Lec.1/ Matrices, Inverse matrices by elementary row

- **Matrices**
- When a system of equations has more than two equations, it is more convenient to use matrices and vectors in solution.
- The size of the matrix is described by the number of its rows and columns. A matrix of n rows and m columns is represented by (n x m) matrix.
- $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}_{n\,x\,m} \quad , \quad i = 1, 2, \dots, n\,; \quad j = 1, 2, \dots, m$

- Types of matrices:
- **Square matrix**: it is a matrix that includes number of rows equals to number of columns (n=m).
- $A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}_{2x2}, B = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 2 & 1 \\ 1 & 8 & 0 \end{bmatrix}_{3x3}$
- Diagonal matrix: it is a square matrix which all of its elements are zeros except the elements on the main diagonal.
- $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

- **Identity matrix:** it is a diagonal matrix but the elements on the main diagonal are equal to 1 and it is denoted by $I_{n.}$
- $I_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- **Transpose matrix: Transpose of A is denoted by A^T which means writing the rows of A as columns in A^T.**
- A= $\begin{bmatrix} 9 & 7 & 5 \\ 1 & 2 & 4 \end{bmatrix}_{2x3}$, $A^T = \begin{bmatrix} 9 & 1 \\ 7 & 2 \\ 5 & 4 \end{bmatrix}_{3x2}$

- Matrix addition: if $A = [a_{ij}], B = [b_{ij}]$ and both A&B are m x n matrices, then
- $A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$
- Ex:
- $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$

Note: for any scalar (number) c , it can multiply the matrix A by c as follows:

cA = c$[a_{ij}] = [ca_{ij}]$

Ex:

$3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix}$

- Note:
- The matrix with only one column, m x 1 in size is called a column vector, while with only one row, 1 x n in size is called a row vector.
- Matrix multiplication: let A be m x k matrix and B be k x n matrix then C=AB is an m x n matrix, where
- $C_{ij} = \sum_{t=1}^{k} a_{it}\, b_{tj}$
- i = 1, 2, …., m , j = 1, 2, …., n

- Ex:
- A= $\begin{bmatrix} 3 & 7 & 1 \\ -2 & 1 & -3 \end{bmatrix}_{2x3}$, $B = \begin{bmatrix} 5 & -2 \\ 0 & 3 \\ 1 & -1 \end{bmatrix}_{3x2}$
- $AB = \begin{bmatrix} 16 & 14 \\ -13 & 10 \end{bmatrix}_{2x2}$
- $BA = \begin{bmatrix} 19 & 33 & 11 \\ -6 & 3 & -9 \\ 5 & 6 & 4 \end{bmatrix}_{3x3}$