



(a) Resistor, (b) Circuit symbol for resistance

$$R = \rho \frac{\ell}{A}$$

Where ρ is known as the *resistivity* of the material in ohm-meters. Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities. Table 2.1 presents the values of ρ for some common materials and shows which materials are used for conductors, insulators, and semiconductors.

TABLE 2.1 Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

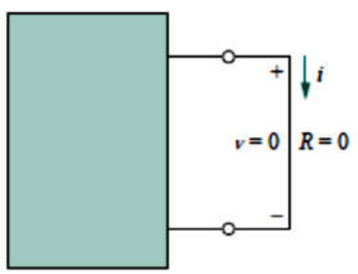
Ohm's law states that the voltage v across a resistor is directly proportional to the current flowing through the resistor.

$$\boxed{v = iR} \qquad v \propto i$$

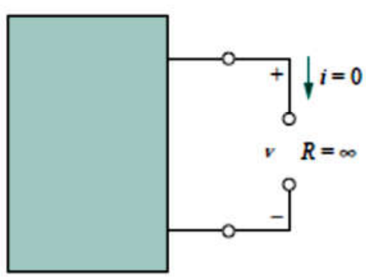
The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω). **Since the value of R can range from zero to infinity, it is important that we consider the two extreme possible values of R. An element with**
$R = 0$ is called a *short circuit*, as shown in Fig.(a)

.A short circuit is a circuit element with resistance approaching zero.

$$v = iR = 0$$



(a)



(b)

(a) Short circuit *($R = 0$)*,
(b) Open circuit *($R = \infty$)*.

An open circuit is a circuit element with resistance approaching infinity.

A useful quantity in circuit analysis is the reciprocal of resistance R, known as *conductance* and denoted by G:

$$\boxed{G = \frac{1}{R} = \frac{i}{v}}$$

The unit of conductance is the *mho* (ohm spelled backward) or reciprocal ohm, with symbol ℧ the inverted omega. Although engineers often use the mhos, in this book we prefer to use the Siemens (S), the SI unit of conductance

$$1\ \text{S} = 1\ ℧ = 1\ \text{A/V}$$

Conductance is the ability of an element to conduct electric current; it is measured in mhos ℧ or Siemens (S).

The same resistance can be expressed in ohms or Siemens. For example, 10 Ω is the same as 0.1 S.

$$i = Gv$$

The power dissipated by a resistor can be expressed in terms of *R*. Using

$$p = vi = i^2R = \frac{v^2}{R}$$

The power interms of ***G***

$$p = vi = v^2G = \frac{i^2}{G}$$

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since *R* and *G* are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy

Example

An electric iron draws 2 A at 120 V. Find its resistance.

Solution:
From Ohm's law,

$$R = \frac{v}{i} = \frac{120}{2} = 60\ \Omega$$

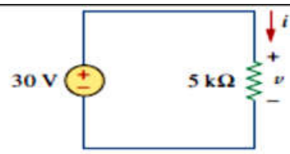
Practice Problem

The essential component of a toaster is an electrical element (a resistor) that converts electrical energy to heat energy. How much current is drawn by a toaster with resistance 15 Ω at 110 V?

Answer: 7.333 A.

Example

In the circuit shown in Fig. 2.8, calculate the current *i*, the conductance *G*, and the power *p*.



Solution:
The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways using either Eqs. (1.7), (2.10), or (2.11).

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

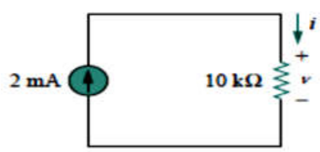
$$p = i^2R = (6 \times 10^{-3})^2 5 \times 10^3 = 180 \text{ mW}$$

or

$$p = v^2G = (30)^2 0.2 \times 10^{-3} = 180 \text{ mW}$$

Practice Problem

For the circuit shown in Fig., calculate the voltage *v*, the conductance *G*, and the power *p*



Example

A voltage source of $20 \sin \pi t$ V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

$$i = \frac{v}{R} = \frac{20 \sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

NODES, BRANCHES, AND LOOPS

A branch represents a single element such as a voltage source or a resistor.

In otherwords, a branch represents any two-terminal element. The circuit in Fig. has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

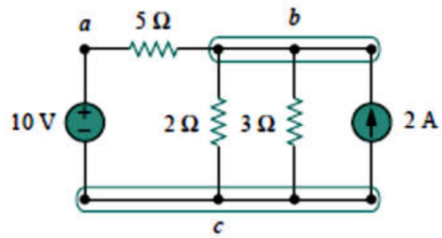
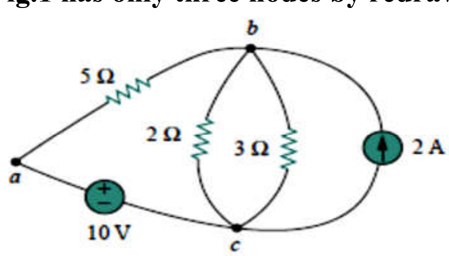


Figure.1 **Nodes, branches, and loops.**

A node is the point of connection between two or more branches.

A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig..1 has three nodes *a, b*, and *c*. Notice that the three points that form node *b* are connected by perfectly conducting wires and therefore constitute a single point. The same is true of the four points forming node *c*. We demonstrate that the circuit in Fig.1 has only three nodes by redrawing the circuit in Fig. 2.



The three-node circuit of Fig. 2. is redrawn

A loop is any closed path in a circuit.

A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node A network with *b* branches, *n* nodes, and *l* independent loops will satisfy the fundamental theorem of network topology

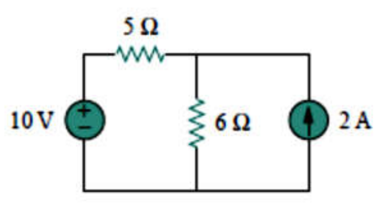
$$\boxed{b = l + n - 1}$$

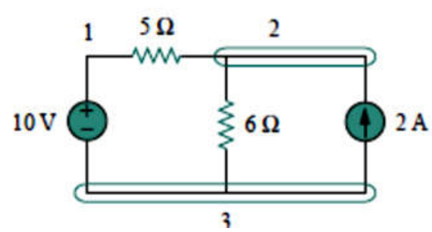
Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.
Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Example

Determine the number of branches and nodes in the circuit shown in Fig. 3. Identify which elements are in series and which are in parallel.

<u>Solution</u>:
Since there are four elements in the circuit, the circuit has four branches: 10 V, 5 Ω, 6 Ω, and 2 A. The circuit has three nodes as identified in Fig. 3. The 5Ω resistor is in series with the 10-V voltage source because the same current would flow in both. The 6Ωresistor is in parallel with the 2-A current source because both are connected to the
same nodes 2 and 3.





(3)The three nodes in the circuit

KIRCHHOFF'S LAWS

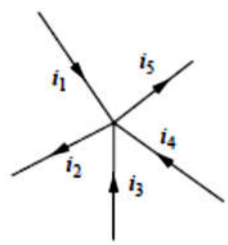
Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

Mathematically, KCL implies that

$$\boxed{\sum_{n=1}^{N} i_n = 0}$$

Where *N* is the number of branches connected to the node and *in* is *n*th current entering (or leaving) the node. By this law, currents entering node may be regarded as positive, while currents leaving the node maybe taken as negative or vice versa Consider the node in Fig.4. Applying KCL gives

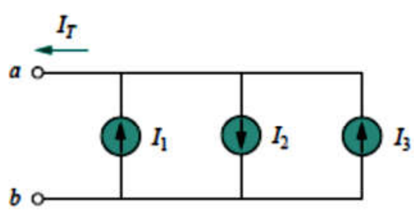


(4) Currents at a node illustrating KCL.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

The sum of the currents entering a node is equal to the sum of the currents leaving the node.



(a)

$$I_T = I_1 - I_2 + I_3$$

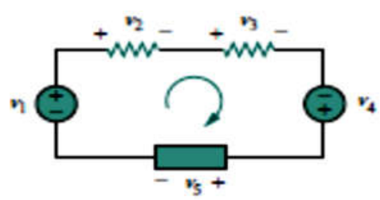
Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\boxed{\sum_{m=1}^{M} v_m = 0}$$

Where *M* is the number of voltages in the loop (or the number of branches in the loop) and $v_m$is the *m*th voltage

The sign oneach voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be$-v_1$,$+v_2$,$+v_3$,$-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have$+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields



$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4$$

which may be interpreted as

$$\boxed{\text{Sum of voltage drops} = \text{Sum of voltage rises}}$$

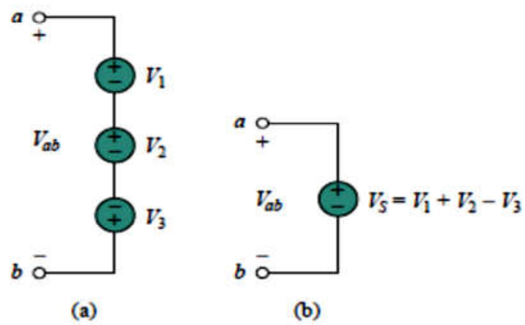
When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources. For example, for the voltage sources shown in Fig. 2.20(a), the combined or equivalent voltage source in Fig. 2.20(b) is obtained by applying KVL.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

or

$$V_{ab} = V_1 + V_2 - V_3 \tag{2.23}$$

To avoid violating KVL, a circuit cannot contain two different voltages V_1 and V_2 in parallel unless $V_1 = V_2$.



Voltage sources in series:
(a) original circuit, (b) equivalent circuit.

Example

For the circuit in Fig. 2.21(a), find voltages v_1 and v_2.

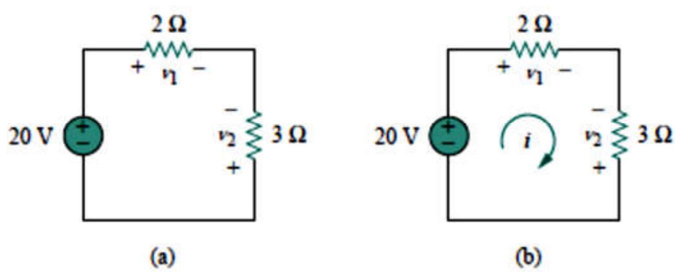


Figure 2.21 For Example 2.5.

Solution:

To find v_1 and v_2, we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \qquad ($$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \qquad ($$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

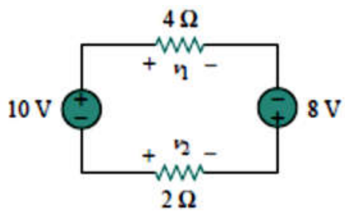
$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20 \qquad \Longrightarrow \qquad i = 4\text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8\text{ V}, \qquad v_2 = -12\text{ V}$$

Practice Problem

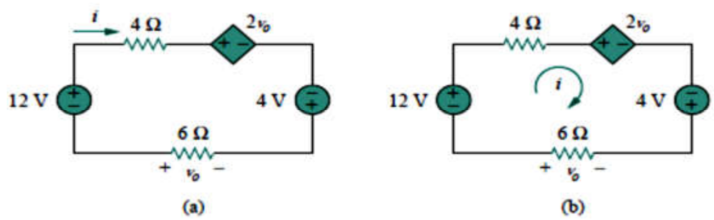
Find v_1 and v_2 in the circuit of Fig



Answer: 12 V, −6 V.

Example

Determine v_o and i in the circuit shown in Fig



Solution:
We apply KVL around the loop as shown in Fig

$$-12 + 4i + 2v_o - 4 + 6i = 0$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i$$

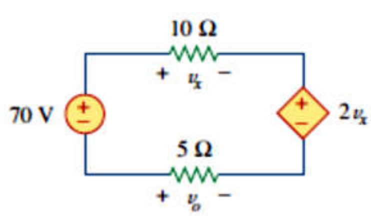
Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \qquad \Longrightarrow \qquad i = -8 \text{ A}$$

and $v_o = 48$ V.

Practice Problem

Find v_x and v_o in the circuit of



Answer: 20 V, −10 V.

Example

Find current i_o and voltage v_o in the circuit shown in Fig.

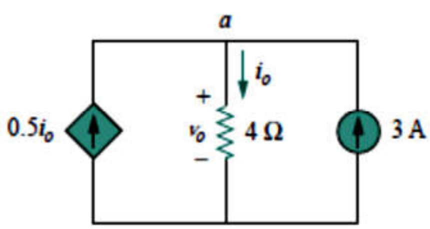
Solution:

Applying KCL to node a, we obtain

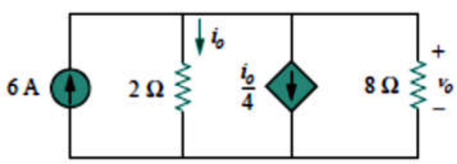
$$3 + 0.5i_o = i_o \qquad \Longrightarrow \qquad i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$



Practice Problem

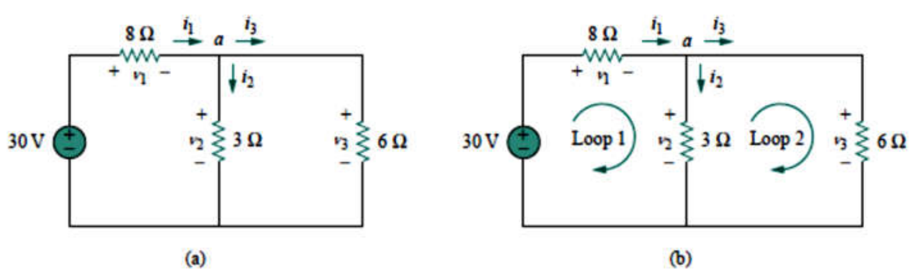


Find v_o and i_o in the circuit of Fig..

Answer: 8 V, 4 A.

Example

Find the currents and voltages in the circuit shown in Fig



Solution:

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Longrightarrow \qquad v_3 = v_2$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2

$$6i_3 = 3i_2 \qquad \Longrightarrow \qquad i_3 = \frac{i_2}{2}$$

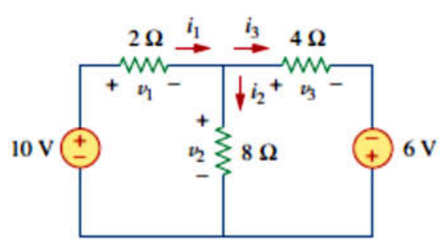
Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$$i_1 = 3\text{ A}, \qquad i_3 = 1\text{ A}, \qquad v_1 = 24\text{ V}, \qquad v_2 = 6\text{ V}, \qquad v_3 = 6\text{ V}$$

Practice Problem

Find the currents and voltages in the circuit shown in Fig



Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 = 1.25$ A.