Interpolation

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A- NEWTON'S DIVIDED-DIFFERENCE INTERPOLATING POLYNOMIAL

As stated above, there are a variety of alternative forms for expressing an interpolating polynomial. Newton's divided-difference interpolating polynomial is among the most popular and useful forms. Before presenting the general equation, we will introduce the first- and second-order versions because of their simple visual interpretation.

1- Linear Interpolation

The simplest form of interpolation is to connect two data points with a straight line. This technique, called linear interpolation.

Given (x_0, y_0) and (x_1, y_1)

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{f_1(x) - y_0}{x - x_0}$$

$$f_1(x) = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

$f_1(x)$: first order interpolation

A smaller interval, i.e., $|x_1 - x_0|$ closer to zero, leads to better approximation.

Example: Given $\ln 1 = 0$, $\ln 6 = 1.791759$, use linear interpolation to find $\ln 2$.

Solution:

$f_1(2) = \ln 2 = \ln 1 + \frac{\ln 6 - \ln 1}{6-1} \times (2-1) = 0.3583519$

True solution: $\ln 2 = 0.6931472$.

$\epsilon_t = |\frac{f_1(2) - \ln 2}{\ln 2}| \times 100\% = |\frac{0.3583519 - 0.6931472}{0.6931472}| \times 100\% = 48.3\%$

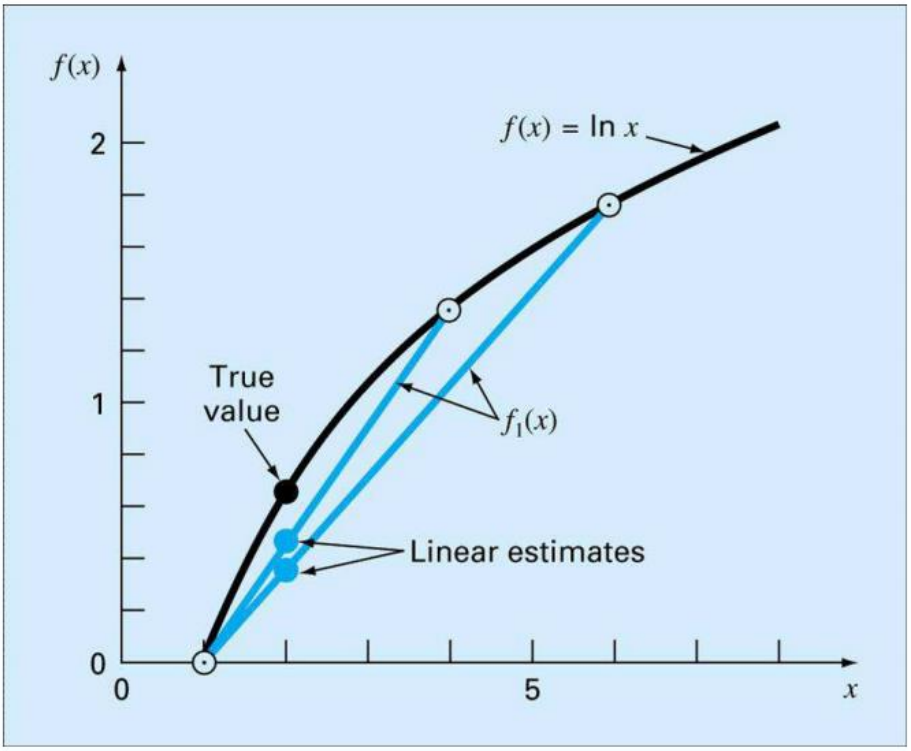

Figure 2: A smaller interval provides a better estimate

2- Quadratic Interpolation

An error maybe resulted from our approximating a curve with a straight line. Consequently, a strategy for improving the estimate is to introduce some curvature into the line connecting the points. If three data points

are available, this can be accomplished with a second-order polynomial (also called a quadratic polynomial or a parabola.

Given 3 data points, (x_0, y_0), (x_1, y_1), and (x_2, y_2), we can have a second order polynomial

$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$

$f_2(x_0) = b_0 = y_0$

$f_2(x_1) = b_0 + b_1(x_1 - x_0) = y_1, \rightarrow b_1 = \frac{y_1 - y_0}{x_1 - x_0}$

$f_2(x_2) = b_0 + b_1(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1) = y_2, \rightarrow b_2 = \frac{\frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0}}{x_2 - x_0}$ (*)

Proof (*):

$$
\begin{aligned}
b_2 &= \frac{y_2 - b_0 - b_1(x_2 - x_0)}{(x_2 - x_0)(x_2 - x_1)} = \frac{y_2 - y_0 - \frac{(y_1 - y_0)(x_2 - x_0)}{x_1 - x_0}}{(x_2 - x_0)(x_2 - x_1)} \\
&= \frac{(y_2 - y_0)(x_1 - x_0) - (y_1 - y_0)(x_2 - x_0)}{(x_2 - x_0)(x_2 - x_1)(x_1 - x_0)} \\
&= \frac{y_2(x_1 - x_0) - y_0 x_1 + y_0 x_0 - (y_1 - y_0)x_2 + y_1 x_0 - y_0 x_0}{(x_2 - x_0)(x_2 - x_1)(x_1 - x_0)} \\
&= \frac{y_2(x_1 - x_0) - y_1 x_1 + y_1 x_0 - (y_1 - y_0)x_2 + y_1 x_1 - y_0 x_1}{(x_2 - x_0)(x_2 - x_1)(x_1 - x_0)} \\
&= \frac{(y_2 - y_1)(x_1 - x_0) - (y_1 - y_0)(x_2 - x_1)}{(x_2 - x_0)(x_2 - x_1)(x_1 - x_0)}
\end{aligned}
$$

Comments: In the expression of $f_2(x)$,

- $b_0 + b_1(x - x_0)$ is linear interpolating from (x_0, y_0) and (x_1, y_1), and
- $+b_2(x - x_0)(x - x_1)$ introduces second order curvature.

Example: Given $\ln 1 = 0$, $\ln 4 = 1.386294$, and $\ln 6 = 1.791759$, find $\ln 2$.

Solution:

$(x_0, y_0) = (1, 0)$, $(x_1, y_1) = (4, 1.386294)$, $(x_2, y_2) = (6, 1.791759)$

$b_0 = y_0 = 0$

$b_1 = \frac{y_1 - y_0}{x_1 - x_0} = \frac{1.386294 - 0}{4 - 1} = 0.4620981$

$b_2 = \frac{\frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0}}{x_2 - x_0} = \frac{\frac{1.791759 - 1.386294}{6 - 4} - 0.4620981}{6 - 1} = -0.0518731$

$f_2(x) = 0.4620981(x - 1) - 0.0518731(x - 1)(x - 4)$

$f_2(2) = 0.565844$

$\epsilon_t = |\frac{f_2(2) - \ln 2}{\ln 2}| \times 100\% = 18.4\%$

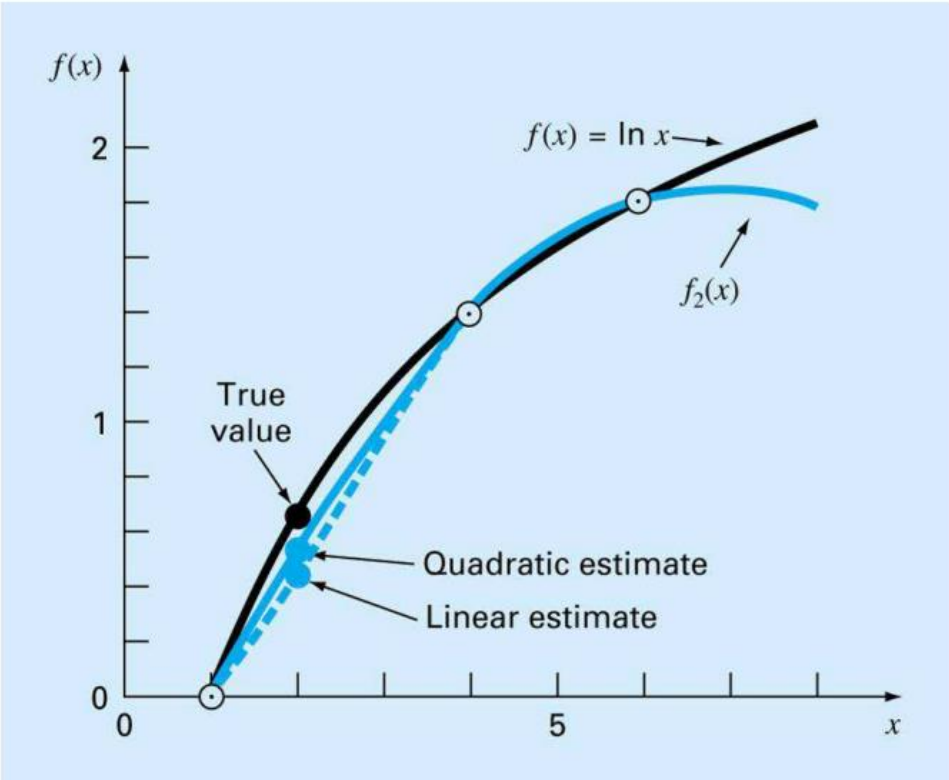

Figure 3: Quadratic interpolation provides a better estimate than linear interpolation

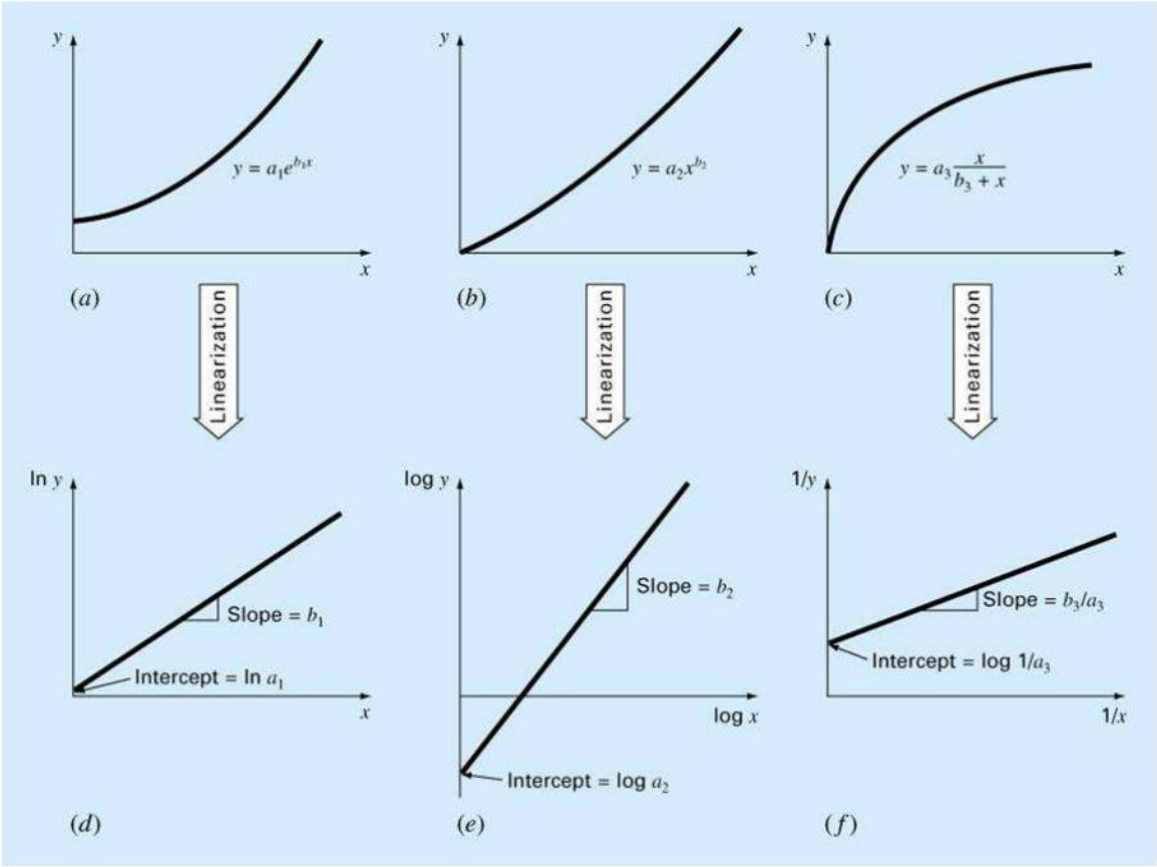

Figure 4: Linearization of nonlinear relationships