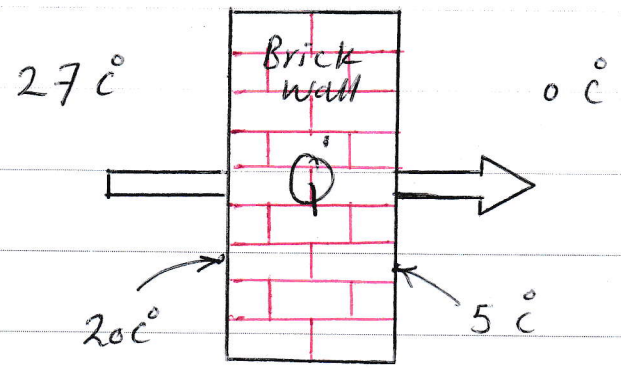
of exergy destruction in the wall, and the rate of total exergy destruction associated with this heat transfer process.

Assumptions:

1- The process is steady, and thus the rate of heat transfer through the wall is constant.

2- Heat transfer through the wall is one-dimensional.

Applying the rate form of the exergy balance to the wall gives



$$\dot{X}_{in} - \dot{X}_{out} - \dot{X}_{destroyed} = \cancel{dX_{system}/dt}^{\,0} = 0$$

$$\dot{Q}\left(1-\frac{T_0}{T}\right)_{in} - \dot{Q}\left(1-\frac{T_0}{T}\right)_{out} - \dot{X}_{destroyed} = 0$$

$$(1035)\left(1-\frac{273}{20+273}\right) - (1035)\left(1-\frac{273}{5+273}\right) - \dot{X}_{destroyed} = 0$$

$$\therefore \dot{X}_{destroyed} = 52.0 \text{ W}$$

Then, the rate of total exergy destruction is

$$(1035)\left(1-\frac{273}{27+273}\right)-(1035)\left(1-\frac{273}{0+273}\right)-\dot{X}_{destroyed}=0$$

$$\therefore \dot{X}_{destroyed}=93-2W$$

The of exergy destruction can also be determined as,

$$\dot{X}_{destroyed}=T_0\cdot\dot{S}_{generated}$$

Ex: Steam enters a turbine steadily at 3 Mpa and 450 c° at a rate of 8 kg/s and exits at 0.2 Mpa and 150 c°. The steam is losing heat to the surrounding air at 100 kpa and 25 c° at a rate of 300 kw, and the kinetic and potential energy changes are negligible. Determine

(a) the actual power output,

(b) the maximum possible power output,

(c) the second-law efficiency,

(d) the exergy destroyed,

(e) the exergy of the steam at the inlet conditions.

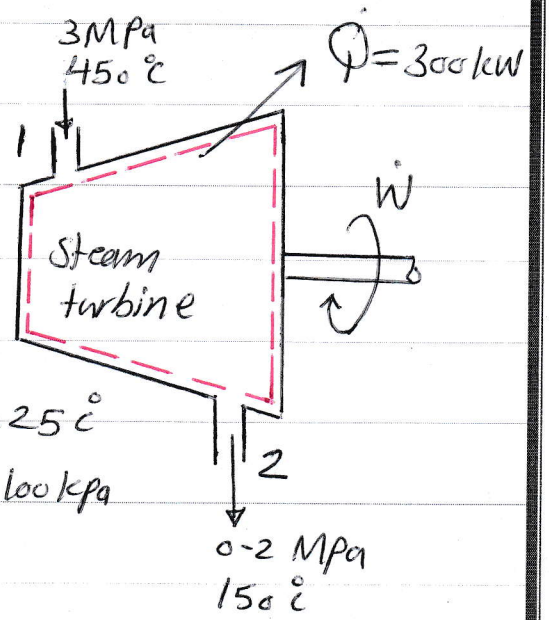
Solution:-

Assumptions:-

1- Steady flow process

2- The kinetic and potential energies are negligible.

We take the turbine as the system.



The properties of the steam at the inlet and exit states and the state of the environment are

$T_0 = 25\,°C$
$P_0 = 100\,kPa$

Inlet state: $P_1 = 3\,MPa$, $T_1 = 450\,°C$ $\Big\}$ $h_1 = 3344.9\,kJ/kg$, $s_1 = 7.0856\,kJ/kg\,K$

Exit state: $P_2 = 0.2\,MPa$, $T_2 = 150\,°C$ $\Big\}$ $h_2 = 2769.1\,kJ/kg$, $s_2 = 7.2810\,kJ/kg\,K$

Dead state: $P_0 = 100\,kPa$, $T_0 = 25\,°C$ $\Big\}$ $h_0 = h_{f\,at\,25°C} = 104.3\,kJ/kg$, $s_0 = s_{f\,at\,25°C} = 0.3672\,kJ/kg\,K$

(a)

$$\dot{E}_{in} - \dot{E}_{out} = \cancel{dE_{system}/dt}^{\,0\,(Steady)} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{out} + \dot{Q}_{out} + \dot{m}h_2 \qquad (ke = pe = 0)$$

$$\therefore \dot{W}_{out} = \dot{m}(h_1 - h_2) - \dot{Q}_{out}$$

$$\dot{W}_{out} = (8)(3344.9 - 2769.1) - 300 = 4306 \text{ kW}$$

(b) The maximum power output (reversible power) is determined from the rate form of the exergy balance applied on the extended system (system + immediate surroundings), whose boundary is at the environment temperature of T_0, and by setting the exerg destruction term equal to zero,

$$\dot{X}_{in} - \dot{X}_{out} - \cancel{\dot{X}_{destroyed}}^{\,0\,(reversible)} = \cancel{dX_{system}/dt}^{\,0\,(steady)} = 0$$

$$\dot{m}\psi_1 - \dot{m}\psi_2 - \cancel{\dot{X}_{heat}}^{\,0} - \dot{X}_{work} = 0$$

$$\therefore \dot{m}(\psi_1 - \psi_2) - \dot{W}_{rev,out} = 0$$

$$\dot{W}_{rev,out} = \dot{m}(\psi_1 - \psi_2)$$
$$= \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2) - \cancel{\Delta ke}^{\,0} - \cancel{\Delta pe}^{\,0}]$$

$$\therefore \dot{W}_{rev,out} = 8[(3344.9 - 2769.1) - (25 + 273)(7.0856 - 7.2811)]$$
$$= 5072 \text{ kW}$$

Note that exergy transfer with heat is zero when the

temperature at the point of transfer is the environment temperature T_0.

(c) The second-law efficiency of a turbine is the ratio of the actual work delivered to the reversible work,

$$\eta_{II} = \frac{\dot{W}_{out}}{\dot{W}_{rev,out}} = \frac{4306}{5072} = 84.9\%$$

(d)

$$\dot{X}_{destroyed} = \dot{W}_{rev,out} - \dot{W}_{out} = 5072 - 4306 = 776 \text{ kW}$$

(e)

$$\psi_1 = (h_1 - h_0) - T_0 (s_1 - s_0) + \cancel{\frac{V_1^2}{2}}^{\,0} + \cancel{gz_1}^{\,0}$$

$$= (h_1 - h_0) - T_0 (s_1 - s_0)$$

$$= (3344.9 - 104.83) - (25 + 273)(7.0856 - 0.3672)$$

$$= 1238 \text{ kJ/kg}$$

EX: water at 200 kpa and 10°C enters a mixing chamber at a rate of 150 kg/min, where it is mixed steadily with steam entering at 200 kpa and 150°C. The mixture leaves the chamber at 200kpa and 70°C, and heat is being lost to the surrounding air at $T_0 = 20°C$ at a rate of 190 kJ/min. Neglecting the changes in kinetic and potential energies, determine the reversible power and the rate of exergy destruction for this process.

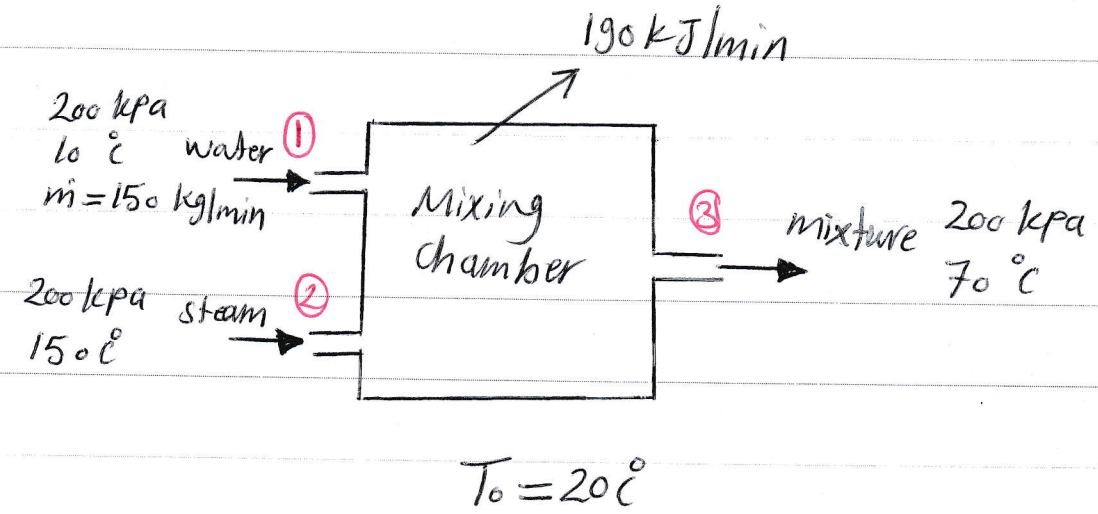
Solution:-

Assumptions:-

1- This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{cv} = 0$, $\Delta E_{cv} = 0$, $\Delta S_{cv} = 0$

2- There are no work interactions involved.

3- The kinetic and potential energies are negligible, ke = 0 and pe = 0.



Mass balance:

$$\dot{m}_{in} - \dot{m}_{out} = \cancel{dm_{system}/dt}^{\,0\,(steady)} = 0$$

$$\therefore \dot{m}_{in} = \dot{m}_{out} \Rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\dot{E}_{in} - \dot{E}_{out} = \cancel{dE_{system}/dt}^{\,0\,(steady)} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{Q}_{out} \quad (\dot{W}=0,\ ke=0,\ pe=0)$$

Combining the mass and energy balances gives

$$\dot{Q}_{out} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

The desired properties at the specified states are determined from the steam tables.

state 1: $\left.\begin{array}{l} P_1 = 200\,kPa \\ T_1 = 10\,^\circ C \end{array}\right\}$ $\begin{array}{l} h_1 = h_{f\,at\,10^\circ C} = 42.022\ kJ/kg \\ s_1 = s_{f\,at\,10^\circ C} = 0.1511\ kJ/kg\,K \end{array}$

state 2: $\left.\begin{array}{l} P_2 = 200\,kPa \\ T_2 = 150\,^\circ C \end{array}\right\}$ $\begin{array}{l} h_2 = 2769.1\ kJ/kg \\ s_2 = 7.281\ kJ/kg\,K \end{array}$

State 3: $\left.\begin{array}{l} P_3 = 200\,kPa \\ T_3 = 70\,^\circ C \end{array}\right\}$ $\begin{array}{l} h_3 = h_{f\,at\,70^\circ C} = 293.07\ kJ/kg \\ s_3 = s_{f\,at\,70^\circ C} = 0.9551\ kJ/kg\,K \end{array}$

$$\therefore \dot{Q}_{out} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$190 = (150)(42.022) + \dot{m}_2 (2769.1) - (150 + \dot{m}_2)(293.071)$$

$$\therefore \dot{m}_2 = 15.29 \text{ kg/min}$$

The maximum power output (reversible power) is determined from the rate form of the exergy balance applied on the extended system (system + immediate surroundings), whose boundary is at the environment temperature of T_0, and by setting the exergy destruction term equal to zero,

$$\dot{X}_{in} - \dot{X}_{out} - \cancelto{0 \text{ (reversible)}}{\dot{X}_{destroyed}} = \cancelto{0 \text{ (steady)}}{dX_{system}/dt} = 0$$

$$\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2 - \dot{W}_{rev,out} - \cancelto{0}{\dot{X}_{heat}} - \dot{m}_3 \psi_3 = 0$$

$$\therefore \dot{W}_{rev,out} = \dot{m}_1 \psi_1 + \dot{m}_2 \psi_2 - \dot{m}_3 \psi_3$$

Note that exergy transfer by heat is zero when the temperature at the point of heat transfer is the environment temperature T_0, and the kinetic and potential energies are negligible. Therefore,

$$\therefore \dot{W}_{rev,out} = \dot{m}_1\left[(h_1-h_0)-T_0(s_1-s_0)\right]$$

$$+\dot{m}_2\left[(h_2-h_0)-T_0(s_2-s_0)\right]$$

$$-\dot{m}_3\left[(h_3-h_0)-T_0(s_3-s_0)\right]$$

$$\dot{W}_{rev,out} = \dot{m}_1(h_1-T_0s_1)-\dot{m}_1(h_0-T_0s_0)$$
$$+\dot{m}_2(h_2-T_0s_2)-\dot{m}_2(h_0-T_0s_0)$$
$$-\dot{m}_3(h_3-T_0s_3)+\dot{m}_3(h_0-T_0s_0)$$

Re-arranging gives

$$\dot{W}_{rev,out} = \dot{m}_1(h_1-T_0s_1)+\dot{m}_2(h_2-T_0s_2)$$
$$-\dot{m}_3(h_3-T_0s_3)-(h_0-T_0s_0)\cancelto{0}{(\dot{m}_1+\dot{m}_2-\dot{m}_3)}$$

$$=(150)\left[42.022-(293)(0.1511)\right]$$
$$+(15.29)\left[2769.1-(293)(7.281)\right]$$
$$-(165.29)\left[293.07-(293)(0.9551)\right]$$

$$=7197 \text{ kJ/min}$$

The exergy destroyed is determined from

$$\dot{X}_{destroyed} = \dot{W}_{rev,out}-\cancelto{0}{\dot{W}_u} = T_0\dot{S}_{gen} = 7197 \text{ kJ/min}$$

$\dot{W}_u = 0$, since there is no actual work produced during the process.

sheet No 2

Q1: A piston-cylinder device contains 8 kg of refrigerant-134a at 0.7 MPa and 60 °C. The refrigerant is now cooled at constant pressure until it exists as a liquid at 20 °C. If the surroundings are at 100 kpa and 20 °C, determine

(a) the exergy of the refrigerant at the initial and the final states

(b) the exergy destroyed during this process.

Q2: A 30 kg copper block initially at 105 °C is dropped into an insulated tank that contains 0.035 m^3 of water at 17 °C. Determine,

(a) the final equilibrium temperature

(b) the work potential wasted during this process. Assume the surroundings to be at 17 °C.

Q3: Air is compressed steadily by an 8 kW compressor from 100 kpa and 17 °C to 600 kPa and 167 °C at a rate of 2.1 kg/min. Neglecting the changes in kinetic and potential energies, determine

(a) the increase in the exergy of the air,

(b) the rate of exergy destroyed during this process,
(c) the second-law efficiency of the compressor.
Assume the surroundings to be at 17°C.

Q4: Cold water ($c_p = 4.18$ kJ/kg.°C) leading to a shower enters a well-insulated, thin-walled, double-pipe, counter flow heat exchanger at 15°C at a rate of 0.25 kg/s and is heated to 45°C by hot water ($c_p = 4.19$ kJ/kg.°C) that enters at 100°C at a rate of 3 kg/s. Determine,
(a) the rate of heat transfer,
(b) the rate of exergy destruction in the heat exchanger
(c) the second-law efficiency of the heat exchanger. Take $T_0 = 25$°C.

Thermodynamic Property Relations:-

It is known that some properties such as temperature, pressure, volume, and mass can be measured directly. Other properties such as density and specific volume can be determined from these using some simple relations. However, properties such as internal energy, enthalpy, and entropy are not so easy to determine because they cannot be measured directly or related to easily measurable properties through some simple relations between commonly encountered thermodynamic properties and express the properties that cannot be measured directly in terms of easily measurable properties.

Partial Differential Relations:-

Let three variables are represented by x, y and z. Their function relationship may be expressed in the following forms:

$$f(x, y, z) = 0 \qquad (1)$$

Let x is a function of two independent variables y and z

$$X = X(y, z) \qquad (2)$$

Then the differential of the dependent variable X is given by

$$dX = \left(\frac{\partial X}{\partial y}\right)_z dy + \left(\frac{\partial X}{\partial z}\right)_y dz, \qquad (3)$$

Where dX is called an exact differential,

If $\left(\frac{\partial X}{\partial y}\right)_z = M$ and $\left(\frac{\partial X}{\partial z}\right)_y = N$

Then $$dX = M\,dy + N\,dz \qquad (4)$$

Partial differentiation of M and N with respect to z and y, respectively, gives

$$\frac{\partial M}{\partial z} = \frac{\partial^2 X}{\partial y \partial z} \quad \text{and} \quad \frac{\partial N}{\partial y} = \frac{\partial^2 X}{\partial z \partial y}$$

or, $$\frac{\partial M}{\partial z} = \frac{\partial N}{\partial y} \qquad (5)$$

dX is a perfect differential when Eq (5) is satisfied for any function X.

Similarly if

$$y = y(x, z) \quad , \quad z = z(x, y) \qquad (6)$$

Then from these two relations, we have

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz \qquad (7)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dx \qquad (8)$$

Substituting Eq (8) in Eq (7) gives

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x \left[\left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy\right]$$

$$= \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y\right] dx$$

$$+ \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$\therefore \cancel{dy} = \left[\left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y\right] dx + \cancel{dy}$$

$$\therefore \left(\frac{\partial y}{\partial x}\right)_z + \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = 0$$

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -\left(\frac{\partial y}{\partial x}\right)_z \qquad (9)$$

$$\therefore \left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{(\partial y/\partial x)_z} \qquad (10)$$

The above equation is called reciprocity relation.

Re-arranging Eq (9) gives

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \qquad (11)$$

Eq (11) is called the cyclic relation, and it frequently used in thermodynamics. However, Eq (11) can be written in terms of p, v and T,

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial v}{\partial T}\right)_p = -1 \qquad (12)$$

The Maxwell Relations:-

The equations that relate the partial derivatives of properties p, v, T and s of a simple compressible system to each orther are called the Maxwell relations. They are obtained from the four Gibbs equations by exploiting the exactness of the differentials of

thermodynamic properties. Two of the Gibbs relations can be expressed as

$$du = Tds - pdv \quad (13)$$
$$dh = Tds + vdp \quad (14)$$

The orther two Gibbs relations are based on two new combination properties the Helmholtz function a and the Gibbs function g, defined as

$$a = u - Ts \quad (15)$$
$$g = h - Ts \quad (16)$$

Differentiating, we get

$$da = du - Tds - sdT \quad (17)$$
$$dg = dh - Tds - sdT \quad (18)$$

Simplifying the above relations by using Eqs (13) and (14), we obtain the other two Gibbs relations for simple compressible systems:

$$da = -sdT - pdv \quad (19)$$
$$dg = -sdT + vdp \quad (20)$$

Examination of the four Gibbs relations reveals that are of the form

$$dz = M\,dx + N\,dy \qquad (21)$$

with

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \qquad (22)$$

Since u, h, a, and g are properties and thus have exact differentials. Applying Eq(22) to each of them, we obtain

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v \qquad (23)$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial v}{\partial s}\right)_P \qquad (24)$$

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \qquad (25)$$

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P \qquad (26)$$

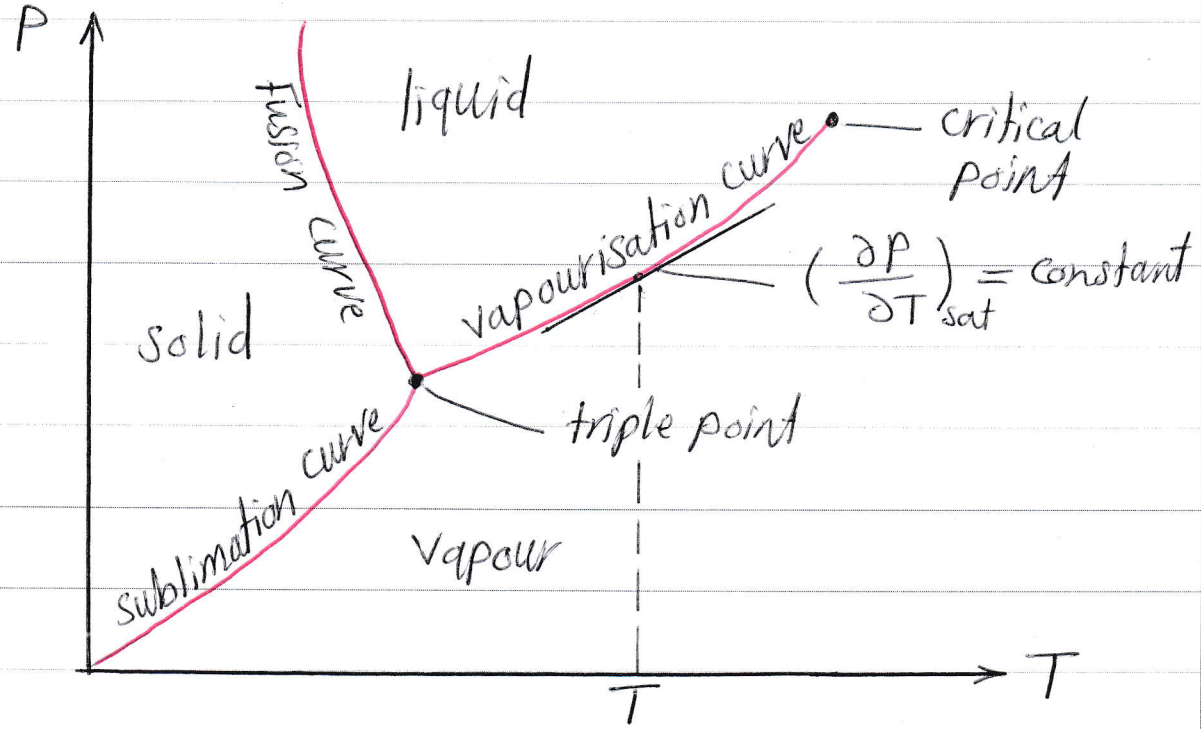
The Eqs (23), (24), (25) and (26) are called the Maxwell relations. They are extremely valuable in thermodynamics because they provide a means of determining the change in entropy, which cannot be

measured directly, by simply measuring the changes in properties p, v and T.

The Clapeyron Equation:-

Clapeyron equation is a relationship between the saturation pressure, temperature, the enthalpy of evaporation and the specific volume of the two phases involved. This equation provides a basis for calculations of properties in a two-phase region. Consider the third Maxwell relation, Eq (25):

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \qquad (27)$$



During a phase-change process, the pressure is the saturation pressure, which depends on the temperature only and is independent of the specific volume. That is,

$$P_{sat} = f(T_{sat})$$

Therefore, the partial derivative $(\partial p/\partial T)_v$ can be expressed as a total derivative $(dp/dT)_{sat}$, which is the slope of the saturation curve on a p-T diagram at a specified saturation state. This slop is independent of the specific volume, and thus it can be treated as a constant during the integration of Eq(27) between two saturation states at the same temperature. For an isothermal liquid-vapour phase change process, for example, the integration yields

$$\int_f^g ds = \left(\frac{dP}{dT}\right)_{sat} \int_f^g dv$$

$$\therefore \quad s_g - s_f = \left(\frac{dP}{dT}\right)_{sat} (v_g - v_f) \qquad (28)$$

or,

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{s_{fg}}{v_{fg}} \qquad (29)$$