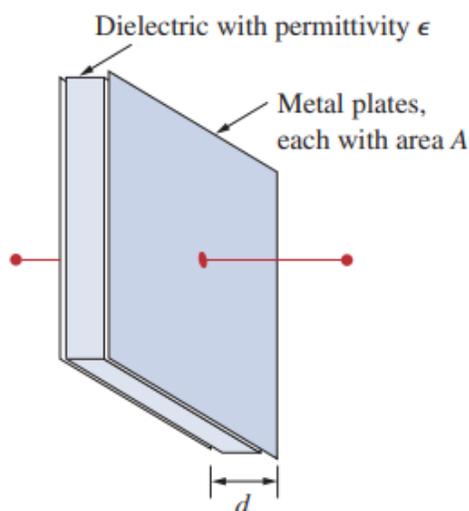


# Capacitors and Inductors

## Capacitors:

A capacitor is a passive element designed to store energy in its electric field. Besides resistors, capacitors are the most common electrical components.

Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems. A capacitor is typically constructed as depicted in Fig. 1



**Figure 1 A typical capacitor.**

## Definition 1:

A capacitor consists of two conducting plates separated by an insulator (or dielectric).

**Capacitor Stores:**

When a voltage source is connected to the capacitor, as in Fig. 2, the source deposits a positive charge  $q$  on one plate and a negative charge on the other. The capacitor is said to store the electric charge.

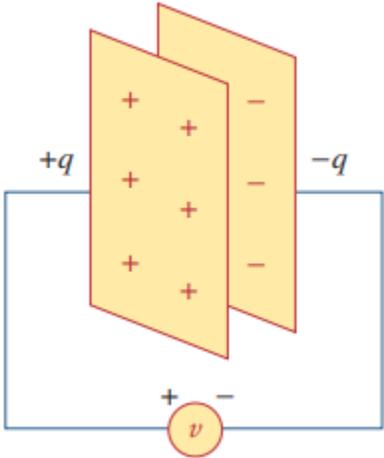


Figure 2: A capacitor with applied voltage  $v$ .

**Description of Capacitor:**

The amount of charge stored, represented by  $q$ , is directly proportional to the applied voltage so that

$$q = cv \dots\dots\dots(1)$$

where  $C$ , the constant of proportionality, is known as the capacitance of the capacitor.

The unit of capacitance is the farad (F),

1 farad = 1 coulomb/volt.

capacitors have values in the Picofarad (pF) to microfarad (μF).

. From Eq. (1), we may derive the following definition.

**Definition 2 :**

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

it does not depend on  $q$  or  $v$  . It depends on the physical dimensions of the capacitor.

the capacitance is given by

$$C = \frac{\epsilon A}{d} \dots\dots\dots(2)$$

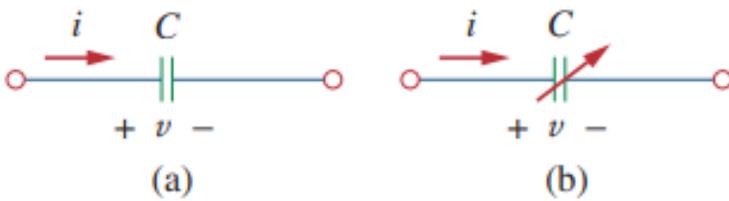
where  $A$  is the surface area of each plate,  $d$  is the distance between the plates, and  $\epsilon$  is the permittivity of the dielectric material between the plates.

Three **factors** determine the value of the capacitance:

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the permittivity, the greater the capacitance.

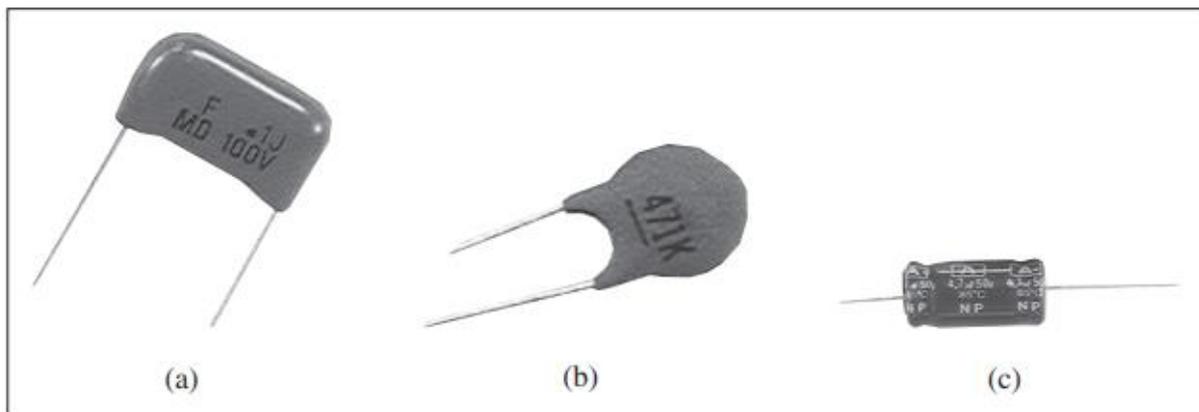
**Capacitor fixed and variable:**

Figure 3 shows the circuit symbols for *fixed* and *variable capacitors*.



**Figure 3: Circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor.**

Note that according to the passive sign convention, if  $v > 0$  and  $i > 0$  or if  $v < 0$  and  $i < 0$  the capacitor is being charged, and if  $v \cdot i < 0$  the capacitor is discharging.



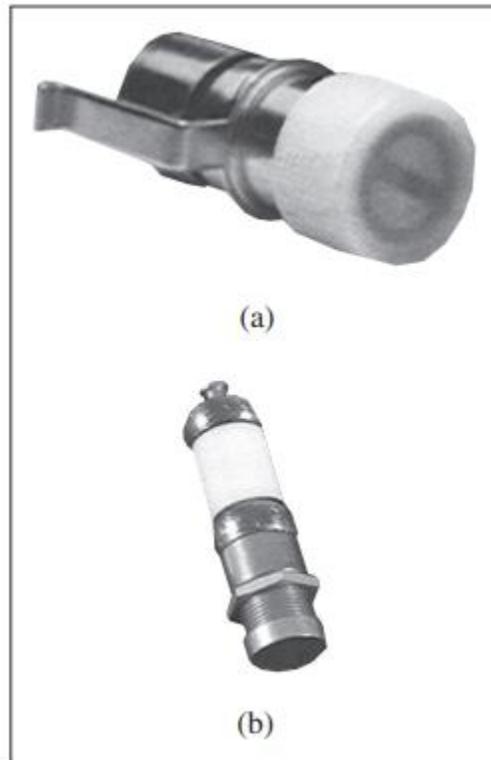
**Figure 4 : Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.**

**Polyester capacitors**: are light in weight, stable, and their change with temperature is predictable.

**Film capacitors**: are rolled and housed in metal or plastic films

**Electrolytic capacitors**: produce very high capacitance.

Figure 5 shows the most common types of ***variable capacitors***:



**Figure 5 Variable capacitors: (a) trimmer capacitor, (b) filmtrim capacitor.**

The capacitance of a trimmer (or padder) capacitor is often placed in parallel with another capacitor so that the equivalent capacitance can be varied slightly.

capacitors are used to block dc, pass ac, shift phase, store energy, start motors, and suppress noise.

**current-voltage relationship:**

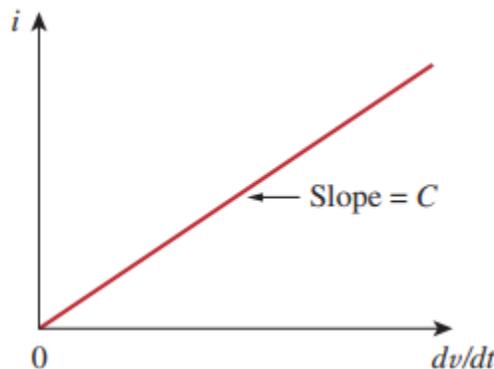
To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of Eq. (1). Since:

$$i = \frac{dq}{dt} \dots\dots\dots(3)$$

differentiating both sides of Eq. (1) gives

$$i = c \frac{dv}{dt} \dots\dots\dots(4)$$

This is the current-voltage relationship for a capacitor, assuming the passive sign convention. The relationship is illustrated in Fig. 6 for a capacitor whose capacitance is independent of voltage.



**Figure 6 Current-voltage relationship of a capacitor.**

## Liner & Nonlinear Capacitor:

Capacitors that satisfy Eq. (4) are said to be *linear*.

If the plot of the current-voltage relationship is not a straight line is said *Nonlinear*.

## The power and energy:

The voltage-current relation of the capacitor can be obtained by integrating both sides of Eq. (4). We get

$$v = \frac{1}{c} \int_{-\infty}^t i(\tau) d\tau \dots\dots\dots(5)$$

or

$$v = \frac{1}{c} \int_{-\infty}^t i(\tau) d\tau + v(t_o) \dots\dots\dots(6)$$

where  $v(t_o) = q(t_o)/C$

Equation (6) shows that capacitor voltage depends on the past history.

The instantaneous power delivered to the capacitor is

$$P = vi = Cv \frac{dv}{dt} \dots\dots\dots(7)$$

The energy stored in the capacitor is therefore

$$W = \int_{-\infty}^t P(\tau) d\tau$$

$$= C \int_{-\infty}^t v \frac{dv}{d\tau} d\tau = C \int_{v(-\infty)}^{v(t)} v dv = \frac{1}{2} C v^2 | \dots \dots \dots (8)$$

$v(-\infty)=0$  because the capacitor was un charged at  $t = -\infty$ , thus

$$W = \frac{1}{2} C v^2 \dots \dots \dots (9)$$

Using Eq. (1), we may rewrite Eq. (9) as:

$$W = \frac{q^2}{2C} \dots \dots \dots (10)$$

Equation (9) or (10) represents the energy stored in the electric field that exists between the plates of the capacitor.

the word **capacitor** is derived from this element's capacity to store energy in an electric field.

**Important properties of a capacitor:**

1-The voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero (Eq.4). Thus

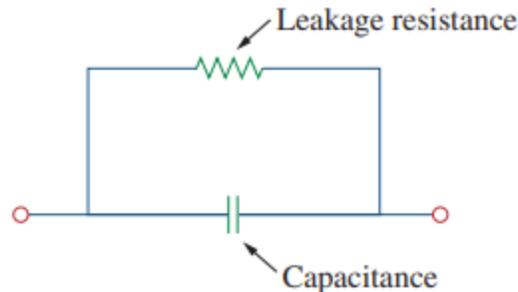
*A capacitor is an open circuit to dc.*

2-The voltage on the capacitor must be continuous.

*The voltage on a capacitor cannot change abruptly.*

3-The ideal capacitor does not dissipate energy.

4-A real, non-ideal capacitor has a parallel-model leakage resistance.  
(Fig.7).



**Figure 7: Circuit model of a non-ideal capacitor.**

**Example 1:** (a) Calculate the charge stored on a 3-pF capacitor with 20 V across it. (b) Find the energy stored in the capacitor.

**Solution:**

$$(a) \quad q = Cv = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

$$(b) \quad W = \frac{1}{2} Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

**H.W :**

What is the voltage across a capacitor if the charge (4.5  $\mu\text{F}$ ) on one plate is 0.12 mC? How much energy is stored?

**Answer: 26.67 v, 1.6 m J.**

**Example 2:** The voltage across a (5  $\mu$ F) capacitor is

$$v = 10 \cos(6000 t) \text{ v.}$$

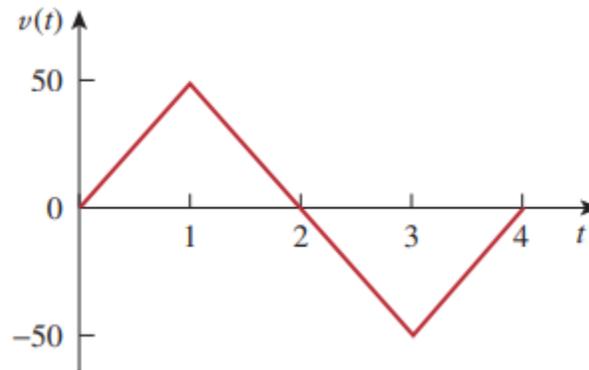
Calculate the current through it.

**Solution:**

**By definition, the current is**

$$\begin{aligned} i &= c \frac{dv}{dt} = 5 \times 10^{-6} \times \frac{d}{dt} \times 10 \cos(6000 t) \\ &= -5 \times 10^{-6} \times 6000 \times 10 \sin(6000 t) = -0.3 \sin(6000 t) \text{ A} \end{aligned}$$

**Example 3:** Determine the current through a  $200 \mu\text{F}$  capacitor whose voltage is shown in Figure



**Solution:**

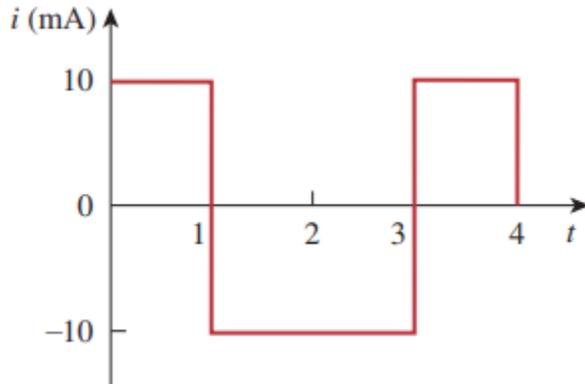
$$v(t) = \left\{ \begin{array}{ll} 50 t V & 0 < t < 1 \\ 100 - 50 t V & 1 < t < 3 \\ -200 + 50 t V & 3 < t < 4 \\ 0 & \text{otherwise} \end{array} \right\}$$

Since  $i = C \frac{\Delta v}{\Delta t}$  or  $i = C \frac{dv}{dt}$  and  $C = 200 \mu\text{F}$ , we take the derivative or the slope of  $v$  to obtain

$i(t)$

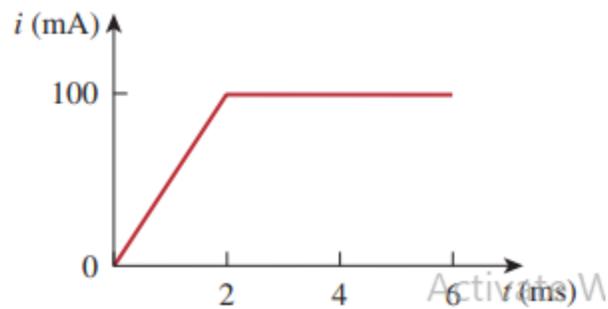
$$= \left\{ \begin{array}{ll} C \text{ slope} = C \frac{\Delta v}{\Delta t} = C \times \frac{50 - 0}{1 - 0} = 50 C = 10 \text{ mA} & 0 < t < 1 \\ C \text{ slope} = C \frac{\Delta v}{\Delta t} = C \times \frac{-50 - 50}{3 - 1} = -50 C = -10 \text{ mA} & 1 < t < 3 \\ C \text{ slope} = C \frac{\Delta v}{\Delta t} = C \times \frac{50 - 0}{4 - 3} = 50 C = 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{array} \right\}$$

Thus the current waveform is as shown in Figure below:



**H.W:** An initially uncharged 1-mF capacitor has the current shown in Figure across it. Calculate the voltage across it at  $t=2$  ms and  $t=5$  ms

**Answer: 100 mV, 400 mV.**



Reference:

Fundamentals\_of\_Electric\_Circuits\_5th\_Edition\_by\_C.\_Alexander\_M.\_Sadiku, 2010.