CHAPTER 1 GENERAL PRINCIPLES

1. INTRODUCTION

Mechanics is the branch of *physics* (classic) *which describes and predicts the conditions of rest or motion of bodies under the action of forces*. It is divided into three parts:

Mechanics of *rigid bodies,*

Mechanics of *deformable bodies,*

Mechanics of *fluids*.

The mechanics of rigid bodies is subdivided into ***statics*** and ***dynamics***, the former dealing with bodies at rest or moving at constant velocity, the latter with bodies in motion of accelerated bodies.

In this part of the study of mechanics, bodies are assumed to be perfectly rigid. Actual structures and machines, however, are never absolutely rigid and deform under the loads to which they are subjected. But these deformations are usually small and do not appreciably affect the conditions of equilibrium or motion of the structure under consideration. They are important, though, as far as the resistance of the structure to failure is concerned and are studied in ***mechanics of materials***, *which is a part of the mechanics of deformable bodies*.

The *third* division of mechanics, the ***mechanics of fluids***, is subdivided into the study of ***incompressible fluids*** and of ***compressible fluids***. An important subdivision of the study of incompressible fluids is ***hydraulics***, which deals with problems involving water.

2. FUNDAMENTAL CONCEPTS

The basic concepts used in mechanics are *space, time, mass,* and *force*. These concepts cannot be truly defined; they should be accepted on the basis of our intuition and experience and used as a mental frame of reference for our study of mechanics.

The concept of ***space*** is associated with the notion of the position of a point *P*. The position of *P* can be defined by three lengths measured from a certain reference point, or *origin,* in three given directions. These lengths are known as the *coordinates* of *P*.

To define an event, it is not sufficient to indicate its position in space. The ***time*** of the event should also be given.

The concept of ***mass*** is used to characterize and compare bodies on the basis of certain fundamental mechanical experiments. Two bodies of the same mass, for example, will be attracted by the earth in the same manner; they will also offer the same resistance to a change in translational motion.

A ***force*** represents the action of one body on another. It can be exerted by actual contact or at a distance, as in the case of gravitational forces and magnetic forces. A force is characterized by its *point of application,* its *magnitude,* and its *direction;* a force is represented by a *vector*.

Idealizations. Due to the enormous complexity of real systems it is necessary to implement idealized models that resemble adequately the actual system.

Particle. A particle has mass but negligible size. Thus, the geometry of the body becomes irrelevant.

Rigid Body. A body can be considered as a combination of a large number of particles, in which all the particles remain at a fixed distance from each other both before and after applying a load.

Concentrated Force. A concentrated force represents the effect of a loading which is assumed to act at a point on a body.

NEWTON'S LAWS OF MOTION

The mechanics of a rigid–body is governed by the three laws of motion introduced by Newton.

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

Second Law: A particle acted upon by an unbalanced force $\boldsymbol{F}$ experiences an acceleration $\boldsymbol{a}$ that has the same direction as the force and a magnitude that is directly proportional to the force. $\boldsymbol{F = ma}$

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.

Newton's Law of Gravitational Attraction - *Weight* is the effect of the force of attraction exerted by the earth on everybody on or near its surface. The weight (W) of a body is

$$\begin{aligned} \boldsymbol{W} &= \boldsymbol{mg} \\ &= (1\text{ kg})\,(9.81\text{ m/s}^2) \\ &= 9.81\text{ N} \end{aligned}$$

m: Mass of the body.

g: Acceleration of gravity (9.81 m/s^2 or 32.2 ft/s^2).

3. UNITS OF MEASUREMENT

Systems and Fundamental Units

International System (SI)
- kg → mass
- m → length
- s → Time

English system (USC)
- slug → mass
- ft → length
- s → Time

Derived Units. Derived units result of the combination of fundamental units. When a quantity is defined in terms of some fundamental quantities, its units will be also defined in terms of fundamental units. For example, the velocity of a moving object is defined as the distance traveled by that object in a given time; therefore the units for velocity are m/s.

Units for Force. According to Newton's Second Law

$F = m\,a\ [kg.\ m/s^2]$ $F = m\,a\ [slug.\ ft/s^2]$

$Kg.\ m/s^2$ = N (newton) $slug.\ ft/s^2 = lbf$ *(pound force)*

Conversion Factors:

$1\ ft = 0.3048\ m$

$1\ slug = 14.5938\ kg$

$$1lb = 1\frac{slug \cdot ft}{s^2} \times \frac{14.5938\,kg}{1\,slug} \times \frac{0.3048\,m}{1\,ft} = 4.4482\,N$$

4. PREFIXES

Multiple	Exponential Form	Prefix	SI Symbol
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n

Example 1: Convert 2 km/h to m/s. How many ft/s is this?

Example 2: Convert the quantities 300 lb • s and 52 slug/ft^3 to appropriate SI units.

Example 3: Evaluate each of the following and express with SI units having an appropriate prefix: (a) (50 mN) (6 GN), (b) (400 mm) $(0.6\ MN)^2$, (c) 45 MN^3/900 Gg.

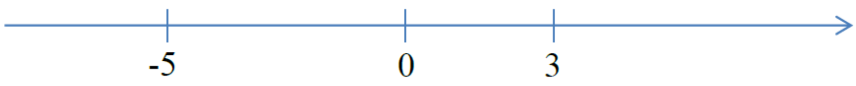
CHAPTER 2 STATICS OF PARTICLES

5. SCALARS AND VECTORS

A ***scalar*** is a quantity represented by a positive or negative number. It has magnitude (its absolute value) and sign (positive or negative). Examples include length, temperature and time, surface, volume, work, mass, mass flow rate, density.

A ***vector*** is any physical quantity that requires both a magnitude and a direction for its complete description, sometimes called ***one-dimensional vectors***. Examples include force, position, velocity, acceleration, and moment.

A vector is graphically shown by an arrow. The length of the arrow represents the magnitude of the vector. The direction often is represented by the angle between the vector and a fixed axis.



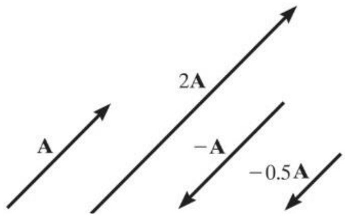
6. VECTOR OPERATIONS

Multiplication or Division of a Vector by a Scalar. Given a scalar ***a*** and a vector ***A*:**

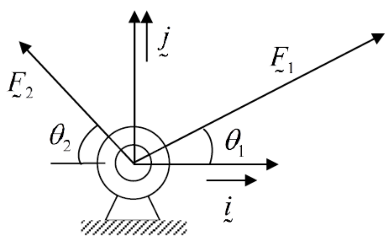
$$\vec{A} = 2\hat{i} - 6\hat{j} + 3\hat{k} \qquad a = -3.5$$

$$a\vec{A} = -7\hat{i} + 21\hat{j} - 10.5\hat{k}$$

$$\frac{\vec{A}}{a} = -0.571\hat{i} + 1.714\hat{j} - 0.857\hat{k}$$

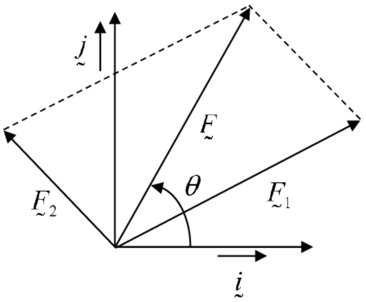


Vector Addition. To add two (or more) vectors is necessary to add each component of the first vector with the corresponding component the second vector.

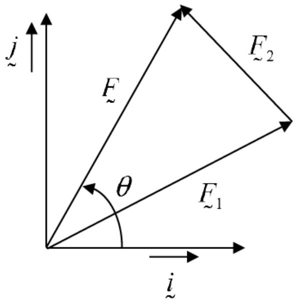


- The most basic way to add two vectors is a ***geometric approach*** called the ***parallelogram law of vector addition***.
- The ***sum*** of the two vectors is called the ***resultant***.
- Consider the two forces $\underset{\sim}{F}1$ and $\underset{\sim}{F}2$ acting on the support as shown. The total force acting on the support is found by adding the two vectors.
- By placing the tails of the vectors at the same point, we can ***construct*** the resultant by forming a ***parallelogram*** as shown on the left. By placing the tail of the second vector at the head of the first, we can construct the resultant by forming a ***triangle*** as shown on the right.

- In general, the triangle formed by $\underset{\sim}{F}1$, $\underset{\sim}{F}2$, and $\underset{\sim}{F}$ is a ***non-right triangle***. The lengths and angles within this triangle can be studied using the ***law of cosines*** and the ***law of sines***.



Parallelogram formed by $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$

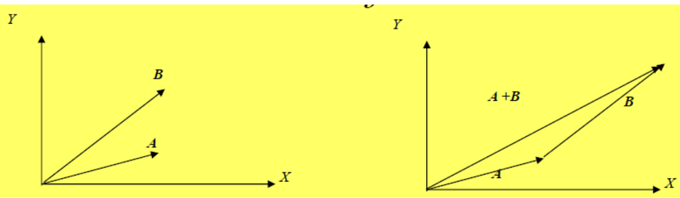


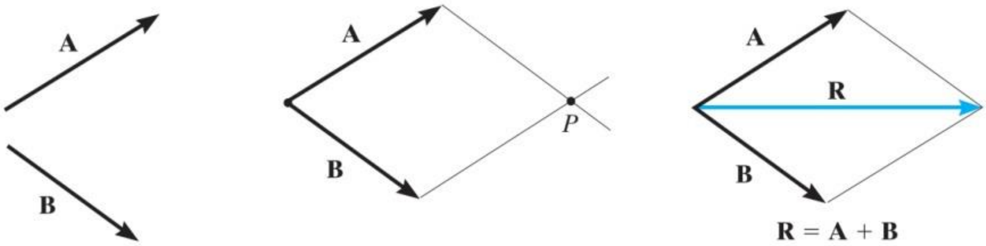
Triangle formed by $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$

$$\vec{A} = 3\hat{i} + \hat{j} \qquad \vec{B} = 4\hat{i} + 4\hat{j}$$

$$\vec{A} + \vec{B} = 3\hat{i} + \hat{j} + 4\hat{i} + 4\hat{j}$$

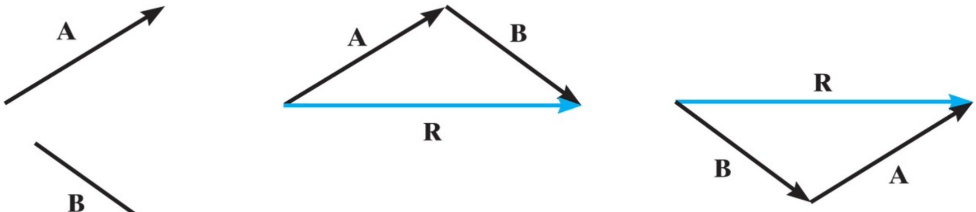
$$\vec{A} + \vec{B} = 7\hat{i} + 5\hat{j}$$





R = **A** + **B**

Parallelogram law

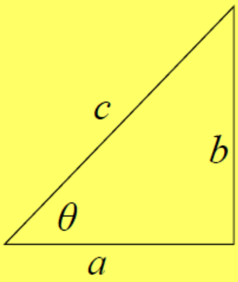


R = **A** + **B**

Triangle rule

R = **B** + **A**

Triangle rule

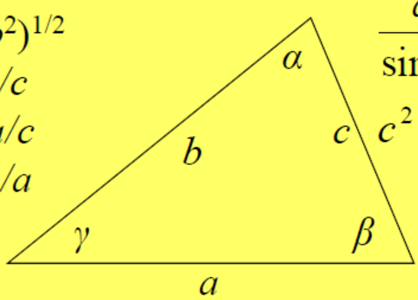


$c=(a^2+b^2)^{1/2}$

$\sin\theta = b/c$

$\cos\theta = a/c$

$\tan\theta = b/a$



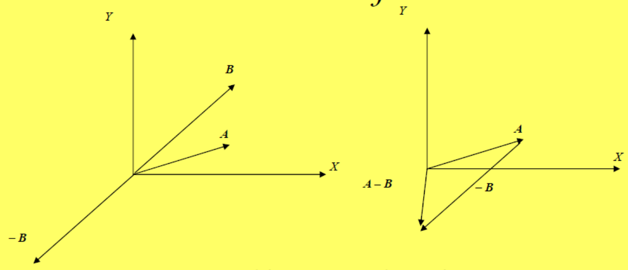
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

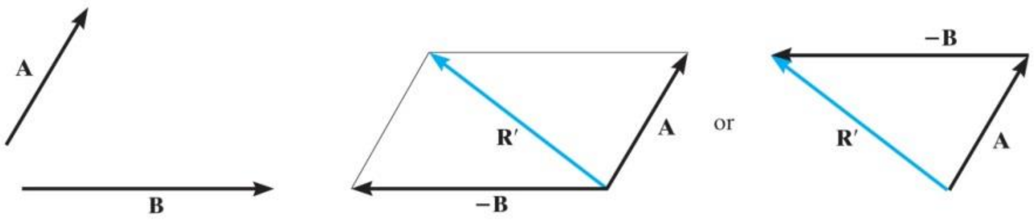
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Vector Subtraction. The difference of two vectors is done by subtracting each component of the subtrahend vector from the corresponding components of the minuend vector.

$$\vec{A} = 3\hat{i} + \hat{j} \qquad \vec{B} = 4\hat{i} + 4\hat{j}$$

$$\vec{A} - \vec{B} = -\hat{i} - 3\hat{j}$$



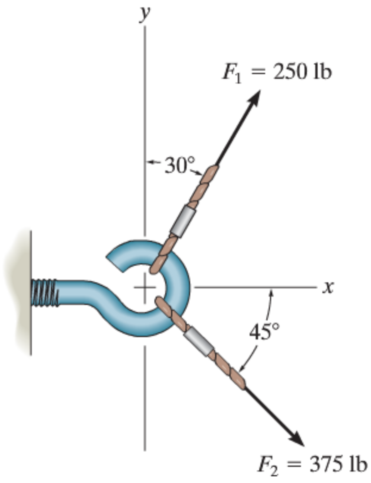


Parallelogram law

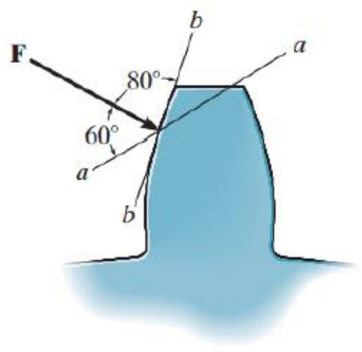
Triangle construction

Vector subtraction

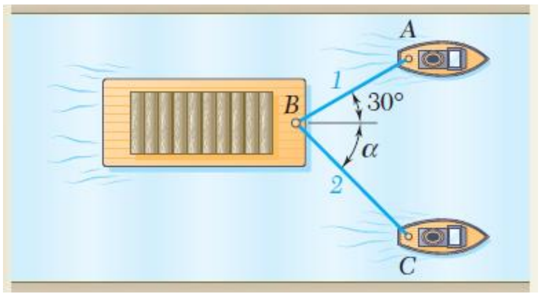
Example 1: Determine the magnitude of the resultant force $F_R = F_1 + F_2$ and its direction, measured counterclockwise from the positive x axis.



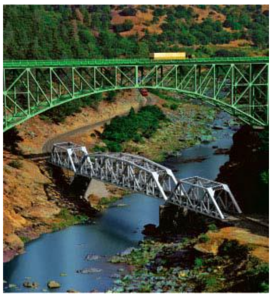
Example 2: The force acting on the gear tooth is $F = 20\ lb$. Resolve this Force into two components acting along the lines *aa* and *bb*.



Example 3: A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (a) the tension in each of the ropes knowing that $\alpha = 45°$, (b) the value of α for which the tension in rope 2 is minimum.



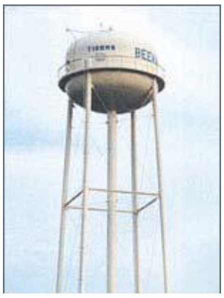
7. APLICATIONS



The forces within the every member of these bridges must be determined to ensure a proper design.

The maximum capacity of a tower crane must be properly specified to determine the forces that every member will support.

When a pressure vessel is designed, it is necessary to determine the center of gravity of its component parts, calculate its volume and surface area, and reduce three-dimensional distributed loadings to their resultants.

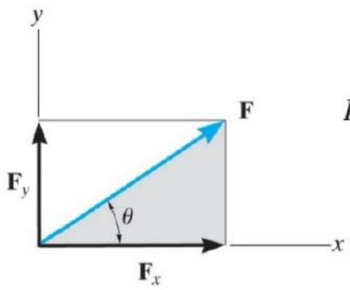


8. ADDITION OF A SYSTEM OF COPLANAR FORCES

For a system of coplanar forces, each force will have only two components.

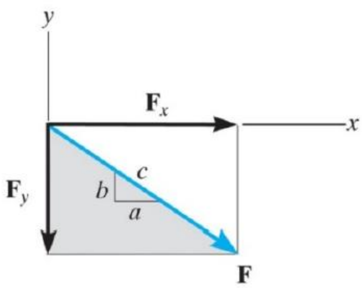
For analytical work, we can represent these components using either ***scalar notation*** or ***Cartesian vector notation***.

Scalar Notation



$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$



$$F_x = F\left(\frac{a}{c}\right)$$

$$F_y = -F\left(\frac{b}{c}\right)$$

Cartesian Vector Notation

Represent the x and y components of a force in terms of Cartesian unit vectors **i** and **j**. They are called unit vectors because they have a dimensionless magnitude of 1, and so they can be used to designate the directions of the x and y axes.

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

Components for Cartesian coordinates in terms of x, y, and z components

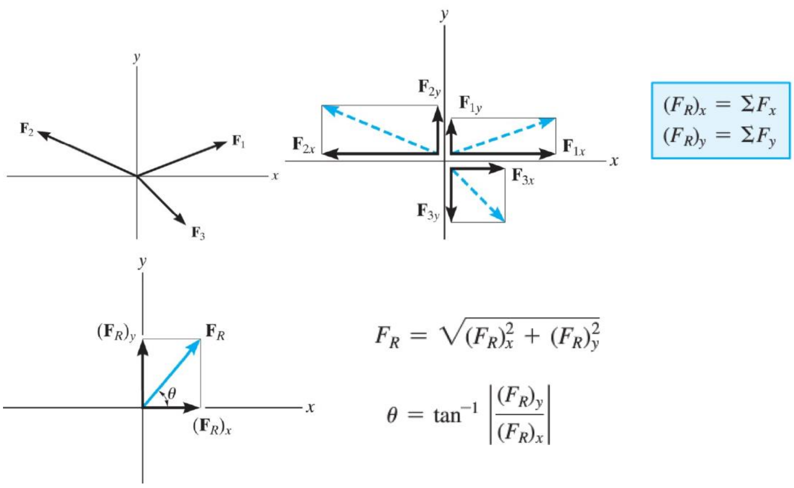
$$F = F_x i + F_y j + F_z k$$

Fx, Fy, Fz, are the components of F in the x, y, z directions

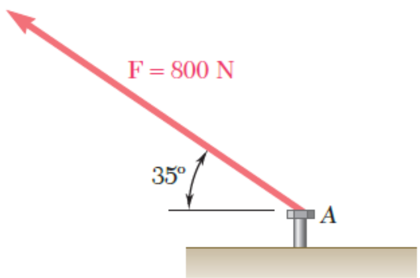
i, j, and k are the unit vectors for the x, y, z directions

Coplanar Force Resultant

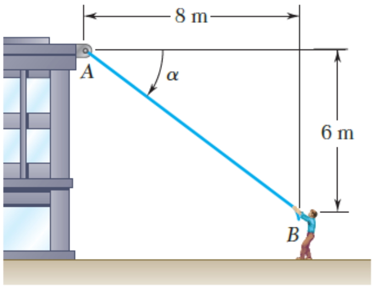
We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e.



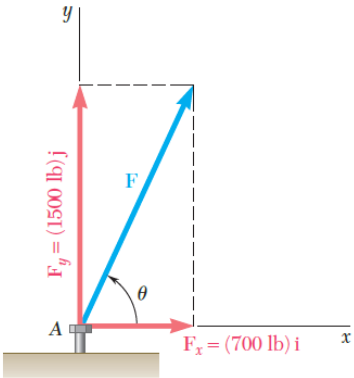
Example 6: A force of 800 N is exerted on a bolt A as shown in Fig. Determine the horizontal and vertical components of the force.



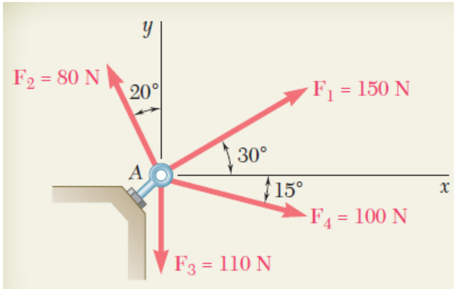
Example 7: A man pulls with a force of 300 N on a rope attached to a building, as shown in Fig. What are the horizontal and vertical components of the force exerted by the rope at point A?



Example 8: A force F = (700 lb) **i** + (1500 lb) **j** is applied to a bolt A. Determine the magnitude of the force and the angle θ it forms with the horizontal.



Example 9: Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.

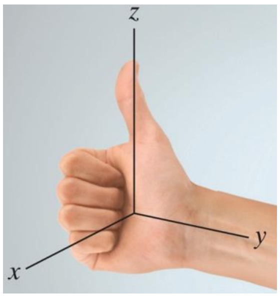


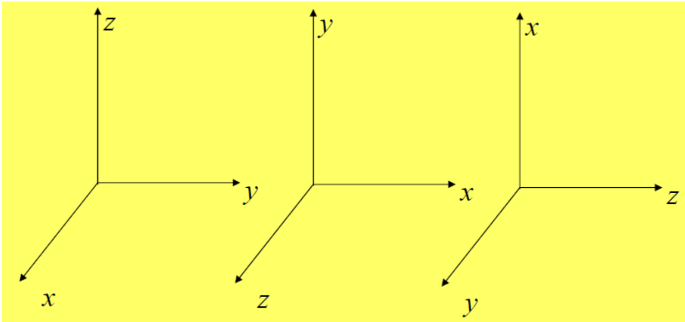
9. CARTESIAN VECTORS

The operations of vector algebra, when applied to solving problems in ***three dimensions***, are greatly simplified if the vectors are first represented in Cartesian vector form.

• *Cartesian system of coordinates (x, y, z)*

Right Handed Coordinate System

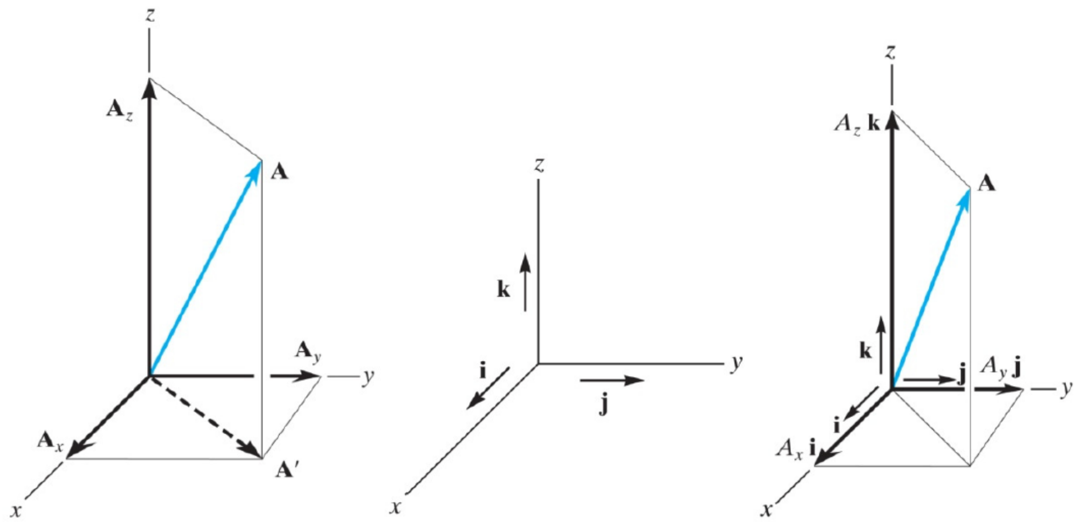




Several combinations of the right handed Cartesian coordinate system.

• *Cartesian Vector Representation*

A vector can have up to three components, one for each axis. In three dimensions, the set of Cartesian unit vectors, **i**, **j**, **k**, is used to designate the directions of the *x*, *y*, *z* axes, respectively.

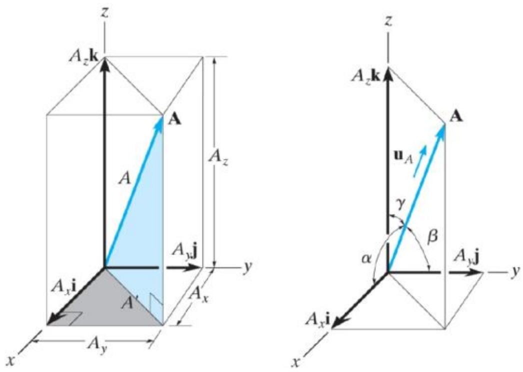


$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}}$$

• *Magnitude and Direction of a Cartesian Vector*

Magnitude: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

The direction of a vector is defined by the ***coordinate direction angles*** measured between the tail of the vector and the positive *x*, *y*, *z* axes provided they are located at the tail of the vector.



$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

α, β, γ are between 0˚ and 180˚

• *Unit Vectors*

An easy way of obtaining these direction cosines is to form a ***unit vector*** $\mathbf{u}_A$ in the direction of **A**.

$\mathbf{u}_A$ has a magnitude of one and the direction of **A**.

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k} \qquad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Here we can see that if only two of the coordinate angles are known, the third angle can be found using this equation.

One the other hand, if the magnitude and coordinate direction angles of **A** are known, then **A** may be expressed as

$$\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}$$

•*Additional Unit Vector Relations*

The addition (or subtraction) of two vectors that are expressed in terms of Cartesian coordinates is achieved by adding (or subtracting) term by term the corresponding components of each vector.

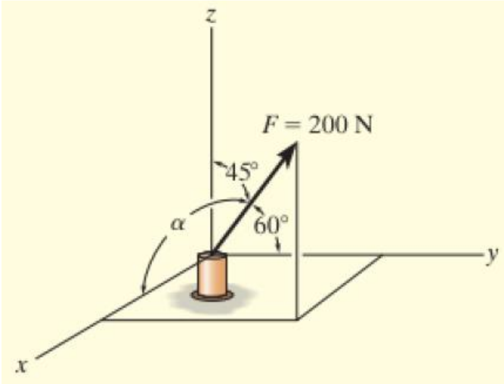
$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$$A = A_x i + A_y j + A_z k \qquad B = B_x i + B_y j + B_z k$$

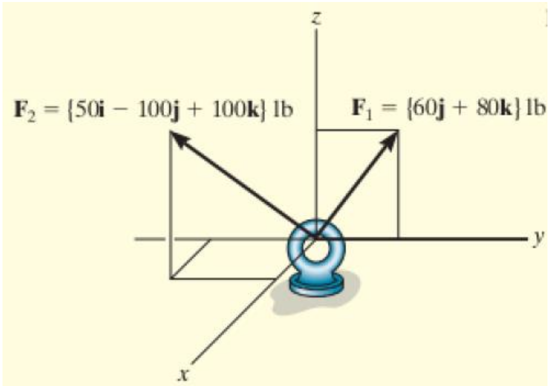
$$R = A + B = (A_x + B_x)i + (A_y + B_y)j + (A_z + B_z)k$$

$$R' = A - B = (A_x - B_x)i + (A_y - B_y)j + (A_z - B_z)k$$

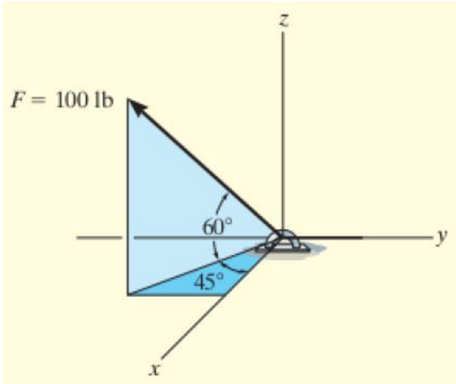
Example 10: Express the force F shown in Figure as a Cartesian vector.



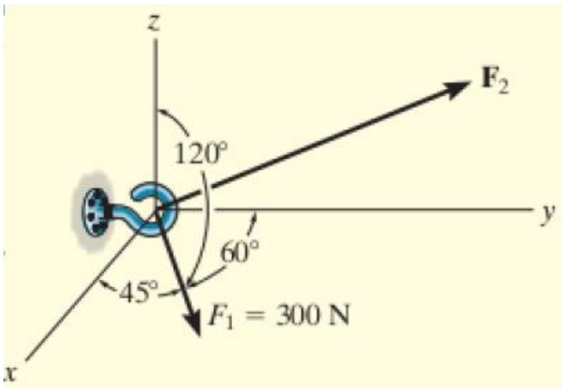
Example 11: Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring in Fig.



Example 12: Express the force F shown in Figure as a Cartesian vector.



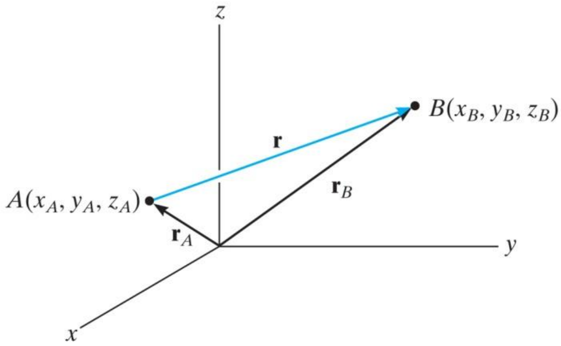
Example 13: Two forces act on the hook shown in Fig. Specify the magnitude of F_2 and its coordinate direction angles of F_2 that the resultant force F_R acts along the positive y-axis and has a magnitude of 800 N.



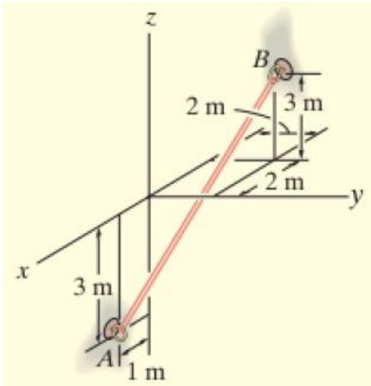
10. POSITION VECTORS

A ***Position Vector*** is a vector that locates a point in space with respect to another point.

$$\boxed{\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}$$

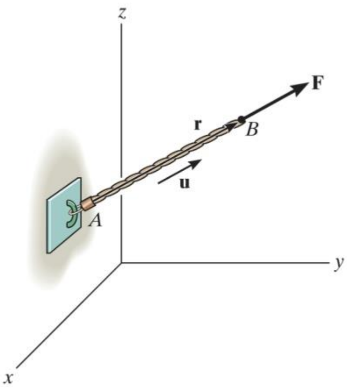


Example 14: An elastic rubber band is attached to points A and B as shown in Fig. Determine its length and its direction measured from A toward B.



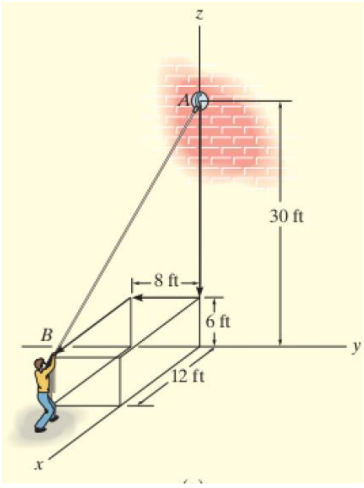
11. FORCE VECTOR DIRECTED ALONG A LINE

When the direction of a force is specified by two points through which its line of action passes, the vector force can be described by multiplying the unit vector of the position vector of these two points times the magnitude of the force exerted.

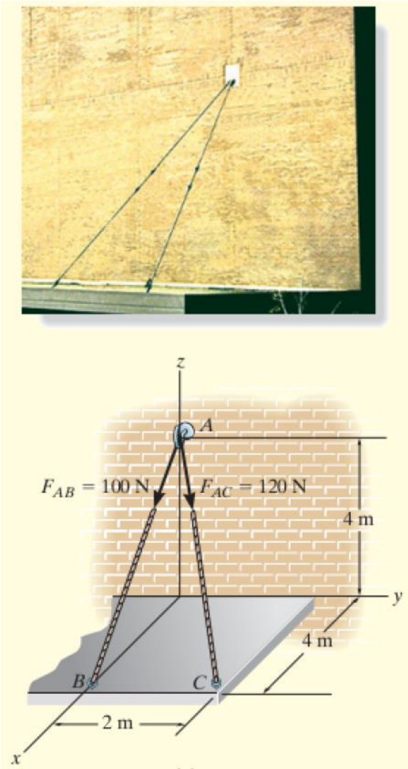


$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

Example 15: The man shown in Figure pulls on the cord with a force of 70 lb. Represent this force acting on the support A as a Cartesian vector and determine its direction.

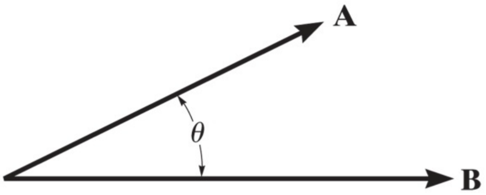


Example 16: The roof is supported by cables as shown in the photo. If the cables exert forces F_{AB} = 100 N and F_{Ac} = 120 N on the wall hook at *A* as shown in Figure, determine the resultant force acting at A. Express the result as a Cartesian vector.



12. DOT PRODUCT

When we need to determine the angle between two 3-D vectors, ***Dot Product*** (often referred as scalar product) is the most adequate way of approaching the problem.



$$\boxed{\mathbf{A}\cdot\mathbf{B} = AB\cos\theta} \quad 0^\circ \le \theta \le 180^\circ$$

Laws of Operation

1. Commutative law: $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
2. Multiplication by a scalar: $a(\mathbf{A}\cdot\mathbf{B}) = (a\mathbf{A})\cdot\mathbf{B} = \mathbf{A}\cdot(a\mathbf{B})$
3. Distributive law: $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$

Cartesian Vector Formulation

$$\mathbf{i}\cdot\mathbf{i} = (1)(1)\cos 0^\circ = 1 \qquad \mathbf{i}\cdot\mathbf{j} = (1)(1)\cos 90^\circ = 0$$

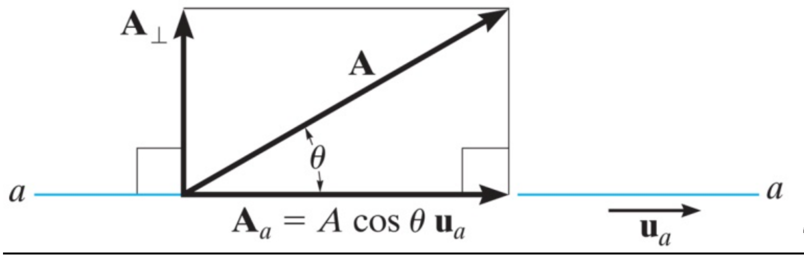
$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$

Applications

Application 1: Finding the angle formed between two vectors or intersecting lines

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) \quad 0^\circ \le \theta \le 180^\circ$$

Application 2: Finding the components of a vector parallel and perpendicular to a line



$$A_a = A\cos\theta = \mathbf{A}\cdot\mathbf{u}_a$$

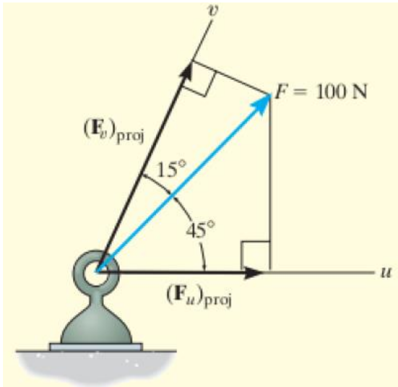
The scalar projection of **A** along a line is determined from the dot product of **A** and the unit vector $\mathbf{u}_a$ which defines the direction of the line.

For the component of **A** that is perpendicular to the line,

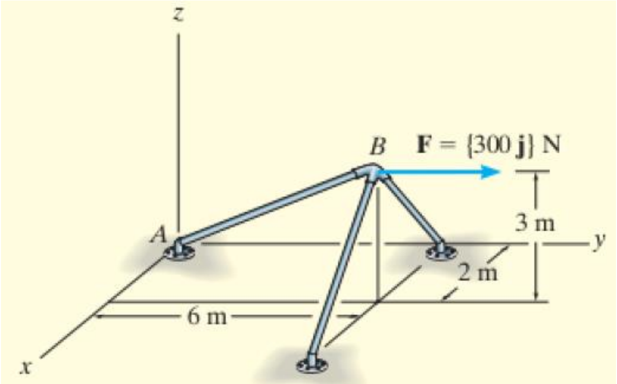
$$\mathbf{A} = \mathbf{A}_a + \mathbf{A}_\perp \Rightarrow \mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_a$$

$$A_\perp = A\sin\theta \qquad A_\perp = \sqrt{A^2 - A_a{}^2}$$

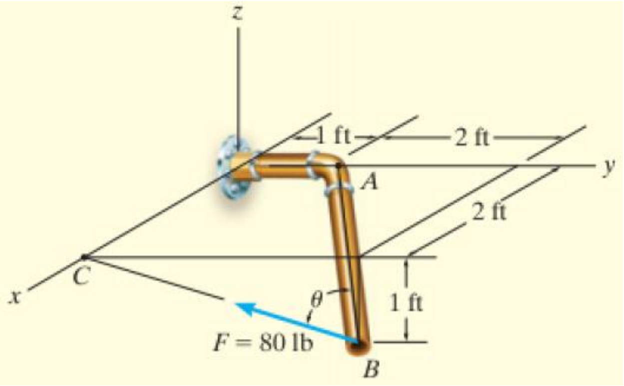
Example 17: Determine the magnitudes of the projection of the force F in Figure onto the *u* and *v* axes.



Example 18: The frame shown in Figure is subjected to a horizontal force F= 300j. Determine the magnitude of the components of this force parallel and perpendicular to member AB.

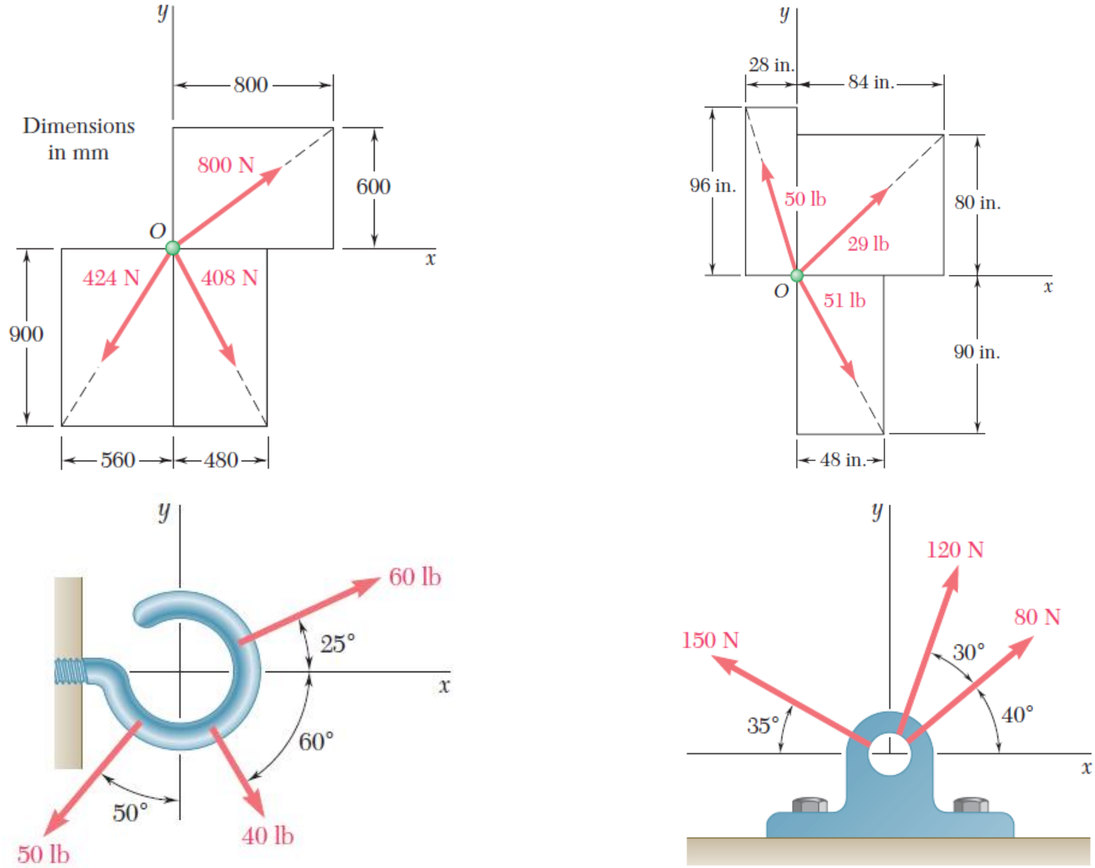


Example 19: The pipe in Figure is subjected to the force of *F -80* lb. Determine the angle θ between F and the pipe segment *BA* and the projection of F along this segment.

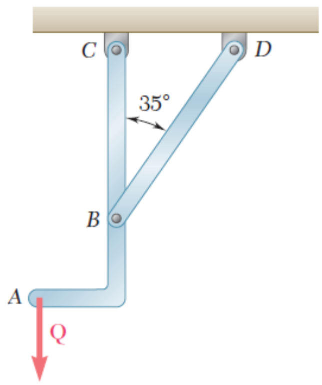


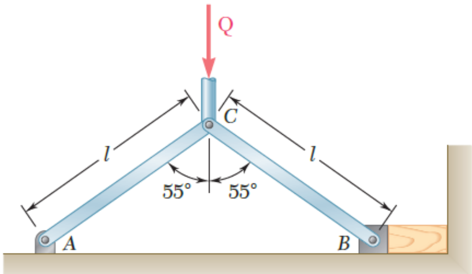
Sheet No. 1

Q 1: Determine the x and y components of each of the forces shown.



Q 2: Member BD exerts on member ABC a force P directed along line BD. Knowing that P must have a 300-lb horizontal component, determine (a) the magnitude of the force P, (b) its vertical component.

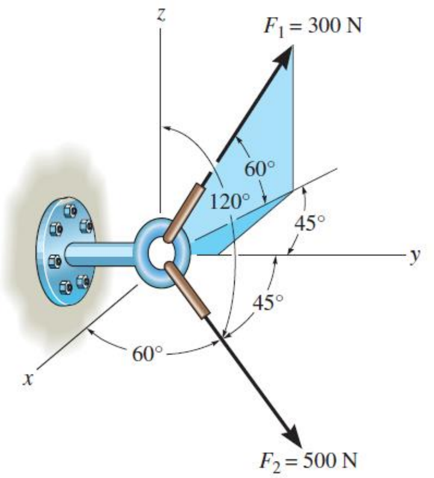


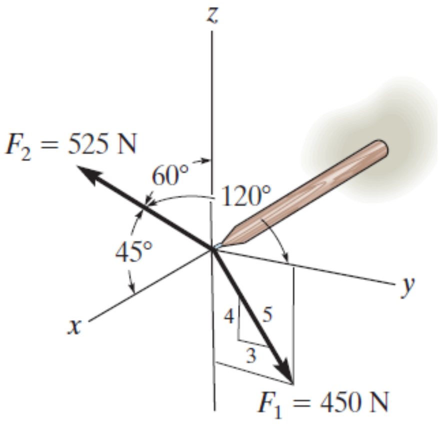


Q 3: Member CB of the vise shown exerts on block B a force P directed along line CB. Knowing that P must have a 1200-N horizontal component, determine (a) the magnitude of the force P, (b) its vertical component.

Q 4: Determine the resultant of the three forces of **Q 1.**

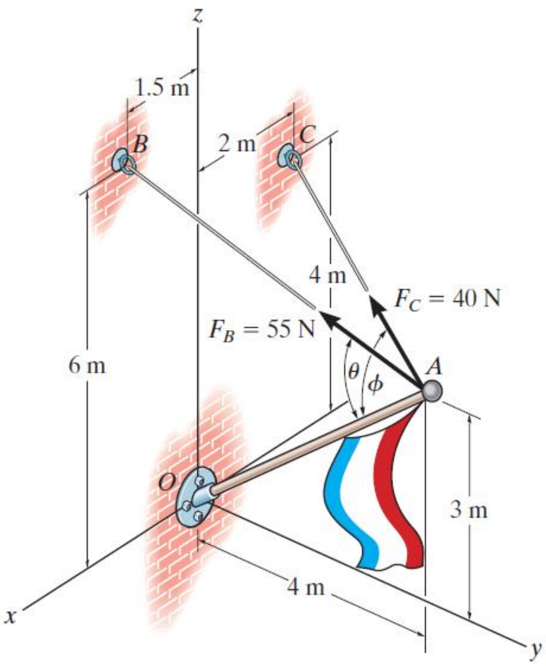
Q 5: The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.





Q 6: Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.

Q 7: Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



CHAPTER 3 EQUILIBRIUM OF A PARTICLE

13. CONDITION FOR THE EQUILIBRIUM OF A PARTICLE

The term ***"equilibrium"*** or ***"static equilibrium"*** is used to describe an object at rest. To maintain equilibrium, it is necessary to satisfy Newton's first law of motion, which requires the *resultant of all the forces acting on the particle to be equal to zero*. Mathematically,

$$\Sigma \mathbf{F} = \mathbf{0}$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

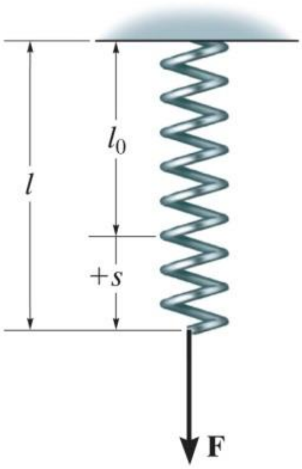
14. FREE BODY DIAGRAM

A ***free–body diagram*** *(FBD)* is a sketch that shows all the forces acting on a particle. This diagram is simply a drawing of the particle **isolated** from its surroundings with all the forces that are acting on it.

Before presenting a formal procedure on how to draw a FBD, we firstly consider two types of connections.

Springs and pulleys are two elements that are frequently used as connections.

Springs are usually employed to support loads or restraint the motion of bodies. A property that defines the "elasticity" of a spring is the *spring constant or stiffness k*. In a large number of springs the displacement and the force applied to the element have a linear relation:



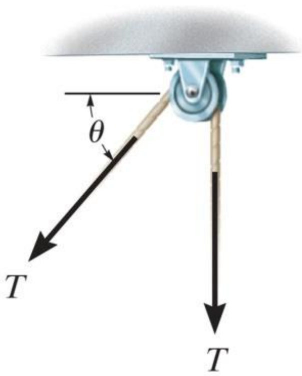
$$\boxed{F = ks}$$

$$s = l - l_0$$

Cables and *Pulleys* are elements used to transmit force or power. In general, they are considered weightless. It is also commonly assumed that cables and ropes cannot stretch.

A cable can support only tension and this force acts *always* in the direction of the element.

Cable supported by frictionless pulley subjected to tension T.

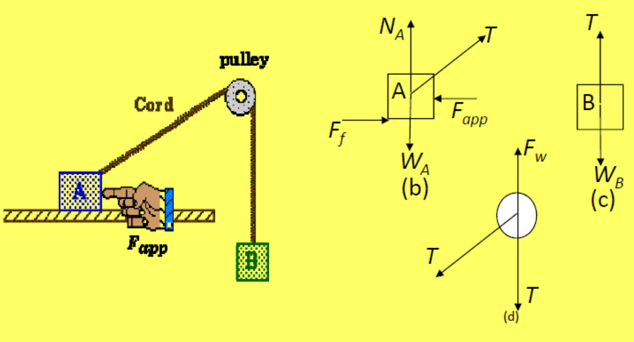


Procedures for drawing a FBD:

Step 1: Draw outlined shape. Imaging the particle to be isolated or cut "free" from its surroundings.

Step 2: Show all forces that act on the particle.

Step 3: Identify each force. Label the forces that are known.



Example of free body diagram.

15. COPLANAR FORCE SYSTEMS

When a particle is subjected to a system of coplanar forces, each force can be projected onto the x and y axes of the plane. Thus, if the forces acting on the particle are both coplanar and in equilibrium then:

$$\sum \vec{F} = 0$$

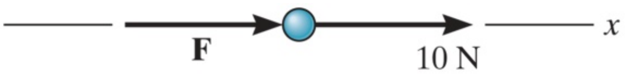
Becomes

$$\sum F_x \hat{i} + \sum F_y \hat{j} = 0$$

That is:

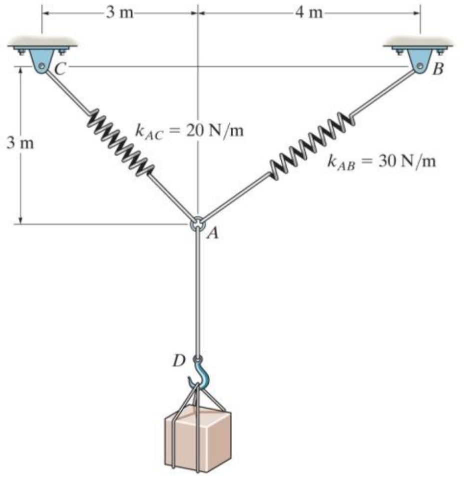
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

When applying each of the two equations of equilibrium, we must account for the sense of direction of any component by using an algebraic sign which corresponds to the arrowhead direction of the component along the x or y axis. It is important to note that if a force is unknown, then the arrowhead sense of the force on the FBD can be *assumed*. Then if the solution yields a negative scalar, this indicates that the sense of the force is opposite to that which was assumed.

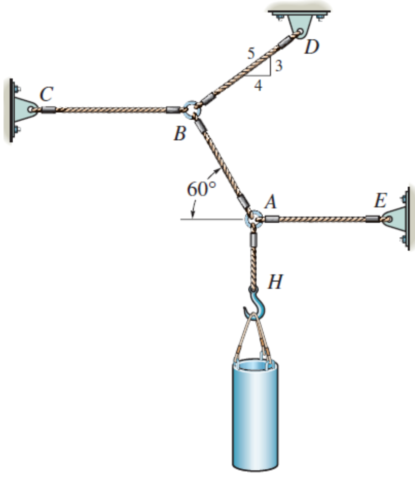


$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad +F + 10\text{ N} = 0$$

Example 1: The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



Example 2: The 30-kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium.


D
5
3
4
C
B
60°
A
E
H

16. THREE-DIMENSIONAL FORCE SYSTEMS

The expression that must be satisfied to ensure the equilibrium of a particle is

$$\sum \vec{F} = 0$$

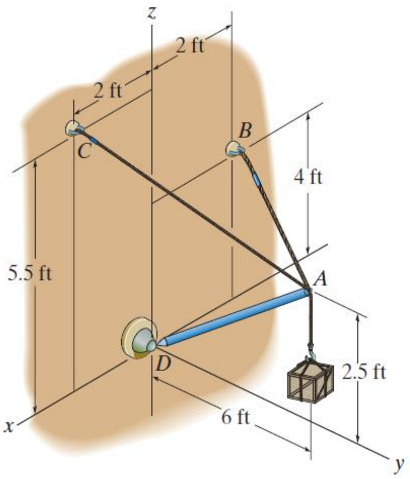
Extending this expression to three dimensions results into:

$$\sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k} = 0$$

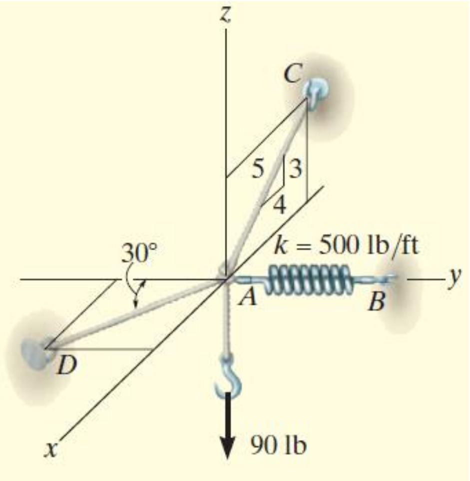
This expression can also be presented as:

$$\Sigma \mathbf{F} = \mathbf{0} \Rightarrow \boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_y &= 0 \end{aligned}}$$

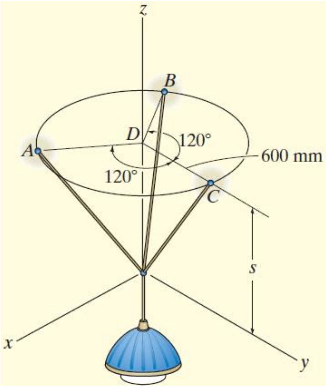
Example 3: If the tension developed in each of the cables cannot exceed 300 lb, determine the largest weight of the crate that can be supported. Also, what is the force developed along strut AD?



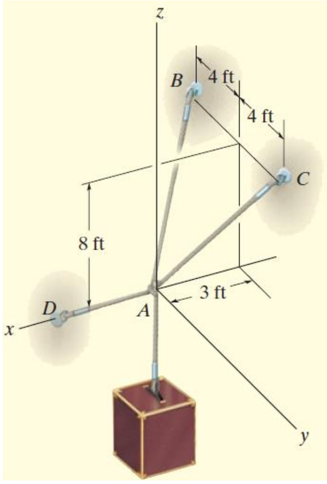
Example 4: A 90-lb load is suspended from the hook shown in Fig. If the load is supported by two cables and a spring having a stiffness k = 500 lb/ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the *x*–*y* plane and cable AC lies in the *x*–*z* plane.



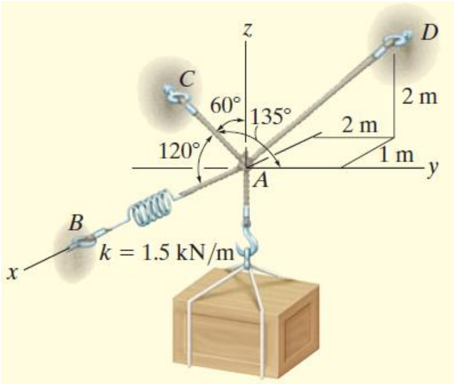
Example 5: The 10-kg lamp in Figure is suspended from the three equal-length cords. Determine its smallest vertical distance s from the ceiling if the force developed in any cord is not allowed to exceed 50 N.



Example 6: Determine the force in each cable used to support the 40-lb crate shown in Fig.

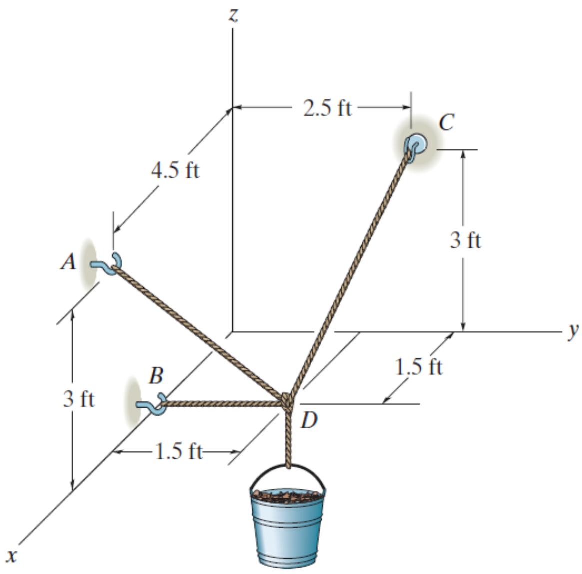


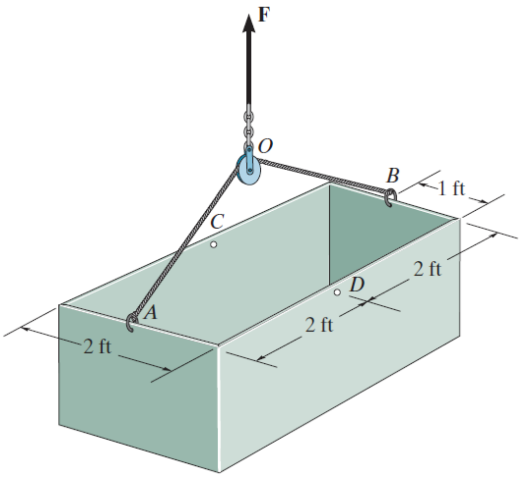
Example 7: Determine the tension in each cord used to support the 100-kg crate shown in Fig.



Sheet No. 1

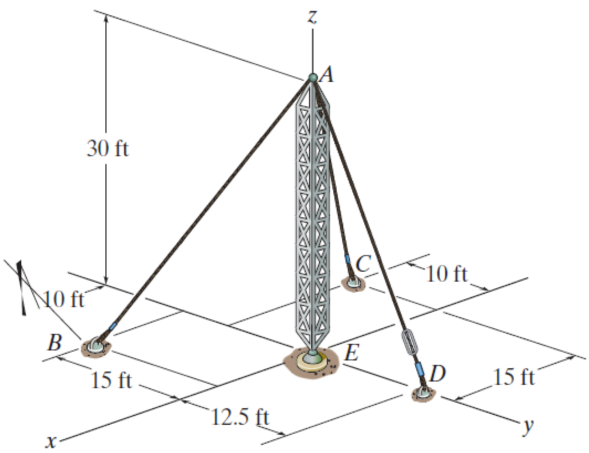
Q 1: If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables DA, DB, and DC.





Q 2: The 200-lb uniform tank is suspended by means of a 6-ftlong cable, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cable. What is this tension?

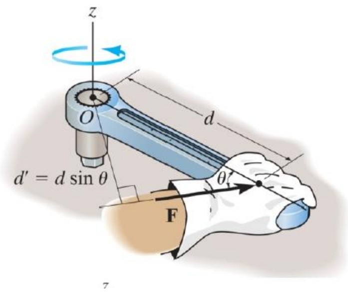
Q 3: If cable AD is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables AB and AC and the force developed along the antenna tower AE at point A.



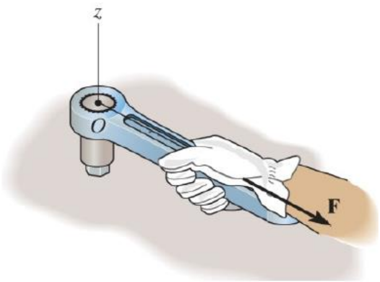
CHAPTER 4 FORCE SYSTEM RESULTANTS

17. MOMENT OF A FORCE – SCALAR FORMULATION

The ***moment*** of a force about a point or axis provides a measure of the tendency of the force to produce rotation on a body about the point or axis.



In the figure, **F** acts on the wrench and is located at a distance *d* from the point *O*. **F** tends to turn the bolt about point *O*. The perpendicular distance from the axis at point *O* to the line of action of the force is called ***moment arm***. The larger the force or the longer the moment arm, the greater the moment or turning effect. This tendency of rotation caused by **F** is called *torque*, *moment of a force* or *moment*. The *moment axis* (*z*) is perpendicular to the plane formed by the vector of the force and the vector of the distance *d*.

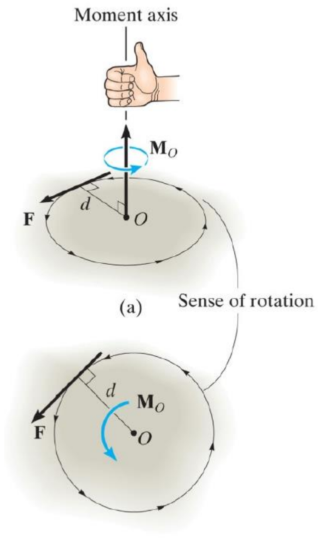


If a force **F** is applied to the wrench, as shown left, no moment is produced about point *O*. This is because the line of action of the force passes exactly through *z*–axis and therefore the distance (moment arm) that creates the moment is zero.

Magnitude and direction of a moment

The moment of a force has both magnitude and direction and thus it is also a *vector quantity*. The magnitude of the moment of a force *F* that has a *moment arm* of *d* units is

$$\boxed{M_O = Fd}$$



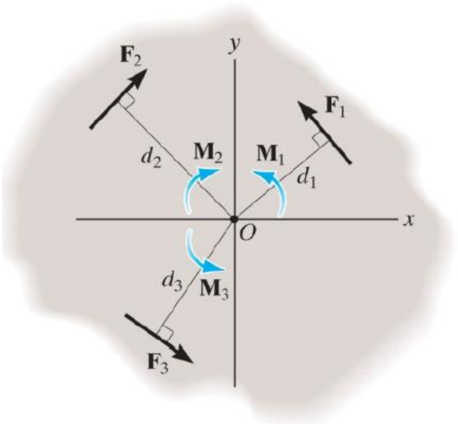
(a)
(b)

The *moment arm* is the perpendicular distance from the axis at point *O* to the line of action of the force.

The units for the moment are N×m in the SI system and lb×ft in the English system.

The direction of the moment **M***O* is defined by its moment axis and specified using the right hand rule, in which the curling direction of the fingers of the right hand, as they are drawn towards the palm, represents the tendency for rotation caused by the moment while the thumb will give the direction of the moment.

Resultant Moment for two-dimensional problems

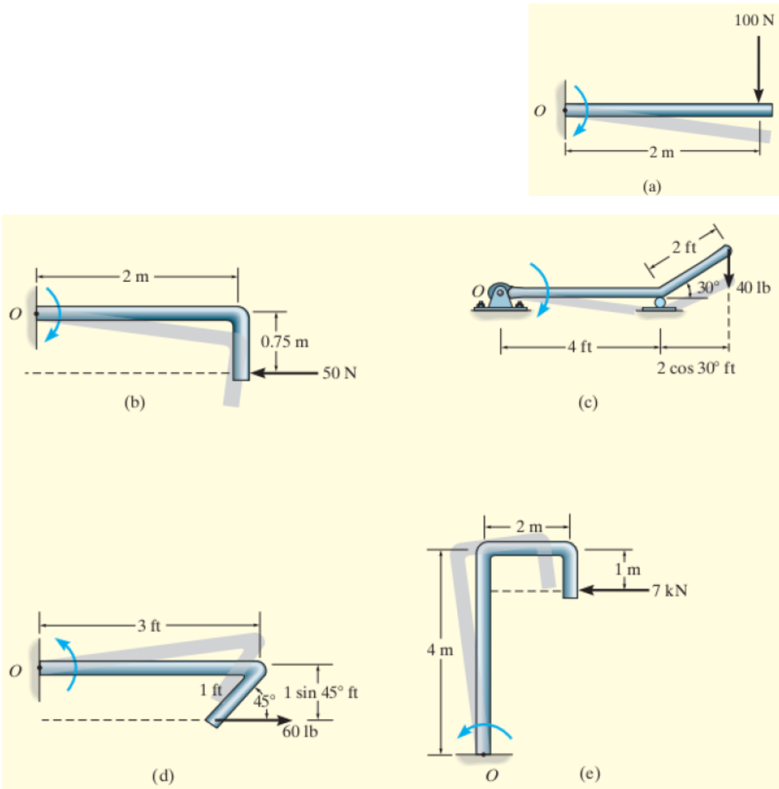


$$\curvearrowleft +(M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1d_1 - F_2d_2 + F_3d_3$$

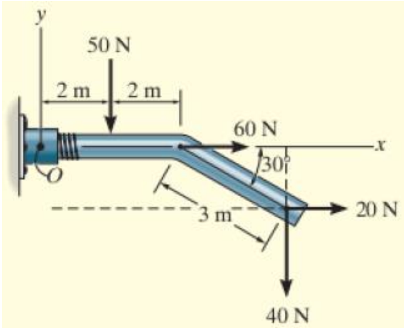
If a system of forces lies in the plane *x–y*, then, the moment produced by each force about point *O* will be directed along axis *z*. The resultant moment $\mathbf{M}_{RO}$ of the system can be determined by simply adding the moments of all the forces *algebraically* since all the moment vectors are collinear.

The counterclockwise curl shown at the left of the equation expresses that by the scalar sign convention, the moment of any force will be *positive* if it is directed along the +*z* axis (counterclockwise), and *negative* if it is directed along the –*z* axis (clockwise).

Example 1: For each case illustrated in Figure, determine the moment of the force about point 0.



Example 2: Determine the resultant moment of the four forces acting on the rod shown in Figure about point 0.

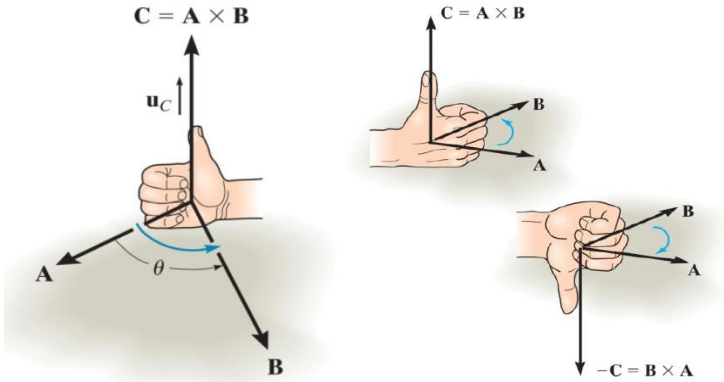


18. CROSS PRODUCT

The *cross product* of two vectors **A** and **B** yields a third vector **C** that is perpendicular to **A** and **B**, which is written as

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

Vector **C** has a *direction* that is perpendicular to the plane containing **A** and **B** such that **C** is specified by the right hand rule.



The *magnitude* of **C** is defined as the product of the magnitudes of **A** and **B** and the sine of the angle θ between their tails ($0^o \leq \theta \leq 180^o$):

$$C = AB \sin \theta$$

Knowing both the magnitude and direction of **C**, we can write

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin \theta)\mathbf{u}_C$$

Where the scalar $AB\sin\theta$ defines the magnitude of **C** and the unit vector $\mathbf{u}_C$ defines the direction of **C**.

Laws of Operation

The commutative law is not valid; i.e., $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$.Rather, $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$

Multiplication by a scalar: $a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$

Distributive law: $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$

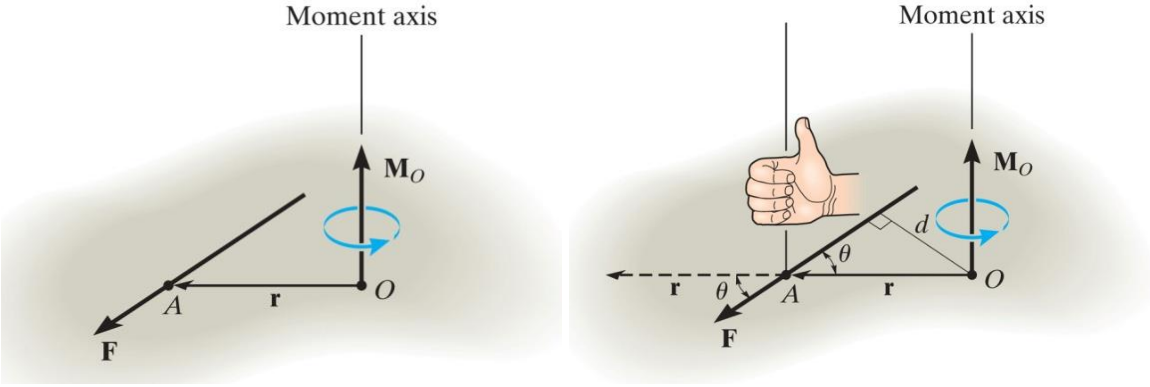
Cartesian Vector Formulation

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

19. MOMENT OF A FORCE – VECTOR FORMULATION

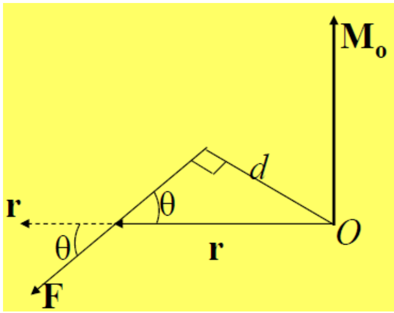


$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Where **r** represents **a position vector going from *O* to *any point* lying on the line of action of F.**

The cross product operation is often used in three dimensions since the perpendicular distance or moment arm from point *O* to the line of action of the force is not needed.

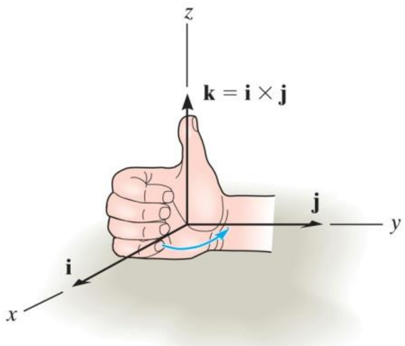
Magnitude and direction of a moment



The magnitude of the moment Mo is given by the definition of the cross product of two vectors

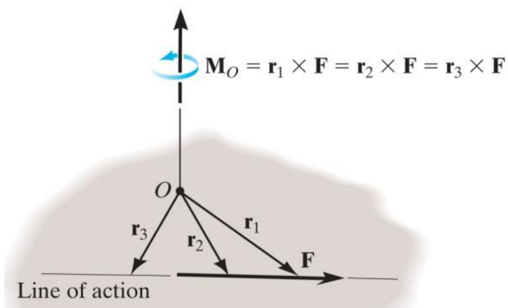
$$M_O = rF \sin\theta = F(r \sin\theta) = Fd$$

θ is measured between the tails of the two vectors.



The direction of **M***o* are determined by the right hand rule. The direction is perpendicular to the plane containing **r** and **F**. The curl of the fingers indicates the sense of rotation caused by the force.

Principle of Transmissibility



We can use any position vector **r** measured from point *O* to **any point** on the line of action of the force **F**. Since **F** can be applied at any point along its line of action and still create this same moment about point *O*, F can be considered a ***sliding vector***. This property is called the ***principle of transmissibility*** of a force.

Cartesian Vector Formulation

If a force and a vector are expressed in Cartesian coordinates the moment vector produced by the force can be found as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

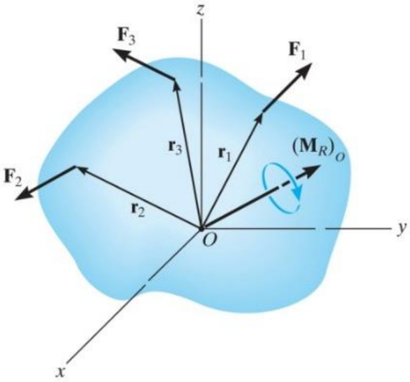
r_x, r_y, and r_z are the components of the position vector drawn from the point O to any point on *the line of action of the force* $\mathbf{F}$ whose components are F_x, F_y, and F_z. Expansion of the determinant results in

$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$

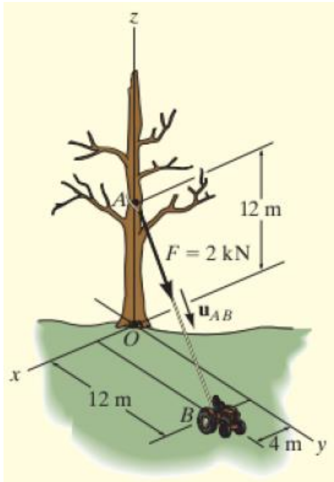
Resultant Moment of a System of Forces

If a system of forces is acting on a body as shown, the resultant moment of the forces about the point O can be determined by vector addition of the moment produced by each force.

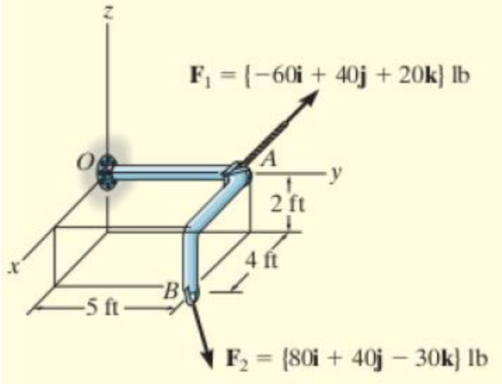
$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$



Example 3: Determine the moment produced by the force F in Fig. about point 0. Express the result as a Cartesian vector.



Example 4: Two forces act on the rod shown in Fig. Determine the resultant moment they create about the flange at 0. Express the result as a Cartesian vector.



20. PRINCIPLE OF MOMENTS

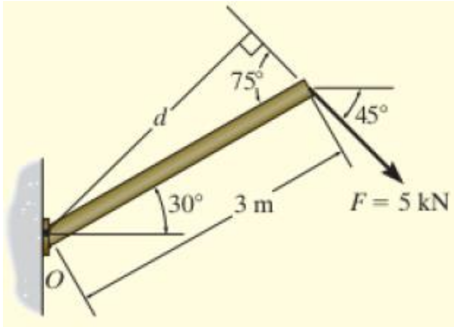
The *principle of moments* often named as *Varignon's theorem* (developed by the French mathematician Varignon (1654–1722)) states that:

The moment of a force about a point is equal to the sum of the moments of the force's components about the point.

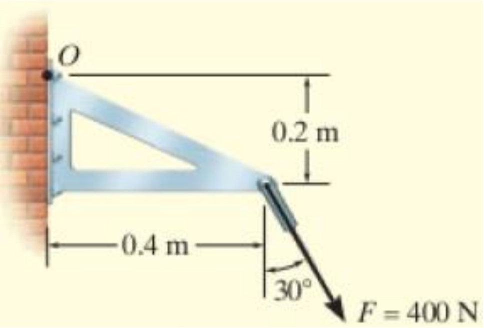
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

For two-dimensional problems, we often use this principle of moments by resolving the force into its rectangular components and then determine the moment using the scalar analysis. The method is generally easier than finding the same moment by directly finding the moment arm *d*.

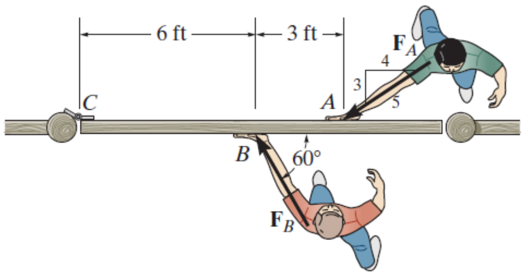
Example 5: Determine the moment of the force in Figure about point *0*.



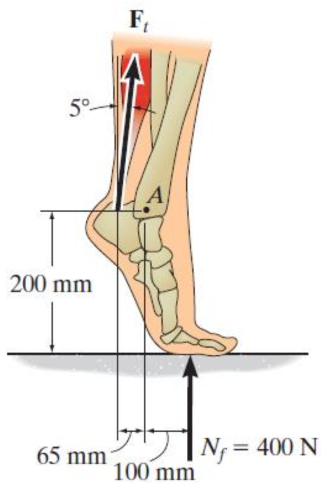
Example 6: Force F acts at the end of the angle bracket shown in Fig. Determine the moment of the force about point 0.



Example 7: Two boys push on the gate as shown. If the boy at B exerts a force of F_B = 30 Ib, determine the magnitude of the force the boy at A must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

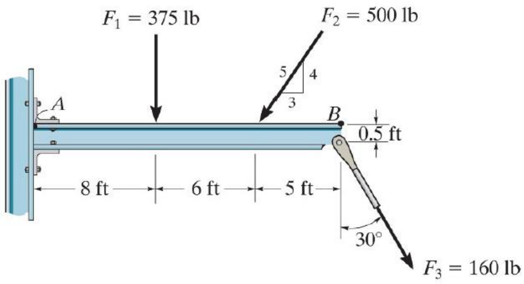


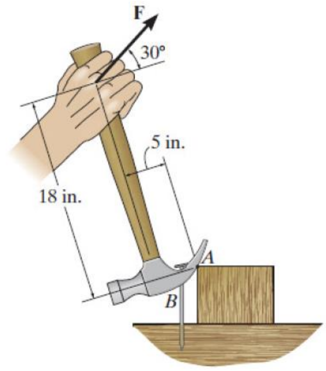
Example 8: The Achilles tendon force of F_t = 650 N is mobilized when the man tries to stand on his toes. As this is done, each of his feet is subjected to a reactive force of N_f = 400 N. Determine the resultant moment of F_t and N_f about the ankle joint A.



Sheet No. 1

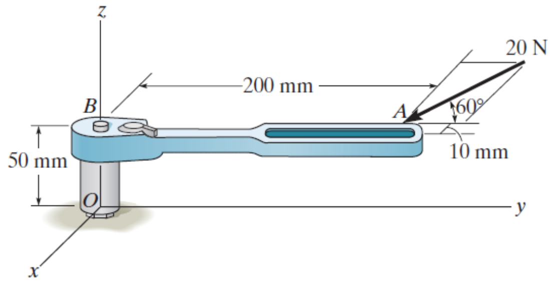
Q 1: Determine the moment about point A of each of the three forces acting on the beam.

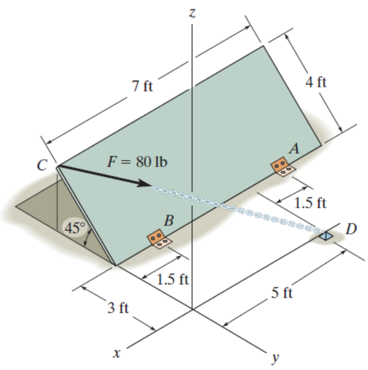




Q 2: In order to pull out the nail at B, the force F exerted on the handle of the hammer must produce a clockwise moment of 500 lb.in about point A. Determine the required magnitude of force F.

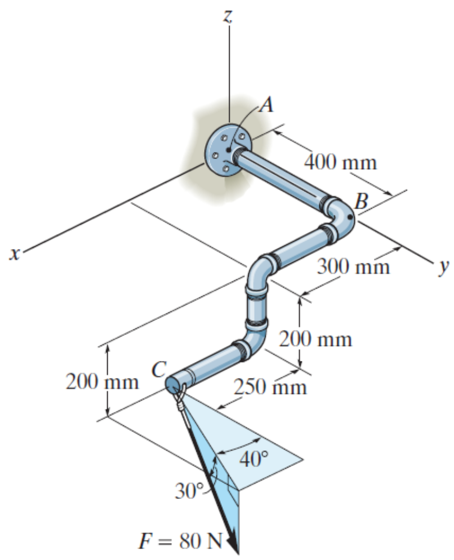
Q 3: The 20-N horizontal force acts on the handle of the socket wrench. What is the moment of this force about point B. Specify the coordinate direction angles a, b, g of the moment axis.





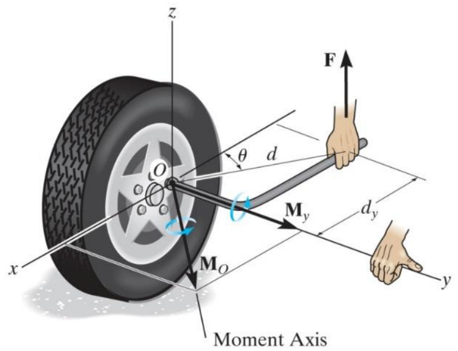
Q 4: Determine the moment of the force F about the door hinge at A. Express the result as a Cartesian vector.

Q 5: The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.



21. MOMENT OF A FORCE ABOUT A SPECIFIED AXIS

Sometimes, the moment produced by a force about a specified axis must be determined, such as shown in the left figure. To determine the turning effect, only the *y* component of moment **M***o* is needed, and the total moment produced is not important.



To determine the component of this moment along a specified axis that passes through the point, we can use two approaches: Scalar analysis and Vector analysis.

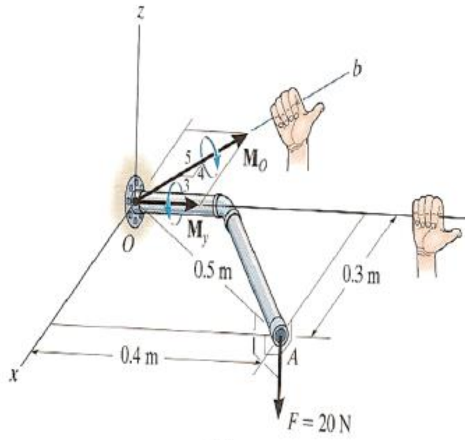
Scalar Analysis

In general, for any axis *a*, the moment of a force *F* about the axis *a* is

$$\boxed{M_a = Fd_a}$$

Where d_a is the **perpendicular distance** between the axis *a* and the line of action of the force. In the above example, the moment of **F** about the *y* axis is $M_y = F\,d_y = F\,(d \cos \theta)$. If the specified axis is along Cartesian axis, this method is convenient because the perpendicular distance is easy to find. Note that a force that ***parallel*** *to* a coordinate axis or has a line of action that ***passes through*** the axis ***does not produce any moment*** about the axis.

The force of 20 N that is applied to the pipe on the *x*–*y* plane produces a total moment M_o.



The force is directed in the negative *z*–direction and it is applied at a location of 0.3 m in the *x*–direction and 0.4 m the *y*–direction. This point of application produces a double effect.

1) Due to the location in the *x*–direction the force tends to unscrew the pipe.

2) Due to the moment arm in the *y*–direction, the force generates a tendency for the pipe to rotate around the *x*–axis.

F = 20 N. Force produces a moment with two components:

1) Moment in the *y*–direction: $M_y = F\,d_x$ = (20 N) (0.3 m) = 6 N–m (+)

2) Moment in the *x*–direction: $M_x = F\,d_y$ = (20 N) (0.4 m) = 8 N–m (–)

Thus, the moment that produces tendency to unscrew the pipe is M_y

Vector Analysis

Previously we learned that the moment of **F** about a point *O* on the axis is $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$. Now we just need to project this moment onto the axis *a*, Recall using dot product to find the projected component of a vector along a line, we can figure out this projected component of $\mathbf{M}_O$ along axis *a* is $\mathbf{M}_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$. The magnitude of $\mathbf{M}_a$ is

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

r here is the position vector **from *any point* on the *a* axis to any point on the line of action of the force**.

M_a could be positive or negative. Positive means $\mathbf{M}_a$ have the same sense as $\mathbf{u}_a$, negative means $\mathbf{M}_a$ will act opposite to $\mathbf{u}_a$. Once *Ma* is determined, we can then express $\mathbf{M}_a$ as a Cartesian vector, $\mathbf{M}_a = M_a\,\mathbf{u}_a$

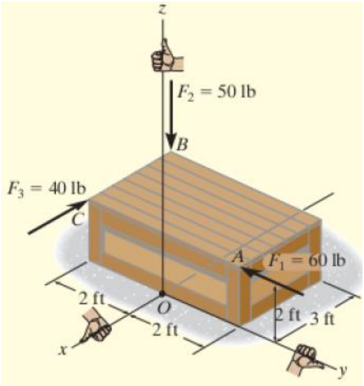
This same problem can be analyzed using vector algebra directly. First we need to determine the position vector ***r*** form the origin to point A and also notice that the force is expressed in vector form as ***F* = *-20k***.

$$r = 0.3\,i + 0.4\,j$$

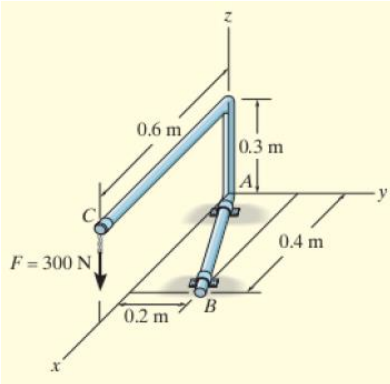
$$M = r \times F = (0.3i + 0.4j) \times (-20k)$$

$$M = 6j - 8i\ [\mathrm{N\text{-}m}]$$

Example 9: Determine the resultant moment of the three forces in Figure about the x axis, the y axis, and the z axis.

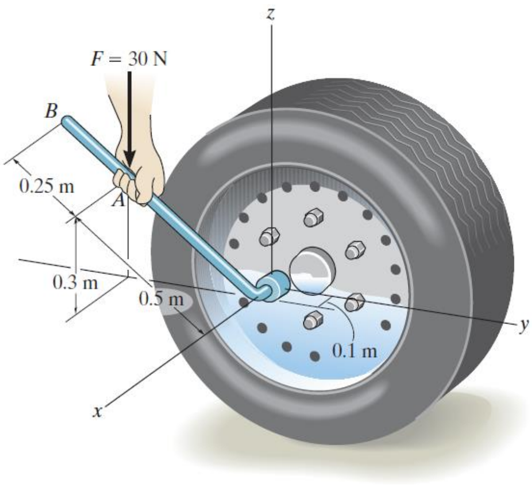


Example 10: Determine the moment NIA B produced by the force F in Figure, which tends to rotate the rod about the AB axis.

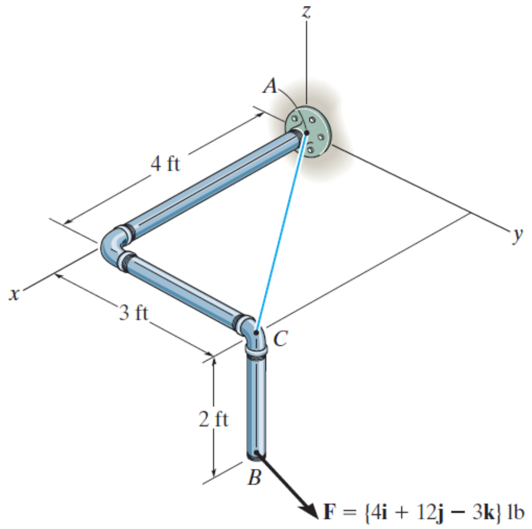


Sheet No. 2

Q 1: The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force of $F = 30\ N$ at A. Determine if this force is adequate, provided 14 N.m of torque about the x axis is initially required to turn the nut. If the 30-N force can be applied at A in any other direction, will it be possible to turn the nut?



Q 2: Solve **Q1** if the cheater pipe AB is slipped over the handle of the wrench and the 30-N force can be applied at any point and in any direction on the assembly.

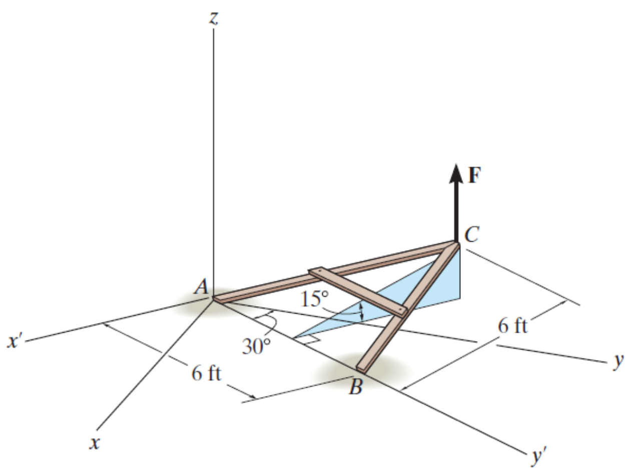


Q 3: Determine the magnitude of the moments of the force F about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach.

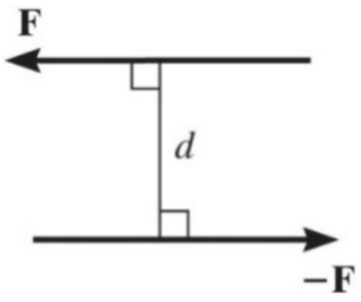
(b) Using a scalar approach.

(c) Determine the moment of the force F about an axis extending between A and C. Express the result as a Cartesian vector.

Q 4: The A-frame is being hoisted into an upright position by the vertical force of $F = 80\ Ib$. Determine the moment of this force about the y axis when the frame is in the position shown.

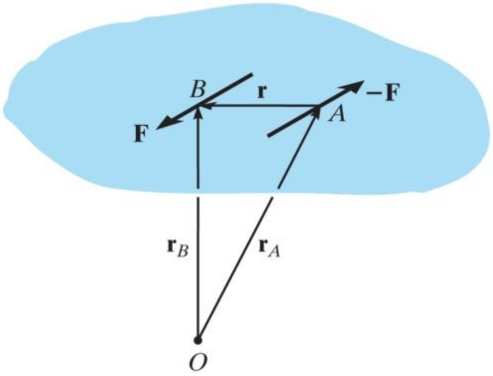


22. MOMENT OF A COUPLE



A couple is defined as ***two parallel forces*** that have the ***same magnitude***, but ***opposite directions***, and are separated by a perpendicular distance *d*.

The resultant force is zero, thus the only effect of the couple is to produce rotation or tendency of rotation in a specific direction.



The moment produced by a couple is named ***couple moment*** and its value can be determined by finding the sum of the moments of both couple forces at any arbitrary point:

$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$:

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

This result indicates that a couple moment is a ***free vector***, i.e., ***it can act at any point*** since **M** depends only upon the position vector **r** directed between the forces. It is different from the moment of force, which requires a point or axis about which moments are determined.

Moment of a couple scalar formulation:

$$\boxed{M = Fd}$$

where *d* is the perpendicular distance between the forces. The direction can be determined by the right-hand rule.

Moment of a couple vector formulation:

$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$

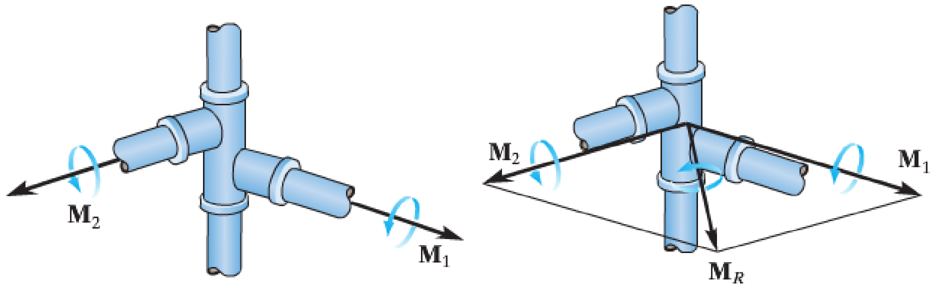
where **r** is a position vector directed from −**F** to **F**. This equation can be easily remembered if one thinks of taking the moments of both forces about a point lying on the line of action of one of the forces.

Equivalent Couples

Two couples are said to be equivalent if they produce the ***same moment***. Since the moment produced by a couple is always perpendicular to the plane containing the couple forces, it is therefore necessary that ***the forces of equal couples lie in the same plane or in parallel planes***.

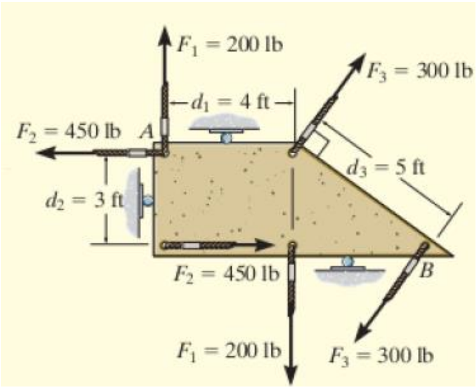
Resultant Couple Moment

Since couple moments are free vectors, they may be applied to any point *P* on a body and added vectorially.

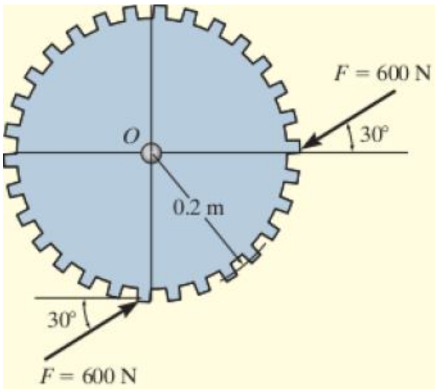


$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$

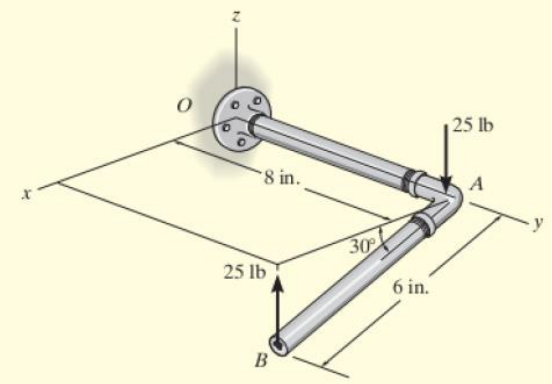
Example 11: Determine the resultant couple moment of the three couples acting on the plate in Figure.



Example 12: Determine the magnitude and direction of the couple moment acting on the gear in Figure.



Example 13: Determine the couple moment acting on the pipe shown in Fig. Segment AB is directed 30^0 below the *x*-*y* plane.



23. EQUIVALENT SYSTEM

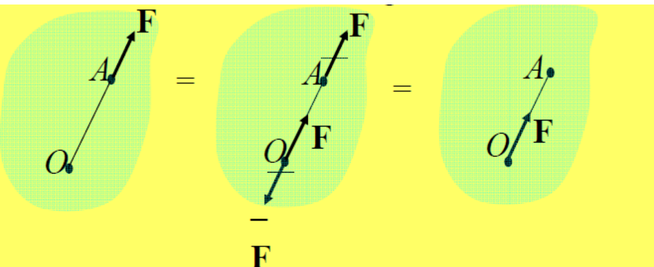
A force applied on a body has two effects, it can produce:

•Translation

•Rotation

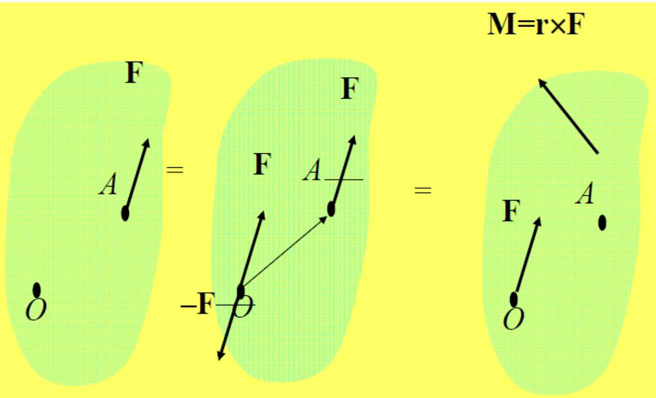
The degree of these effects depends on where and how the force is applied.

1) Line of Action of Force Passes through Point O



Reduction of a force acting on a point without *moment arm* with respect to *O*.

2) Line of Action of Force Produces a Moment about Point O.



Reduction of a force acting on a point with *moment arm* with respect to *O*.

Two systems of forces and couples are called ***equivalent systems*** if the two systems have the same resultant force and the same resultant moment about any point.

The weight of these traffic lights can be replaced by their equivalent resultant force $W_R = W_1 + W_2$ and a couple moment $(M_R)_O = W_1d_1 + W_2d_2$ at the support, O.

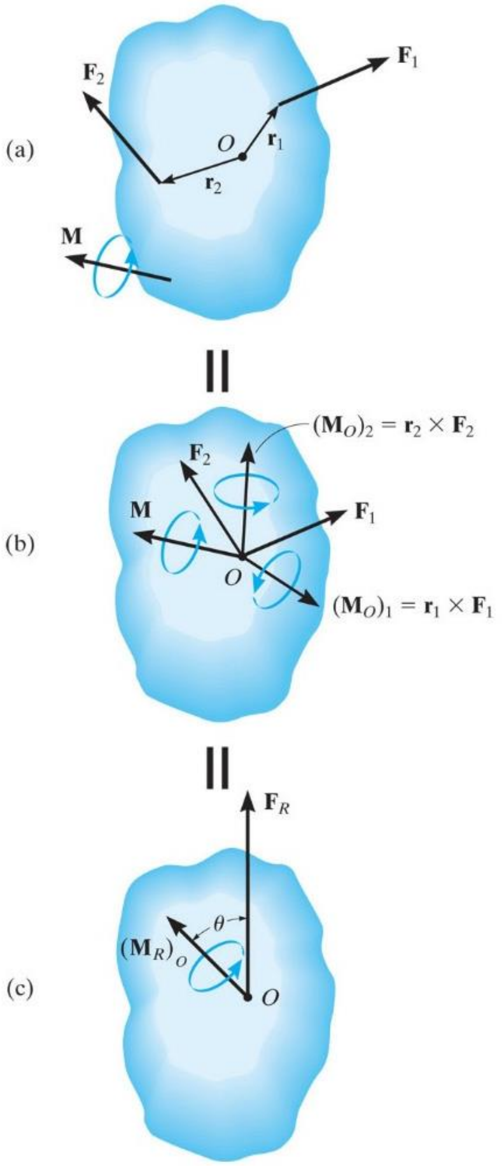
24. RESULTANT OF A FORCE AND A COUPLE SYSTEM

We can use this idea to reduce a system of forces and couples to a single force-couple system acting at some point *O*. The single force is the resultant force of the system, and the single couple moment is the sum of the moments of all the forces about *O* plus the sum of all the couple moments.

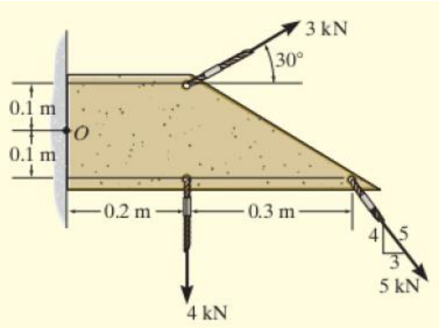
$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

For two dimensional problems,

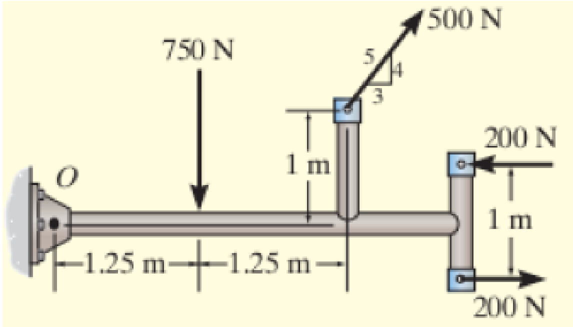
$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$



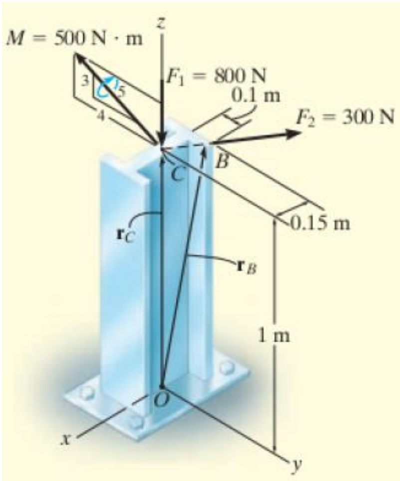
Example 14: Replace the force and couple system shown in Figure by an equivalent resultant force and couple moment acting at point 0.



Example 15: Replace the force and couple system acting on the member in Figure by an equivalent resultant force and couple moment acting at point 0.

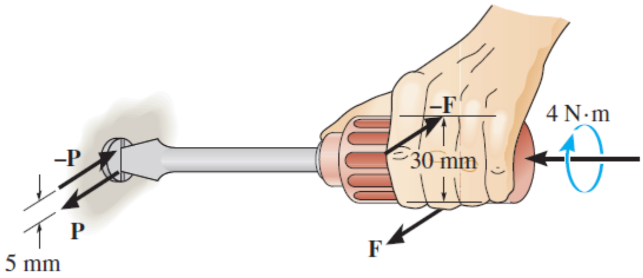


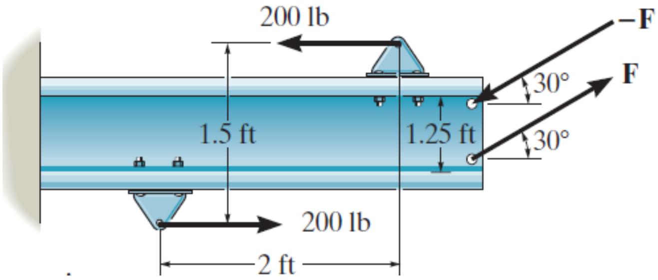
Example 16: The structural member is subjected to a couple moment M and forces F_1 and F_2 in Fig. Replace this system by an equivalent resultant force and couple moment acting at its base, point 0.



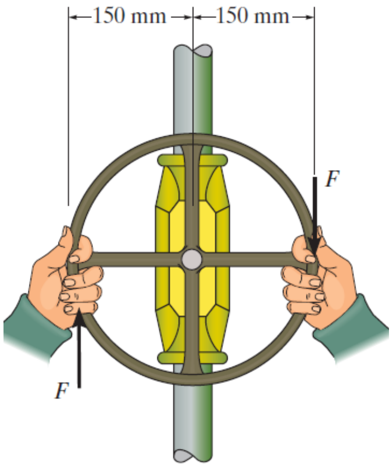
Sheet No. 3

Q 1: A twist of 4 N. m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces F exerted on the handle and P exerted on the blade.

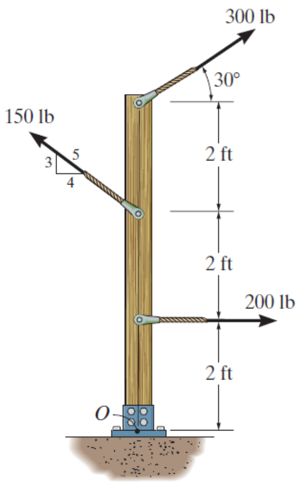




Q 2: Two couples act on the beam. If *F = 125 lb*, determine the resultant couple moment.

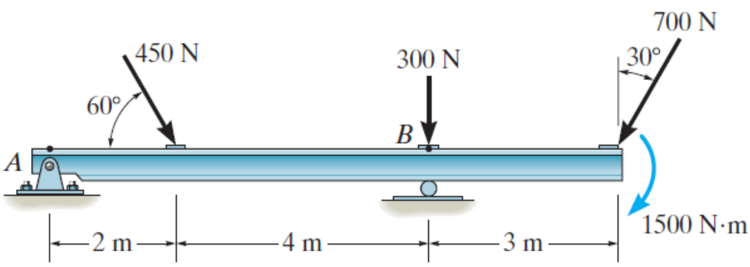


Q 3: The man tries to open the valve by applying the couple forces of F = 75 N to the wheel. Determine the couple moment produced.



Q 4: Replace the force system acting on the post by a resultant force and couple moment at point O.

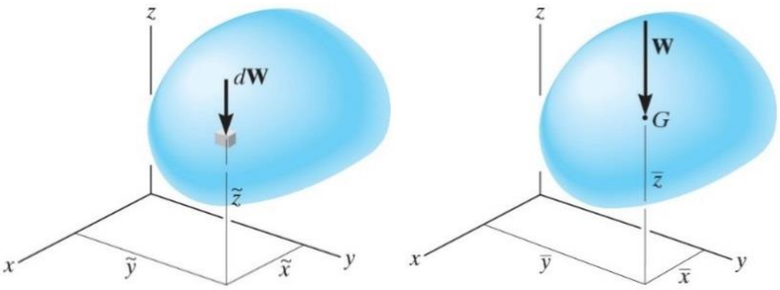
Q 5: Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from B.



CHAPTER 5 CENTER OF GRAVITY AND CENTROID

25. CENTER OF GRAVITY, MASS AND THE CENTROID OF A BODY

Center of Gravity. The center of gravity G is the point where the resultant weight of a system of particles is located.



$$+\downarrow F_R = \Sigma F_z; \qquad W = \int dW$$

$$(M_R)_y = \Sigma M_y; \qquad \bar{x}W = \int \tilde{x}\,dW$$

$$(M_R)_x = \Sigma M_x; \qquad \bar{y}W = \int \tilde{y}\,dW$$

Since the weight of the body is the sum of the weights of all of its particles, and a rigid body is assumed to be continuous, the coordinates of the ***center of gravity*** are

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW} \quad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW} \quad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

Since $dW = g\,dm$ and g is constant, the location of ***center of mass*** of a body is

$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm} \quad \bar{y} = \frac{\int \tilde{y}\,dm}{\int dm} \quad \bar{z} = \frac{\int \tilde{z}\,dm}{\int dm}$$

Since $dm = \rho\, dV$, and assume the body is made from a ***homogeneous materials (ρ is constant)*** the ***centroid*** or ***geometric center*** of the body is

$$\bar{x} = \frac{\int_V \tilde{x}\,dV}{\int_V dV} \quad \bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} \quad \bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV}$$

These equations represent a balance of the moment of the volume of the body, so the centroid of the body lies in any axis of symmetry, if they exist, of the body.

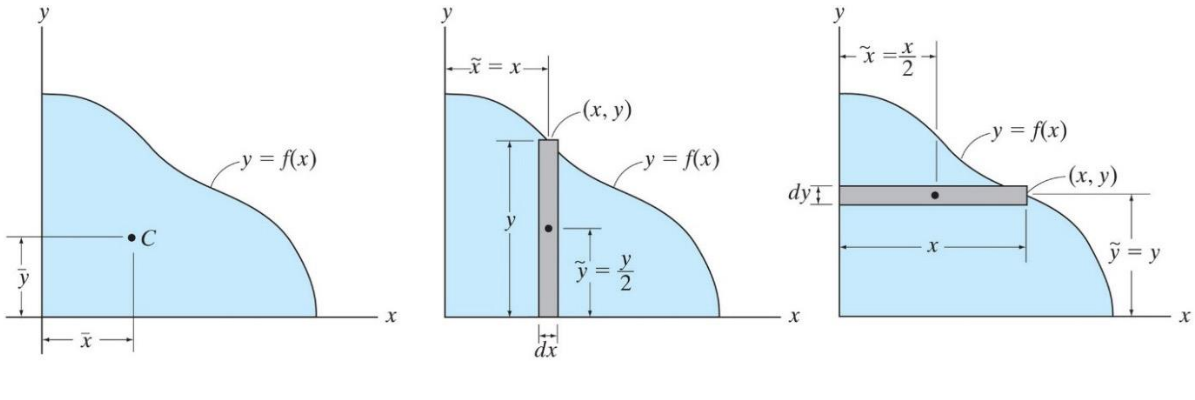
Centroid of an area

If an area lies in the x-y plane and is bounded by the curve $y = f(x)$, then its centroid will be in this plane and can be determined from

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA}$$

Same as centroid of a body, centroid of an area lies in any axis of symmetry of the area.

This integrals can be evaluated by performing a single integration if we use a rectangular strip.



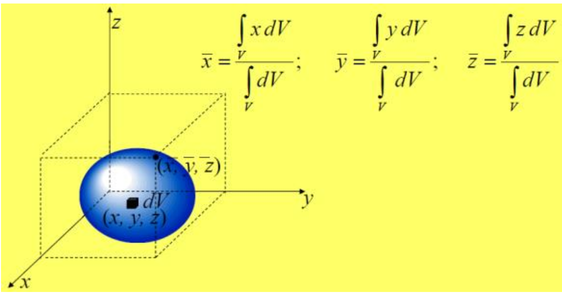
(a) (b) (c)

$\tilde{x}$, $\tilde{y}$ are the coordinates of centroid of the strip.

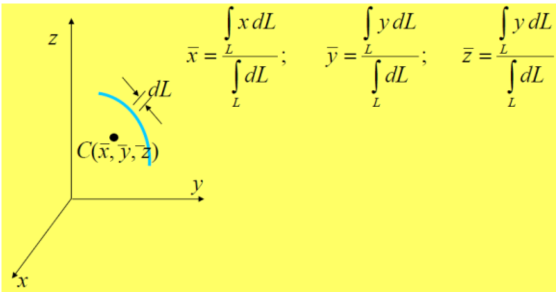
If a vertical strip is used as shown in (b), the area of the strip $dA = ydx$, and $\tilde{x} = x$, $\tilde{y} = y/2$

If a horizontal strip is used as shown in (c), the area of the strip $dA = xdy$, and $\tilde{x} = x / 2$, $\tilde{y} = y$

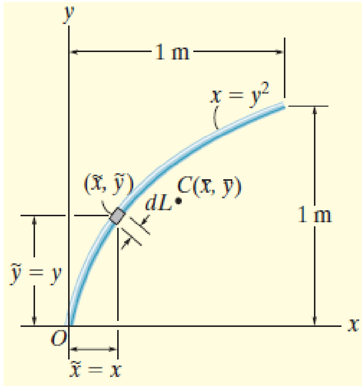
Volume. When a body is subdivided into volume elements dV, the location of the centroid for the volume of the object can be determined by computing the “moments” of the elements about each of the coordinate axes.



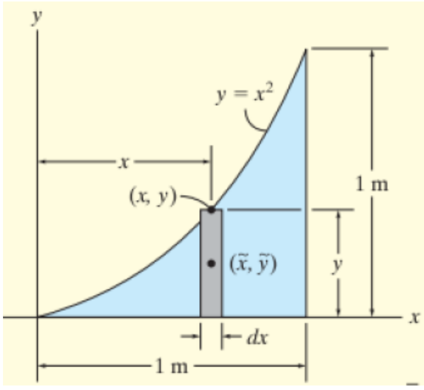
Line. If the geometry of the studied body is such as a thin rod or wire, it can be approached to a line. The balance of moments of the differential elements dL about each coordinate axis results in.



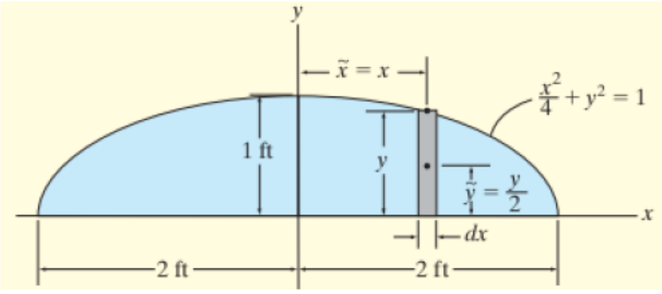
Example 1: Locate the centroid of the rod bent into the shape of a parabolic arc as shown in Fig.



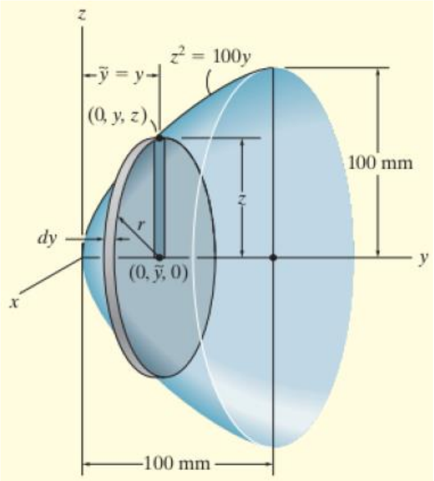
Example 2: Locate the centroid of the area shown in Fig.



Example 3: Locate the centroid of the semi-elliptical arc a shown in Fig.

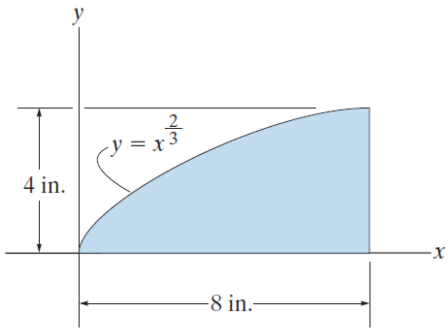


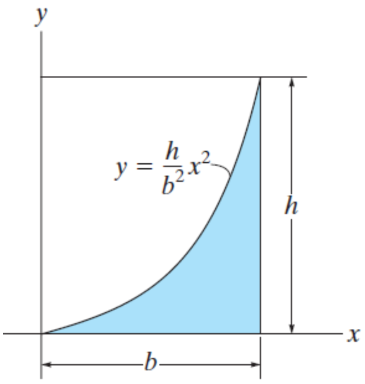
Example 4: Locate the $\bar{y}$ centroid for the paraboloid of revolution, shown in Fig.



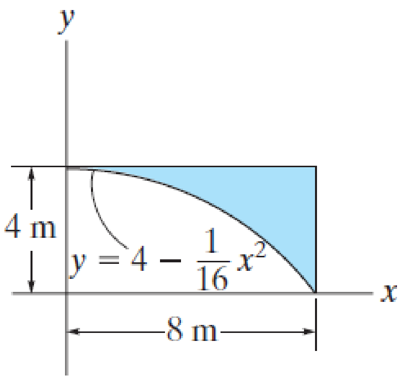
Sheet No. 1

Q 1: Locate the centroid ȳ of the area.

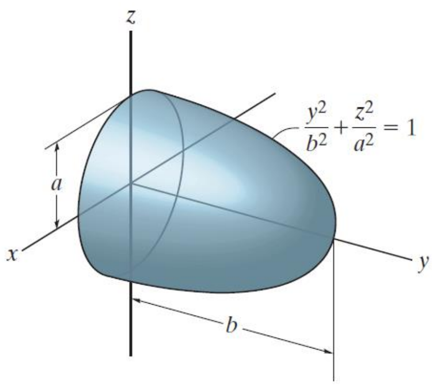




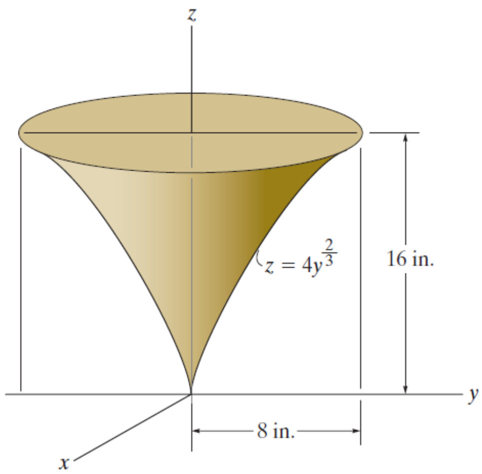
Q 2: Locate the centroid ȳ of the shaded area.



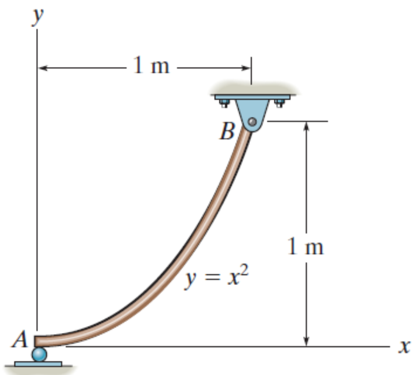
Q 3: Locate the centroid x̄ of the shaded area.



Q 4: Locate the centroid of the ellipsoid of revolution.



Q 5: Locate the center of gravity z̄ of the solid.



Q 6: Locate the center of gravity x̄ of the homogeneous rod. If the rod has a weight per unit length of 100 N/m, determine the vertical reaction at A and the x and y components of reaction at the pin B.

26. COMPOSITE BODIES

A ***composite body*** consists of a series of connected "simpler" shaped bodies, which may be rectangular, triangular, semicircular, etc. A composite body can often be sectioned or divided into simpler parts and if the weight and location of the center of gravity of each of these parts are known, it is possible to determine the center of gravity of the whole body from this information instead of carrying out the integration of the relevant equations.

To find the center of gravity of a composite body each of the different "components" of the body is treated as a particle.

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \qquad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} \qquad \bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W}$$

$\bar{x}, \bar{y}, \bar{z}$ represent the coordinates of the center of gravity G of the composite body.

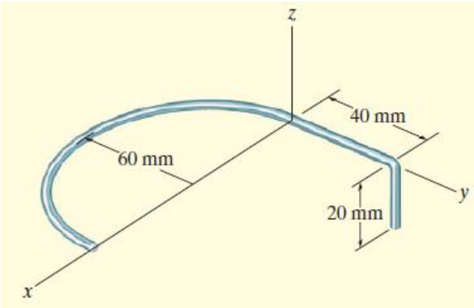
$\tilde{x}, \tilde{y}, \tilde{z}$ represent the coordinates of the center of gravity of each composite part of the body.

ΣW is the sum of the weights of all the composite parts of the body, or simply the total weight of the body.

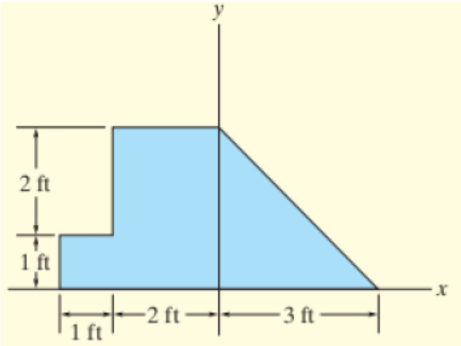
Similarly, a ***composite area*** consists of a series of connected "simpler" shaped areas, which may be rectangular, triangular, semicircular, etc. A composite area can often be sectioned or divided into simpler parts and if the centroid of each of these parts is known, it is possible to determine the centroid of the whole area without carrying out integration.

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} \qquad \bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

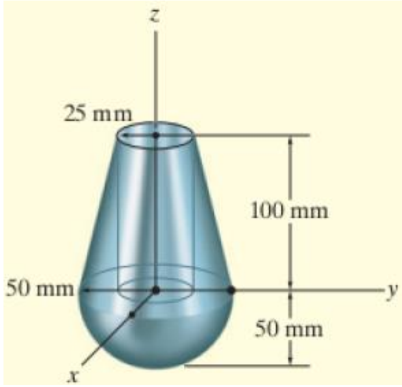
Example 5: Locate the centroid of the wire shown in Fig.



Example 6: Locate the centroid of the plate area shown in Fig.

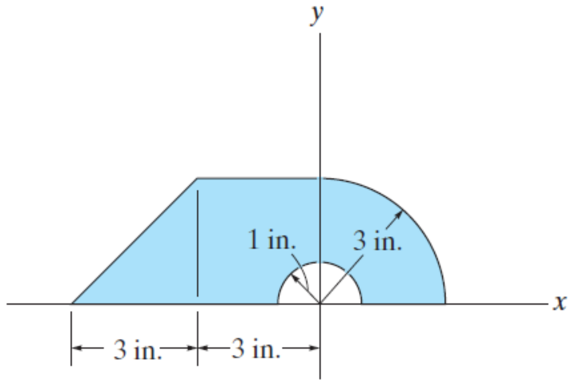


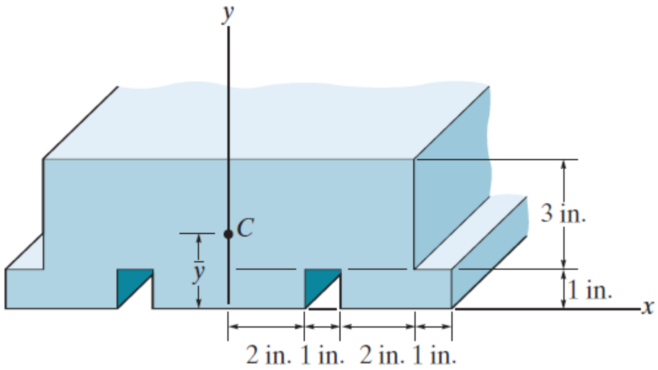
Example 7: Locate the center of mass of the assembly shown in Fig. The conical frustum has a density of ρ_c = 8 Mg/m^3, and the hemisphere has a density of ρ_h = 4 Mg/m^3. There is a 25-mm-radius cylindrical hole in the center of the frustum.



Sheet No. 2

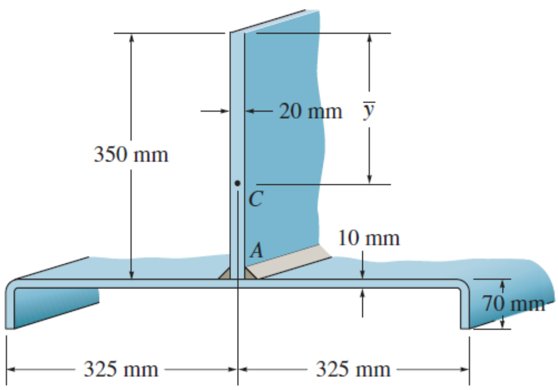
Q 1: Locate the centroid (x, y) of the area.

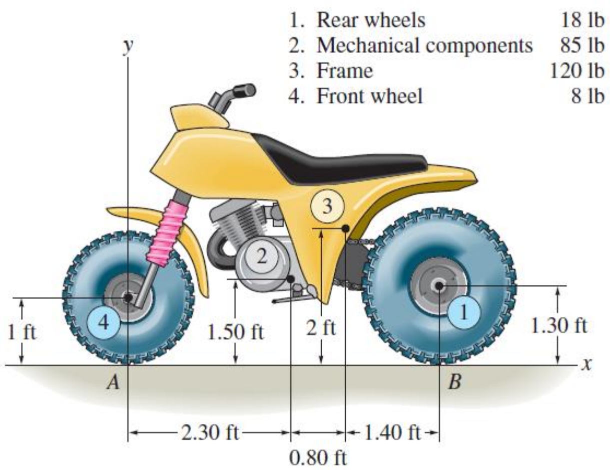




Q 2: Determine the location of the centroid C for a beam having the cross-sectional area shown. The beam is symmetric with respect to the y axis.

Q 3: Locate the centroid ȳ of the cross-sectional area of the beam constructed from a channel and a plate. Assume all corners are square and neglect the size of the weld at A.





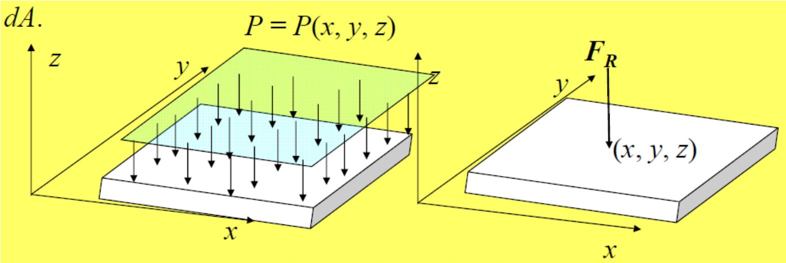
Q 4: Determine the location (x̄, ȳ) of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the x–y plane, determine the normal reaction each of its wheels exerts on the ground.

27. RESULTANT OF A GENERAL DISTRIBUTED LOAD

Pressure distribution on a surface. The plate shown is being subjected to the pressure distribution expressed by $P = P(x, y)$.

If $P = P(x, y)$ is known, the force $d\boldsymbol{F}$ acting on the differential area Da can be computed as $d\boldsymbol{F} = P(x, y)\, dA$.

Thus, the entire loading on the plate represents then a system of *parallel forces* infinite in number and each acting on a separate differential area



Determination of the resultant of a distributed load.

Magnitude of the resultant force. The magnitude $\boldsymbol{F_R}$ is determined by summing each of the differential forces $d\boldsymbol{F}$ acting on the entire surface area of the plate.

$$F_R = \int_A P(x, y)\, dA$$

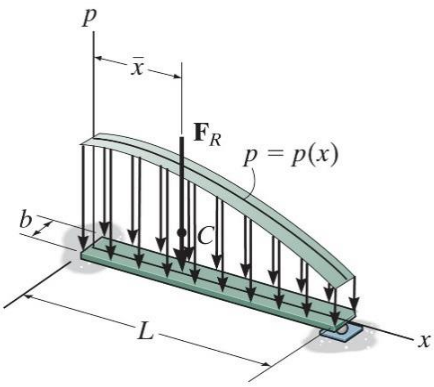
Location of the resultant force. The location of the centroid ($\bar{x}$, $\bar{y}$) of $\boldsymbol{F_R}$ is calculated by setting the moments of $\boldsymbol{F_R}$ equal to the moments of all the forces $d\boldsymbol{F}$ about the respective y–and x–axes

$$\bar{x} = \frac{\int_A xP(x, y)\, dA}{\int_A P(x, y) dA}; \qquad \bar{y} = \frac{\int_A yP(x, y)\, dA}{\int_A P(x, y) dA}$$

The *line of action of the resultant force passes through the geometric center or centroid of the volume under the distributed–loading diagram.*

28. REDUCTION OF A SIMPLE DISTRIBUTED LOADING

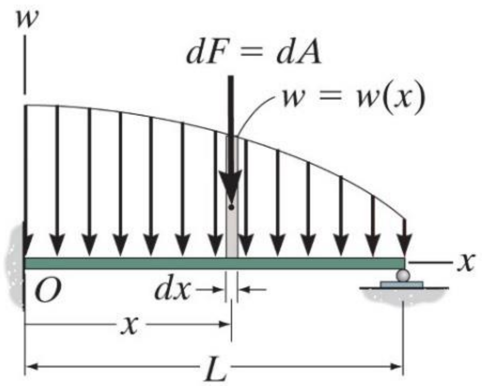
Sometimes, a body may be subjected to a loading that is distributed over its surface. For example, the pressure of the wind on the face of a sign, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all ***distributed loadings***. The pressure exerted at each point on the surface indicates the intensity of the loading. It is measured using pascals (Pa or N/m^2) in SI units or psi (pound per square inch). Also lb/ft^2 is a unit frequently used to define pressure in the U.S. Customary system.



The most common type of distributed loading encountered in engineering practice can be represented along a single axis. It contains only one variable *x*, and for this reason, we can also represent it as a *coplanar distributed load*.

Consider the beam shown in the right figure that has a constant width and is subjected to a pressure loading $p = p(x)$ N/m^2. Multiply the loading function by the width of the beam, b, we get the coplanar distributed load $w(x) = p(x)\, b$ N/m.

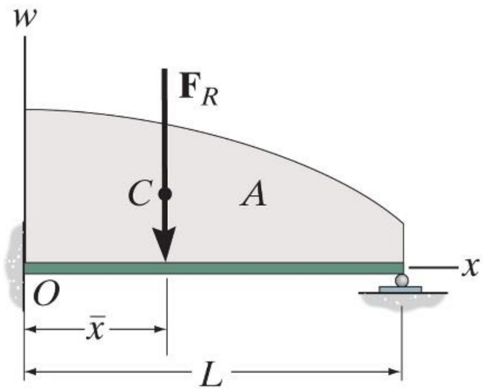
Magnitude of Resultant Force



$$F_R = \int_L w(x)\, dx = \int_A dA = A$$

***The magnitude of the resultant force is equal to the area A under the loading diagram*.**

Location of Resultant Force

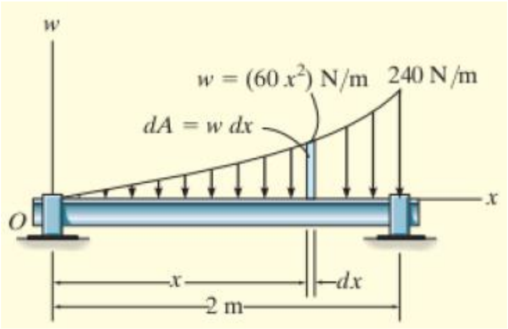


$$\bar{x} = \frac{\displaystyle\int_L x w(x)\, dx}{\displaystyle\int_L w(x)\, dx} = \frac{\displaystyle\int_A x\, dA}{\displaystyle\int_A dA}$$

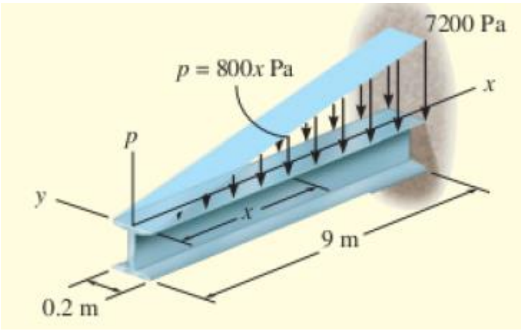
The resultant force has a line of action which passes through the centroid C (geometric center) of the area under the loading diagram.

When the distributed-loading diagrams is in the shape of a rectangle, triangle, or some other simple geometric form, the centroid can be obtained directly from the tabulation given on the inside back cover of the textbook.

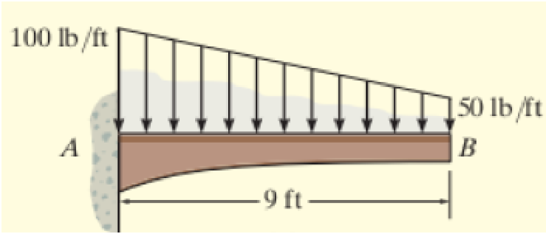
Example 8: Determine the magnitude and location of the equivalent resultant force acting on the shaft in Fig.



Example 9: A distributed loading of p = (800*x*) Pa acts over the top surface of the beam shown in Fig. Determine the magnitude and location of the equivalent resultant force.

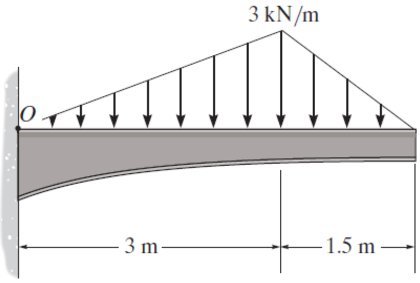


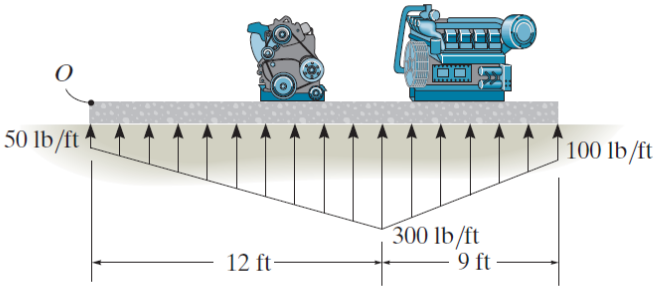
Example 10: The granular material exerts the distributed loading on the beam as shown in Fig. Determine the magnitude and location of the equivalent resultant of this load.



Sheet No. 3

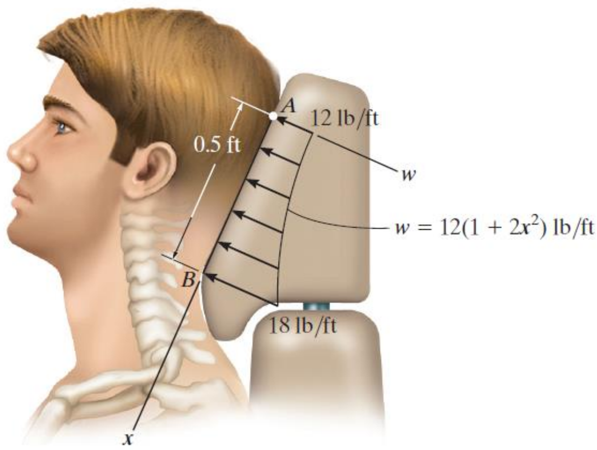
Q 1: Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point O.

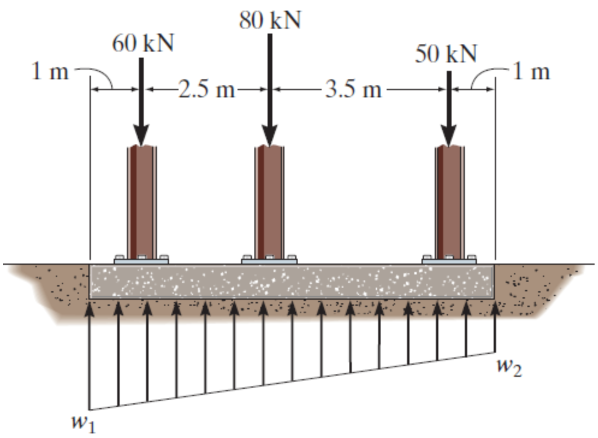




Q 2: The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.

Q 3: Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point A.





Q 4: If the soil exerts a trapezoidal distribution of load on the bottom of the footing, determine the intensities w_1 and w_2 of this distribution needed to support the column loadings.