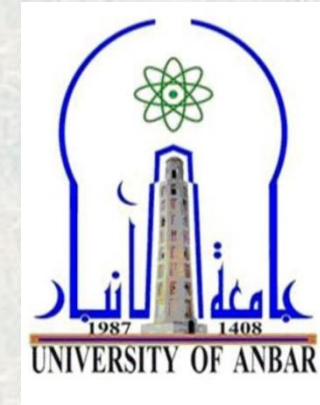




# Chapter Eight

## Hypothesis Testing



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2. Steps in Hypothesis Testing—Traditional Method
3. z Test for a Mean
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5. z Test for a Proportion
6.  $X^2$  Test for a Variance or Standard Deviation

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# Chapter Eight

## Hypothesis Testing

### 1. Preface:

- **Statistical Hypothesis Testing** is a decision-making process for evaluating claims about a population.
- In hypothesis testing, the researcher must define the population under study, state the particular hypotheses that will be investigated, give the significance level, select a sample from the population, collect the data, perform the calculations required for the statistical test, and reach a conclusion.
- **For Example:** a scientist might want to know whether the earth is warming up. A physician might want to know whether a new medication will lower a person's blood pressure. An educator might wish to see whether a new teaching technique is better than a traditional one. A production of concert factory is within the standard requirement. Automobile manufacturers are interested in determining whether seat belts will reduce the severity of injuries caused by accidents.

- Three methods used to test hypotheses are:

1. The traditional method
2. The P-value method
3. The confidence interval method

## 2. Steps in Hypothesis Testing—Traditional Method

- ❑ A **statistical hypothesis** is a conjecture about a population parameter. This conjecture may or may not be true.
- ❑ There are two types of statistical hypotheses for each situation
  1. The **null hypothesis ( $H_0$ )** is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.
  2. The **alternative hypothesis ( $H_1$ )**, is a statistical hypothesis that states an existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.



**Example 1:** A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the automobile battery without the additive is 36 months, then her hypotheses are

$$H_0: \mu = 36 \quad \text{and} \quad H_1: \mu > 36$$

In this situation, the chemist is interested only in increasing the lifetime of the batteries, so her alternative hypothesis is that the mean is greater than 36 months. The null hypothesis is that the mean is equal to 36 months. This test is called right-tailed, since the interest is in an increase only.

**Example 2:** A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is \$78, her hypotheses about heating costs with the use of insulation are

$$H_0: \mu = \$78 \quad \text{and} \quad H_1: \mu < \$78$$

This test is a *left-tailed test*, since the contractor is interested only in lowering heating costs. To state hypotheses correctly, researchers must translate the *conjecture* or *claim* from words into mathematical symbols. The basic symbols used are as follows:

<i>Equal to</i>	=	<i>Greater than</i>	>
<i>Not equal to</i>	≠	<i>Less than</i>	<

- The null and alternative hypotheses are stated together, and the null hypothesis contains the equals sign, as shown (where  $k$  represents a specified number).

Two-tailed test	Right-tailed test	Left-tailed test
$H_0: \mu = k$	$H_0: \mu = k$	$H_0: \mu = k$
$H_1: \mu \neq k$	$H_1: \mu > k$	$H_1: \mu < k$

### Hypothesis-Testing Common Phrases

<p style="text-align: center;"><math>&gt;</math></p> <p>Is greater than Is less than Is above Is below Is higher than Is lower than Is longer than Is shorter than Is bigger than Is smaller than Is increased Is decreased or reduced from</p>	<p style="text-align: center;"><math>&lt;</math></p> <p>Is greater than Is less than Is above Is below Is higher than Is lower than Is longer than Is shorter than Is bigger than Is smaller than Is increased Is decreased or reduced from</p>
<p style="text-align: center;"><math>=</math></p> <p>Is equal to Is not equal to Is the same as Is different from Has not changed from Has changed from Is the same as Is not the same as</p>	<p style="text-align: center;"><math>\neq</math></p> <p>Is equal to Is not equal to Is the same as Is different from Has not changed from Has changed from Is the same as Is not the same as</p>

### Example 3:

1. A researcher thinks that if expectant mothers use vitamin pills, the birth weight of the babies will increase. The average birth weight of the population is 8.6 pounds.
2. An engineer hypothesizes that the mean number of defects can be decreased in a manufacturing process of compact disks by using robots instead of humans for certain tasks. The mean number of defective disks per 1000 is 18.
3. A psychologist feels that playing soft music during a test will change the results of the test. The psychologist is not sure whether the grades will be higher or lower. In the past, the mean of the scores was 73.

### Solution

1.  $H_0: m = 8.6$  and  $H_1: m > 8.6$
  2.  $H_0: m = 18$  and  $H_1: m < 18$
  3.  $H_0: m = 73$  and  $H_1: m \neq 73$
- A statistical test uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected. The numerical value obtained from a statistical test is called the **test value**.



- **Note:**

1. In this type of **statistical test**, the mean is computed for the data obtained from the sample and is compared with the population mean. Then a decision is made to reject or not reject the null hypothesis on the basis of the value obtained from the statistical test. If the difference is significant, the null hypothesis is rejected. If it is not, then the null hypothesis is not rejected.
2. In the hypothesis-testing situation, there are **four possible outcomes**
  - A **type I error** occurs if you reject the null hypothesis when it is true.
  - A **type II error** occurs if you do not reject the null hypothesis when it is false.

	$H_0$ true	$H_0$ false
Reject $H_0$	<b>Error Type I</b>	<i>Correct decision</i>
Do not reject $H_0$	<i>Correct decision</i>	<b>Error Type II</b>

**For an example:** In a jury trial, there are four possible outcomes. The defendant is either guilty or innocent, and he or she will be convicted or acquitted.

$H_0$ : The defendant is innocent

$H_1$ : The defendant is not innocent (i.e., guilty)

	$H_0$ true (innocent)	$H_0$ false (not innocent)
Reject $H_0$ (convict)	Type I error 1.	Correct decision 2.
Do not reject $H_0$ (acquitt)	Correct decision 3.	Type II error 4.

- **The level of significance**

The **level of significance** is the maximum probability of committing a type I error. This probability is symbolized by  $\alpha$  (Greek letter **alpha**).

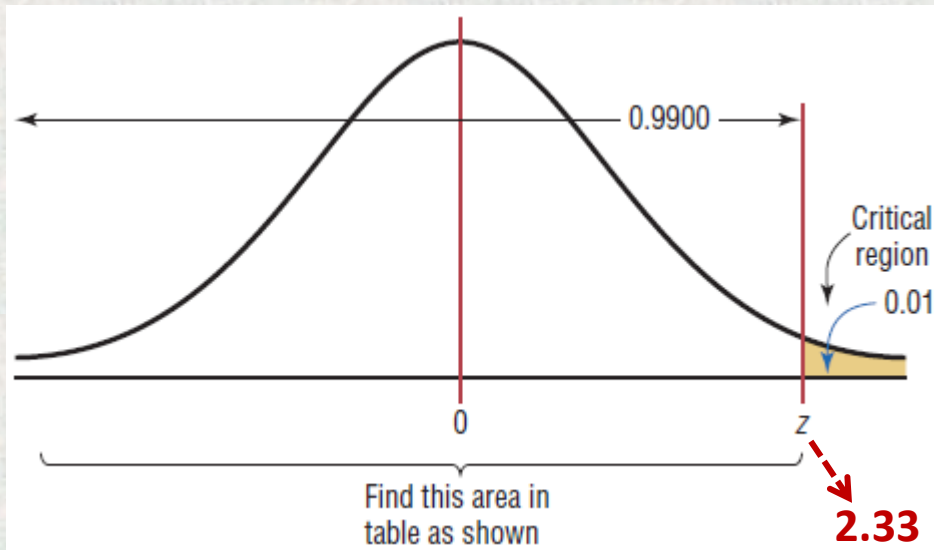
- Statisticians generally agree on using three arbitrary significance levels: the 0.10, 0.05, and 0.01 levels. In other words, when  $\alpha = 0.10$ , there is a 10% chance of rejecting a true null hypothesis; when  $\alpha = 0.05$ , there is a 5% chance of rejecting a true null hypothesis; and when  $\alpha = 0.01$ , there is a 1% chance of rejecting a true null hypothesis.
- Rejection and acceptance (not rejection) region depends on the critical value (CV) which can be determined based on the table of normal distribution (z value). The **critical value** separates the critical region from the noncritical region. CV can be either to the right side of the mean or to the left side of the mean (one or two -tailed).
- The **critical** or **rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.
- The **noncritical** or **non-rejection region** is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected (should be accepted).



- A **one-tailed test** indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean. A one-tailed test is either a **right tailed test** or **left-tailed test**, depending on the direction of the inequality of the alternative hypothesis.

**Example 4:** Finding the Critical Value for  $\alpha = 0.01$  (Right-Tailed Test).

**Solution:** Using z table.

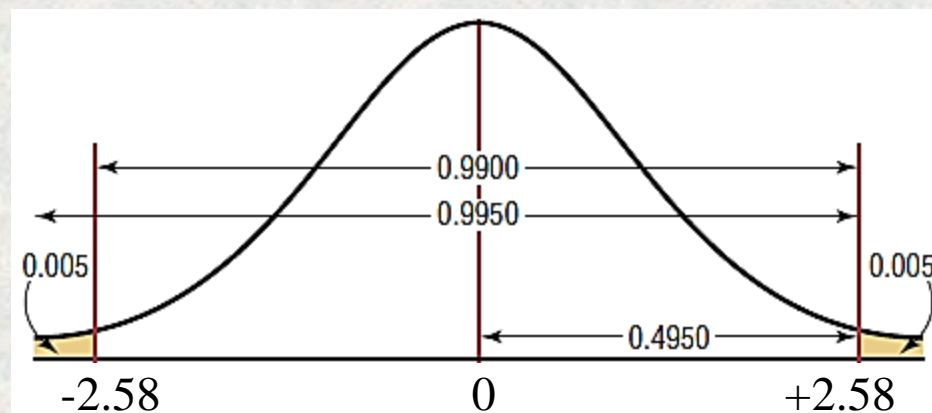
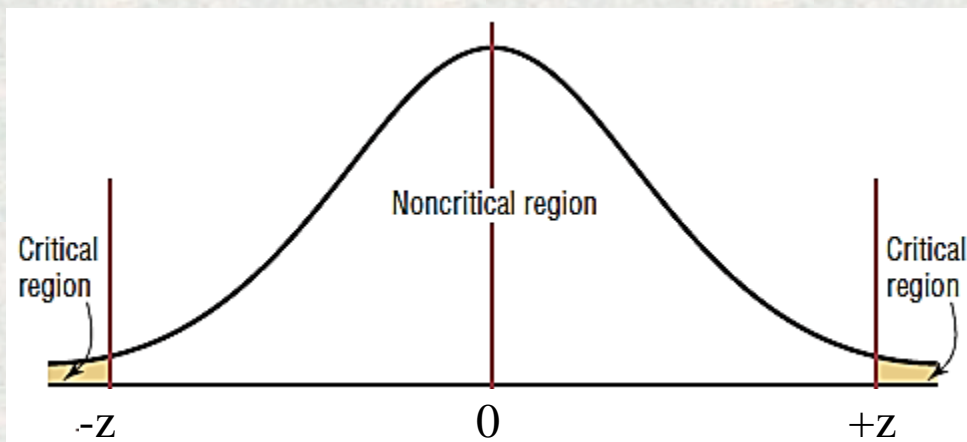


z	0.00	0.01	0.02	0.03	0.04	0.05	...
0.0							
0.1							
0.2							
0.3							
⋮							
2.1							
2.2							
2.3				0.9901			
2.4							
...							

Closest value to 0.9900

- In a **two-tailed test**, the null hypothesis should be rejected when the test value is in either of the two critical regions.

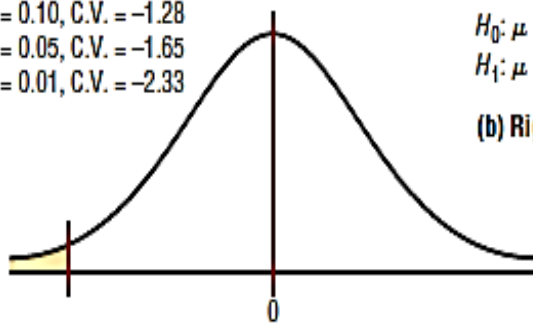
**For example**, a two-tailed test, the critical region must be split into two equal parts. If  $\alpha = 0.01$ , then one-half of the area, or 0.005, must be to the right of the mean and one half must be to the left of the mean, as shown in the figure below.



## Summery

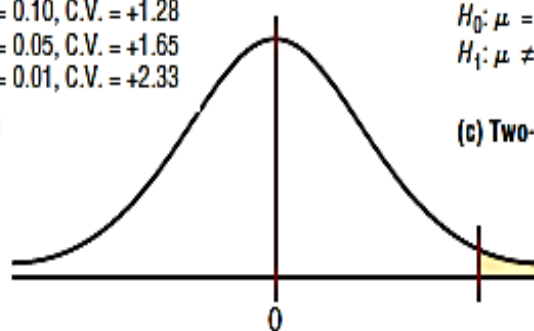
$$\begin{aligned} H_0: \mu &= k \\ H_1: \mu &< k \end{aligned} \begin{cases} \alpha = 0.10, \text{C.V.} = -1.28 \\ \alpha = 0.05, \text{C.V.} = -1.65 \\ \alpha = 0.01, \text{C.V.} = -2.33 \end{cases}$$

(a) Left-tailed



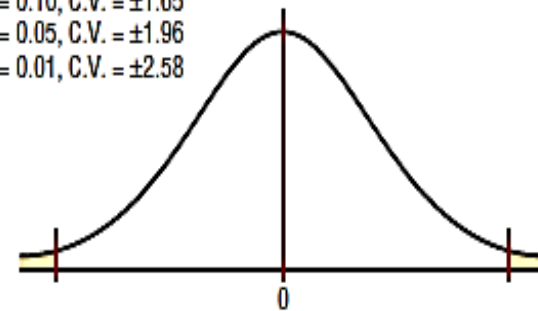
$$\begin{aligned} H_0: \mu &= k \\ H_1: \mu &> k \end{aligned} \begin{cases} \alpha = 0.10, \text{C.V.} = +1.28 \\ \alpha = 0.05, \text{C.V.} = +1.65 \\ \alpha = 0.01, \text{C.V.} = +2.33 \end{cases}$$

(b) Right-tailed



$$\begin{aligned} H_0: \mu &= k \\ H_1: \mu &\neq k \end{aligned} \begin{cases} \alpha = 0.10, \text{C.V.} = \pm 1.65 \\ \alpha = 0.05, \text{C.V.} = \pm 1.96 \\ \alpha = 0.01, \text{C.V.} = \pm 2.58 \end{cases}$$

(c) Two-tailed



## Procedure Table

Finding the Critical Values for Specific  $\alpha$  Values, Using Table z values.

### Step 1

**Draw the figure and indicate the appropriate area.**

1. If the test is left-tailed, the critical region, with an area equal to  $\alpha$ , will be on the left side of the mean.
2. If the test is right-tailed, the critical region, with an area equal to  $\alpha$ , will be on the right side of the mean.
3. If the test is two-tailed,  $\alpha$  must be divided by 2; one-half of the area will be to the right of the mean, and one-half will be to the left of the mean.

### Step 2

1. For a left-tailed test, use the z value that corresponds to the area equivalent to  $\alpha$  in Table z values.
2. For a right-tailed test, use the z value that corresponds to the area equivalent to  $(1 - \alpha)$ .
3. For a two-tailed test, use the z value that corresponds to  $\alpha/2$  for the left value. It will be negative. For the right value, use the z value that corresponds to the area equivalent to  $1 - \alpha/2$ . It will be positive.

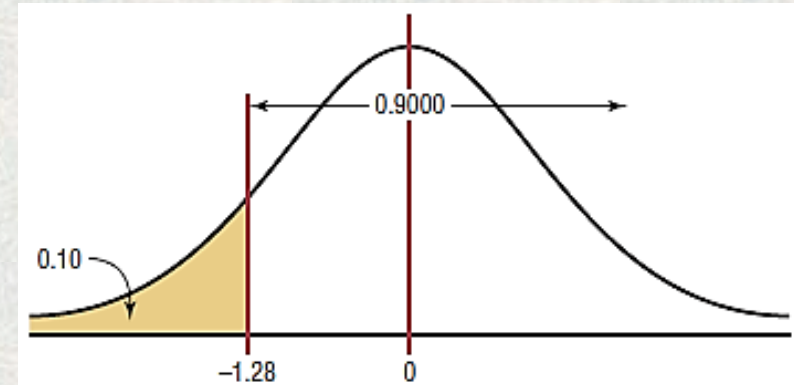


**Example 5:** Using Table E in Appendix C, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region. **a.** A left-tailed test with a 0.10. **b.** A two-tailed test with a 0.02. **c.** A right-tailed test with a 0.005.

**Solution**

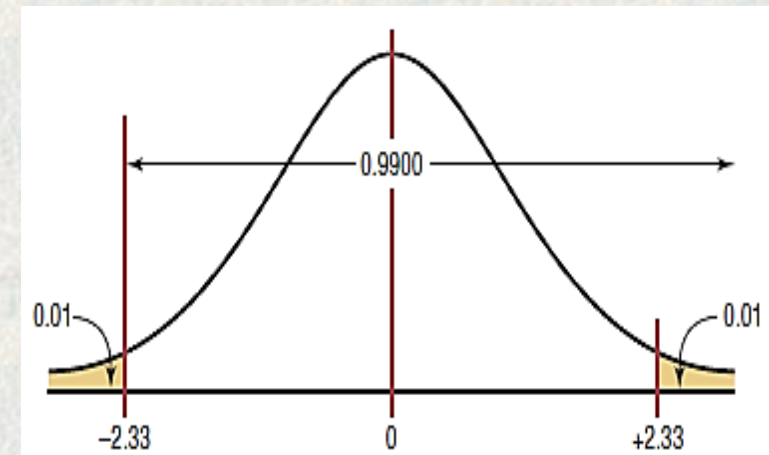
a). **Step 1:** Draw the figure and indicate the appropriate area. The area of 0.10 is located in the left tail, as shown in Figure below.

**Step 2:** Find the area closest to 0.1 from z's table. In this case, it is 0.1003. Find the z value that corresponds to the area 0.1003. It is 1.28. See Figure below.



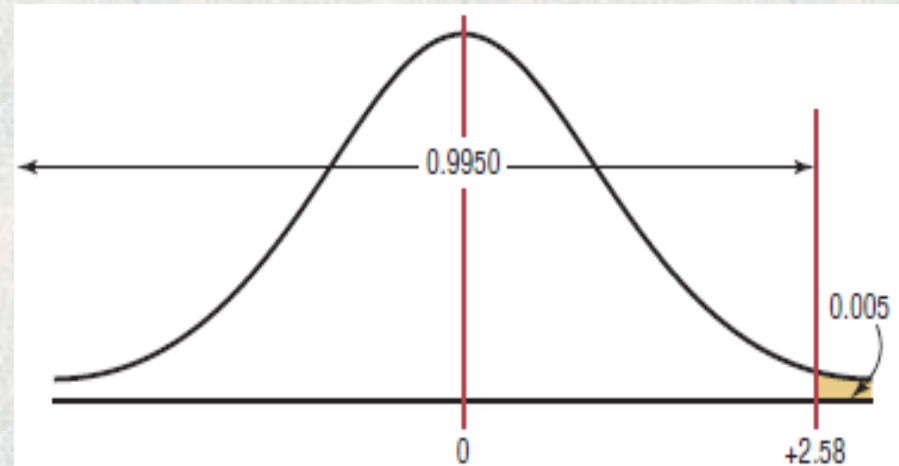
b). **Step 1:** Draw the figure and indicate the appropriate area. In this case, there are two areas equivalent to  $\alpha/2$  or  $0.02/2 = 0.01$ .

**Step 2:** For the left z critical value, find the area closest to  $\alpha/2$ , or  $0.02/2 = 0.01$ . In this case, it is 0.0099. For the right z critical value, find the area closest to  $1 - \alpha/2$ , or  $1 - 0.02/2 = 0.9900$ . In this case, it is 0.9901. Find the z values for each of the areas. For 0.0099,  $z = -2.33$ . For the area of 0.9901,  $z = 0.9901$ ,  $z = +2.33$ .



c). **Step 1:** Draw the figure and indicate the appropriate area. Since this is a right-tailed test, the area 0.005 is located in the right tail, as shown in Figure below.

**Step 2:** Find the area closest to  $1-\alpha$ , or  $1 - 0.005 = 0.9950$ . In this case, it is 0.9949 or 0.9951. The two z values corresponding to 0.9949 and 0.9951 are 2.57 and 2.58. Since 0.9500 is halfway between these two values, find the average of the two values  $(2.57 + 2.58) / 2 = 2.575$ . However, 2.58 is most often used.



### **❑ In hypothesis testing, the following steps are recommended for Traditional Method**

**Step 1** State the hypotheses (null and alternative) and identify the claim.

**Step 2** Find the critical value(s) from the appropriate table.

**Step 3** Compute the test value.

**Step 4** Make the decision to reject or not reject the null hypothesis.

**Step 5** Summarize the results.

### 3. z Test for a Mean

- The **z test** is a statistical test for the mean of a population. It can be used when  $n \geq 30$ , or when the population is normally distributed and  $\sigma$  is known. The formula for the z test is
- Where:  $\bar{X}$  = sample mean;  $\mu$  = hypothesized population mean;  $\sigma$  = population standard deviation;  $n$  = sample size
- **Assumptions for the z Test for a Mean When  $\sigma$  Is Known**
  1. The sample is a random sample.
  2. Either  $n \geq 30$  or the population is normally distributed if  $n < 30$ .
- **Procedure Steps: there are five steps for solving *hypothesis-testing* problems:**
  - Step 1** State the hypotheses and identify the claim.
  - Step 2** Find the critical value(s).
  - Step 3** Compute the test value.
  - Step 4** Make the decision to reject or not reject the null hypothesis.
  - Step 5** Summarize the results.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$



**Example 6:** A researcher wishes to see if the mean number of days that a basic, low-price, small automobile sits on a dealer's lot is 29. A sample of 30 automobile dealers has a mean of 30.1 days for basic, low-price, small automobiles. At  $\alpha = 0.05$ , test the claim that the mean time is greater than 29 days. The standard deviation of the population is 3.8 days.

**Solution:**

**Step 1** State the hypotheses and identify the claim.

$$H_0: \mu = 29 \quad \text{and} \quad H_1: \mu > 29 \text{ (claim)}$$

**Step 2** Find the critical value. Since  $\alpha = 0.05$  and the test is a right-tailed test, the critical value is  $z = 1.65$  from Table.

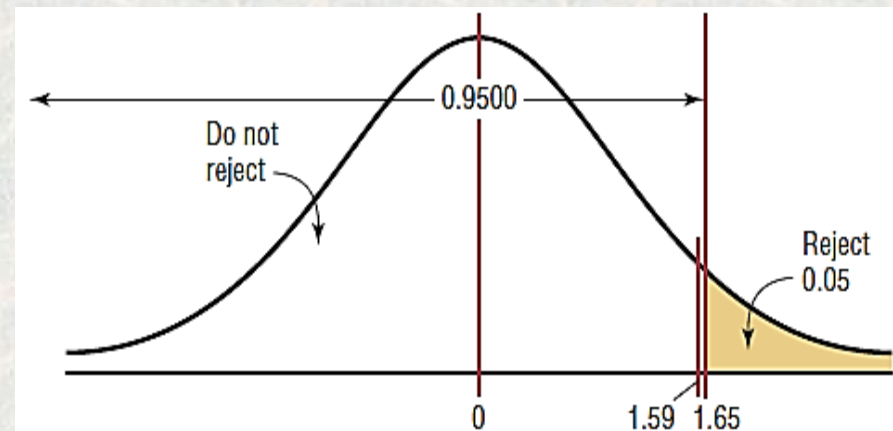
**Step 4** Make the decision. Since the test value, 1.59, is less than the critical value, 1.65, and is not in the critical region, the decision is to not reject the null hypothesis.

As shown in the figure

**Step 5** Summarize the results. There is not enough evidence to support the claim that the mean time is greater than 29 days.

**Step 3** Compute the test value.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{30.1 - 29}{3.8 / \sqrt{30}} = 1.59$$



**Example 7:** The School Rehabilitation Foundation reports that the average cost of rehabilitation for a primary school for each 10 years is \$24,672. To see if the average cost of rehabilitation is different at a particular school, a researcher selects a random sample of 35 schools to find that the average cost of their rehabilitation is \$26,343. The standard deviation of the population is \$3251. At  $\alpha = 0.01$ , can it be concluded that the average cost of rehabilitation at a particular school is different from \$24,672?

**Solution**

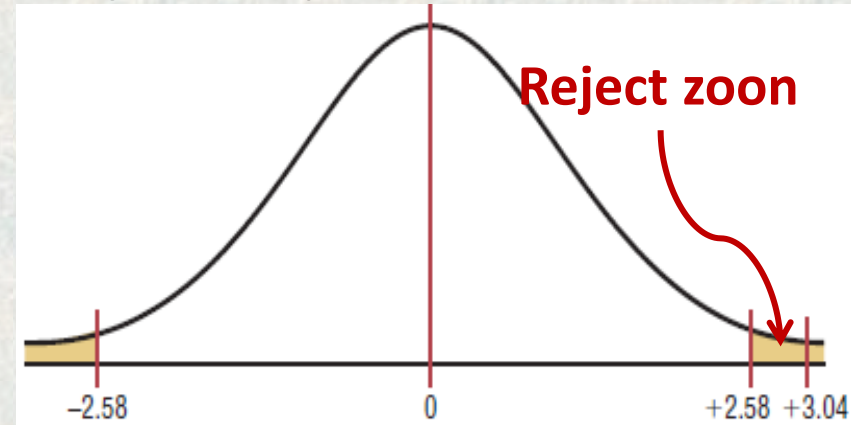
**Step 1** State the hypotheses and identify the claim.  $H_0: \mu = \$ 24,672$  and  $H_1: \mu \neq \$24,672$  (claim)

**Step 2** Find the critical value. Since  $\alpha = 0.01$  and the test is a two-tailed test, the critical value is  $z = \pm 2.58$ .

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{26,343 - 24,672}{3251 / \sqrt{35}} = 3.04$$

**Step 4** Make the decision. Reject the null hypothesis, since the test value falls in the critical region, as shown in Figure.

**Step 3** Compute the test value.

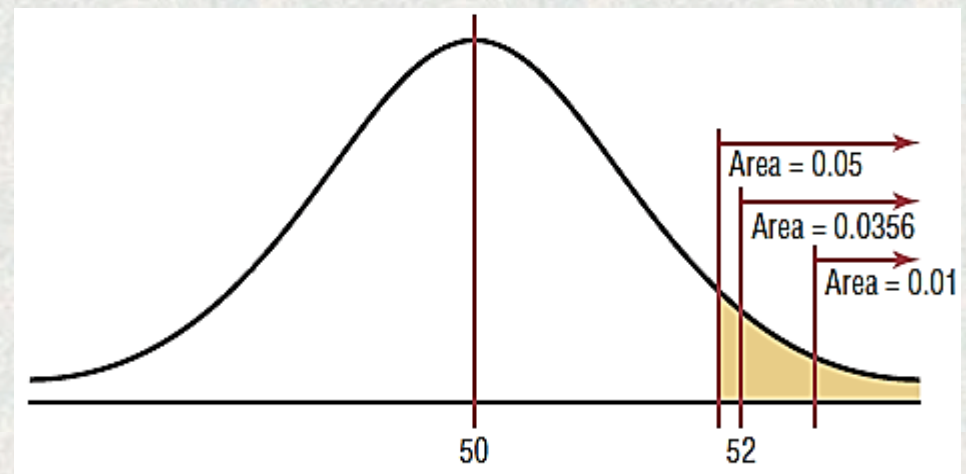


**Step 5** Summarize. There is enough evidence to support the claim that the average cost of rehabilitation at the particular school is different from \$24,672.

## 4. P-Value Method for Hypothesis Testing

- ❑ The **P-value** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true. In other words, the P-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample statistic or a more extreme sample statistic occurring if the null hypothesis is true.
- ❑ For example, suppose that an alternative hypothesis is  $H_1: \mu > 50$  and the mean of a sample is  $\bar{X} = 52$ . If the P-value = 0.0356 for a statistical test, then the probability of getting a sample mean of 52 or greater is 0.0356 if the true population mean is 50.

The relationship between the P-value and the  $\alpha$  value can be explained in this manner. For  $P = 0.0356$ , the null hypothesis would be rejected at  $\alpha = 0.05$  but not at  $\alpha = 0.01$





- **Procedure for Solving Hypothesis-Testing Problems (P-Value Method)**

**Step 1** State the hypotheses and identify the claim.

**Step 2** Compute the test value.

**Step 3** Find the  $P$ -value.

**Step 4** Make the decision.

**Step 5** Summarize the results.

**Example 8:** A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At  $\alpha = 0.05$ , is there enough evidence to reject the claim? Use the  $P$ -value method.

**Solution**

**Step 1** State the hypotheses and identify the claim.  $H_0: \mu = 8$  and  $H_1: \mu \neq 8$ (claim)

**Step 2** Compute the test value.

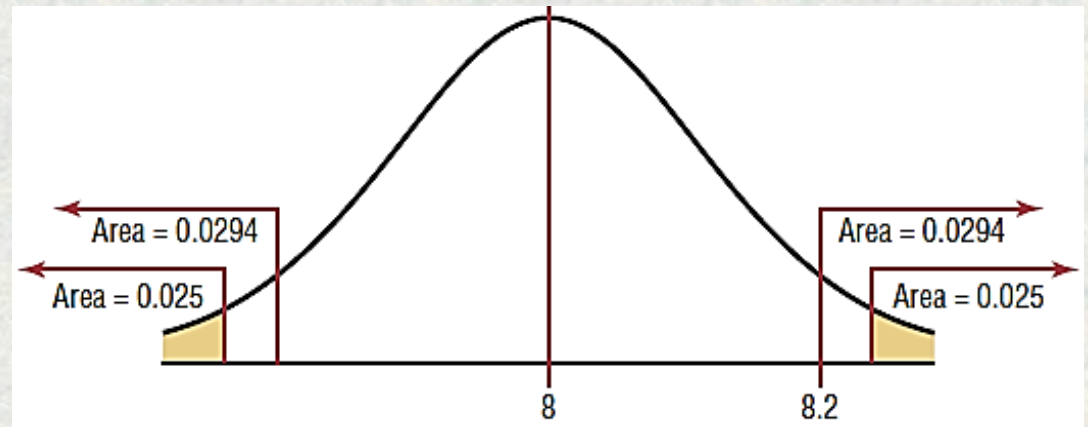
$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{8.2 - 8.0}{0.6 / \sqrt{32}} = 1.89$$

**Step 3** Find the  $P$ -value. Using N.D. Table, find the corresponding area for  $z = 1.89$ . It is 0.9706. Subtract the value from 1.0000. ( $1.0000 - 0.9706 = 0.0294$ ).

Since this is a two-tailed test, the area of 0.0294 must be doubled to get the  $P$ -value.  $\rightarrow (2(0.0294) = 0.0588)$ .

**Step 4** Make the decision. The decision is to not reject the null hypothesis, since the P-value is greater than 0.05. As shown in the figure.

**Step 5** Summarize the results. There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.



**Example 9:** A producer bricks wishes to test the claim that the average of compressive strength of the product is greater than 5700 psi. He selected a random sample of 36 bricks and finds the mean to be 5950 psi. The population standard deviation is 659 psi. Is there evidence to support the claim at  $\alpha = 0.05$ ? Use the *P*-value method.

### **Solution**

**Step 1** State the hypotheses and identify the claim.  $H_0: \mu = 5700$  and  $H_1: \mu > 5700$  (claim)

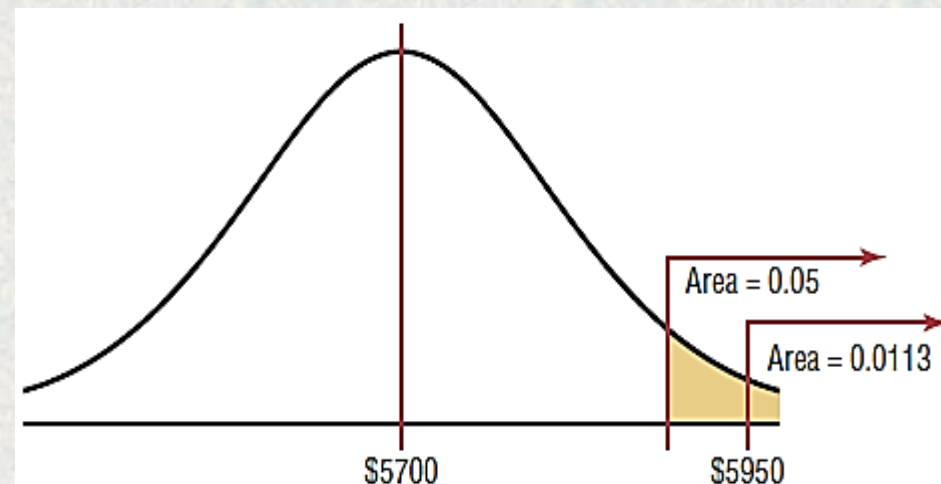
**Step 2** Compute the test value.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{5950 - 5700}{659 / \sqrt{36}} = 2.28$$

**Step 3** Find the  $P$ -value. Using N.D. Table, find the corresponding area for  $z = 2.28$ . It is 0.9887. Subtract the value from 1.0000. to find the area in the right tail.  $(1.0000 - 0.9887 = 0.0113) = P$ -value.

**Step 4** Make the decision. Since the  $P$ -value is less than 0.05, the decision is to reject the null hypothesis. As shown in the figure.

**Step 5** Summarize the results. There is enough evidence to support the claim that the compressive strength is greater than 5700 psi.



## 4. $t$ - Test for a Mean

- The  $t$  test is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, and  $s$  is unknown.
- The formula for the  $t$  test is.

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

The degrees of freedom are d.f.  $n = 1$ .

**Note:** The formula for the  $t$  test is similar to the formula for the  $z$  test. But since the population standard deviation  $\sigma$  is unknown, the sample standard deviation  $s$  is used instead.



**Example 10:** Find the critical  $t$  value for  $\alpha = 0.05$  with d.f. = 16 for a right-tailed  $t$  test.

**Solution**

Find the 0.05 column in the top row and 16 in the left-hand column. Where the row and column meet, the appropriate critical value is found; it is 1.746. as shown in the figure.

	One tail, $\alpha$	0.25	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, $\alpha$	0.50	0.20	0.10	0.05	0.02	0.01
1							
2							
3							
4							
5							
⋮							
14							
15							
16				1.746			
17							
18							
⋮							

**t – Distribution table**

**Example 11:** Find the critical  $t$  value for  $\alpha = 0.01$  with d.f. 22 for a left-tailed test.

**Solution**

Find the 0.01 column in the row labeled One tail, and find 22 in the left column. The critical value is 2.508 since the test is a one-tailed left test.

**Example 12:** Find the critical values for  $\alpha = 0.10$  with d.f. = 18 for a two-tailed  $t$  test.

**Solution**

Find the 0.10 column in the row labeled Two tails, and find 18 in the column labeled d.f. The critical values are +1.734 and -1.734.

**Example 13:** Find the critical value for  $\alpha = 0.05$  with d.f. 28 for a right-tailed  $t$  test.

**Solution**

Find the 0.05 column in the One-tail row and 28 in the left column. The critical value is 1.701.

- **Assumptions for the  $t$  Test for a Mean When  $\sigma$  Is Unknown**

1. The sample is a random sample.
2. Either  $n \geq 30$  or the population is normally distributed if  $n < 30$ .

- **Test hypotheses using the  $t$  test (traditional method), is the same procedure as for the  $z$  test, except use the table of  $t$  distribution.**

**Step 1** State the hypotheses and identify the claim.

**Step 2** Find the critical value(s) from Table F.

**Step 3** Compute the test value.

**Step 4** Make the decision to reject or not reject the null hypothesis.

**Step 5** Summarize the results.

**Example 14:** An engineer of soil investigation claims that the average bearing capacity of soil is 16.3 Ton. A random sample of 10 samples had a mean bearing capacity is 17.7 ton. The sample standard deviation is 1.8 Ton. Is there enough evidence to reject the engineer's claim at  $\alpha = 0.05$ ?

**Solution**

**Step 1:**  $H_0: \mu = 16.3$  and  $H_1: \mu \neq 16.3$  (claim)

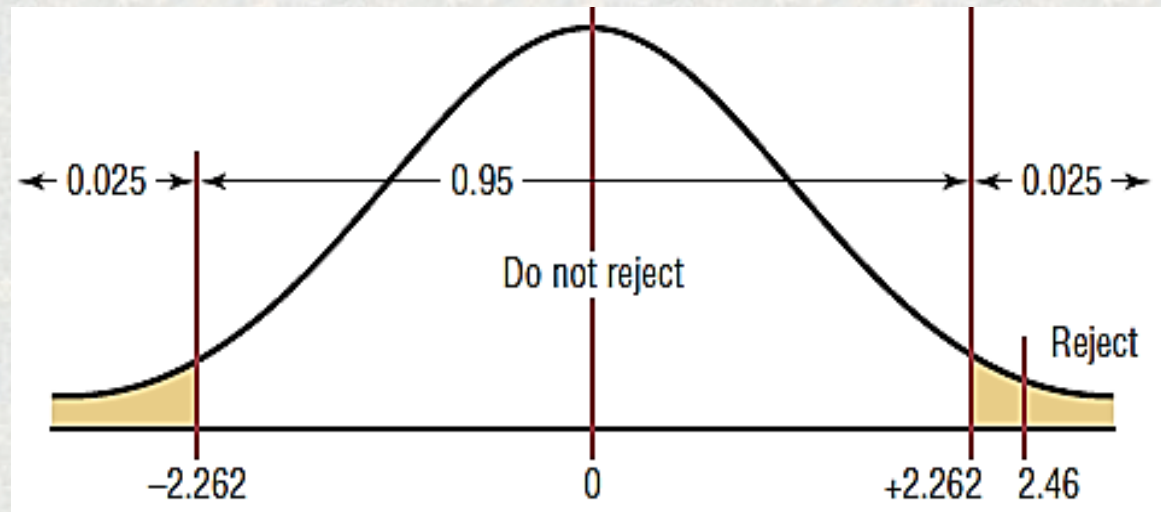
**Step 2:** The critical values are +2.262 and -2.262 for  $\alpha = 0.05$  and d.f. = 9.

**Step 3** The test value is

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

**Step 4** Reject the null hypothesis since  $2.46 > 2.262$ . As shown in the figure.

**Step 5** There is enough evidence to reject the claim that the average bearing capacity is 16.3 Ton.





**Example 15:** An engineer evaluated the production of a concrete factory for high strength. he claimed that the average compressive strength of the products is less than 60 MPa. A random sample of eight samples are selected as shown below. Is there enough evidence to support the engineer's claim at  $\alpha = 0.10$ ?

**Compressive strength: 60, 56, 60, 55, 70, 55, 60, 55**

**Solution**

**Step 1:**  $H_0: \mu = 60$  and  $H_1: \mu < 60$  (claim)

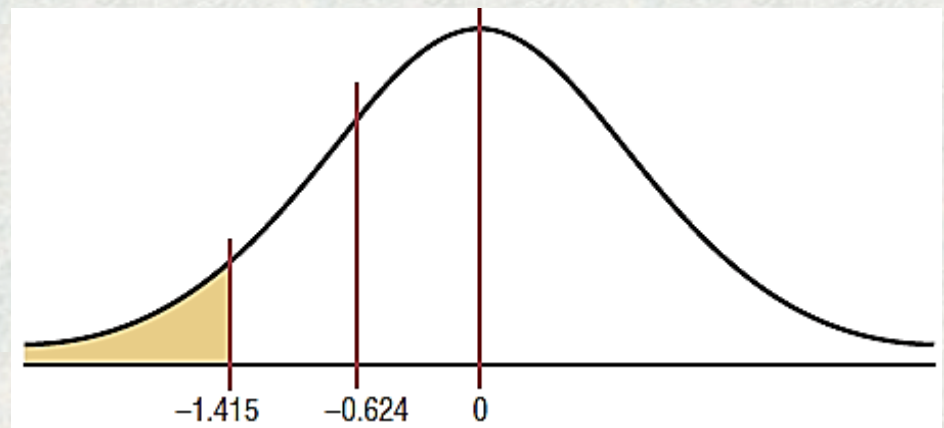
**Step 2:** At  $\alpha = 0.10$  and d.f. = 7. the critical value is - 1.415.

**Step 3:** To compute the test value, the mean and standard deviation must be found.  $\bar{X} = 58.88$  and  $S = 5.08$ , find t.

**Step 4:** Do not reject the null hypothesis since -0.624 falls in the noncritical region. As shown in the figure.

**Step 5:** There is not enough evidence to support the engineer's claim that the average compressive strength is less than 60 MPa.

$$t = \frac{\bar{X} - \mu}{S / \sqrt{n}} = \frac{58.88 - 60}{5.08 / \sqrt{7}} = -0.624$$



## 5. z Test for a Proportion

A normal distribution can be used to approximate the binomial distribution (proportion) when  $np \geq 5$  and  $nq \geq 5$ , the standard normal distribution can be used to test hypotheses for proportions.

### Formula for the z Test for Proportions

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Where:  $\hat{p} = \frac{x}{n}$  (sample proportion)

$p$  = population proportion

$n$  = sample size

### Assumptions for Testing a Proportion

1. The sample is a random sample.
2. The conditions for a binomial experiment are satisfied.
3.  $np \geq 5$  and  $nq \geq 5$ .

**Note:** The steps for hypothesis testing are the same as those used to find critical values and *P*-values.

**Example 16:** A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At  $\alpha = 0.05$ , is there enough evidence to reject the dietitian's claim?

## Solution

**Step 1:** State the hypothesis and identify the claim.

$$H_0: p = 0.60 \text{ (claim)} \quad \text{and} \quad H_1: p \neq 0.60$$

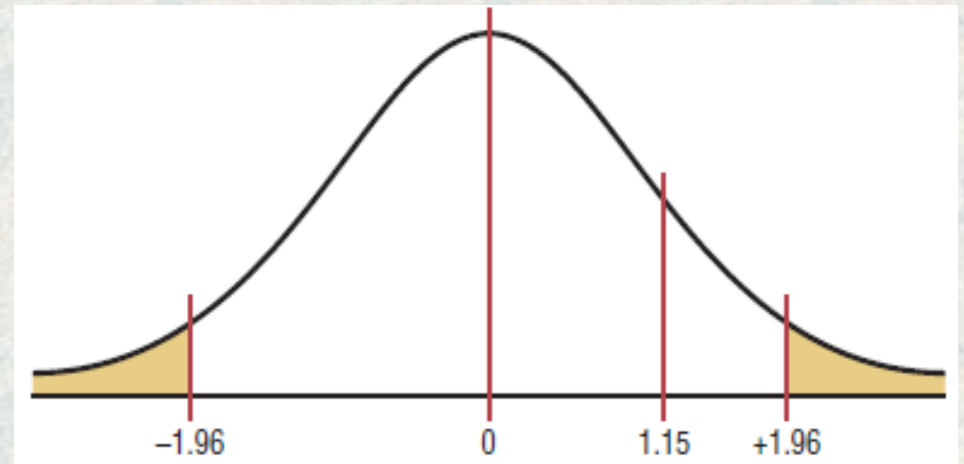
**Step 2:** Find the critical values. Since  $\alpha = 0.05$  and the test value is two-tailed, the critical values are  $\pm 1.96$ .

**Step 3:** Compute the test value. First, it is necessary to find  $\hat{p}$ .

$$\hat{p} = \frac{X}{n} = \frac{125}{200} = 0.64$$

$$P = 0.6 \rightarrow q = 1 - 0.6 = 0.4$$

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.64 - 0.6}{\sqrt{(0.6)(0.4)/200}} = 1.15$$



**Step 4:** Make the decision. Do not reject the null hypothesis since the test value falls outside the critical region, as shown in figure.

**Step 5** Summarize the results. There is not enough evidence to reject the claim that 60% of people are trying to avoid trans fats in their diets.



**Example 17:** An engineer claims that more than 25% of all construction company advertise. A sample of 200 companies in a certain city showed that 63 had used some form of advertising. At  $\alpha = 0.05$ , is there enough evidence to support the engineer's claim? Use the  $P$ -value method.

### **Solution**

**Step 1:** State the hypothesis and identify the claim.

$H_0: p = 0.25$  and  $H_1: p > 0.25$  (claim)

**Step 2:** Compute the test value.

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.315 - 0.25}{\sqrt{(0.25)(0.315)/200}} = 2.12$$

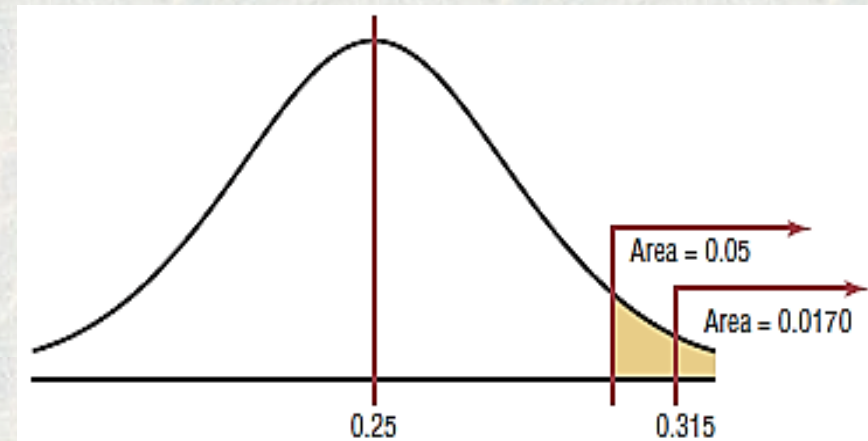
**Step 3** Find the  $P$ -value. The area under the curve for  $z = 2.12$  is 0.9830. Subtracting the area from 1.0000, you get  $1.0000 - 0.9830 = 0.0170$ . The  $P$ -value is 0.0170.

**Step 4:** Reject the null hypothesis, since  $0.0170 < 0.05$  (that is,  $P$ -value  $< 0.05$ ). As shown in the figure

**Step 5** There is enough evidence to support the attorney's claim that more than 25% of the lawyers use some form of advertising.

$$\hat{p} = \frac{X}{n} = \frac{63}{200} = 0.315$$

$$P = 0.25 \rightarrow q = 1 - 0.25 = 0.75$$



## 6. $\chi^2$ Test for a Variance or Standard Deviation

In Chapter 7, the chi-square distribution was used to construct a confidence interval for a single variance or standard deviation. This distribution is also used to test a claim about a single variance or standard deviation.

- **Formula for the  $\chi^2$  Test for a Single Variance**

$$\chi^2 = \frac{(n - 1)S^2}{\sigma^2}$$

with degrees of freedom equal to  $n-1$

$n$  = sample size

$S^2$  = sample variance

$\sigma^2$  = population variance

- **Assumptions for the Chi-Square Test for a Single Variance**

1. The sample must be randomly selected from the population.
2. The population must be normally distributed for the variable under study.
3. The observations must be independent of one another

- **The traditional method for hypothesis testing:**

**Step 1** State the hypotheses and identify the claim.

**Step 2** Find the critical value(s).

**Step 4** Make the decision.

**Step 3** Compute the test value.

**Step 5** Summarize the results.

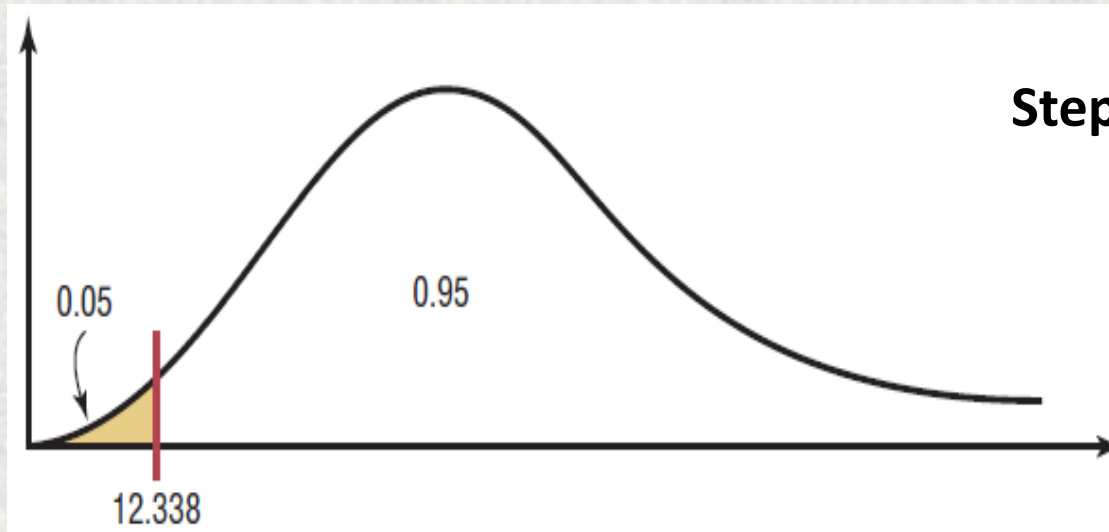
**Example18:** An engineer wishes to see whether the variation in the experience of construction for 23 companies is less than the variance of the population ( $\sigma^2 = 225$ ). The variance of the companies is 198. Is there enough evidence to support the engineer's claim that the variation of the companies is less than the population variance at  $\alpha = 0.05$ ? Assume that the scores are normally distributed.

### Solution

**Step 1:** State the hypotheses and identify the claim.

$H_0: s^2 \geq 225$  and  $H_1: s^2 < 225$  (claim)

**Step 2:** Find the critical value. Since this test is left-tailed and  $\alpha = 0.05$ , use the value  $1 - 0.05 = 0.95$ . The degrees of freedom are  $n - 1 = 23 - 1 = 22$ . Hence, the critical value is 12.338. Note that the critical region is on the left, as shown in figure.

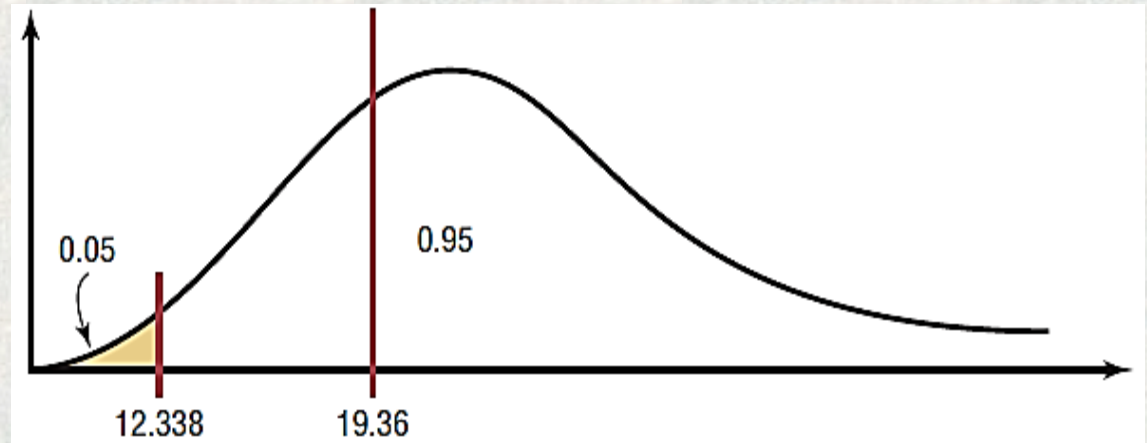


**Step 3:** Compute the test value.

$$\begin{aligned}\chi^2 &= \frac{(n-1)s^2}{\sigma^2} \\ &= \frac{(23-1)(198)}{(225)} \\ \chi^2 &= 19.36\end{aligned}$$



**Step 4** Make the decision. Since the test value 19.36 falls in the noncritical region, as shown the figure, the decision is to not reject the null hypothesis.



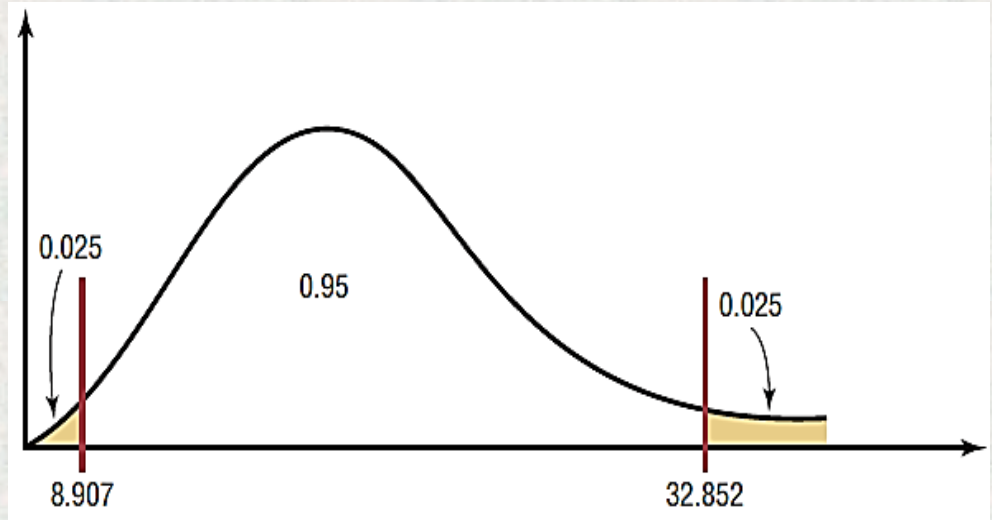
**Step 5** Summarize the results. There is not enough evidence to support the claim that the variation in test scores of the engineer's claim is less than the variation in scores of the population.

**Example 19:** An petrol engineer wishes to test the claim that the variance of the lead content of the fuel is 0.644. Lead content is measured in milligrams, and assume that it is normally distributed. A sample of 20 bottles has a standard deviation of 1.00 milligram. At  $\alpha = 0.05$ , is there enough evidence to reject the manufacturer's claim?

### **Solution**

**Step 1:** State the hypotheses and the claim.  $H_0: \sigma^2 = 225$  and  $H_1: \sigma^2 = 225$  (claim)

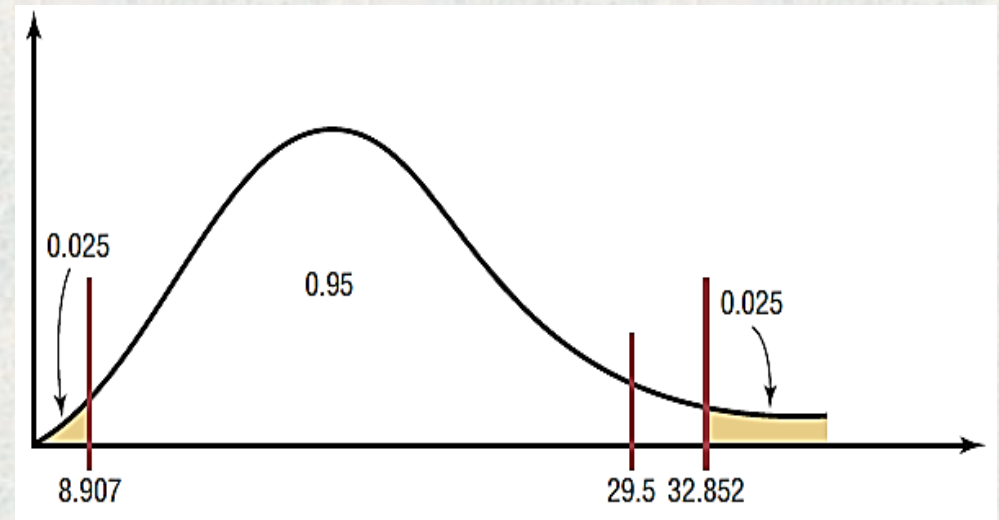
**Step 2:** Find the critical value. Since this test is two-tailed and  $\alpha = 0.05$ , the critical values for 0.025 and 0.975 must be found. d.f. =  $20 - 1 = 19$ ; hence, the critical values are 32.852 and 8.907, respectively. The critical or rejection regions are shown in the figure.



**Step 3:** Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(1.0)^2}{(0.644)} = 29.5$$

**Step 4:** Make the decision. Do not reject the null hypothesis, since the test value falls between the critical values ( $8.907 < 29.5 < 32.852$ ) and in the noncritical region, as shown in the figure.



**Step 5:** Summarize the results. There is not enough evidence to reject the engineer's claim that the variance of the lead content of the fuel is equal to 0.644.

*Thank you, the end of Ch. 8*