3-Rotation

In rotation, the object is rotated Ó about the origin. The convention is that the direction of rotation is counterclockwise if Ó is a positive angle and clockwise if Ó is a negative angle.

3-1 Rotation about the origin

The rotation matrix to rotate an object about the origin in anticlockwise direction is :

Cos Ó	Sin Ó	0
- Sin Ó	Cos Ó	0
0	0	1

Or, in equation :

Xnew= X * CosÓ - Y * SinÓ
Ynew= Y *Cos Ó + X * Sin Ó

When Ó=90, the matrix that cause a rotation through an angle of 90 (Π/2) is

0	1	0
-1	0	0
0	0	1

When Ó=180

-1	0	0
0	-1	0
0	0	1

When Ó=270

0	-1	0
1	0	0
0	0	1

When Ó=360

1	0	0
0	1	0
0	0	1

Example 1: rotate the line P1(1,4) and P2(3,1) anticlockwise 90 degree.
Solution:

1	4	1
3	1	1

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0	1	0
-1	0	0
0	0	1

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-4	1	1
-1	3	1

3-2 Rotate about a specific point (XP,YP)

We need three steps:

First: translate the points (and the object) so that the point (XP,YP) lies on the origin

XP1= X - XP
YP1= Y - YP

Second: rotate the translated point (and the translated object) by Ó degree about the origin to obtain the new point (XP2,YP2)

XP2=XP1 * Cos Ó - YP1 * Sin Ó
YP2=YP1 * Cos Ó + XP1 * Sin Ó

Third : Back translation
XP3= XP2 + XP
YP3= YP2 + YP

Note:: Rotation in clockwise direction :

In order to rotate in clockwise direction we use a negative angle, and because :

Cos (-Ó)= Cos Ó
Sin (-Ó)= - sin Ó

So the matrix will be (for clockwise rotation):

Cos Ó	-Sin Ó	0
Sin Ó	Cos Ó	0
0	0	1

In equations: Xnew=X * Cos Ó + Y * Sin Ó
Ynew= Y * Cos Ó - X * Sin Ó

Example 2: Rotate the square (2,1),(4,1),(2,3),(4,3) counterclockwise with Ǿ=45 around the point (2,1)

Solution : First: translate the square by TX=-2 and TY=-1

(2,1) ===> (0,0)
(4,1) ===> (2,0)
(2,3) ===> (0,2)
(4,3) ===> (2,2)

Second: Rotate by Ǿ=45

(0,0) ===> (0 , 0)
(2,0) ===> (1.414 , 1.414)
(0,2) ===> (-1.414 , 1.414)
(2,2) ===> (0 , 2.828)

Third: Back translation by TX= 2 and TY= 1

(0 , 0) ===> (2 , 1)
(1.414 , 1.414) ===> (3.414 , 2.414)
(-1.414 , 1.414) ===> (0.586 , 2.414)
(0 , 2.828) ===> (2 , 3.828)