Solution of system of linear equations using

Gauss Jordan elimination method

- In the Gaussian elimination method, we write simpler equivalent augmented matrices, where each row of an augmented matrix represents an equation that can perform the row operations on the augmented matrix.
- Steps:
- 1) Construct the augmented matrix (A:B).
- 2) Applying row operations including (adding or subtracting two rows, interchange two rows, multiplying any row by any constant except zero.

- Let A be a matrix, X a column vector, B a column vector then the system of linear equations is denoted by $\boldsymbol{AX} = \boldsymbol{B}$
- The solution to a system of linear equations starts by the augmented matrix as shown for the following system:
- $x - 2y = -5$
- $3x + y = 6$

- ***Note: Number of variables equals to the number of equations***

- Depends on the coefficients of x, y and the constants on the right-hand side of the equation. The matrix of coefficients for this system is 2 x 2 matrix

$$\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$$

- If we insert the constants from the right-hand side of the system into matrix of coefficients, we get the 2x3 matrix.

$$\left[\begin{array}{cc|c} 1 & -2 & -5 \\ 3 & 1 & 6 \end{array}\right]$$

- We use a vertical line between the coefficients and the constants to represent the equal signs. This matrix is augmented matrix of the system also it can be written as:

$$\begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

- **Application of Gauss Jordan elimination method to solve the system (AX = B)**
- Ex1: use Gaussian elimination method to solve the following system of equations
- $x - 3y = 11$
- $2x + y = 1$
- Sol: (note: no. of variables = no. of equations =2)
- The augmented matrix:
- $\begin{bmatrix} 1 & -3 & | & 11 \\ 2 & 1 & | & 1 \end{bmatrix}$

Now we want to get this matrix $\begin{bmatrix} 1 & 0 & | & x \\ 0 & 1 & | & y \end{bmatrix}$ by applying row operations as follows:

- $\begin{bmatrix} 1 & -3 & | & 11 \\ 2 & 1 & | & 1 \end{bmatrix} \rightarrow R_2' = -2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -3 & | & 11 \\ 0 & 7 & | & -21 \end{bmatrix} \rightarrow R_2' = \frac{1}{7}R_2$

- $\begin{bmatrix} 1 & -3 & | & 11 \\ 0 & 1 & | & -3 \end{bmatrix} \rightarrow R_1' = 3R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & -3 \end{bmatrix}$
- The solution to the system is (x = 2 and y = -3)
- To check the result, substitute the values of x & y in any equation, such as in ($x - 3y = 11$)

$2 - 3(-3) = 11$
$2 + 9 = 11$

$11 = 11 \rightarrow$ L.H.S = R.H.S

- Ex2: use Gaussian elimination method to solve the following system of equations
- $2x - y + z = -3$
- $x + y - z = 6$
- $3x - y - z = 4$

- $\begin{bmatrix} 2 & -1 & 1 & & -3 \\ 1 & 1 & -1 & | & 6 \\ 3 & -1 & -1 & & 4 \end{bmatrix} \rightarrow$ we want to get this matrix $\begin{bmatrix} 1 & 0 & 0 & & x \\ 0 & 1 & 0 & | & y \\ 0 & 0 & 1 & & z \end{bmatrix}$

- $R_1 \leftrightarrow R_2 \rightarrow \begin{bmatrix} 1 & 1 & -1 & & 6 \\ 2 & -1 & 1 & | & -3 \\ 3 & -1 & -1 & & 4 \end{bmatrix},$
- $R_3' = -3R_1 + R_3$ & $R_2' = -2R_1 + R_2 \rightarrow \begin{bmatrix} 1 & 1 & -1 & & 6 \\ 0 & -3 & 3 & | & -15 \\ 0 & -4 & 2 & & -14 \end{bmatrix}$

- $R_2' = -\frac{1}{3}R_2 \rightarrow \begin{bmatrix} 1 & 1 & -1 & | & 6 \\ 0 & 1 & -1 & | & 5 \\ 0 & -4 & 2 & | & -14 \end{bmatrix}$
- $R_1' = -R_2 + R_1$ & $R_3' = 4R_2 + R_3 \rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 1 \\ 0 & 1 & -1 & | & 5 \\ 0 & 0 & -2 & | & 6 \end{bmatrix}$
- $R_3' = -\frac{1}{2}R_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & -1 & | & 5 \\ 0 & 0 & 1 & | & -3 \end{bmatrix}$
- $R_2' = R_3 + R_2 \rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & -3 \end{bmatrix}$
- The solution to the system is (x = 1, y = 2 and z = -3)

- To check the result, substitute the values of x=1 , y=2 & z=-3 in any equation, such as in equation $(2x - y + z = -3)$

$2(1) - 1(2) + (-3) = -3$
$2 - 2 - 3 = -3$
$-3 = -3 \rightarrow$ L.H.S = R.H.S

- **Homework 2:** Solve the following equations by Gauss-Jordan Elimination Method, and check the results.

1) $2x - y = 18$
   $3x + y = 2$

2) $3x - 2y + 8z = 9$
   $-2x + 2y + z = 3$
   $x + 2y - 3z = 8$